IB Extended Essay

Mathematics

What is the Mathematical Relationship Between Economic Growth and Population Growth?

Session: May 2019

Word Count: 3993

Table of Contents

What is the Mathematical Relationship Between Economic Growth and Population Growth?

1. Introduction

The human population has been growing exponentially for thousands of years, culminating in today's whopping 7 billion. This number is huge by any metric. Not only that, but it is also expected that we will be reaching 10 billion[1] in the next 30 years. So, if we do the simple math, that is a 3 billion increase. To put things into context, the world population in 1960 was 3 billion[2]. Think about it. It is predicted our humanoid ancestors came to being 6 million years ago. What we call homo sapiens, modern humans, roamed the Earth for the first time 200 thousand years ago[3], and only after all those years were we able to hit 3 billion in 1960. Now, we can increase our numbers by that amount in 30 years.

So, human population is increasing in a very rapid manner. Sure, the amount of children a mother gives birth to is vastly reduced, but that does not change the fact that we are multiplying at an unsustainable rate. What I want to see is why are we still trying to reproduce in this manner. Our surroundings certainly have a hand in it. One of the main reasons behind the intent to increase the population other than the natural human instinct of reproduction (and the reason behind why many countries are encouraging increased population growth) is the idea that more workers equal more production. This used to hold true exactly 250 years ago,

1 Our World in Data. (2018). *Future Population Growth*. [online] Available at: https://ourworldindata.org/future-population-growth.

2 "World Population By Year - Worldometers". *Worldometers.Info*, 2018, http://www.worldometers.info/world-population/world-population-by-year/.

3 Howell, Elizabeth. “How Long Have Humans Been On Earth?” *Universe Today*, 23 Dec. 2015, www.universetoday.com/38125/how-long-have-humans-been-on-earth/.

which I will explore in a more detailed manner over the 4th section, but today, that does not exactly have to be the situation. Whether if it is the case is what I wish to uncover with this research.

2. Aim

The main goal of this paper is to find the relationship between economic growth and population growth when total factor production, physical capital, output elasticies of labor and physical capital, working population, education level of workers, average working hours, and capacity utilization rate are kept constant. (Unfamiliar terms are addressed in the next section) With this research, it is hoped that the relationship found will act as a guide to promote or discourage population growth according to its contribution or damage to economic growth.

3. Key Terms

This section should be referred to in case one of the terms which is not specifically explained causes confusion. To make it easier, each of these terms will be bolded the first time they appear after the key terms section.

Population Growth: Positive percentage difference between population values of consecutive years. Aside from birth and death rates, immigration is also included in this value.

Gross Domestic Product (GDP): GDP of a country is the aggregate worth of all goods and services that were produced annually within that country's borders. Also, the nationality of the producer doesn't matter in this context; the production is counted towards the GDP of the country in which the production happens. This value is used as a measure for determining the economic health of a country.[4]

Inflation: Percentage decrease in the amount of goods and services a unit of currency can buy.[5] For example, inflation will cause the value of a currency to fall.

Nominal GDP: Used to refer to raw GDP value of a country. There are no alteration to the original number whatsoever.[6]

Real GDP: This one is also used to express the GDP of a country, but this time the effect inflation has on the value is removed. Inflation will cause devaluation in the currency, automatically raising the price of all goods. This means that because the populace has to spend more to buy something, the production value will look as if it has increased, resulting in the nominal GDP being less accurate. This is then converted to real GDP for this value to reflect how much this country produces and not how much they spend on consumption and investing.[7]

GDP Growth (Economic Growth): When economic growth is mentioned, this is what it refers to. It is the percentage change in GDP within a quarter of the year.[8]

Geometric Growth: Increasing or decreasing number pattern where the difference between consecutive numbers changes with a constant exponential value over the variable.

---

4 Kenton, Will. “Gross Domestic Product - GDP.” *Investopedia*, Investopedia, 13 Dec. 2018, www.investopedia.com/terms/g/gdp.asp.

5 Kenton, Will. “Purchasing Power.” *Investopedia*, Investopedia, 13 Dec. 2018, www.investopedia.com/terms/p/purchasingpower.asp.

6 Amadeo, Kimberly. “Gross Domestic Product: Understanding What a Country Produces.” *The Balance Small Business*, The Balance, www.thebalance.com/what-is-gdp-definition-of-gross-domestic-product-3306038.

7 Amadeo, Kimberly. “Gross Domestic Product: Understanding What a Country Produces.” *The Balance Small Business*, The Balance, www.thebalance.com/what-is-gdp-definition-of-gross-domestic-product-3306038.

8 Amadeo, Kimberly. “Gross Domestic Product: Understanding What a Country Produces.” *The Balance Small Business*, The Balance, www.thebalance.com/what-is-gdp-definition-of-gross-domestic-product-3306038.

Arithmetic Growth: Increasing or decreasing number pattern where the difference between consecutive numbers is constant.

Labor: Labor is the amount of mental and physical effort put into production and services by human agents. Its value increases as the education of the workers, working population, organizational efficiency, etc. increase. This value cannot be directly reflected with a number but it can still be shown as a factor of a function's output. [9]

Physical Capital: Used to refer to any tangible financial asset used for making production possible. For example, trucks rented or bought to transport goods are a form of capital. Also, something does not have to be contributing directly to the production process to be counted as capital. So, air conditioners in a workplace are also a form of capital.[10]

Total Factor Production: Total factor production represents the role played by science, technology, and innovation. An uncountable number of factors affect it, ranging from environmental factors to quality of the equipment. However, the most important contributor to it is machinery efficiency. A high total factor production value will correspond to cutting edge equipment and facilities, wide-spread automated production systems, and efficient machinery. [11]

Returns to Scale: This term is used to describe how much the output of a function will be affected when all the input values are changed proportionally. If the output is changed more than the proportional change in inputs, then it is said there is increasing returns to scale. If the

---

9 Amadeo, Kimberly. "Why Your Work Is Critical to the Economy." *The Balance Small Business*, The Balance, www.thebalance.com/labor-definition-types-and-how-it-affects-the-economy-3305859.

10 "Capital - Overview, Guide, Examples, Types of Capital." *Corporate Finance Institute*, corporatefinanceinstitute.com/resources/knowledge/finance/capital/.

11 Comin, Diego. *Total Factor Productivity*. Aug. 2006, www.people.hbs.edu/dcomin/def.pdf.

output gets altered proportionally, then there is constant returns to scale. If the output is scaled less than the proportional change, then there is decreasing returns to scale.

Output Elasticity: Within the context of economics, output elasticity of an input variable is percent change in output for every one percent change in that input. It is used to observe how much can change in one variable affect the output. When the output elasticities of all variables are added together, it can be understood what kind of returns to scale the function has. More than 1, exactly 1, and less than 1 output elasticities correspond to increasing, constant, and decreasing returns to scale respectively.

Capacity Utilization Rate: This value is a ratio of actual output to the output that is the physical maximum. It is primarily used to reflect factors like organizational efficiency (how competent the administration is), non-routine events (i.e. strikes or natural events), worker efficiency, and infrastructure (i.e. slow internet, power outages).Since maximum output is always a number larger than actual output, $0 < capaciity\ utilization\ rate < 1$.[12]

4. Historical Context

When **population growth** and its various effects on production is thought, the first person who should be mentioned is Thomas Malthus (1766 - 1834). He was the first person to look for a correlation between population growth and production. Specifically, food production. He theorized that while population was **growing in a geometric manner**, the food supply was **increasing with an arithmetic pattern**. Plus, thanks to the fact that rather than machines and robots, the workers had to attend to everything required for production, **economic growth**

12 Egilmez, M. (2012, March 28). Kapasite Kullanımı Nedir, Nasıl Olculur, Ne Ise Yarar? Retrieved from http://www.mahfiegilmez.com/2012/03/kapasite-kullanm-nedir-nasl-olculur-ne.html

always needed an increase in population. This was due to the fact that before the industrial revolution the amount of land worked on was directly proportional to production. Quality of tools, organizational efficiency, and worker expertise played a part in production, but their role was small, and these commodities were really hard to improve. Land expansion and population growth was the only effective way to increase production (which explains the countless expansionist wars of the pre-Industrial Revolution era). All these circumstances were the main arteries of the Malthusian economic model[13].

Malthus' theory quickly become obsolete after the Industrial Revolution. This is because factories and the tools which came with the Revolution were able to yield the same amount of production with less human input, meaning increase in food yield could keep up with population growth as long as the efficiency increasing technologies kept advancing. What it also meant was that production not only increased with manual **labor**, but also from machinery. Advances in sciences, technology, and innovation started to contribute to all manners of production much more significantly thanks to their positive effect on machinery efficiency. Worker expertise became a must for most jobs as they had to produce technologically advanced goods often. As workplaces grew bigger and logistics became more important, the need for faster and larger transport vehicles became clear. Thus, the contribution of total factor production to an economy was magnified drastically, and human labor, while still fundamental, ceased to be the only approach to increasing production. Malthusian economics left its place to contemporary economic models, where total factor production plays an important role alongside labor and physical capital.

---

13 "Thomas Malthus | Biography, Theory, Books, & Facts". *Encyclopedia Britannica*, 2018, https://www.britannica.com/biography/Thomas-Malthus.

5. Methodology

5.1 Nominal GDP Calculation

First, a way to calculate the nominal GDP of a country must be designated to be able to bring population to the field of economic growth. As simple as it may be, combining four types of budget expenditure in a nation results in its nominal GDP:

$$GDP = C + G + P + E - I$$ [14]

C: General consumption (food, clothing, utilities, luxury goods, etc.)

G: Governmental expenditure (public schools hospitals, government employee salaries, etc.)

P: Investment made to buy **physical capital**

E: All exports

I: All imports

Because all these values represent all the places where production can go to, it is considered to be an accurate way to calculate GDP. Since imports are also products used by the country in question but they are not produced by that country, they are subtracted from the rest of the equation.

5.2 Real GDP Calculation

Now, this nominal GDP must be converted to real GDP for the magnifying effect of the **inflation** to go away. For this process, GDP deflator values for each country must be

14 "GDP Formula - How to Calculate GDP, Guide and Examples." *Corporate Finance Institute*, corporatefinanceinstitute.com/resources/knowledge/economics/gdp-formula/.

examined. GDP deflator is a number which reflects how much the price for every good and service has increased. A country designates a base year and that year's GDP deflator value becomes 100. Later years, this value increases. For example, if next GDP deflator becomes 125, then it can be said that there is a general increase of 25% in the prices of every good and service in that country. Due to the fact that there are millions of different goods and services, it is impossible to calculate this value within the context of this paper and thus it must be acquired from other sources. Once the nominal GDP is multiplied by GDP deflator of the base year and then divided by GDP deflator of the current year, real GDP is calculated:

$$Nominal\ GDP \times \frac{Base\ Year\ GDP\ Deflator}{Current\ Year\ GDP\ Deflator} = Real\ GDP$$

For example, it is given that Country A has the nominal GDP of $1000 in 2018. This country uses year 2004 as its base year where GDP deflator was 75. In 14 years the overall price of all goods and services in Country A increased by 33.3% due to inflation, so its GDP deflator in 2018 is 100. To calculate the real GDP of Country A in 2018, nominal GDP of 2018 ($1000) is multiplied with the 2004 GDP deflator (75) and divided by GDP deflator of 2018 (100). The result is $750, the real GDP of 2018 for Country A.

$$1000 \times \frac{75}{100} = 750$$

This means that while Country A's production is actually worth $750, it looks as if it is worth $1000 due to inflation.

5.3 Cobb-Douglass Production Function

As the starting point of this research, Cobb-Douglass Production Function will be used. This function, just like other production functions, is used to show the relationship between the outputs and the inputs. However, Cobb-Douglass is the ideal one because it is specifically made to be used for larger economies, namely countries. It uses labor and physical capital as the parameters of the function, even though other values can be changed as well. Another reason it is widely used is because of its inclusion of total factor production, meaning effects of technology and innovation can be observed directly. It is as follows:

$$f(L,K) = AL^{\alpha}K^{\beta} = Y$$

f(L,K): The function used to determine GDP of a country.

A: A positive constant representing total factor production.

L: Labor

K: Physical Capital

α:Output elasticity of labor

β: Output elasticity of physical capital

Y: GDP

In this function, L and K are variables and rest are held constant. **Output elasticities** α and β are relatively foreign concepts here. Both of them are bigger than 0. Because output elasticity is percent change in output for every one percent change in the input in question, α goes through a process like this ("δ" letter is the symbol for the change in the variable next to it, which is why $\frac{\delta y}{\delta x}$ is a derivation expression):

$$\alpha = \frac{Percent\ chang\ in\ GDP}{Percent\ change\ in\ labor} = \frac{\frac{\delta Y}{Y} \times 100}{\frac{\delta L}{L} \times 100} = \frac{\frac{\delta Y}{Y}}{\frac{\delta L}{L}}$$

$$\alpha = \frac{\frac{\delta Y}{Y} \times \frac{L}{\delta L}}{\frac{\delta L}{L} \times \frac{L}{\delta L}} = \frac{\delta Y}{Y} \times \frac{L}{\delta L}$$

$$\alpha = \frac{\delta Y}{\delta L} \times \frac{L}{Y}$$

For β, the equation is the same, except instead of L for labor, there is K for capital:

$$\beta = \frac{\delta Y}{\delta K} \times \frac{K}{Y}$$

α and β can be thought of as L and K's efficiency at contributing to economic growth, respectively. This investigation and all the postulated theories will be based on Cobb - Douglass Production Function.

6. Theory

6.1 Assumptions

Output Elasticities are Constant: To be able to observe the relationship between only one input and one output, all other values must be constant. While α and β are not variables, they are not constant with their current form either. This is because while L is increasing (as it will be discussed later) $\frac{\delta^2 Y}{\delta L^2} < 0$ and vice versa, meaning $\frac{\delta Y}{\delta L} \times \frac{L}{Y}$ and $\frac{\delta Y}{\delta K} \times \frac{K}{Y}$ will be constantly changing. It is not possible to express α and β as $\frac{\delta Y}{\delta L} \times \frac{L}{Y}$ and $\frac{\delta Y}{\delta K} \times \frac{K}{Y}$ since they both include δY, the change in GDP. Therefore, α and β letters used to denote output elasticities of labor and physical capital respectively, will be assumed constant.

Constant returns to scale (α+β=1): It will be assumed when physical capital and labor are multiplied by a coefficient, GDP will also change proportionally.

Output elasticity of labor is 0.5 or more: As output elasticity goes down, positive changes in labor, and thus population, will have a decreased effect on economic growth. However, because population growth inevitably increases consumption rates, the funds allocated for investment will be lower, meaning growth of physical capital will be slowed down. In this case, population growth would have a negative effect on economic growth. As a result, it will be assumed labor has an output elasticity equal to or more than output elasticity of physical capital. So, $0.5 \leq \alpha \leq 1$, since $\alpha + \beta = 1$.

Population and Working Population Grows Proportionally: Later in the research, unemployment rate will be used as a constant value and population change will be used as a variable. For unemployment to stay constant, this precaution is taken.

6.2 Relation Between Population and Labor

To be able to find the relationship between economic and population growth, relation between labor and population must be found. This will make it possible to write labor in terms of population. To be able to do that however, labor must be separated into its factors within a function separate from Cobb-Douglass Production Function. In the light of the research made for this paper, the following function is formulated for a much clearer interpretation process. However, just like Cobb-Douglass function, this function is a model, not a formula. It cannot be used to calculate an exact number, but it is useful for seeing the general effect the constants or the variables have over the output. Here, the most important factors of labor were identified, and their justification as to why they are here are explained below:

$$f(P) = P(1 - U)(365H)EC = L$$

f(P): Function with variable "P".

P: Population, the key variable

L: Labor

U: Unemployment rate. It is very important that this value is written in its percent form (i.e. If unemployment rate is 12%, U should be $\frac{12}{100}$). Because it is assumed population and working population grow proportionally, U will stay as a constant, which means further calculations regarding variable P will not affect this constant. (1-U) is multiplied with variable P to include only the working population to the function. Since unemployment rate is always a smaller value than 1 and cannot be 0 or a negative value, $0 < (1 - U) < 1$.

H: Average labor hours per day. It has the coefficient 365 to carry it to the scale of one year. It is multiplied with P because it represents how much each worker works per day, and thus is directly related to labor. Worker efficiency drops due to long working hours are accounted in **capacity utilization rate**, the last constant. Because H is hours per day, 0 < H < 24.

E: Education level of the workforce. This is a coefficient similar to total factor production. As research shows[15], it is known that it definitely affects labor, but it may not be possible to properly express this constant with a number. Regardless, it is multiplied with P since it is an input capable of enhancing the quantity and quality of output, labor. Because labor is possible without education, E > 1.

C: (Refer to "Key Terms" for further information) Capacity utilization ratio is included because it accounts for other miscellaneous factors.[16] Since C is the ratio of actual output to maximum possible output, it is multiplied with P to paint a more accurate picture of labor. Capacity utilization rate's definition dictates: 0 < C < 1.

15 *EDUCATIONAL ATTAINMENT OF THE LABOUR FORCE*. OECD, www.oecd.org/els/emp/3888221.pdf.

16 Santacreu, Ana M. *Employment and Capacity Utilization Over the Business Cycle*. Economic Synopses, 12 Sept. 2016, files.stlouisfed.org/files/htdocs/publications/economic-synopses/2016-09-12/employment-and-capacity-utilization-over-the-business-cycle.pdf.

It is important to note that these factors of labor, with the help of research on contemporary labor economics, are postulated for the purposes of this investigation. Now it is possible to put the population variable into the Cobb-Douglass Production Function.

6.3 Relation Between Population and GDP

Let us first recall the original function, implement the newly found labor equation and isolate population variable. (Reminder: Y: GDP, A: Total Factor Production, L: Labor, K: Physical Capital, α and β: Output Elasticities)

$$Y = AL^{\alpha}K^{\beta}$$

$$Y = A[P(1-U)(365H)EC]^{\alpha}K^{\beta}$$

$$Y = A[(1-U)(365H)EC]^{\alpha}P^{\alpha}K^{\beta}$$

To simplify the equation, let "c" replace constants "$(1-U)$, E, 365H, C":

$$Y = Ac^{\alpha}P^{\alpha}K^{\beta}$$

Now, it is possible to take the derivative of Y according to P:

$$\frac{\delta Y}{\delta P} = Ac^{\alpha}\alpha P^{\alpha-1}K^{\beta}$$

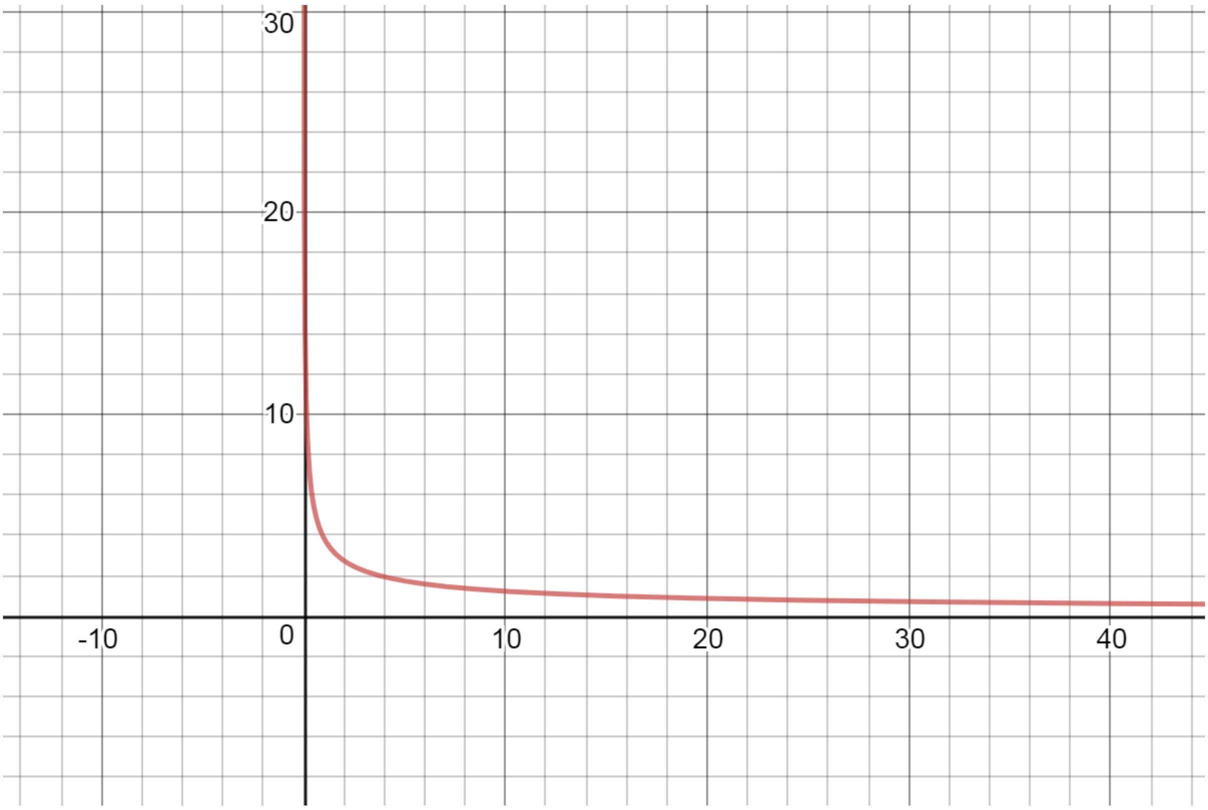
6.4 Analysis

It is already apparent that $\frac{\delta Y}{\delta P}$ will be equal to change in GDP (economic growth) per one unit change in population (population growth). However, further analysis of this equation will lead to interesting trends in economic growth. Let us start with writing "$P^{\alpha-1}$"as"$P^{\alpha} \times P^{-1}$" and then multiplying both the dividend and divisor with P^{1}:

$$\frac{\delta Y}{\delta P} = Ac^{\alpha}\alpha P^{\alpha}P^{-1}K^{\beta}$$

$$\frac{\delta Y}{\delta P} = \frac{Ac^{\alpha}\alpha P^{\alpha}P^{-1}K^{\beta}P^{1}}{P^{1}} = \frac{Ac^{\alpha}\alpha P^{\alpha}K^{\beta}}{P} = Ac^{\alpha}\alpha K^{\beta} \times \frac{P^{\alpha}}{P}$$

Since $\beta > 0$ and $\alpha + \beta = 1$, α is smaller than 1, meaning $\frac{P^{\alpha}}{P}$ will get smaller as population increases. Because the rest of the equation is kept constant, this will also result in $\frac{\delta Y}{\delta P}$ getting infinitely smaller as P becomes larger, meaning the effect of population growth on economic growth will become minuscule after population reaches a certain point. To have a better understanding, a graph of δY (y-axis) to δP (x-axis) will be analyzed.



Graph 1: Numerical values on the axes are irrelevant.

The resulting graph is an isoquant. As it can be seen, $\frac{\delta Y}{\delta P}$ is infinitely large at the start of the graph, 0 on the x-axis. However, as P becomes infinitely larger, $\frac{\delta Y}{\delta P}$ gets closer and closer to 0 of the y-axis. It is also important to note that the curve is within area 1, meaning y and x values are always positive. With these obervations, two postulations can be made:

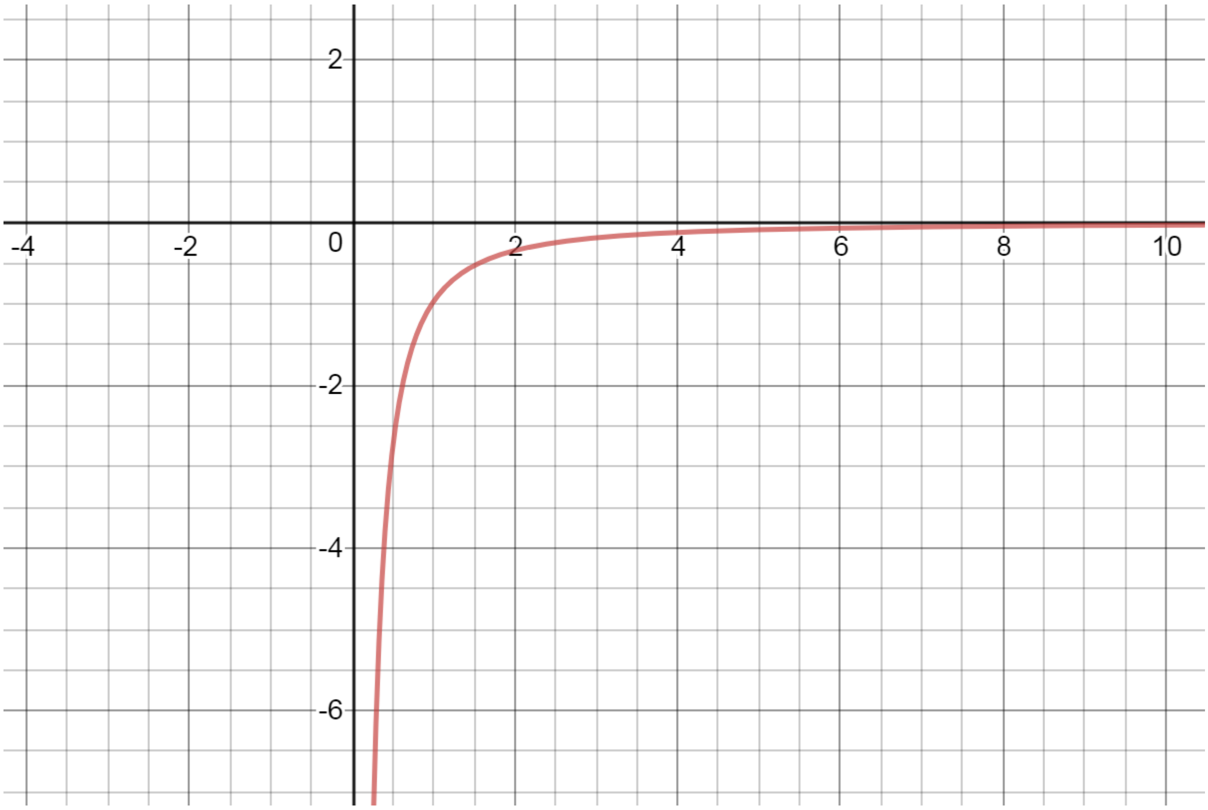
1. $\lim_{P \to \infty} \left(Ac^{\alpha} \alpha P^{\alpha-1} K^{\beta} \right) = 0$

2. $\frac{\delta Y}{\delta P} > 0$

To further analyze the $\frac{\delta Y}{\delta P}$ equation, $\frac{\delta^2 Y}{\delta P^2}$ should be looked into. This will provide better clues for the trend Graph 1 follows. Let us start with taking the derivative of $\frac{\delta Y}{\delta P}$:

$$\frac{\delta^2 Y}{\delta P^2} = Ac^{\alpha} \alpha(\alpha - 1) P^{\alpha-2} K^{\beta} = Ac^{\alpha} \alpha^2 (-\alpha) P^{\alpha-2} K^{\beta}$$

Since $\alpha - 1$ is a negative numbers and since there are not any other negative numbers, the result of $\frac{\delta^2 Y}{\delta P^2}$ will be negative. This was expected considering $\frac{\delta Y}{\delta P}$ was always decreasing. To have a better understanding, a graph of $\delta^2 Y$ (y-axis) to δP^2 (x-axis) will be analyzed.



Graph 2: Numerical values on the axes are irrelevant

Graph 2 is also an isoquant. It is very similar to Graph 1. The most significant difference is the place of Graph 2, which is area 4. As a result, $\frac{\delta^2 Y}{\delta P^2}$ values are always negative. Also, as P becomes infinitely large, rate of change of $\frac{\delta Y}{\delta P}$ gets very close to 0. When the equation and the graph are observed, another two postulations can be made:

1. $\lim_{P \to \infty} \left(Ac^{\alpha} \alpha^2 (-\alpha) P^{\alpha-2} K^{\beta} \right) = 0$

2. $\frac{\delta^2 Y}{\delta P^2} < 0$

7. Conclusion

7.1 Postulations

1. $\frac{\delta Y}{\delta P} = A[(1-U)(365H)EC]^{\alpha}\alpha P^{\alpha-1}K^{\beta} = Ac^{\alpha}\alpha P^{\alpha-1}K^{\beta}$

2. $\lim_{P\to\infty}\left(Ac^{\alpha}\alpha P^{\alpha-1}K^{\beta}\right) = 0$

3. $\frac{\delta Y}{\delta P} > 0$

4. $\lim_{P\to\infty}\left(Ac^{\alpha}\alpha^{2}(-\alpha)P^{\alpha-2}K^{\beta}\right) = 0$

5. $\frac{\delta^2 Y}{\delta P^2} < 0$

7.2 Discussion

The postulations make a very interesting point. Population is an essential ingredient for an economy as it can be understood from the infinitely large $\frac{\delta Y}{\delta P}$ at the start of the curve, but what is fascinating comes after that. Once population reaches a certain point, population growth loses its ability to meaningfully alter economic growth. This means that for a highly populated country it would have damaging consequences to have a population growth greater than the amount required to sustain the population. Not only would this growth fail to positively increase economic growth, but by increasing consumption it would decrease the amount of production allocated for increasing physical capital stock. As a result, population growth would slow down economic growth instead of hastening it.

One misunderstanding may arise from how population growth could possibly be affecting economic growth while $\frac{\delta Y}{\delta P}$ is always positive. Could consumption come to a position where it can have such a great impact? Apparently, it does not have to. The economic growth as a result of increasing population will eventually be very close 0, effectively nullifying population growth's influence. Consumption on the other hand will always increase proportionally to population growth. As a result, there exists a point in the function where negative impact of consumption will trump the positive impact of population growth. Before that point however, population growth will definitely have a significant positive effect on economic growth.

7.3 Further Research

In this research, output elasticities were kept constant to have δY and δP kept isolated at one side of the equation to keep the investigation as focused as possible. If a research was conducted where there is no such concern and output elasticities were freely replaced with $\frac{\delta Y}{\delta L} \times \frac{L}{Y}$ and $\frac{\delta Y}{\delta K} \times \frac{K}{Y}$, some interesting results might be found.

It was also assumed that the output elasticity of labor was equal to or more than 0.5, otherwise population growth would not have a meaningful effect on economic growth. A new investigation can be carried where population is kept constant and physical capital would be used as the variable. After that, an analogy between physical capital centered calculations and a scenario where population growth was not constant can be made to see how much population growth could have possibly damaged economic growth.

It was stipulated that returns to scale was constant. However, this is not necessarily the situation every time. A case-by-case study can be made to calculate returns to scale of selected economies. Then, the average of all those values can be taken to see whether it is close to 1, the value required for returns to scale to be constant.

Lastly, the existence of a critical point in population where the beneficial effect of population growth rapidly decreases is mentioned several times but it was never discussed how this point can be discovered. Via data analysis of several economies, this critical point can be found on a case-by-case basis.

8. Bibliography

1. Kenton, Will. “Factors Of Production.” *Investopedia*, Investopedia, 13 Dec. 2018, www.investopedia.com/terms/f/factors-production.asp.

2. “Neoclassical Economics.” *Mill, On Liberty, Chapter 1 | Library of Economics and Liberty*, www.econlib.org/library/Enc1/NeoclassicalEconomics.html.

3. “Inada Conditions.” *The Columbia Encyclopedia, 6th Ed*, Encyclopedia.com, 2019, www.encyclopedia.com/social-sciences/applied-and-social-sciences-magazines/inada-conditions.

4. Kenton, Will. “Elasticity.” *Investopedia*, Investopedia, 13 Dec. 2018, www.investopedia.com/terms/e/elasticity.asp.