# TED Ankara College Foundation Private High School

## International Baccalaureate

## Extended Essay

# Surface Area's Effect to Air Drag

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May 2015

Physics

Word Count: 3977

# 1. Abstract

The objective of this experiment is to investigate the answer of the following question: "How do the surface area of a rotating propeller of a helicopter affect the average velocity of its free fall when it is released from a constant 4m height?" Velocity of the helicopters are affected by air drag. Air drag doesn't have a stable formula. As the speed of the objects that are exposed to air resistance changes, components that affect the magnitude of air drag force changes. For example, for low speeds the air resistance is directly proportional to the velocity, but for high speeds the air resistance is directly proportional to the square of the velocity. Since I didn't know if the velocity of the helicopter models will be high or low at the beginning of the experiment, I controlled elements that affect the air friction in both high and low speeds. A helicopter model was used to measure the effect of cross-sectional area on the velocity. I tried to make the model's features as similar as possible to a real helicopter. There were 2 propellers and there were masses under the propellers. The surface area of the propellers of the helicopter model were decreased by decreasing the length and width of the propellers proportionally and models were released from the certain height, 4m. The landing times were recorded. The height was divided by the times needed for landing and the average velocity was found. The graph of area vs velocity was sketched. The effect of surface area on air friction and eventually velocity was found by interpreting the graph. The graph showed that the cross-sectional area of the propellers is inversely proportional to velocity. From this it can be understood that the models are in the category of the low speed vehicles.

Word Count: 299

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## 2. Introduction

The scope of the work is to test the effect of air drag to propellers of the helicopters. Helicopters are widely used today because of them being the most practical air vehicle. They don't need huge areas to take of or land. They can elevate vertically. This practicality is caused by their propellers which rotate and make the helicopters stay or move freely in the air.

Throughout flying there is a drag force affecting the helicopter. This drag force which can be referred as air resistance, opposes the direction of the helicopter, making moving harder for the vehicles. The objective of the experiment is to find the effect of the cross-sectional areas of these propellers to the air resistance.

I chose to investigate this subject because I was amazed by the air vehicles since I was a children. My father took me to a show in which planes and helicopters are displayed when I was four years old. After the exhibition there was a show of plane acrobatics. This still was the best memory of my life. I had always been curious about the physics of air and this essay was a good opportunity for me to start to learn about one of the simplest phenomenon relating to air. I chose helicopters as the subject while it is easier to observe the effect of drag force to its propellers than the wings of planes.

## 3. Background Information

### 3.1 What is a Helicopter?

A helicopter is a vehicle that can fly by its rotating propellers. These propellers are turned by the power supplied by the motors. The helicopters can rise vertically because of the propellers and so it doesn't need much area to land or take off unlike planes. These properties of the helicopters allows it to be used at remote areas where there are no proper places to land or take off for normal aircrafts. These vehicles are great for emergencies because it is fast both in air and take off/land.

Figure1: An ambulance helicopter that is used for emergencies.

### 3.2 What are Propellers?

Propellers are widely used to generate power. They are used in wind turbines. In the wind turbines, there is a box in the center called nacelle. There are 3 propellers connected to it. The kinetic energy of the wind causes propellers to rotate generating a mechanical energy. The magnetic rotor inside the generator rotates inside loops of copper wire. Electrons in the copper flows and electrical energy is generated.[1]

Propellers are in the shape of a blade and mostly used in helicopters as the source of power as well. They help to stay in the air by producing a pressure difference between front and back surfaces, accelerating the fluid. (In our experiment the fluid is air.) In this experiment the electric generating quality of the propellers will not be taken into account since the propellers are rotated by the helicopters.

### 3.3 Rotational Motion in the Experiment

The motion of the helicopter models when falling down can be referred as rotation around a fixed axis which is a case of rotational motion. Euler's Rotation Theorem shows that when one object is undergoing a rotational motion, it can only have one stationary axes.[2] It is impossible to undergo circular motion with more than 1 axis. This means that the axis of rotation of the helicopter model which is center of the propellers in the experiment will always be at the same place and the helicopter model will land directly without swinging.

### 3.4 What is Angular Displacement?

Angular displacement is the angle through which a point or line has been rotated about a specified axis. It is usually given in radians.[3]

### 3.5 What is Angular Velocity?

Angular velocity is the rate of change of angular displacement. It is a vector quantity which means that it is described by both direction and magnitude. The SI unit of angular velocity is rad/s. However, it can be measured in degrees per second or degrees per hour. It is usually represented by the symbol ω.[4]

### 3.6 What is air friction?

Air friction (air drag) is a kind of fluid friction. Normally, the friction forces change according to surface area. However, this model of friction depend on the velocity. The components that affect the drag force changes with the changing speed. For low speeds drag force is proportional to velocity of the object and a constant that depends on the features of fluid and the dimensions of object.

This drag force can be calculated as:

$F_{drag}$: -bv

b: constant which depends on the features of the fluid (in our case air) and the dimensions of the object(kg/s)

v: Velocity(m/s)

There is a negative sign because drag force is always directly opposite to the velocity.[5]

For higher speeds the drag force is proportional to the square of the velocity, air density, drag coefficient and cross-sectional area. Drag force can be calculated as:

$F_{drag}$: $-C\rho Av^2$

C: Drag coefficient

$\rho$: Air density($kg/m^3$)

A: Cross-sectional area($m^2$)

v: Velocity($m^2/s^2$) [6]

### 3.6.1 What is Drag Coefficient?

The drag coefficient (denoted as $C_d$) is a quantity which quantifies the resistance (drag) of an object in a fluid environment. It is always associated with surface area. In the experiment the fluid is air and the surface area is the cross-sectional areas of the propellers. As the drag coefficient decreases, the object will encounter less resistance.[7]

### 3.6.2 What is Air density?

The air density is the mass per unit volume of Earth's Atmosphere. Like air pressure, it is inversely proportional to altitude. Temperature and humidity affects its value.[8]

Air density is used in a lot of branches of science such as air conditioning, aeronautics, atmospheric research and meteorology.

The air density differs in dry air and humid air. The dry air is directly proportional to air pressure but inversely proportional to specific gas constant and temperature

$\rho_{\text{dry air}} = P / (R*T)$

where; $\rho_{\text{dry air}}$ = Density of dry air ($kg/m^3$)

P = Air Pressure (Pa) ($kgm^{-1}s^{-2}$)

R = Specific gas constant for dry air (approximately $287.05Jkg^{-1}K^{-1}$)

T = Temperature (K) [9]

The dry air density is a theoretical value since air contains some moisture in real world. The moist air is less dense than the dry air because there is an addition of water vapor. The humid air density can be found by summing the dry air density and water vapor density with their partial pressures. It can be calculated as;

$\rho_{\text{humid air}} = P_{\text{dry air}} / (R_{\text{dry air}} * T) + P_{\text{water vapor}} / (R_{\text{water vapor}} * T)$

where;

- $\rho_{\text{humid air}}$: density of humid air ($kg/m^3$)
- $P_{\text{dry air}}$: the pressure of dry air(Pa)($kgm^{-1}s^{-2}$)
- $P_{\text{water vapor}}$: the pressure of water vapor(Pa)($kgm^{-1}s^{-2}$)

- $R_{dry\ air}$: the specific gas constant for dry air (approximately 287.05Jkg$^{-1}$K$^{-1}$)
- $R_{water\ vapor}$: the specific gas constant for water vapor (approximately 461.49 Jkg$^{-1}$K$^{-1}$)
- T: Temperature(K) [10]

Total Pressure that is measured by the barometer is the sum of the pressure of dry air and pressure of water vapor. Pressure of water vapor at 22°C is 19.8 tor or 2645.302Pa.[11]

### 3.6.2.1 What is Gas Constant?

The gas constant is the measure of the weight of the molecules in a gas. It is denoted by the symbol R and it is a constant which is used in ideal gas law and Nernst equation. Boltzmann Constant can be found by dividing the gas constant(R) by Avogadro's Constant. Avogadro's constant is $6.022*10^{23}$.[12] The Boltzmann Constant is $1.3806488*10^{-23}$J/K.[13] Unlike Boltzmann constant which is expressed in units of energy per temperature per particle, gas constant is described in units of energy per temperature inclement per mole. It shows that the gas constant is a constant which relates energy scale to temperature scale, when considering a mole of particles at the stated temperature. The value of the gas constant is 8.3144621Jmol$^{-1}$K$^{-1}$.[14] The value of the gas constant of a specific gas can be found by dividing gas constant by the molar mass of the gas.[15]

$R_{specific} = R / N$

$R_{specific\ of\ water\ vapor} = 8.3144621$ / Molar mass of $H_2O$(kg)

Mass of 1 mol of O = 16g [16]

Mass of 1 mol of H = 1g [17]

Mass of 1 mol of $H_2O$ = 16 + 2*1 =18g

18g = 0.018kg

$R_{\text{specific of water vapor}} = 8.3144621 / 0.018$

$R_{\text{specific of water vapor}} = 461.914\ \text{Jkg}^{-1}\text{K}^{-1}$

$R_{\text{specific of dry air}} = 8.3144621 / \text{Molar mass of dry air(kg/mol)}$

Molar mass of dry air(kg/mol) = 0.0289644 kg/mol [18]

$R_{\text{specific of dry air}} = 8.3144621 / 0.0289644$

$R_{\text{specific of dry air}} = 287.058\ \text{Jkg}^{-1}\text{K}^{-1}$

## 4. DESIGN

**4.1 Research Question:** How does the surface area of a rotating propeller of a helicopter affect the average velocity of its free fall when it is released from a constant 4m height?

**4.2 Hypothesis:** An object which is free falling experiences an air drag. Things that affect air friction changes with changing speeds. For high speeds, the drag coefficient, air density, the cross-sectional area of the object and the square of the object's velocity is directly proportional to drag force. For low speeds a constant, which is a combination of the features of the fluid and dimensions of the object, and velocity is directly proportional to the drag force according to Stokes' Law. In the experiment, the air drag, air density and the drag coefficients will be stable and I think the helicopter models speed will be in the category of low speed. Therefore, average velocity of the helicopter models' is inversely proportional to the cross-sectional areas. As cross sectional area of propellers increases, the velocity of the helicopter decreases and the helicopter slows down. It lasts longer for it to land.

### 4.3 Variables

**Independent Variable:** Surface area of the propellers

**Dependent Variable:** Time required for the helicopter to land

**Controlled Variables:**

1) Height of which the helicopter models are released

Height at which the models are released should be controlled since the different heights conclude in different falling times. Because of this difference, there will be errors finding the velocity. Therefore, all of the helicopter models will be released from a height of 4m.

2) Air Flow

Air flow cause models to fluctuate. This fluctuation may cause errors while they can either decrease the time of falling of model if the air flow is in the same direction with the falling of the model or it can increase the falling time of model when the air flow is directed to opposite direction of the falling. Moreover, it can increase the falling time because flow will cause the helicopter model to fluctuate in air. The experiment will be done in my apartment which is a closed building. This will minimalize the air flow affecting the models.

3) Mass

Falling time of models changes with changing mass because as the helicopters get heavier, a larger gravitational force will act on them, causing them to fall faster. Therefore, all helicopters should have the same mass. First model will be made up of a rectangle of length 46.2 cm and width of 16.5cm. Other models will be made by cutting some parts of this rectangle. The part that is cut to lessen the surface area will be added to the compartment of the model. Therefore, all models will contain the same volume of paper and all will have the same mass.

4) Type of material the helicopter is made of

Different types of materials have different drag coefficients. Because of different drag coefficients, models will encounter different air resistances. Therefore, all models will be made up of paper.

5) Helicopter model

Differences in the creating process of helicopters leads to some errors while different helicopter models may lead to different air resistances. All models will be prepared using the same method, only changing the width and length of propellers. So, they will have same central axis of rotational motion which is the center of propellers.

6) Temperature

Temperature is inversely proportional to the air density and so air drag. Therefore, all of the trials will be made in the same temperature. Temperature will be stabilized to 22°C by special air conditioning.

7) Air pressure

Air pressure is directly proportional to air density. Differences in air pressure will lead to differences in air resistance. Therefore, Pressure/Flow Controllers will be used to stabilize the air conditioning to 693mmHg. With constant pressure and temperature, pressure of water vapor and dry air will be stabilized too.

8) Drag Coefficient

Drag coefficient is proportional to the air drag, so different values of drag coefficients will lead to different air resistances. Drag coefficients will be controlled by using the same material (paper) and using the same technique in the making process of helicopter models.

9) Ratio of Propellers Length to Width

All helicopter models' propeller's ratio must be equal in order to get the same drag coefficient. Therefore all propellers ratio of length to width will be 2.80.

## 4.4 Materials:

- 25 A3 paper
- Staples
- Scissors
- Meter Stick (±0.01[1])
- 4m height (±0.01[2])
- Stopwatch (±0.01[3])
- Pressure/Flow Controllers
- Special air conditioning

## 4.5 Procedure:

1. Stabilize the temperature to 22°C by using special air conditioning.
2. Stabilize air pressure to 693mmHg by using Pressure/Flow Controllers.
3. Take an A3 paper.

---

[1] 0.1 of the smallest division in the meter stick
[2] 0.1 of the smallest division in the meter stick
[3] 0.1 of the smallest division in the stopwatch

4. Cut the paper to form a rectangle with the length of 46.2 cm and width of 16.5cm.

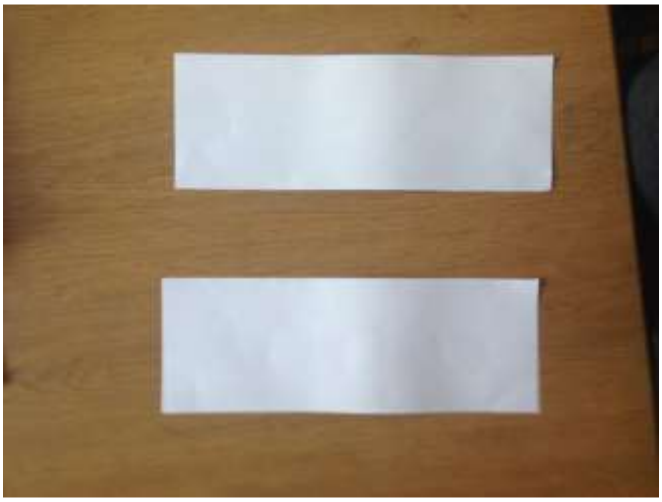


Diagram2: Two pieces of rectangle paper with the length of 46.2 cm and width of 16.5cm.

5. Separate the length of a piece into 11 same space and draw lines through 5th, 6th spaces.
6. Draw a line that separates the upper space of 5th into 2 equal piece.
7. Draw two lines that separate the lower space of 6th to 3 equal piece. Your paper will look like this.

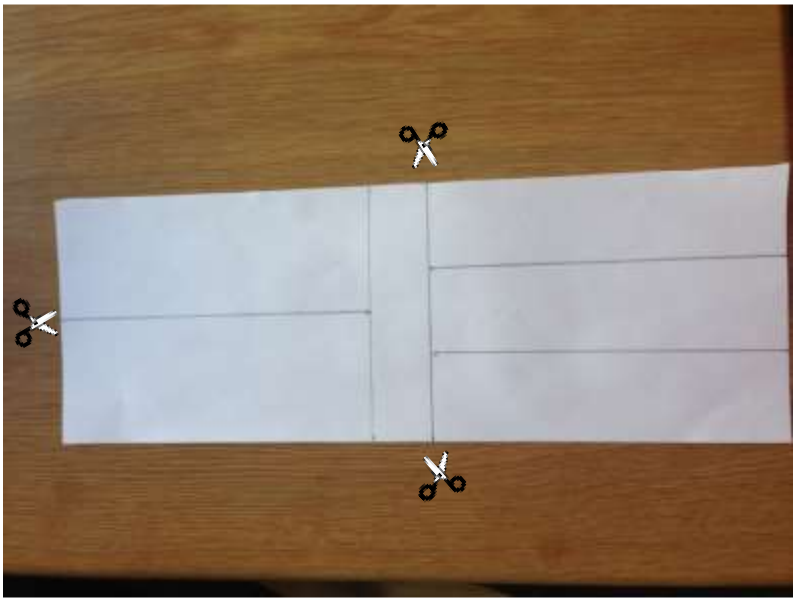


Diagram3: The look of paper after 6th step.

**8.** Cut the lines with a scissors as shown;

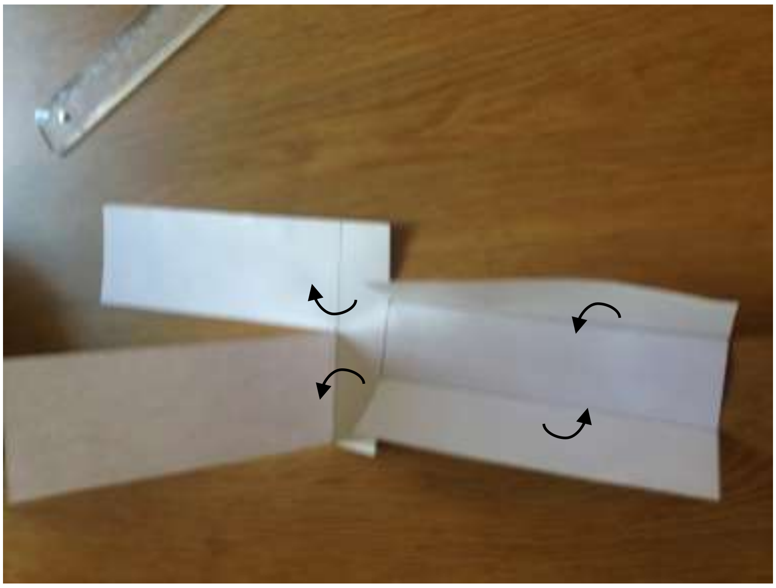


Diagram4: The lines which are cut.

**9.** Fold right lower part into left and left lower part into right.

**10.** Fold left upper part to behind and right upper part into front.

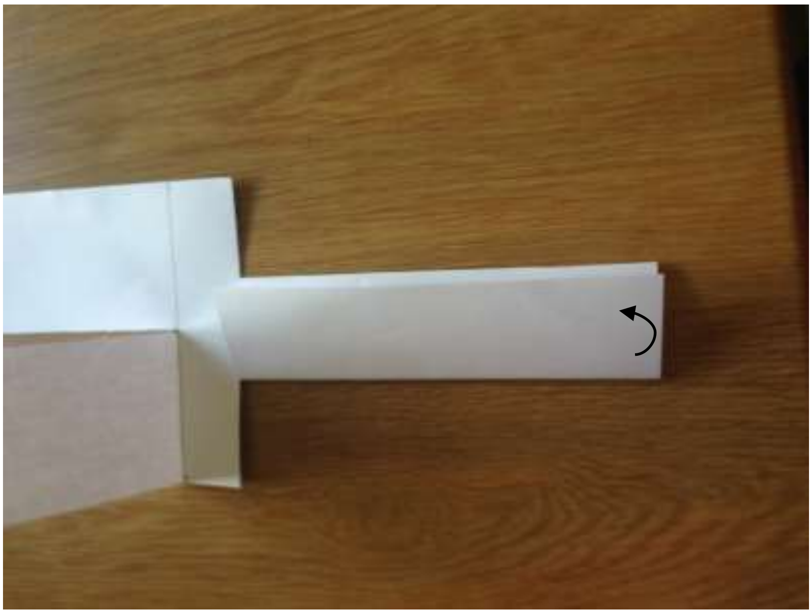


Diagram5: The paper after step 9.

**11.** Fold 11th space of the model, which you separate in step 4, to up.

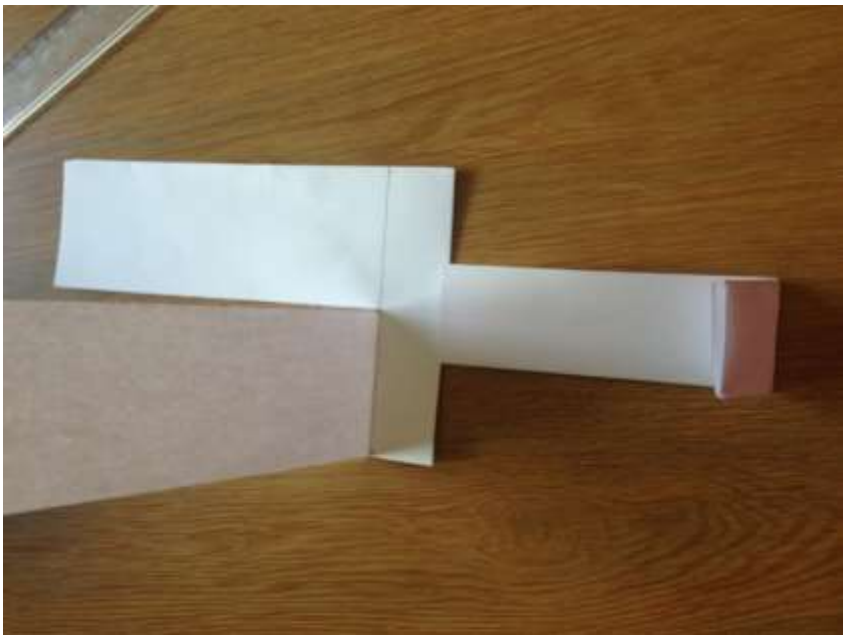


Diagram6: The model after step 10.

**12.** Staple the folded part.

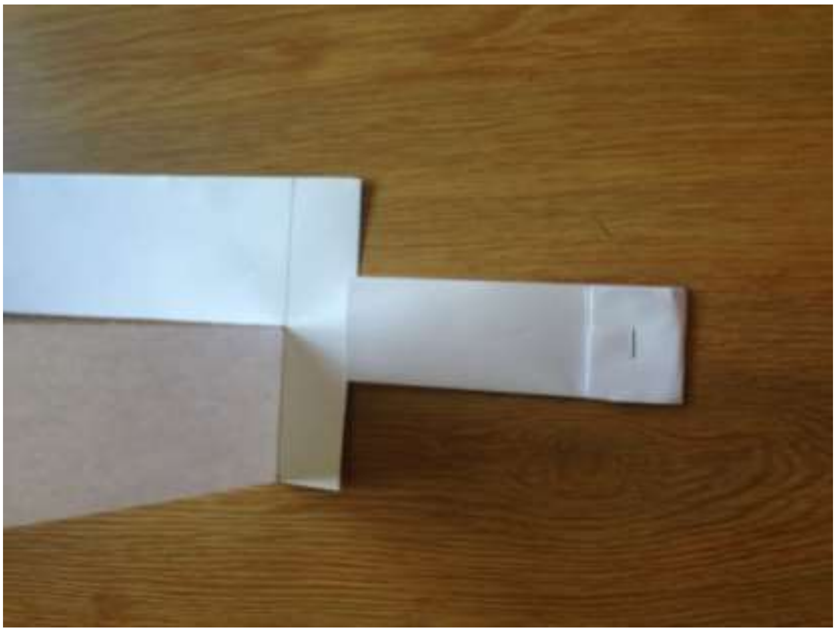


Diagram7: The model after step 11.

**13.** Hold the model from propellers, release it from 4m height and measure the time it lands.

Diagram8: The holding position of the model before it is been released.

**14.** Repeat the steps 3-13 with rectangles of;

| | Length(cm) ±(0.01) | Width (cm) ±(0.01) |
|---|---|---|
| 1 | 46.20 | 16.49 |
| 2 | 42.90 | 15.32 |
| 3 | 39.60 | 14.14 |
| 4 | 36.30 | 12.96 |
| 5 | 33.00 | 11.78 |
| 6 | 29.70 | 10.61 |
| 7 | 26.40 | 9.43 |
| 8 | 23.10 | 8.25 |
| 9 | 19.80 | 7.07 |
| 10 | 16.50 | 5.89 |

Table1: The length and width of the propellers of the helicopter models which are determined by ratio of length to width. Length/width must be 2.80 in all helicopter models.

Don’t forget to staple the parts you cut from the first rectangular (length: 46.2; width: 16.5) to make other rectangles in 11th space.

**15.** Release all the models 4 more times from the 4m height.

## 5. DATA COLLECTION AND PROCESSING

### 5.1 Raw Data

| Propeller's | | Trials | Time to Land(s)(±0.001) |
|---|---|---|---|
| Length(cm)±(0.01) | Width(cm)±(0.01) | | |
| 46.20 | 16.50 | 1 | 15.780 |
| | | 2 | 15.620 |
| | | 3 | 14.980 |
| | | 4 | 15.450 |
| | | 5 | 15.840 |
| 42.90 | 15.30 | 1 | 14.820 |
| | | 2 | 14.360 |
| | | 3 | 14.980 |
| | | 4 | 15.160 |
| | | 5 | 14760 |
| 39.60 | 14.10 | 1 | 11.980 |
| | | 2 | 12.580 |
| | | 3 | 12.920 |
| | | 4 | 12.450 |
| | | 5 | 12.370 |
| 36.30 | 12.90 | 1 | 10.430 |
| | | 2 | 10.670 |
| | | 3 | 10.620 |
| | | 4 | 10.550 |
| | | 5 | 10.480 |

| | | | |
|---|---|---|---|
| 33.00 | 11.80 | 1 | 8.680 |
| | | 2 | 8.750 |
| | | 3 | 8.320 |
| | | 4 | 8.460 |
| | | 5 | 9.010 |
| 29.70 | 10.60 | 1 | 3.440 |
| | | 2 | 3.520 |
| | | 3 | 4.680 |
| | | 4 | 4.230 |
| | | 5 | 3.960 |
| 26.40 | 9.40 | 1 | 3.570 |
| | | 2 | 3.870 |
| | | 3 | 3.670 |
| | | 4 | 3.770 |
| | | 5 | 3.360 |
| 23.10 | 8.20 | 1 | 3.460 |
| | | 2 | 2.900 |
| | | 3 | 3.160 |
| | | 4 | 3.230 |
| | | 5 | 2.980 |
| 19.80 | 7.10 | 1 | 3.220 |
| | | 2 | 3.260 |
| | | 3 | 2.810 |
| | | 4 | 3.380 |
| | | 5 | 3.060 |
| 16.50 | 5.90 | 1 | 2.260 |
| | | 2 | 2.560 |
| | | 3 | 2.400 |
| | | 4 | 2.550 |
| | | 5 | 2.300 |

Table2: Length and width of the propellers of the helicopter model’s and their time for landing with the trials.

## 5.2 Data Processing

- 1m = 100cm

  Length of Rectangle(cm) / 100 = Length of Rectangle(m)

  =46.2 / 100

  =0.462m

- Percentage Uncertainty of Length of Rectangle=

  =(Maximum Percentage Uncertainty of Length of Rectangle(cm) + Minimum Percentage Uncertainty of Length of Rectangle(cm)) / 2

  =(0.01 / 46.2 * 100) + (0.01 /16.5 * 100)

  =(0.02 + 0.06)

  =%0.08

  =%0.04

- Width of Rectangle(cm) / 100=Width of Rectangle(m)

  =16.5 / 100

  =0.165m

- Percentage Uncertainty of Width of Rectangle=

  =[Maximum Percentage Uncertainty of Width of Rectangle(cm) + Minimum Percentage Uncertainty of Width of Rectangle(cm)] / 2

  =(0.01 / 16.5 * 100) + (0.01 /5.9 * 100)

  =%0.06 + %0.17

  =%0.23

  =%0.12

- Length of 1 of 11 spaces in the rectangle=

  Length of rectangle / 11

  =0.462 / 11

  =0.042m

- Length of Propellers=

  Length of 1 of 11 spaces in the rectangle * 5

  =0.042 * 5

  =0.21m

- Width of propellers=

  =Width of Rectangle / 2

  =0.165 / 2

  =0.0825m

- Area of one of the propellers=

  Width of propellers * Length of Propellers

  =0.0825 * 0.21

  =0.0173m$^2$

- Cross-sectional area of the helicopter

  = Area of one of the propellers * 2 (because there are two propellers in each helicopter)

  =0.017 * 2

  =0.034m$^2$

- Uncertainty of area of one of the propellers=

  =Percentage Uncertainty of Length of Rectangle + Percentage Uncertainty of Length of Rectangle

  =%0.04 + %0.12

  =%0.16

- Time in which helicopter landed

  =(trial1 + trial2 + trial3 + trial4 + trial5) / 5

  =(15.78 + 15.62 + 14.98 + 15.45 + 15.84) / 5

  =77.67 / 5

  =15.53s

- Uncertainty of time=

  Uncertainty of trial1 + Uncertainty of trial2 + Uncertainty of trial3 + Uncertainty of trial4 + Uncertainty of trial5

  =0.001 + 0.001 + 0.001 + 0.001 + 0.001 = 0.005

  =(Maximum Percentage Uncertainty of Time + Minimum Percentage Uncertainty of Time) / 2

  =[(0.005 / 2.41 * 100) + (0.005 / 15.53 * 100)] / 2

  =(0.21 + 0.03) / 2

  =0.24 / 2

  =%0.12

- Average Velocity(m/s) = Height(m) / Time in which helicopter landed(s)

  =4 / 15.53

  =0.26m/s

- Uncertainty of Average Velocity =

  = Percentage Uncertainty of Height + Percentage Uncertainty of Time

  Uncertainty of Height is 0.01 because it is the 0.1 of the smallest division in meter

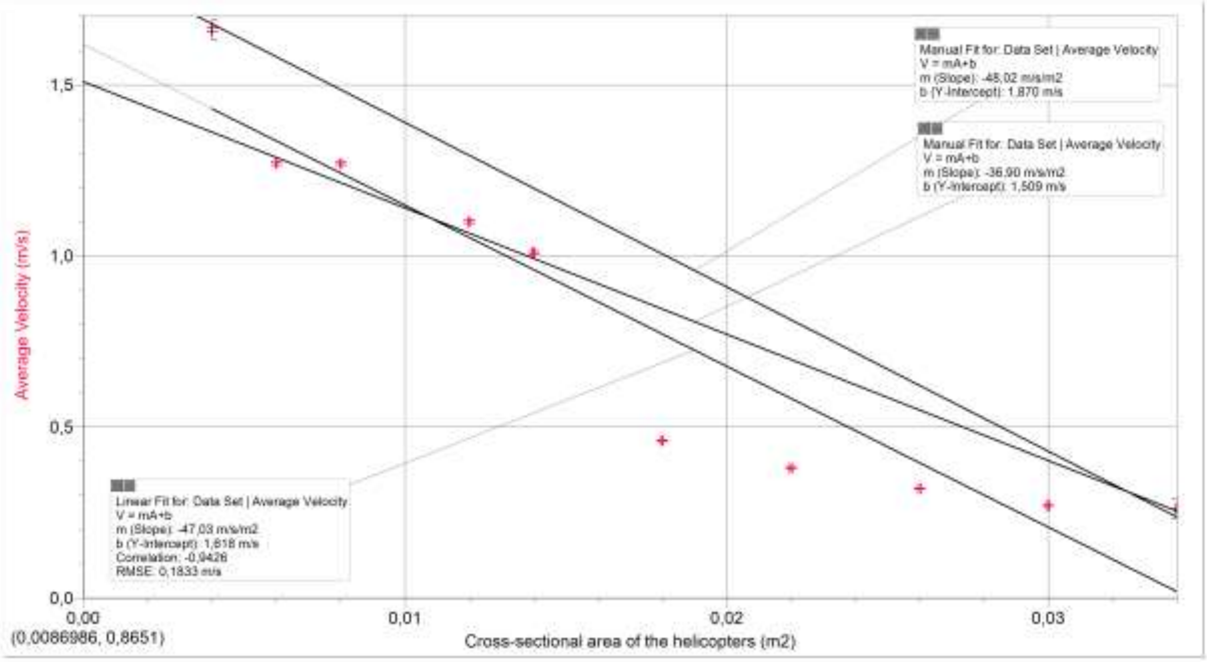
  = (0.01 / 4 * 100) + (%0.12)

  = %0.25 + %0.12

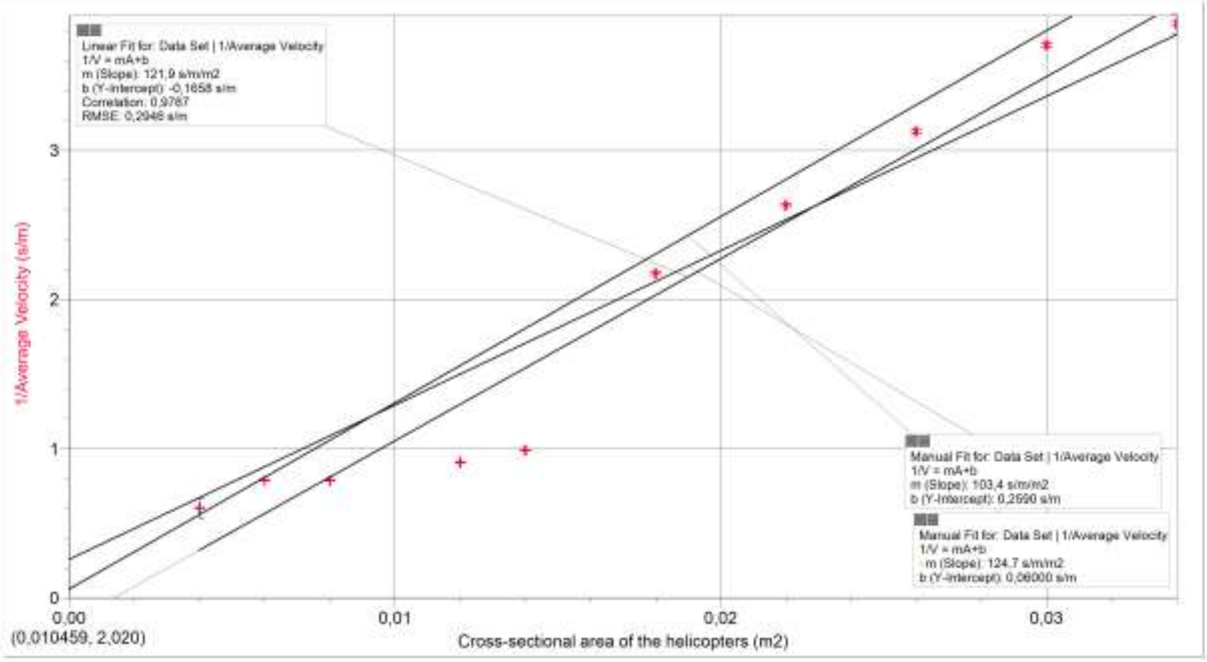
  =%0.37

| Length of Rectangle (m) ± (%0.04) | Width of Rectangle (m) ± (%0.12) | Length of 1 of 11 spaces in the rectangle (m) ± (%0.04) | Length of Propellers (m) ± (%0.04) | Width of propellers (m) ± (%0.12) | Area of one of the propellers ($m^2$) ± (%0.16) | Cross-sectional area of the helicopters ($m^2$) ± (%0.16) | Time in which helicopter landed (s) ± (%0.12) | Average Velocity(m/s) ± (%0.37) |
|---|---|---|---|---|---|---|---|---|
| 0.462 | 0.165 | 0.042 | 0.210 | 0.082 | 0.017 | 0.034 | 15.53 | 0.26 |
| 0.429 | 0.153 | 0.039 | 0.195 | 0.076 | 0.015 | 0.030 | 14.82 | 0.27 |
| 0.396 | 0.141 | 0.036 | 0.180 | 0.070 | 0.013 | 0.026 | 12.46 | 0.32 |
| 0.363 | 0.129 | 0.033 | 0.165 | 0.064 | 0.011 | 0.022 | 10.55 | 0.38 |
| 0.330 | 0.118 | 0.030 | 0.150 | 0.058 | 0.009 | 0.018 | 8.64 | 0.46 |
| 0.297 | 0.106 | 0.027 | 0.135 | 0.052 | 0.007 | 0.014 | 3.97 | 1.01 |
| 0.264 | 0.094 | 0.024 | 0.120 | 0.046 | 0.006 | 0.012 | 3.65 | 1.10 |
| 0.231 | 0.082 | 0.021 | 0.105 | 0.040 | 0.004 | 0.008 | 3.15 | 1.27 |
| 0.198 | 0.071 | 0.018 | 0.090 | 0.034 | 0.003 | 0.006 | 3.15 | 1.27 |
| 0.165 | 0.059 | 0.015 | 0.075 | 0.028 | 0.002 | 0.004 | 2.41 | 1.66 |

Table3: Processed data of length and width of the rectangle, length of 1 of 11 spaces in the rectangle, length and width of propellers in metres; cross-sectional area of the propellers and the helicopter in $m^2$; time needed for helicopter models to land in seconds and velocity of the helicopter models in meter per seconds with their percentage errors.



Graph1: Cross-sectional area of the helicopter vs Average Velocity in free fall with best line and worst lines.



Graph2: Cross-sectional area of the helicopter vs 1/Velocity in free fall with best line and worst lines.

- Slope of Best Line: 121.9s/m$^3$
- Slope of Minimum Worst Line: 103.4s/m$^3$

- Slope of Maximum Worst Line: 124.7s/m$^3$
- Uncertainty = (Slope of Maximum Worst Line - Slope of Minimum Worst Line) / 2

  = (124.7 - 103.4) / 2

  = 21.3 / 2

  =10.6s/m$^3$
- Percentage Error[19] = (Uncertainty / Slope of Best Line) * 100

  (10.6 / 121.9) * 100

  =0.087 * 100

  =8.7%
- For Graph1: Velocity = m * Area + y-intercept

  General Equation between Velocity and Area $\longrightarrow$ Velocity = -47.03 * Area + 1.618

## 6. Conclusion and Evaluation

My research question was asking the effect of surface area of rotating propellers of a helicopter on its average velocity when free falling. In my hypothesis, I suggested that velocity is inversely proportional to surface area. From the second graph and the minus sign in the general equation “Velocity = -47.03 * Area + 1.618” it can be understood that the velocity decreases as the cross-sectional area increases and the cross-sectional area is directly proportional to 1/velocity. The cross-sectional area is inversely proportional to the velocity for low speeds and square of velocity for high speeds theoretically.[20] Therefore, the helicopters’ velocity in the experiment will be regarded as low speed. The experiment is proper to answer the research question and the hypothesis is confirmed. In both graphs some error bars are seen. This shows that there are some errors. The differences in the raw data show that there are some random errors. For instance, the maximum and minimum values of landing times when the length is 23.1cm and the width is 8.2cm are 3.46 and 2.90. There is a difference of 0.56s. (3.46-2.90=0.56) This may seem small but when we compare it to the mean value of times required for landing which is 3.15, it can be found out that it is 17.8% of the mean value; (0.56/3.15)*100. These kind of big differences in raw data show that there are some random errors.

It can be seen that there are some random errors in this experiment from the difference between trials in the raw data which means that data is not precise. These random errors include errors made during the creating process of the helicopter models and the experimental errors. First of all, there is a possibility that the helicopter models' width and length are different than it was thought because my hand slips when drawing the lines. Also, when I was using the ruler when drawing lines, I put the ruler to the exact points and so the line drawn was 1mm above the where it should be. Besides, in some experiments I could not release the helicopters properly: My hand was a little wet so one propeller of the helicopter stick to my thumb. It was released a little after the other propeller, so the helicopter shook when falling. This may cause an unexpected increase in the time required for the helicopter to land. Furthermore, the ratio of length to width of the propellers must be 2.80. For example, when the length is length is 16.50cm the width is 5.89285714. However, the smallest division of the ruler that is used is 0.01cm. Therefore, the width is arranged as 5.89. This may lead small errors. Another random error is at some trials I didn't hold the helicopters straight so they didn't fall straightly but they swung. Moreover, there are some errors when stopping the time. The experiment was done by two person: one at the bottom with the stopwatch and me at the top releasing the models. I said I dropped to make the other person stop the watch. However, there was a slight difference with my releasing time and the onset of the stopwatch. This may cause some misreading at the time needed for the helicopter to land. In addition to these, the helicopter's model changes when it hits the ground due to the force it encountered by the floor. We use the same models in all experiments and at final trials the models were slightly deformed. Therefore, the helicopters didn't fall straightly. This is a cause in the increase of time in the final trials. In addition to the random errors, there were some systematic errors too. For instance, we staple the parts that were cut to the 11th space to stable the mass of the helicopters. Therefore, there was a difference in the center of gravity of the small and big helicopter models. The mass was gathered in the bottom at the small helicopters but it was spreaded in the bigger ones. Therefore, they fell faster than expected. Additionally, in small models, instead of 1 staple, 2 staples are used because 1 staple didn't hold all the paper that needs to be added. Because of this smaller models are heavier about 0.1g which is the mass of the extra staple. Those kind of systematic errors show that data is not very accurate.

We can get rid of these errors by using a wooden rectangle when drawing the rectangles. The wooden rectangle will be 1mm fewer than the intended width and length of the rectangles. The model rectangle will be placed in A3 paper and lines will be drawn as its perimeter. The 1mm spaces would lead us to get exact rectangles because those 1mm spaces will be filled with lines. That way the length and width will be exactly as intended and the errors that was caused by the creating process of the helicopter models will be prevented. A system is needed for the releasing of the helicopters to block the effects of errors made during the releasing process. It can be something like this: an object will be attached to 4m height. (Preferably wooden because it needs to be something durable and working with metals is harder than working with woods.) A latch will be added to the end of the wood. It will be controlled that wood makes a 90° with the latch and the 4m height. The helicopters will be attached to the latch and released by opening the latch. One must be careful about the latch being humid. Otherwise, it may cause the same error with one propeller sticking to the humid hand. Therefore, new latch must be used in each experiment, each trial. That way there will be no errors when releasing the helicopters; the helicopter will fall straightly and the time of landing will be as expected. Moreover, the stopwatch error can be blocked by this solution too. I would count down from 3 and then open the latch. Therefore, the stopwatch will be started at the exact time that helicopter was released. A ruler with a smaller division should be used and ratio of length to width must be arranged according to the smallest unit ruler can measure. Furthermore, new helicopter models must be made for each experiment, each trial. That way the errors due to the damage of the helicopters will be thwarted. Besides, there should be more trials to minimalize random errors. For instance, it would be better if there were 10 trials for each length and width instead of just 5. The cut parts from the original rectangle must be spread equally to all parts of the helicopter including the propellers. It can be done by stapling some parts to the propellers and the body of the helicopter instead of putting all of the cut parts into the 11th space. Additionally, all helicopters body parts should be stapled twice for all of the models to be equal in terms of masses.

Smaller models had a higher angular velocity but smaller radius of rotation. Bigger models had a lower angular velocity but bigger radius of rotation. Angular velocity and radius was inversely proportional and as it can be seen from the equation;

$$v = \omega r \quad or \quad \omega = \frac{v}{r}$$ [21]

V= Linear Velocity

ω =Angular Velocity

r = Radius

This shows that linear velocity was held constant. Therefore, it didn’t lead to any errors.

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