International Baccalaureate

Physics Extended Essay

Measurements with a Custom-Built Jumping Ring Apparatus and Determination of the Applied Net Force as a Function of Current

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ABSTRACT

In this extended essay, a jumping ring setup has been established in order to find magnetic force applied on a ring as a function of applied current. A basic setup consists of a solenoid, ferromagnetic rod core and non-ferromagnetic electrically well conductive ring put on the core. In this work, however, a programmable microcontroller unit was included into the system so that time of flight of the ring could be measured. Electrical devices were also included for overcurrent protection. A set of time of flight versus jumping distance of an aluminum ring measurement has been made together with voltage and current measurements on the solenoid. Impedance of the solenoid was found using voltage-current linear curve fit. From inductance-impedance relationship, magnetic energy stored in the solenoid was calculated. Ring speeds were calculated to obtain the acceleration and hence the net force applied on the ring. Net force acting on the ring as a function of current was plotted. Finally, it was surprisingly observed that jump height increases as the mass of ring increases within a certain mass range.

1. INTRODUCTION

Thomson's alternating current jumping ring[1] is the first device of its kind based on the Faraday's Electromagnetic Induction law in physics. Elihu Thompson performed the first experiment with the jumping ring device in 1887. It consists of a coil, ferromagnetic rod core and nonferromagnetic electrically well conductive ring put on the core as shown in Figure 1.1[2]. Some modern actuators, such as the contactor used in the experimental system of this thesis, are based on the principle of the Thompson jumping ring.
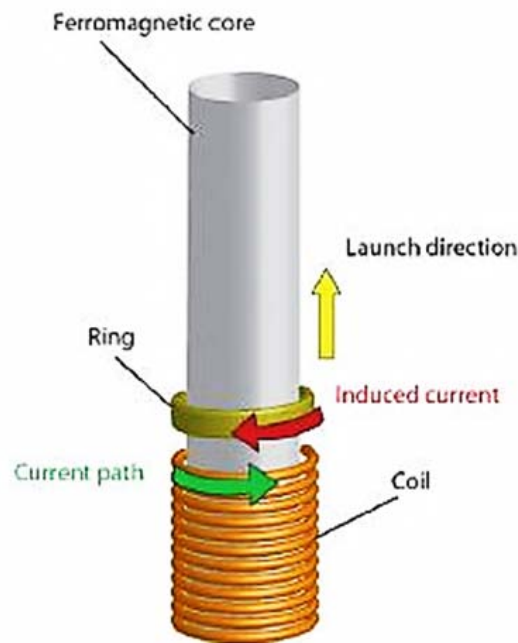

Figure 1.1 Basic jumping ring apparatus.

Faraday's law states that whenever a conductor is placed in a varying magnetic field, electromotive force (emf) is induced. If the conductor circuit is closed, current is also induced. The induced emf is equal to the rate of change of flux linkages (flux linkages is the product of turns, N of the coil, and the magnetic flux associated with it). Because the rate of change of a constant direct current is zero, it does not induce a changing magnetic field. On the other hand, when alternating current is switched on, it produces a magnetic flux in the solenoid, and then a metal ring placed over the ferromagnetic core of a solenoid jumps off. Magnetic flux lines near the end of the solenoid pass through the aluminum ring as the field grows. Change in the magnetic flux induces an emf in the ring and current flows. The ring and the coil repel each other when their current flows in opposite directions. The direction of the induced current in the ring

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is such as to oppose the change inducing it (Lenz's law) and the force on the ring due to current in the field acts to drive the ring out of the field, i.e. upwards. Alternating current generates an alternating force. Although the current direction changes, the force is always such as to push the ring out of the field. This force effect is counteracted by the weight of the ring. This is illustrated in Figure 1.2 [3].
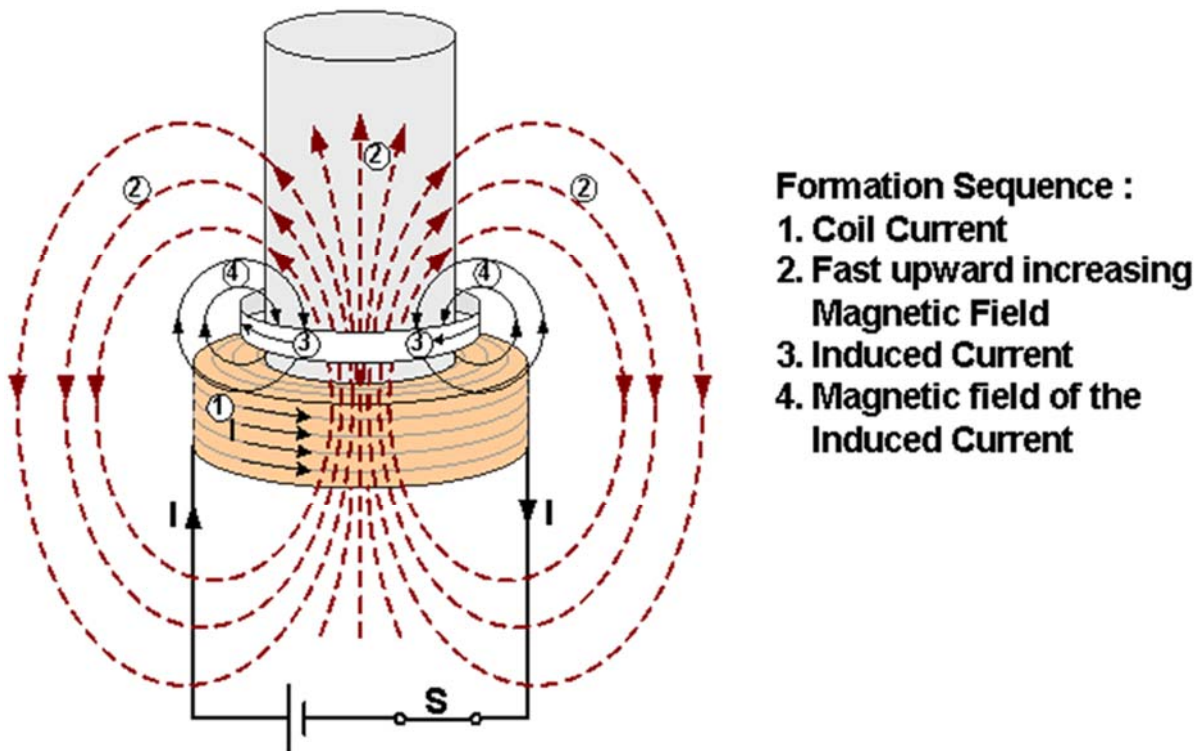
Figure 1.2. Currents and the Magnetic Fields within a Jumping Ring [3].

Experiments with jumping ring has gained much attention not only at scientific levels [4, 5] but also at both university undergraduate [6, 7] and high school level classroom activities [8, 9]. One can find a large number of web pages, articles and videos on the internet. All of the classical jumping ring setups that can be found on the internet as well as in experimental physics books consist of a variable voltage supply, a solenoid, thin iron rods placed into the solenoid and a switch to open and close the circuit. In this study, however, number of additions has been made so that one could measure, display and calculate physical quantities. **To best of my knowledge, the jumping ring apparatus designed for this study is the first apparatus making use of a microcontroller and electrical protection systems.**

Dependent Variables	Applied net force to the ring, jumping speed of aluminum rings
Constant Variables	Temperature, gravitational acceleration, material of the rings, densities of rings
Independent Variables	Voltage, Mass of rings

Table 1.1. The key variables of the experiment.

2. MATERIAL

2.1. Jumping Ring Apparatus

A simple jumping ring can be operated by connecting it directly to 50 Hz, 220 V power line through a push button switch. In order to prevent the solenoid from overcurrent that may burn the wire, a timer relay, a contactor relay and a thermal overcurrent relay were used. A digital voltmeter was connected to solenoid in parallel and a digital ammeter in series to measure the supplied voltage and the electrical current passing through it, respectively. Additionally, optical sensors and a microcontroller board are added to measure the time during the passage of a ring between two sensors and the measured time is displayed on a LCD display. Part list of the system is given in Table 2.1. The jumping ring setup is shown in Figure 2.1. Brief explanation of the system is given below.

2.1.1. Startup Circuit For Electrical Protection

Figure 2.2 gives a schematic illustration of the whole system. The design includes a timer (or timing) relay to limit the operational time in order to prevent excessive heat up of the solenoid in use. A contactor and an overcurrent relay are also included in order to limit the flow of electrical current above a maximum value. Electrical protection system is shown in Figure 2.3. The circuit is on when the push button switch is pressed (part number 11 in Table 2.1 and in Figure 2.1).

Timer relay is used for controlling the time within a system. When supply voltage is applied to the apparatus, timer relay allows the electric current flow until the preset time range is reached. In this work, the time relay was set to one second.

Contactors are used by electrical equipment that is frequently turned on and off. A sudden overcurrent flow through electrical equipment such as motors, coils on a repeated demand may cause them to break easily. In such demanding applications, high current is controlled by a low current through contactors so that overload is prevented in repeated on and off operations.

Operation of a contactor shown in Figure 2.4 is based on the Faraday's law. Coil placed inside acts as an electromagnet and pulls the iron core so that the contact terminals come into close position to let the current flow.

1. Insulated wire to make a solenoid. Diameter of wire without insulator is one milimiter. It can withstand a high current (>20 A) long enough without any overheating damage.
2. Iron wire with 3 mm diameter. The wire is cut to make thin rods each having 65 cm in length. Thin rods are inserted through solenoid.
3. Two optical sensors (Sharp MZ80 Industrial Infrared Sensor). They are placed 20 cm apart from each other. Choose an optical sensor with a reasonable delay time (response time). When the power is turned on, aluminum ring moves upward fast and covers the distance between two sensors within a time frame of fractions of a second.
4. Arduino Uno microcontroller board and AC-to-DC adaptor to supply external input voltage. According to Arduino web page (http://arduino.cc/en/Main/ArduinoBoardUno) recommended range for the adapter is 7-12 V DC. When connected to a computer via USB, it does not require external power supply.
5. Timer Relay (Vendor :Entes, Model: SM-8,www.entes.com.tr).
6. Contactor (Vendor: OAG, Model: OAG-4011).
7. Thermal Overcurrent Relay. (Vendor: OAG, Models: OAG-D59 with 16-25 A and OAG-D59 with 25-40 A). In this setup, there are two overcurrent relays, assembled side by side. They have different range limits on electricity current. One with 16-25 A and the other with 25-40 A. Only one of them is used. At the time of design, solenoid was directly connected to 220V outlet and the current measured with a digital clamp AC ammeter. AC current measurements indicated that we ought to have two overload thermal relays in hand, each having different current limiting values. But only one of them is used at a time.
8. Digital voltmeter (ENTES EVM-3) and ammeter (ENTES EPM-4A CT-25 Included) to measure the voltage applied to solenoid and current passing through it, respectively.
9. Variable AC voltage supply (Variac).
10. PVC or other polymeric type three circular plates. Base plate is 20 cm in diameter and 2 cm in thickness. Solenoid is placed in between two circular plates whose diameters and thicknesses are 13 cm and 1 cm, respectively. Plates are drilled and made holes to place aluminum bars and screwed tightly as shown in Figure 2.1.
11. Push Button Switch
12. Insulation Gloves to protect against electric shocks (**IMPORTANT**).

Table 2.1. List of parts used for jumping ring apparatus.
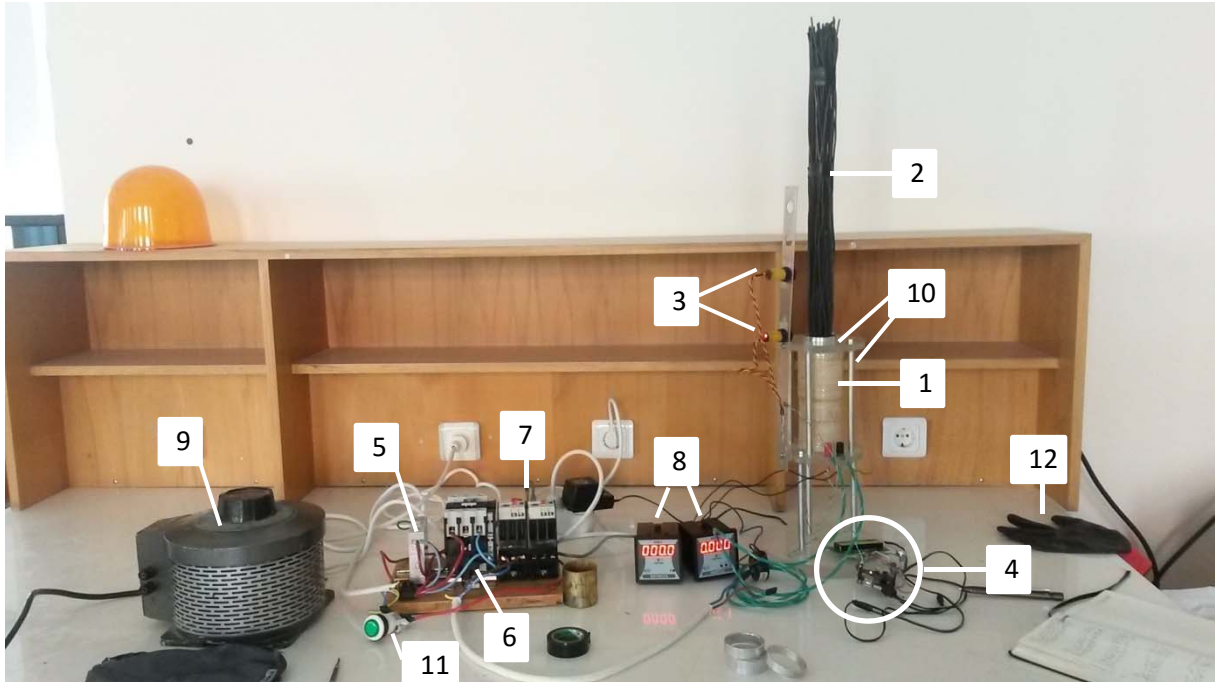
Figure 2.1. Jumping Ring Setup. Numbers indicate the part numbers given in Table 2.1.
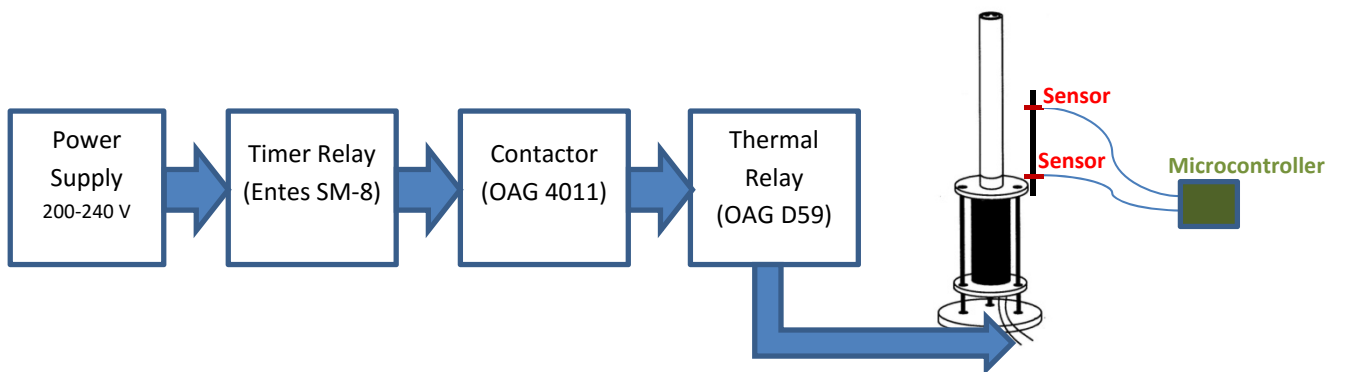
Figure 2.2. Schematic Illustration of the Jumping Ring Apparatus used for the experiment. Manufacturers and the model numbers are given in parenthesis.
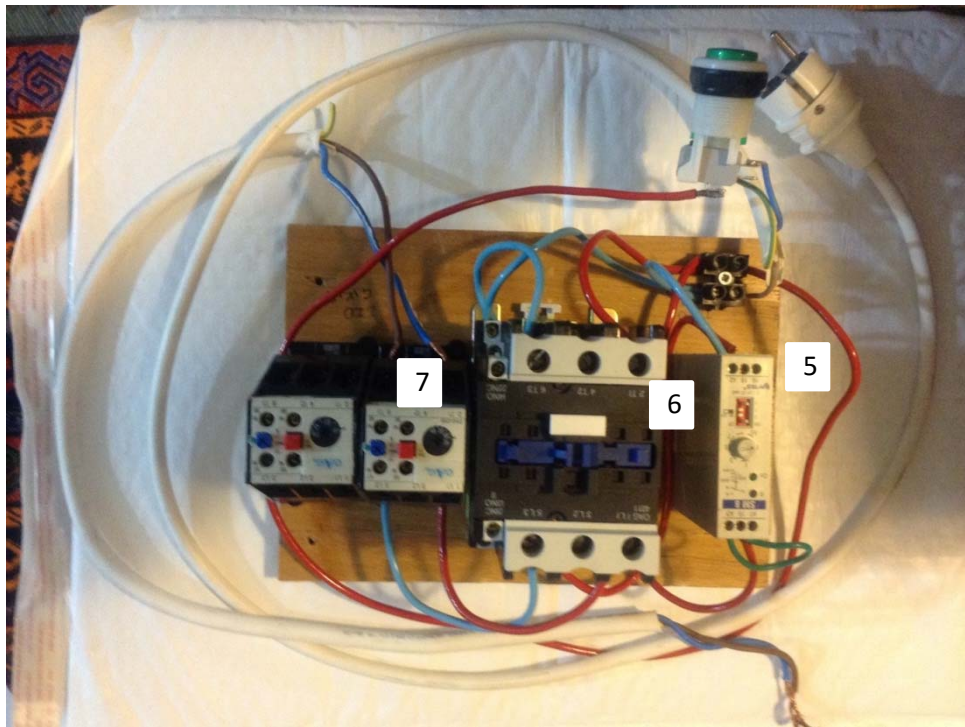
Figure 2.3. Startup Circuit. Because especially contactor connections require experience and knowledge, wiring should be done by an electrical technician. Numbers on the picture indicates the part numbers given in Table 2.1.

Figure 2.4. Magnetic force of the coil pulls the iron core down bringing contacts of the conductor in a closed position (Illustration is from: http://www.youtube.com/watch?v=tMIg24cHqwE).

Thermal overcurrent relay shown in Figure 2.3 (see also Figure 2.1, Part number 7), protects electrical power systems against excessive currents that may be caused by short circuits, ground faults, etc. The overcurrent relay operates when the current in the solenoid exceeds the set value. In this experiment, overcurrent value was adjusted to 25 A on the relay.
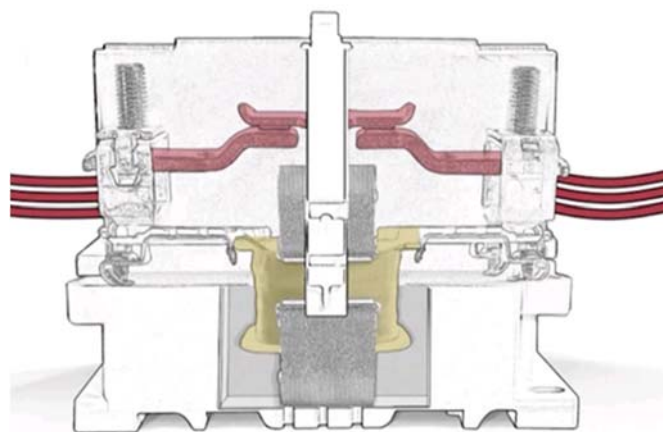
To measure the potential difference, a digital voltmeter was connected to the solenoid in series and ammeter in parallel. Alternative current measurement is done by the induction with the use of a transformer as shown in Figure 2.5. Data sheet of the ammeter specifies the number of turns of the cable that should pass through the transformer with respect to measurement range of the ammeter. Three turns have been chosen to increase the accuracy of measurement up to a three decimal points. This also reduces measurement range of the device down to 40 A from 200 A.

Figure 2.5. Digital ammeter and its transfromer to measure alternative current.

2.1.2. Microcontroller Reading

Reading of measurements is achieved by two optical sensors and a microcontroller unit. Normally sensor send signal as long as the laser light produced by the sensor is detected. If jumping ring (aluminum) interrupts the laser light while moving upward direction, signal produced by the sensor is also interrupted and at the same moment the change is recorded by a single-board programmable microcontroller (Arduino board). The length of time in passing between two sensors is displayed on a LCD display. Its electronic setup is shown in Figure 2.6
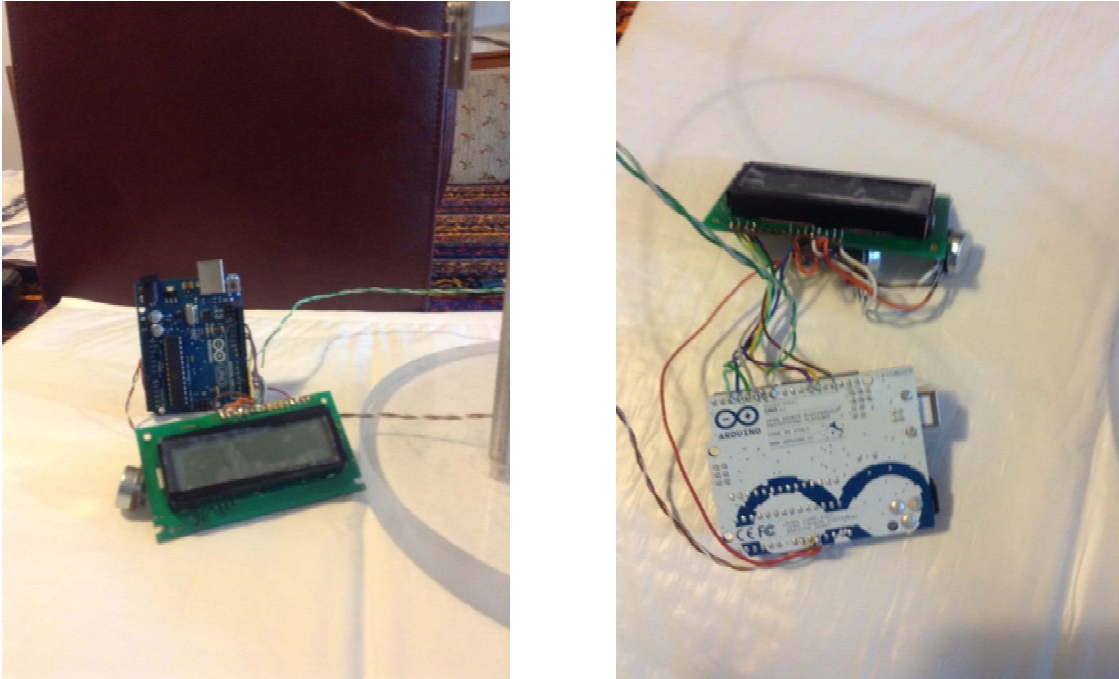
Figure 2.6. Microcontroller and LCD display.

To measure the time a ring starts from rest and reaches the upper sensor, a code has to be written and saved in the memory of the microcontroller (Arduiono Uno). Arduino Uno provides an environment to write, compile and load a program in C language. Entire code that drives the microcontroller and its explanation is presented in Appendix A.

3. METHOD
3.1. Impedance and the Magnetic Energy Stored

In the preliminary part of the experiment, electrical current values corresponding to different applied voltages to the solenoid were measured. Importance of these measurements and the plot produced from them are explained below:

Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference across the two points. Proportionality constant is called resistance. For an alternating current circuit shown in Figure 3.1, however, we talk about electrical impedance instead of resistance. Electrical impedance is the measure of the opposition that an alternating current circuit presents to a current when a voltage is applied. Magnitude of electrical impedance, Z, is given by the quotient of the magnitudes of alternating voltage, V, and electrical current, I,

$$|Z| = \frac{|V|}{|I|} = \sqrt{R^2 + (2\pi f L)^2} \qquad (1)$$
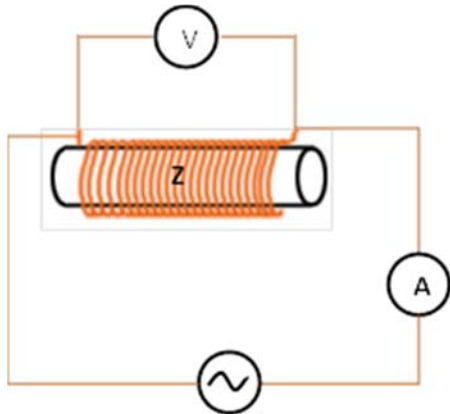
Figure 3.1. Illustration of the circuit with solenoid.

In Eq. (1), f is the frequency (50 Hz), R is the resistance of the circuit and L is the inductance. Inductance is the name given to the property of a component that the change of current flowing through it induces an electromotive force (voltage) within itself that opposes the change of current as a result of their changing magnetic field. *Eq. (1) states that the magnitude of impedance of an alternating current circuit shown in Figure 3.1 is also directly proportional to the potential difference across the solenoid and inversely proportional to the current in the circuit.* This implies that the ratio is linear and we expect that the plot of I-V should give a straight line. The slope gives the magnitude of impedance.
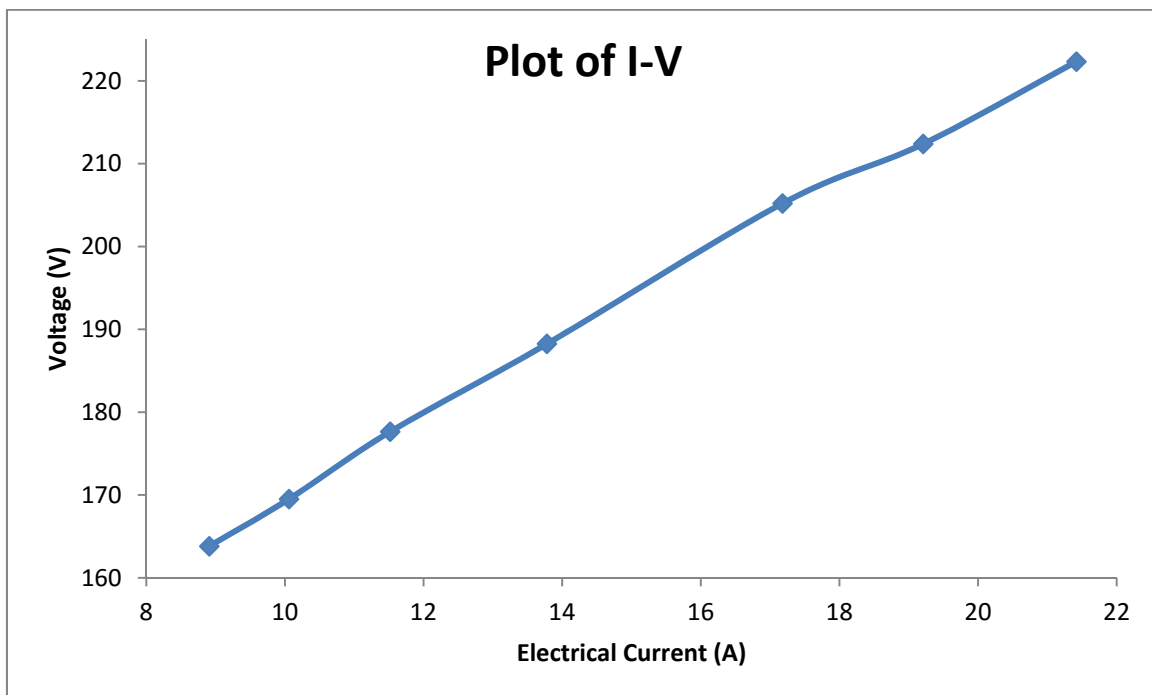
Figure 3.2. Variation of current with respect to applied voltage. Axis for voltage and current are chosen such that the slope gives the magnitude of impedance directly as given by the Equation (1).

Figure 3.2 shows the plot of voltage-current measurements. Linear fit of the data gives

$$V = Z.I + b, \qquad (2)$$

where Z= 4.685 ± 0.09754 Ω, b=123 ± 1.487 V with a 0.9989 correlation. Z is the slope of the line.

| Current (±0.001 A)	Voltage (±0.01 V)
8.91	163.82
10.06	169.52
11.52	177.65
13.78	188.25
17.18	205.21
19.21	212.39
21.42	222.33

Table 3.1. Average current and voltage values used to plot Figure 3.2.

Voltage and current values of Figure 3.2 is listed in Table 3.1. They are the average values of Tables B.1-B.7 given in Appendix B. Time values will be used in the next section. In order to have statistically accurate result, I have collected at least ten measurements for each applied voltage and corresponding current. Small deviation from the straight line in Figure 3.2 is due to fluctuations of the city voltage as well as in the fluctuations of frequency around 50 Hz during the summer time. To calculate the inductance, L, first resistance, R, of the solenoid has been measured to be 1.83 ± 0.001 Ω. Then, using the left hand side of Eq. (1),

$$(4.685)^2 = (1.83)^2 + (2\pi(50)L)^2,$$

and finally we arrive at L= 13.73 ± 0.01903 mH.

Since the magnetic energy stored in the solenoid (inductor) is given by

$$U = \frac{1}{2}LI^2, \qquad (3)$$

The magnetic energy stored corresponding to the current, I, in the solenoid can be approximately estimated. For the current values given in Table 3.1, the stored energies are given in Table 3.2.

Current ±o.oo1 A	U ±0.001 Joule
8.91	0.55
10.06	0.77
11.52	0.91
13.78	1.30
17.18	2.03
19.21	2.53
21.42	3.15

Table 3.2. Magnetic energy stored corresponding to the current in the solenoid.

If the solenoid used for this thesis were an ideal solenoid, we could also calculate the magnitude of the magnetic field, B, using the formula below,

$$U = \frac{B^2}{2\mu}(\pi d^2 l) \qquad (4)$$

where

B = Magnitude of the magnetic field (T, Tesla)

μ = Air Permeability =$k\mu_0$, where k≈1 and μ_0 =$4\pi.10^{-7}$ T/Ampere.m (vacuum).

d = Radius of the solenoid (m),

ℓ = Length of the solenoid (m).

3.2. Determination of the Net Force

In this part of the experiment, the jumping ring apparatus is energized in order to create electromagnetic propulsion that shoots the aluminum ring in upward direction. When the aluminum ring is placed while the power switch button is off, optical sensors at the bottom sends its electrical signal to previously programmed Arduino microcontroller, indicating that the ring obstructs the laser light. When the button is pressed, allowing the current flow, induced current in the ring opposes the current in the long straight iron wires. As a result, the ring is launched in the upward direction. At the time the ring is launched, the light of bottom sensor is no longer obstructed and stops sending signal to the microcontroller. This initiates the count of time. The second optical sensor placed 20 cm above the first one signals when the ring passes in

front of it. At that moment, count of time is terminated. The time measured by the programmed microcontroller is then displayed on the LCD.

A constant magnetic force F_B acting upward and the gravitational force F_g acting downward on the aluminum ring during the measurement. That is, we can assume that motion of the aluminum ring starts from rest and covers the distance $\Delta y = 0.2$ m in t seconds because of a constant net force acting on it. Drag force is not included in the calculation. Equations of the linear motion are given by

$$\Delta y = v_0 t + \frac{1}{2} a t^2 , \quad (5)$$

$$v = v_0 + at, \quad (6)$$

$$F_{net} = m_{ring} a, \quad (7)$$

where,

$$v_0 = 0, \text{ initial speed,}$$

$$\vec{F}_{net} = \vec{F_B} + \vec{F_g} ,$$

$$F_{net} = F_B - F_g ,$$

$$F_g = m_{ring} \, g$$

$$g = 9.8 \text{ m/s}^2 \text{ gravitational acceleration}$$

for an aluminum ring with

Internal Diameter (cm)	4.40±0.01
Outer Diameter (cm)	5.00±0.01
Height (cm)	1.30±0.01
Mass (g)	15.5459±0.00001

and using the Equation 5 through 7 and the values given in Tables B.1 through B.7 in Appendix B, result of calculations are summarized in Table 3.3 and in Figure 3.3. Best fit to the data is given by

$$F_{net} = A i^2 + B i + C , \quad 8.91 \leq i \leq 21.42 \quad (8)$$

where i is the current and F_{net} is the net force, and the constants are

A = -0.004379 ± 0.001939,

B = 0.2167 ± 0.05874,

C = 0.5790 ± 0.4123

Average Voltage (V)	Average Current (A)	Average Time, t (s)	Acceleration at time t (m/s^2)	Speed at time t		Net Force (N)
				m/s	km/h	
163.82	8.91	0.054083	136.75	7.396	26.63	2.13
169.52	10.06	0.050819	154.88	7.871	28.34	2.41
177.65	11.52	0.050449	157.16	7.929	28.54	2.44
188.25	13.78	0.048142	172.59	8.309	29.91	2.68
205.21	17.18	0.044968	197.81	8.895	32.02	3.08
212.39	19.21	0.044709	200.11	8.946	32.21	3.11
222.33	21.42	0.044104	205.64	9.07	32.65	3.20

Table 3.3. Acceleration, speed and the net force on the ring values.
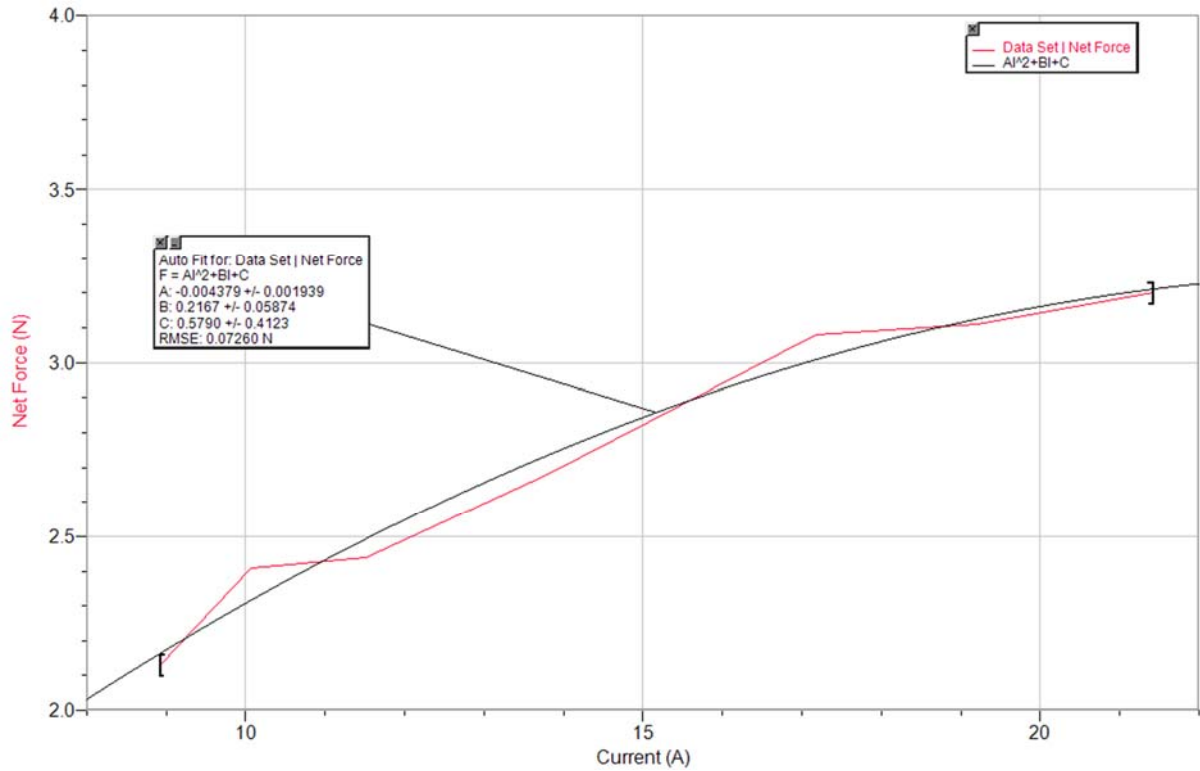
Figure 3.3. Variation of net force applied to ring with AC current.

The range given in Eq. (8) indicates that the fit is only valid between $8.91 \leq i \leq 21.42$ amperes. Because for a full range, there would be no force applied to the ring if there were no current flow. That is, for $i=0$ we have F=0 and therefore the coefficent C in Eq. (8) would have been zero if Eq. (8) were valid in the full range.

3.3. Experiments with different masses

In this part of the experiment, three aluminum rings having the same inner and outer radius but with different heights have been used as shown in Figure 3.4. Masses of three rings are 12.4, 15.5 and 20.1 grams. Durations of jump height of 0.2 m with respect to ring masses for various applied voltages were measured. The results are presented in Figure 3.5.

Figure 3.4. Illustration of a ring

One would expect that as the mass of ring increases, jumping time of a ring for a fixed length becomes longer. However, as can be seen from Figure 3.5, measurements indicate that the jump

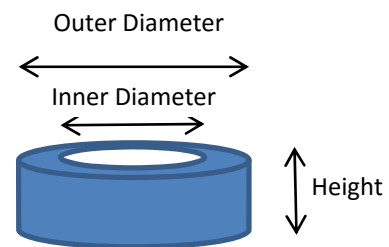time decreases as the mass increases. I should point out that the change in the mass is only because of the increase of heights of rings.

This finding came as a surprise to me. I have brought this issue into the attention of academicians at the Department of Physics Engineering in Hacettepe University. They have provided me two academic publications [5, 6] with some explanations. Physics and mathematics involved in one of these papers is beyond my current knowledge. Therefore, I will summarize their findings.

The provided literature shows that their results are consistent with my findings. In the study[6] given in Figure 3.6, jump heights of rings with respect to masses were presented for two cases; one for cooled rings and the other for rings at room temperature. Internal and external diameter of rings remained fixed just as I did in my experiment. Figure 3.6 states that the mass change is due to increase in thickness (that is "the height" in my study).

Explanation is given in the second academic paper: Rings of different axial lengths (thickness) sliced from a metal pipe with constant wall thickness have a circumferential resistance that varies <u>inversely</u> with the length. However, inductance remains unchanged (see Eq. (1) above). Thus, changing the thickness of the ring and thereby its conduction cross section mainly affects its resistance [5].
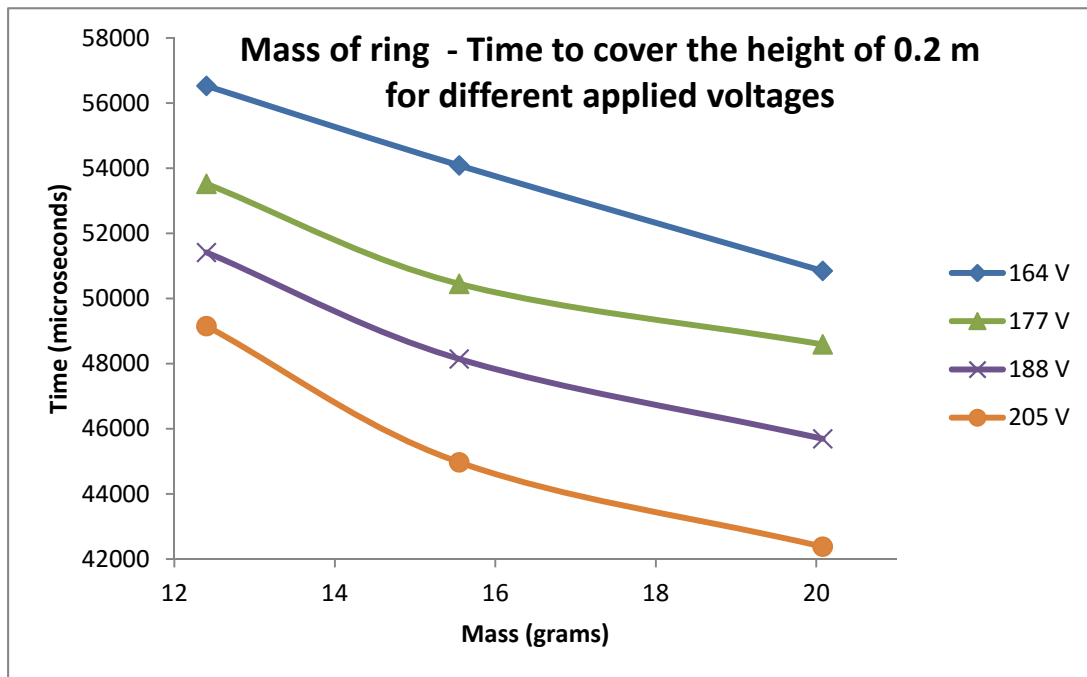

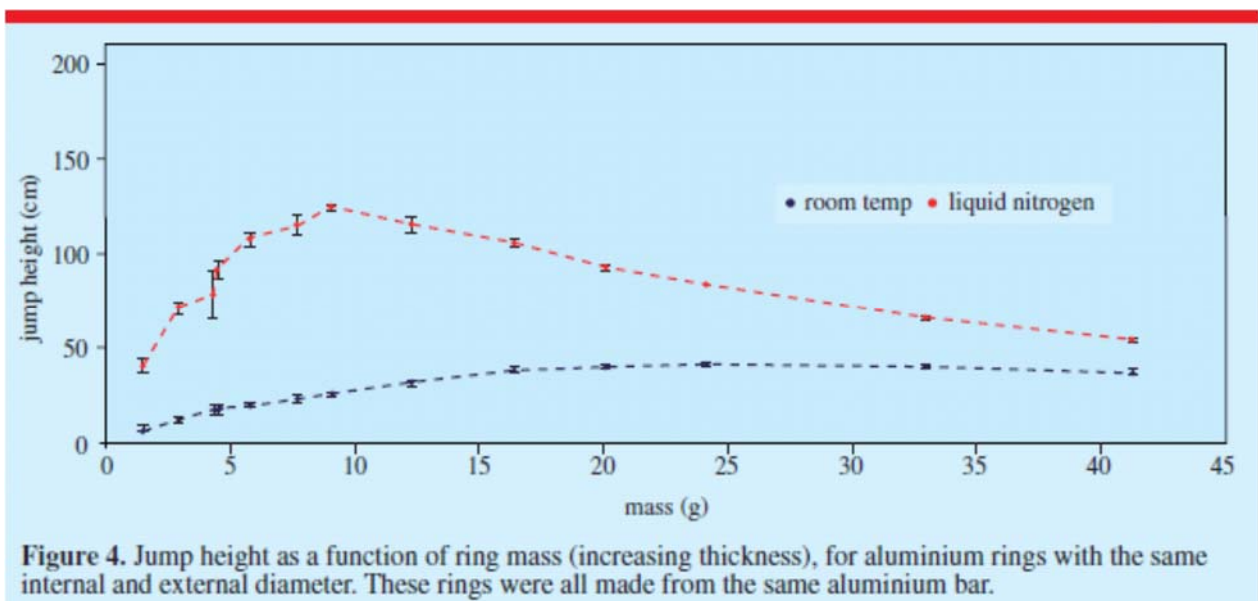Figure 3.5. Duration of jump height of 0.2 m as a function of ring mass.

Figure 4. Jump height as a function of ring mass (increasing thickness), for aluminium rings with the same internal and external diameter. These rings were all made from the same aluminium bar.

Figure 3.6. Finding of a study that shows an increase in jump height as a function of ring mass (From reference [6]).

4. CONCLUSION

Because of the straggling and hysteresis (delay) contributing to the measurements, experimental data fluctuates. There are numbers of reasons for straggling and hysteresis. 220 V power source, that is, the electrical distribution system, has its own inherent fluctuations, especially during the hot summer season of the year 2014 I have performed the experiments. It is inevitable that each additional device introduced into the whole experimental setup also introduces its own error.

I have not been able to make a long straight enough iron wire to place into the solenoid. Steel wires in the market do not have magnetic property. Therefore, I had to use raw iron so that magnetic induction could be achieved. It is difficult to make straight wire from a raw iron. Additionally, the fact that magnetic induction lagging behind the magnetizing force, delays and statistical errors are introduced into the system.

Another systematic error was with the sensors connected to Arduino microcontroller. Although they were sensitive enough to detect the rings, in some trials the lasers were not able to detect the rings due to their high speed. Those non-detected trials were not included in the extended

essay. To overcome this a higher price and quality of sensors could be used, however it would be too pricy to buy them. The voltmeter and ammeter had also their systematic errors.

In spite of above difficulties, calculated magnetic energy stored in the solenoid with respect to the applied current given in section 3.1 provides us the magnitude order of the stored energy. The net force applied to aluminum ring is calculated by the measurements of the microcontroller unit in Section 3.2. The most important part of the experiment is to show that for a certain weight range of the aluminum ring, the jump time becomes surprisingly shorter due to increased magnetic force as presented in Section 3.3 of the thesis.

REFERENCES

[1] Elihu Thomson, "Novel phenomena of alternating currents," The Electrician,London, June 10, 1887.

[2] Source: Matos V, Silva, L and Sena Esteves J, "Induction Ring Launcher," (https://repositorium.sdum.uminho.pt/bitstream/1822/8419/1/InductionRL.pdf).

[3]Source:http://macao.communications.museum/eng/exhibition/secondfloor/moreinfo/2_2_5 _jumpingring.html

[4] Daniel Vachtl, Dobroslav Kovac, Daniel Mayer, "Forces Calculatıon Method Of Thompson Levıtatıng Rıng Durıng Its Non-Harmonic Feeding," Acta Electrotechnica et Informatica No. 2, Vol. **6**, 2006.

[5] Paul J. H. Tjossem, Elizabeth C. Brost, "Optimizing Thomson's jumping ring," American Journal of Physics, Vol. 79 (4), April 2011.

[6] M. Baylie, P. J. Ford, G. P. Mathlin, C. Palmer, "The Jumping Ring Experiment," Physics Education, Vol. **44(1)**, January 2009.

[7] Felix Waschke, Andreas Strunz, Jan-Peter Meyn, "A Safe and Effective Modification of Thomson's Jumping Ring Experiment," European Journal of Physics, Vol. **33**, 2012.

[8] C. S. Schneider, J. P. Ertel, "Understanding the classroom jumping ring," Fall Meeting of the Chesapeake Section of the American Association of Physics Teachers, at the U. S. Naval Academy, Annapolis, MD, 8,NOV,97.

[9] C. S. Schneider, J. P. Ertel, "A classroom jumping ring," American Journal of Physics, Vol. **66(8)**, 1998.

APPENDIX A

Driver code for Arduino Uno is written in C languauge. Explanation of the numbers in circles follows the code.

```
#include <LiquidCrystal.h>                        ①

LiquidCrystal lcd(3, 5, 11, 4, 12, 2);
unsigned long zaman1;
unsigned long zaman2;                             ②
int sen1 = 6;
int sen2 = 7;


int sval1;
int sval2;                                        ③
int k;
int ts=4000;



void setup()
{
 pinMode(sen1,INPUT);
 pinMode(sen2,INPUT);                             ④
 lcd.begin(16, 2);
}
```

```
void loop()
{
 k=1;
 lcd.setCursor(0,1);
 lcd.print("Ready for experiment.   ");
 int sval1 = digitalRead(sen1);

 while(k==1)
 {
  sval1 = digitalRead(sen1);
  if(sval1==1)
 {
  zaman1 = micros();
  lcd.setCursor(0,1);
  lcd.print("Working...");
  int sval2 = digitalRead(sen2);
  while (sval2==1)
  {
   sval2 = digitalRead(sen2);

  }
  zaman2 = micros();
  lcd.clear();
  lcd.setCursor(0,0);
  lcd.print("t=");

  lcd.setCursor(3,0);
  lcd.print(zaman2-zaman1);
  lcd.setCursor(14,0);
  lcd.print("us");
  lcd.setCursor(0,1);
  lcd.print("Ready for experiment. ");

  delay(ts);
  k=0;
  }
 }
}
```

⑤
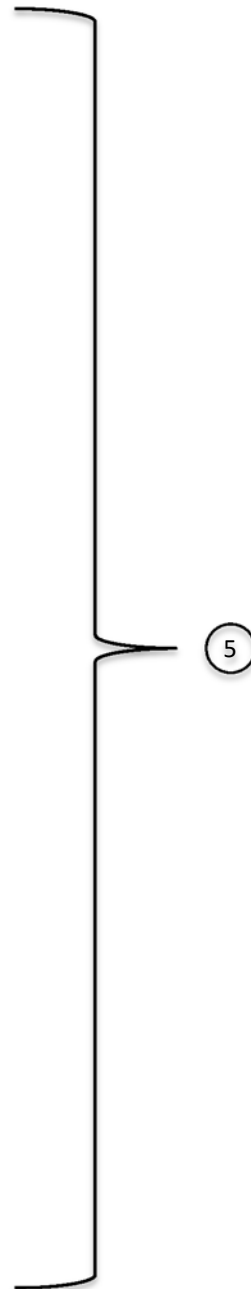
Explanation of the code:

(1)

Standard functions are defined in library functions. The library file "LiquidCrystal.h" needed for the C code is declared. Since Arduino has its own library functions, we don't need to define all of the functions to be used in the program.

(2)

Variables and constants to be used in the program and devices' (sensors and analog screen) pin numbers.

(3)

To measure the time between two optical sensors, we define two variables: sval1 and sval2. The data (one or zero) sent from the sensors are stored in these variables.

Integer variable "ts" is used to store the wait time during experiment. It is the experiment specific constant. The sensors stop reading for 4000 microseconds to allow aluminum ring to fall down after the jump during which no reading is done.

(4)

In function "void setup", the input and output pin connections are defined.

(5)

In this part Arduino starts processing the signals issued by two sensors. Cursor position on the LCD display is set and "Ready for experiment" is displayed. As long as the ring remains at the start position, the first sensor (sen1) sends "0" because its light is obstructed. This information is stored in "sval1". If "sval1" is changed to "1", then the time measurement is started (zaman1). Time measurement continues as long as the digital value (sval2) sent by the upper sensor (sen2) remains to be "1" (sval2==1). When the light of the upper sensor is obstructed, then the value of sval2 is no longer 1, then

Time is recorded (zaman2);

Cursor on the LCD display is positioned;

Elapsed time is calculated and printed;

LCD displays "Ready for experiment.";

Wait for ts seconds to allow the ring to drop back;

k is set to 0, that is, the while loop (k==1) is terminated.

APPENDIX B

	Voltage (V)±0.1	Current (A)±0.001	Time (µs)±1
	164.2	8.893	54180
	161.5	8.786	55996
	164.4	8.906	52884
	163.3	8.906	56028
	164.3	8.92	51540
	164.5	8.933	54368
	163.4	8.846	54312
	164.4	8.92	56940
	163.3	8.986	53864
	164.2	8.92	54468
	164.5	8.986	50340
Average	163.8182	8.909273	54083.64
Std. Deviation	0.906442	0.056717	1945.734

(1)

	Voltage (V)±0.1	Current (A)±0.01	Time (µs)±1
	170.5	10.08	48852
	170.4	10.06	49268
	169.7	10.60	50924
	170.3	10.08	50912
	170.3	9.51	50504
	165.6	10.04	50636
	169.9	9.986	51040
	169.9	10.00	51984
	168.6	10.00	51124
	170.0	10.04	52952
Average	169.52	10.06	50819.6
Std. Deviation	1.480090087	0.259104097	1177.372517

(2)

	Voltage (V)±0.1	Current (A)±0.01	Time (μs)±1
	177.8	11.58	52024
	177.8	11.56	48544
	177.7	11.40	48216
	177.7	11.56	48064
	177.9	11.61	52504
	178.1	11.62	49276
	178.2	11.61	52876
	177.3	11.34	50604
	177.2	11.45	53076
	177.2	11.45	47552
	177.2	11.49	52204
Average	**177.6455**	**11.51545**	**50449.09**
Std. Deviation	**0.367012**	**0.095012**	**2160.353**

(3)

	Voltage (V)±0.1	Current (A)±0.01	Time (μs)±1
	188.6	13.86	51344
	188.7	13.89	46704
	187.9	13.60	47800
	188.4	13.80	49796
	188.3	13.77	46264
	188.2	13.81	47064
	188.3	13.82	51472
	188.3	13.80	50452
	188.3	13.80	45480
	188.0	13.74	48228
	187.8	13.73	44960
Average	**188.2545**	**13.78364**	**48142.18**
Std. Deviation	**0.273363**	**0.076586**	**2313.138**

(4)

	Voltage(V) ±0.1	Current(A) ±0.01	Time (μs) ±1
	205.3	17.6	44240
	205.7	16.84	44192
	205.4	17.49	44884
	205.1	16.73	46612
	205.2	17.46	44832
	205.1	16.46	43836
	204.9	17.46	47112
	205.1	16.73	48116
	205.3	17.54	42168
	205	17.49	43696
Average	**205.21**	**17.18**	**44968.8**
Std. Deviation	**0.228279**	**0.433846**	**1798.348**

(5)

	Voltage (V) ±0.1	Current (A) ±0.01	Time (μs) ±1
	212	19.21	44460
	213.3	19.35	44852
	212.6	19.26	43780
	212.4	19.29	44208
	212.1	18.68	43668
	212	19.18	45640
	211.9	19.17	44604
	212.3	19.26	45512
	212.4	19.28	45076
	213	19.38	44292
	212.3	19.24	45712
Average	**212.390909**	**19.2090909**	
			44709.45
Std. Deviation	**0.434636734**	**0.186839747**	
			1353.18506

(6)

	Voltage (V) ±0.1	Current (A) ±0.01	Time (µs) ±1
	222.5	21.58	44268
	222.2	21.53	45908
	221.8	21.46	43456
	222	21.46	42464
	222.4	20.8	42336
	222.6	21.62	43328
	222.5	21.56	44492
	222.8	21.65	43876
	222.4	21.28	45264
	222.1	21.25	45648
Average	**222.33**	**21.42**	**44104**
Std. Deviation	**0.302030168**	**0.2552319**	**1249.48345**

(7)

Table B.1 through B.7. Statistical values for various voltage and electrical current values. Time values in the last column give the distance (0.2 m) covered by the aluminum ring between two optical sensors.