TED Ankara College Foundation Private High School

International Baccalaureate

Extended Essay

Effect of Counterweight Mass on the Range of a Counterweight Trebuchet

Candidate Name: Hidayet Cem Ozturk

Candidate Number: 001129 - 0147

Supervisor Name: Mehmet Bozkurt

May 2015

Physics

Word Count: 3636

Hidayet Cem Ozturk
001129 - 0147

Abstract

Although it has none or little use today, trebuchets were essential siege weapons in medieval warfare. Especially the counterweight trebuchet is capable of throwing massive objects at great distances. In this investigation, it was intended to examine a counterweight trebuchet's performance over increasing quantity of mass that the counterweight part contains. A mini trebuchet is built out of wooden planks, using a simple mechanism to project a small metal ball. After controlling all the other factors the experiment was carried out to observe the ball's range with different counterweight masses. When the data recorded was plotted on a graph, a linear change was observed with a slope of 3.208 m/kg for a 130.0 g metal spherical ball. According to the hypothesis, a linear change was estimated indeed, only with a slope of 4.630 m/kg. This ratio of error, approximately 30.71%, can be ignored, since estimations on the range were based on a frictionless system, not to mention it's not possible to construct a 100% efficient system or even close to that in terms of thermodynamics. Correlation coefficient of the experimental results is 0.9964, which shows that the results are consistent within the experiment itself. On the other hand, efficiency of the trebuchet gradually decreased with increasing counterweight mass, indicating that the system has an energy capacity and the range would rise only up to a point if further investigation was done. As a conclusion, this experiment provides a brief understanding of principals of pre-gunpowder siege units in general, as well as scopes the particular mechanisms of counterweight trebuchets and methods to optimize a trebuchet in order to maximize its energy efficiency.

Word Count: 271

Contents

Hidayet Cem Ozturk
001129 - 0147

1. INTRODUCTION

1.1 Background Information: Trebuchet is a long-range and powerful siege weapon first used in 12th century to destroy enemy walls and buildings. Due to its efficient engineering, it was a decisive unit in medieval wars and long after the invention of gunpowder and cannon. The mechanism of a trebuchet is based on a firing system that stores potential energy by lifting the weight to a certain height and this potential energy is converted to the kinetic energy of the payload during the release of the counterweight. The most common type of the trebuchet is counterweight trebuchet. The range of the trebuchet is affected by many factors, including the mass of the counterweight. Adjusting the mass changes the stored potential energy, thus changes the initial velocity of the payload. In this experiment, a small sized wooden trebuchet will be built. The payload is chosen as a metal ball. The counterweight's mass is changed and the displacement of the ball will be examined.

1.2 Parts and Functioning of a Trebuchet:

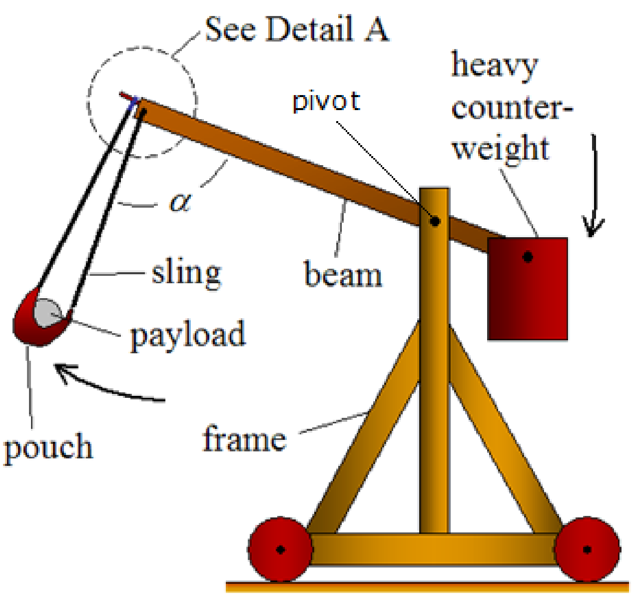


Diagram 1: Parts of a counterweight trebuchet labeled.[1]

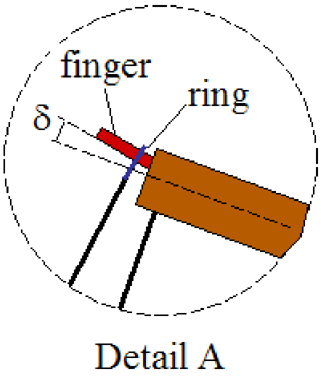


Diagram 2: Detailed view of the firing system.[2]

Frame

The frame is a part of the skeleton of the trebuchet, attached to the base and carries the pivot that the beam can slide on freely. It provides height and stability to the beam. It consists of 4 pieces, 2 on each side that is placed in a slightly inclined position.

Pouch

The half-closed pocket sewed into the middle of the sling is the pouch. It is needed to launch the payload properly, since the pocket must open and let the ball be ejected when the end of the sling attached on the nail detaches. It is made up of a soft fabric.

Beam

The rod that carries the counterweight and the payload in each side and is fixed to the frame is defined as beam. It is mobile to enable the launch of the payload. The arm in counterweight side is shorter compared to that of payload side in order to provide a greater linear speed of the payload. The ratio is taken as 1/3 in this investigation.

Pivot

Pivot is the bar that fixes the two sides of the frame and provides mobility for the beam. It is in a cylinder shape to enable swinging for the beam.

Sling

A thin piece of rope is attached at the end of the payload side of the beam. This rope is called the sling. One end of the rope is fixed to the beam and the other one is attached to the finger on the back of the beam. The sling holds the pouch that carries the payload, the metal ball in this experiment. High initial velocities can be reached via the flexibility of the rope, rather than a solid rod that holds the payload.

Payload

The payload is the projectile that is placed in the pouch and launched when the counterweight is released. In this investigation, it is a metal ball.

Finger and Ring

Finger is the nail that allows the sling to detach and therefore provide space for the pouch to launch the payload. It is nailed in a slightly inclined position so that the sling does not detach too early.

The slipknot that is suspended on the finger is called the ring. It functions in launching process as a trigger. Due to centripetal force acting on it because of the circular motion of the beam, it is released and afterwards the pouch is opened.

Counterweight

In order to obtain a certain potential energy to fly the payload, a mass is needed. Counterweight is one of the factors that affect the range of the payload. Height, mass and gravitational acceleration determine the potential energy of the counterweight, which is transferred to the payload as kinetic energy. Dumbbells with different masses will be used as counterweight in this experiment.

1.3 Terms and Formulae:

What is energy transformation?

Energy transformation can be defined as one type of energy converting to another type. Systems produce an output of the converted energy and a small fraction of energy loss, due to friction and other forces.

What is mechanical energy?

Mechanical energy is calculated by the sum of potential energy and kinetic energy of an object. It is conserved in a frictionless system.

$$E_M = E_P + E_K$$ [3]

E_M: mechanical energy (J)

What is potential energy?

It is the energy stored in a body due to its mass and position. Potential energy is formulated as:

$$E_P = m.g.h$$ [4]

E_P: potential energy (J)
m: mass of the object (kg)
g: gravitation acceleration ($m.s^{-2}$)
h: height of the object (m)

What is kinetic energy?

It is the energy a body has due to its motion. Kinetic energy formula of an object is:

$$E_K = \frac{1}{2} m.v^2$$ [5]

E_K: kinetic energy (J)
m: mass of the object (kg)
v: velocity of the object ($m.s^{-1}$)

Transformation of potential energy to kinetic energy

If it is assumed that the system is ideal and there is no energy loss, the formula:

$$E_{M_c} = E_{M_p} \Rightarrow \frac{1}{2} m_c.v^2 + m_c.g.h = \frac{1}{2} m_p.v^2 + m_p.g.h$$

Can be derived, where:

E_{Mc}: mechanical energy of the counterweight
E_{Mp}: mechanical energy of the payload
m_c: mass of the counterweight (kg)
g: gravitation acceleration ($m.s^{-2}$)
h: height of the counterweight (m)
m_p: mass of the payload (kg)
v: velocity of the payload ($m.s^{-1}$)

- ✓ Velocity of the counterweight, so its kinetic energy is assumed as zero.

What is free fall?

The motion an object does when no other forces than gravity acts on it is called free fall. The object accelerates with a constant acceleration. This value is 9 $m.s^{-2}$ for the Earth. The equation for determining the falling time is:

$$h = \frac{1}{2} g.t^2$$ [6]

h: height of the object (m)
g: gravitational acceleration ($m.s^{-2}$)
t: falling time (s)

What is projectile motion?

An object projected from a surface in an inclined way does projectile motion. The pathway of the object draws a curve due to the gravity. Vertical and horizontal components of velocity of the object determine its range.

What is displacement?

The minimum distance of a projectile from its initial position to final position is defined as displacement or range. The object reaches its maximum range when trajectory angle is 45°. Travelled distance is calculated by the formula:

$$d = (v_0)_x .t$$ [7]

d: displacement (m)
$(v_0)_x$: horizontal component of initial velocity of the object ($m.s^{-1}$)
t: time of flight (s)

What is time of flight?

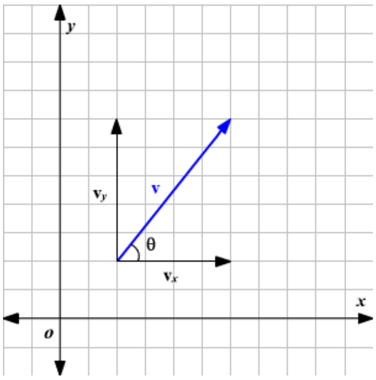
The time elapsed between the launch of a projectile and the its first hit on the ground is called the time of flight. Formula for the time of flight is:

$$t = \frac{2.(v_0)_y}{g}$$ [8]

t: time of flight (s)
$(v_0)_y$: Vertical component of initial velocity of the object ($m.s^{-1}$)
g: gravitational acceleration ($m.s^{-2}$)

1.4 Trigonometric Properties:

Calculating the components of a vector and relevant trigonometric equations



$$v_x = v.\cos\theta$$

$$v_y = v.\sin\theta$$

$$\sin 2\alpha = 2.\sin\alpha.\cos\alpha$$

Diagram 3: Modeling a vector and its components. [9]

2. DESIGN

2.1 Research Question:

How does range of the payload of a counterweight trebuchet change with a linear increase in counterweight mass, under same trajectory angle of the payload?

2.2 Hypothesis:

Range of the payload of a counterweight trebuchet increases linearly with the linear increase in counterweight mass; because mass of the counterweight is directly proportional to velocity of the payload squared, which is directly proportional to the displacement when the time factor is replaced with time of flight formula containing vertical component of velocity of the payload. The process can be summarized as:

$$m_c.g.h = \frac{1}{2} m_p.{v_0}^2 \Rightarrow v_0 = \sqrt{\frac{2.m_c.g.h}{m_p}}$$

$$d = (v_0)_x.t \wedge t = \frac{2.(v_0)_y}{g} \Rightarrow d = \frac{2.(v_0)_x.(v_0)_y}{g}$$

$$(v_0)_x = v_0.\cos\alpha \wedge (v_0)_y = v_0.\sin\alpha \Rightarrow d = \frac{2.{v_0}^2.\sin\alpha.\cos\alpha}{g}$$

$$d = \frac{2.{v_0}^2.\sin\alpha.\cos\alpha}{g} \wedge v_0 = \sqrt{\frac{2.m_c.g.h}{m_p}} \Rightarrow d = \frac{4.\sin\alpha.\cos\alpha.m_c.h}{m_p} = \frac{2.\sin 2\alpha.m_c.h}{m_p}$$

m_c: counterweight mass (kg)

m_p: payload mass (kg)

g: gravitational acceleration ($m.s^{-2}$)

h: change in height of the counterweight (m)

d: travelled distance (m)

v_0: initial velocity of the payload ($m.s^{-1}$)

$(v_0)_x$: horizontal component of initial velocity of the payload ($m.s^{-1}$)

$(v_0)_y$: vertical component of initial velocity of the payload ($m.s^{-1}$)

t: time of flight (s)

α: angle between the surface and the velocity vector of the payload (°)

It can be seen that displacement (d) is directly proportional to counterweight mass (m_c).

2.3 Variables:

Independent variable: Counterweight mass

Dependent variable: Range of the payload

Controlled variables:

1) Initial and releasing height of the counterweight
2) Initial and releasing height of the payload
3) Trajectory angle
4) Payload mass
5) Sling length
6) Counterweight side length of the beam
7) Payload side length of the beam
8) Mass of the beam
9) Pivot height
10) Angle between the finger and the cross section of the beam

Controlling the controlled variables

1) Change in height of the counterweight determines the total energy transformation, thus the initial velocity of the payload. In order to control, a stabilized sack is used to carry the masses. The sack hits the ground, preventing it to swing so that the change in height is constant.

2) The payload also gains potential energy when it is lifted through the air, which affects its launching kinetic energy. To control it, the payload is placed on a surface in first position. The counterweight hits the ground always in the same height so the payload's launching height is constant.

3) According to the formula derived earlier:

$$d = \frac{2.\sin 2\alpha.m_c.h}{m_p}$$

Range depends on the sine of twice of the angle between x-axis and direction of velocity. Height of the counterweight is fixed in order to make the angle stable. A protractor is used to calculate the angle between surface and the beam in launching position.

4) Travelled distance is reversely proportional to mass of the payload mass, according to the formula above. Same plastic ball is used throughout the experiment to control the payload mass.

5) Sling length determines height of the payload and its potential energy when it is released. Same string sling is used throughout the experiment to launch the ball in a constant height.

6) The beam size changes the initial height and potential energy of the counterweight. A hole is drilled on the beam to fix it on the pivot, so that the energy change is constant. Counterweight side is the shorter side of the beam.

7) Long side of the beam changes the both initial height of the counterweight and the releasing height of the payload, since the point of support determines the inclination of the beam. The hole drilled on the beam fixes it on the pivot. Payload side is the longer side of the beam.

8) When the payload is lifted, long side of the beam is also lifted and gains some potential energy. This energy is not equal to the energy that short side of the beam loses when the counterweight falls, since their masses are not equal. Same uniform, wooden plank is used as the beam to get constant energy change.

9) Pivot height is another variable that is influential on inclination of the beam. The pivot is fixed to the frame so its height is constant.

10) It affects the releasing time of the payload, thus the initial height. A nail is used as the finger, which is stationary.

✓ Gravitational acceleration is not taken as a controlled variable.

2.4 Materials:

1) Metal ball (130.0 ± 0.1 g)
2) Dumbbells (1.0 ± 0.1 kg), (2.0 ± 0.1 kg) x 2, (3.0 ± 0.1 kg) x 2
3) String sling (80.0 ± 0.1 cm), (60.0 ± 0.1 cm)
4) Rhombus pouch, made up of leatherette (25.0 ± 0.1 x 12.5 ± 0.1 cm in diagonals)
5) Plastic sack (28.5 ± 0.1 g)
6) Hammer and nails
7) Tape measure (± 1.0 cm)
8) Tapeline (± 0.1 cm)
9) Weighing scale (± 0.1 kg)
10) Precision scale (± 0.1 g)
11) Protractor

2.5 Method:

First of all, parts of the trebuchet (base, plate, frame, pivot and beam) are combined with each other using hammer and nails. After the construction process, one eyehook on each side and a nail on the long side are nailed on the beam. One piece of string is attached on the eyehook nailed on the long side and a slipknot is made on one side of the other, suspending it on the nail. Then, the pouch is attached to these strings. The plastic sack is suspended on the eyehook nailed on the short side. The metal ball is placed in the pouch and the weight (2, 3, 4, 5, 6, 7, 8, 9 kg) is placed in the plastic sack. Angle between the surface and the beam is measured via the protractor in order to compute the components of the velocity vector of the ball. At this point, the trebuchet is ready to fire but the setup is not completed.

Figure 1: Illustrating the current setup of the trebuchet.

In order to set a firing position, the sling is stretched and the pouch is placed on the plate, facing the backside of the trebuchet.

Figure 2: Final firing position of the payload.

Figure 3: Overall view of the trebuchet in firing position.

In this position, the beam is set free and the weight accelerates towards the ground. The pouch is pulled by the beam, so it travels on the plate and loses contact when the beam reaches a certain height. The counterweight hits the ground, with the kinetic energy that beam has gained, the pouch continues to accelerate. While the beam is performing a circular motion, the ring detaches due to centripetal force acting on it, letting the pouch open and the ball eject. The ball starts a projectile motion. When it hits the ground for the first time, the point is marked and the distance is measured from the initial position. 5 trials are done for each mass of the counterweight.

3. DATA COLLECTION AND PROCESSING

3.1 Data Collection:

Raw Data Table

Mass of the counterweight (m_c) (± 0.1 kg)			2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Range of the payload (d) (± 0.01 m)	Trials	1	6.15	10.40	13.81	16.04	20.13	21.42	26.68	29.32	30.78
		2	6.23	9.71	14.42	15.87	20.27	24.73	26.47	30.55	30.73
		3	5.48	9.85	13.60	16.74	19.58	25.26	25.87	28.38	33.19
		4	5.21	7.94	11.80	17.72	18.63	22.11	27.46	27.52	30.05
		5	5.36	10.67	11.62	16.46	20.39	23.61	27.14	28.64	29.66

Table 1: Displacement of the ball, using different masses for counterweight.
Uncertainties of distance arrived from the tape measure used and that of mass of the counterweight from the weighing scale.

These were the experimental results of range of the trebuchet. Now the theoretical values will be calculated and compared to data recorded above.

Mass of the counterweight (m_c) = 2.0 ± 0.1 kg
Mass of the ball (m_p) = 130.0 ± 0.1 g
Initial height of the counterweight (h) = 31.0 ± 0.1 cm
Final height of the counterweight = 0
Initial height of the ball = 6.0 ± 0.1 cm
Releasing height of the ball = 115.3 ± 0.1 cm
Trajectory angle (α) = 37.5 ± 0.1 °

3.2 Processing:

Total potential energy change of the ball:

$$(\Delta E_P)_p = m_p.g.\Delta h = 0.13 \times 9.81 \times 1.093 = 1.394J$$

Some of the potential energy is used to lift the long side of the beam. Energy change in the short side and the first 25 cm's of the long side are equal and cancel each other.

Mass of the beam = 0.7 ± 0.1 kg
Initial height of the beam at 25 cm = 37.6 ± 0.1 cm
Final height of the beam at 25 cm = 61.9 ± 0.1 cm
Initial height of the beam at 75 cm = 6.0 ± 0.1 cm
Final height of the beam at 75 cm = 89.2 ± 0.1 cm

Total potential energy change of the beam:

$$(\Delta E_P)_b = m_b.g.\Delta h = 0.35 \times 9.81 \times 0.50 \times \frac{(0.243 + 0.832)}{2} = 0.923J$$

Energy transformation of the counterweight is calculated:

$$E_P = m_c.g.h = 2.0 \times 9.81 \times 0.31 = 6.082J$$

Potential energy that the ball and the beam have gained is subtracted:

$$(\Delta E_P)_T = (\Delta E_P)_c - \left[(\Delta E_P)_p + (\Delta E_P)_b\right] = 6.082 - (1.394 + 0.923) = 3.765J$$

It is assumed that the remaining energy is converted to kinetic energy of the ball:

$$E_K = 3.765J = \frac{1}{2} m_p.v^2 = 0.065 \times v^2 \Rightarrow v = 7.61 m.s^{-1}$$

$$v_x = 7.61 \times \cos 37.5 = 6.04 m.s^{-1}$$

$$v_y = 7.61 \times \sin 37.5 = 4.63 m.s^{-1}$$

Given the data, the ball performs projectile motion from a height.

Time of flight:

$$t = \frac{v_y}{g} + \frac{\sqrt{2g.h_i + v_y^{\ 2}}}{g} = 1.15s$$

h_i: initial height of the ball in releasing position

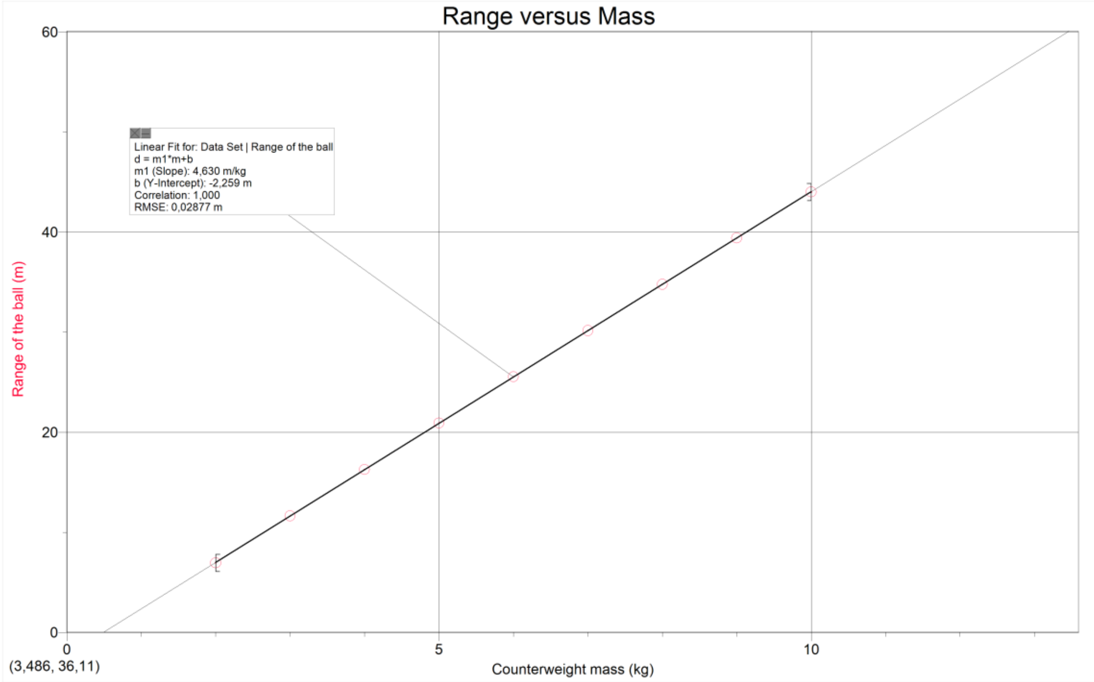
Range:

$$d = v_x.t = 6.95m$$

When the process is applied for all counterweight masses:

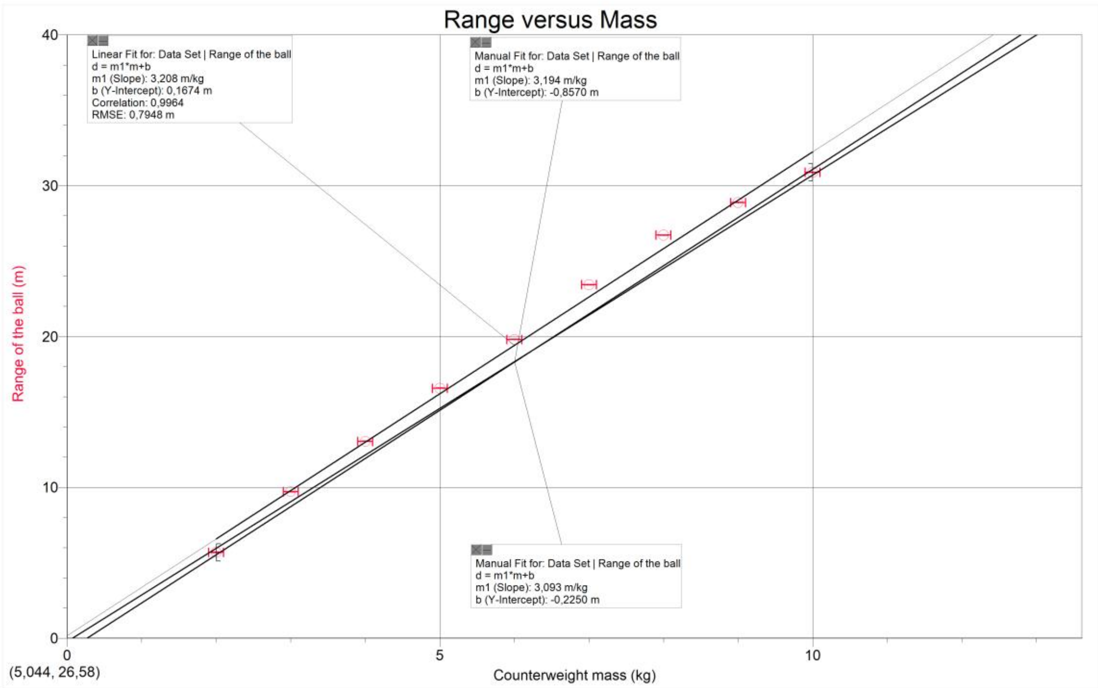
Counterweight mass (m_C) (± 0.1 kg)	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Expected range of the payload (d) (m)	6.95 (±0.51)	11.64 (±1.36)	16.28 (±1.40)	20.92 (±0.92)	25.55 (±0.88)	30.16 (±1.92)	34.78 (±0.80)	39.41 (±1.52)	44.01 (±1.76)
Average range (d) (± 0.01 m)	5.69	9.71	13.05	16.57	19.80	23.43	26.72	28.88	30.88
Uncertainty	0.51	1.36	1.40	0.92	0.88	1.92	0.80	1.52	1.76
Standard deviation	0.47	1.07	1.26	0.73	0.72	1.65	0.61	1.13	1.37
Percentage error	18.13	16.58	19.84	20.79	22.50	22.31	23.17	26.72	29.83

Table 2: Processed data, representing the theoretical and actual values of the range of the payload for given counterweight masses, including statistical error calculations.



Graph 1: Expected values of the range according to the hypothesis.

Equation of the line: $d = 4.630 \times m_c - 2.257$



Graph 2: Experimental results, line of best fit, minimum worst line and maximum worst line of the data.

Equation of the line of best fit: $d = 3.208 \times m_c + 0.1674$

3.3 Error Calculation:

Difference between the slopes of two graphs indicates the deflection of experimental results from the theoretical values. Percentage difference is calculated as:

$$\frac{m_{theoretical} - m_{experimental}}{m_{theoretical}} \times 100 = \frac{4.630 - 3.208}{4.630} \times 100 = 30.71\%$$

5. CONCLUSION AND EVALUATION

5.1 Conclusion:

In this investigation, the aim was to examine the change in range of a payload launched by a counterweight trebuchet when the mass of the counterweight is increased. The prediction made about the range is based on energy transformation: potential energy of the counterweight decreases while it is accelerating towards the ground and this energy is transferred to the ball as kinetic energy, making it move. After the ball is released and becomes independent from the trebuchet, it performs a projectile motion. Range of the ball can be determined with the known values of initial velocity of the ball and the releasing angle of the ball with respect to the ground. While observing this relation, there were various variables that would affect the dependent variable, such as the energy change of other parts of the trebuchet and trajectory angle of the ball. Controlling these properties would set an error margin and provide consistent results with the experimentation. Not all of them were perfectly controlled, considering the error bars in the x-axes (counterweight mass). According to the hypothesis, a linear graph was expected between the range and the mass. The correlation between these two quantities is approved to be in a first degree equation, as the graph shows a straight curve.

However, when the experimental results are plotted on a graph, a magnified deflection can be detected which is reflected on the graph representing the experimental data, with a slightly curved line. This indicates that there were major errors in the procedure or the hypothesis lacked precision, as the percentage error increased along the heavier counterweights reaching 29.83% at peak. A total error of 30.71% in the slope of the counterweight mass versus range graph which is quite high, also confirms the presence of certain random and systematic errors.

5.2 Evaluation:

The most significant error that caused the gap between the experimental and hypothetical values was neglecting friction force. The efficiency of the mechanism depends on how much energy is lost due to the friction force between the pivot and the beam. It is very likely that assuming the system is working with 100% efficiency and there is no energy loss has lead erroneous results. Furthermore, this force increases with increasing masses of the counterweight, shortening the range of the payload. In order to overcome this problem, either friction coefficient of the wooden surface can be determined and involved in the calculations, or oiling the pivot can reduce the coefficient, thus the error. Another solution is to modify an apparatus called a “bearing” between the pivot and the beam that has a low friction coefficient. This apparatus is attached on the both part and each piece is responsible for the contact between the parts. It has several small marbles inside the two pieces; the marbles slide on the surface when one side moves. It enables the beam to move easier and prevents energy loss.

Figure 4: Mechanism of a ball bearing.[10]

However, there are many other parts that were affected by the friction force, such as the pouch (while travelling on the plate), neglected because of their small magnitudes.

It is assumed that all of the potential energy that the counterweight has converts to kinetic energy of the payload. This assumption can be refuted by the velocity that the counterweight has when it hits the ground. With increasing masses, a greater proportion of the total energy it has converts to kinetic energy. Percentage error increase referred in Table 2 also implies this phenomenon. The counterweight performs a circular motion, not a free fall. Decreasing its linear velocity would reduce the kinetic energy it gains. The counterweight side of the beam can be shortened in order to maintain less linear velocity, thus less kinetic energy transformation.

The measurements are taken as the minimum perpendicular distance from the trebuchet; on 1 dimension. Hence, it is assumed that the payload travels on a straight track without any deflection. In practical terms, the beam was mobile both around and on the pivot so the payload could be launched crosswise, or even the sling could direct the pouch in an angular way that would lead to shorter measurements than the actual value. This can explain the differences between the trials that were same counterweight mass were used. The special equipment mentioned in the solutions for friction can be used to solve this problem, which also fixes the position of the beam.

Air resistance was another limitation in experimenting. When the ball is launched and starts its motion, a counter force occurs in the reverse direction of the motion from the air molecules. In order to calculate drag force, value of air density was needed, which requires complex devices to measure. Thus, it was neglected in the calculations and lead higher range predictions for the ball. As an improvement, the experiment area can be vacuumed; there would be no fluid that would cause drag on the ball.

It was formulated that the range is affected by the angle formed between the velocity vector of the payload and the ground. In making predictions about the range, this angle is expected to be constant. In some of the trials, the ring detached earlier and the pouch initiated the payload’s motion in a greater angle than it was assumed. Along with the deflection in the beam’s direction, this error also indicates the possible explanation of high standard deviation values. The only way to solve this complication is to measure each trial’s releasing angle separately, which doesn’t seem feasible at all.

The experiment was done by 2 people: one of them released the arm to initiate the launch and the other one marked the point where the ball dropped in the first place. It is likely to first person to cause direct intervention since the arm is released manually; the velocity of the beam would be decreased. A more complex launching mechanism that includes a trigger can well reduce this type of possible error. Also the marking process depends on the second person’s senses, which could be mistaken easily. One simple solution for this is to paint the ball before the launch so that it leaves a colorful trace to easily detect and do the measurements required.

Although the system was based on energy transformation, energy change was not considered for all parts of the trebuchet. For example, the sack that counterweight was put in or the pouch that payload was put in had certain masses. Their potential and kinetic energy were changed when the payload was launched. These masses were neglected due to the fact that they were small; however they most probably affected the total energy of the payload that was used in its motion and slightly changed its range. Including these quantities in calculations would reduce the error and provide more precise predictions on the range values.

5. BIBLIOGRAPHY

1. "Trebuchet Physics." Real World Physics Problems. N.p., n.d. Web. 25 June 2014. / http://www.real-world-physics-problems.com/trebuchet-physics.html (25.06.2014)
2. "Trebuchet Physics." Real World Physics Problems. N.p., n.d. Web. 25 June 2014. / http://www.real-world-physics-problems.com/trebuchet-physics.html (25.06.2014)
3. "Conservation of Energy." Conservation of Energy. N.p., n.d. Web. 01 July 2014. / http://physics.bu.edu/~duffy/py105/EnergyConservation.html (01.07.2014)
4. "Potential Energy." Potential Energy. N.p., n.d. Web. 13 June 2014. / http://jersey.uoregon.edu/vlab/PotentialEnergy/ (13.07.2014)
5. "Kinetic Energy | Physics." Encyclopedia Britannica. N.p., n.d. Web. 28 July 2014. / http://global.britannica.com/EBchecked/topic/318130/kinetic-energy (28.07.2014)
6. "Equations for a Falling Body." Wikipedia. Wikimedia Foundation, n.d. Web. 04 Aug. 2014. / http://en.wikipedia.org/wiki/Equations_for_a_falling_body (04.08.2014)
7. "Displacement, Velocity, Acceleration." Displacement, Velocity, Acceleration. N.p., n.d. Web. 05 Aug. 2014. / http://www.grc.nasa.gov/WWW/k-12/airplane/disvelac.html (05.08.2014)
8. Tsokos, K. A. "Projectile Motion." Physics for the IB Diploma. Cambridge: Cambridge UP, 2010. 136. Print. (05.08.2014)
9. "Components of a Vector." Components of a Vector. N.p., n.d. Web. 09 Aug. 2014. / http://hotmath.com/hotmath_help/topics/components-of-a-vector.html (09.08.2014)
10. "A&F Bearings." A&F Bearings. N.p., n.d. Web. 11 Oct. 2014. / http://www.aftexas.com/images/bearing_4.gif (11.10.2014)