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Estimating instantaneous velocity of an object with constant acceleration

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Abstract

This study focuses on one dimensional motion of an object moving along an inclined plane under the influence of gravitational force. The purpose is to estimate the instantaneous velocity of the object at the middle point of its travelling path. Instantaneous velocity is the velocity of an object at any instant, which is obtained from the average velocity by taking the time interval zero. This is the mathematical definition of the derivative of position of an object with respect to time. The instantaneous velocity of the object at the middle point of it path is estimated by two different methods based on the theoretical definitions. In the first method, average velocities measured for varying time intervals are collected and the instantaneous velocity is estimated by shrinking the time interval closer and closer to zero. In the second method, position of the object with respect to time is measured and the instantaneous velocity is estimated by finding the slope of the tangent line at time when the object is at the middle. The latter is the usual method to find the instantaneous velocity of an object at any instant. The results obtained from the two methods are compared to test the reliability of the former method. It is found that the instantaneous velocity is well estimated by shrinking the time interval measured for a series of average velocity values, as it is compatible with the one found by the conventional way.

Word count: 240

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1. Introduction

Motion of objects is one of the study areas of physics. By motion of objects, what is meant is how fast they move, how far they move in a specific amount of time, etc. This study specifically deals with the motion where the object moves along a straight line. This is "one dimensional motion".

1.1. Position and Displacement

Position of an object is the location of the object as measured with respect to a chosen reference point. This reference point is generally the origin of an axis, for example 0 point of x axis (Fig. 1).

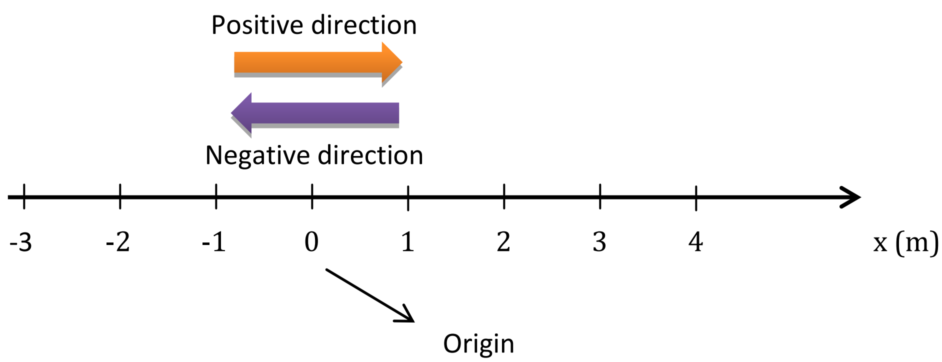


Figure 1. x axis in units of length (in meters)

Positive direction of the axis lies in the direction of increasing numbers and negative direction of the axis lies in the direction of decreasing numbers. For example an object's position might be $x = 2m$, this means the object is 2 meters in the positive direction from the origin. If it were $x = -2m$, again the object's distance is 2 meters from the origin but this time to the negative direction. The change in the position of an object is called displacement. If the object's position is changed from x_1 to x_2, then the displacement, Δx is

$$\Delta x = x_2 - x_1 \tag{1}$$

Here, Δ, the Greek letter delta represents a change in a quantity. For example if a particle moves from $x_1 = 3m$ to $x_2 = -1m$, then the displacement of the object is $\Delta x = x_2 - x_1 = -1 - 3 = -4m$. The negative sign indicates that the motion is in the negative direction. So, displacement of an object includes the information of the direction of the motion in addition to the amount of change in the position. Such a quantity is a vector quantity, a quantity that has both direction and magnitude. What matters for a displacement calculation is only the initial and final position of the object. An object can travel back and forth through the axis, how many meters covered for a trip is irrelevant, displacement is calculated by subtracting the initial position from the final position.

1.2. Average Velocity and Average Speed

In the previous section, initial and final positions of an object are mentioned. These are the positions of an object at the beginning and end of the motion of the object. One can associate several quantities with the phase "how fast", referring the initial and final positions in the time of travel. One of them is average velocity. The average velocity, v_{avg} of an object is defined by the ration of its displacement, Δx, achieved during a particular time interval, Δt [REF]. That is;

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \tag{2}$$

Here t_2 is the time when the object is at x_2, and similarly t_1 is the time when the object is at x_2. On an x vs. t graph, the slope of the straight line connecting two points gives the average velocity between these points. Velocity is also a vector quantity since it is obtained from the displacement. On the other hand, average speed is another way of expressing "how fast" an object is. Velocity is derived from the displacement whereas speed is obtained from the total distance that is travelled by the objects between initial and final times. That is;

$$s_{avg} = \frac{total\ distance}{\Delta t} \tag{3}$$

Average speed does not include direction, it is a scalar quantity.

1.3. Instantaneous Velocity and Average Speed

Average velocity and average speed refer to an information describing how fast an object moves for a particular time interval. However, the question "how fast" might be asked for a particular instant instead of an interval. The velocity at any instant can be obtained using the definition of average velocity. An "instant" refers to a time interval where the initial and final values are the same. That corresponds to the limit when Δt goes to zero[1].

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{4}$$

Instantaneous velocity is the rate at which position is changing with time at a given instant, in other words, velocity is the derivative of position with respect to time. Speed is the magnitude of the velocity.

2. Design and Method

2.1. Research Question

For an object moving along an inclined plane under the influence of gravitational force, can instantaneous velocity at the middle point of the traveling path be estimated by shrinking the time interval closer and closer to zero among a collection of average velocity data measured for varying displacement values?

2.2. Hypothesis

The definition of instantaneous velocity of an object is average velocity in the limit as the associated time interval goes to zero. Average velocity can be measured straightforwardly for any position interval, i.e. displacement. One can suppose that these average velocity measurements are performed in such a way that a particular point is kept at the middle. In fact this point is point where the instantaneous velocity is desired to be estimated. Among this series of average velocity measurements, the closest approximation to the instantaneous velocity at the point of interest is the one corresponding to the minimum displacement. This also corresponds to the minimum time interval for of all measurements. One can obtain an average velocity vs. displacement graph, the vertical interception point of which gives the instantaneous velocity of the object at the point of interest. The vertical interception point is the exact point of limit when the associated time interval goes to zero.

2.3. Variables

2.3.1. Variables for Part I

Independent variables: Positions of the photogate timers from the releasing point

Dependent variable: Average velocity of the object between photogate timers

Controlled variables: Gravity, slope of the track, releasing point, the mass and shape of the glider, photogate timer model, air supply power, and medium temperature

2.3.2. Variables for Part 2

Independent variable: Position of the second photogate timer from the releasing point

Dependent variable: Time required for the object to travel between photogate timers

Controlled variables: Position of the first photogate timer from the releasing point, gravity, slope of the track, releasing point, the mass and shape of the glider, photogate timer model, air supply power, and medium temperature

How to control the controlled variables:

1) Gravity: The gravitational force is the only force that is applied to the glider in this experiment when it is considered that the experiment is conducted on a frictionless track and it needs to be kept constant throughout the process. To keep the gravity constant the experiment will be conduct at the same altitude, in a laboratory placed in İncek , Ankara and the altitude of İncek is around 1200 m.
2) Temperature of the medium: Just like most of the experiments temperature has effects on different components of the experiment. First of all, it is known that temperature changes the intermolecular forces between different molecular species so the friction force changes with the temperature of the medium. Also the geometrical properties and densities of the equipments might get affected by the temperature, too. Although the changes in the equipment due to temperature might be negligible, the changes in the air might be quite important with effecting both resistance caused by air and the pressure that is applied to the system. To keep the temperature constant the laboratory where I will conduct this experiment will have air conditioners that will set the temperature to 25°C.
3) Slope of the track: The experiment will be conducted on an inclined plane and since the force affecting the object parallel to the plane changes directly with the slope of the plane, it must be kept constant. The air track is raised by two wooden blocks with height of 20 cm from under the point 0 cm on track.
4) Mass of the Glider: The mass of the glider is also crucial since it affects all the forces that apply to the glider. The friction force is almost negligible but since it is not totally gone a change in the mass of the object would make it change, too. Also, according to the Newton's 2^{nd} law, a change in the mass would directly affect the gravitational force. To

prevent these changes the same glider with a mass of 170 grams will be used in each trial of the experiment.

5) Type of the glider: The geometrical and the materialistic properties of the glider is also important. The surface type of the glider affects the friction force and the aerodynamic properties effects the aerial resistance that might be applied. To keep these properties constant SF- 6306 type Pasco glider will be used in each trial
6) Power of the air supply: The air supply provides an opposite force to the gravitational force. Although it doesn't affect the force that is parallel to the air track it might change the friction force. The flow of air through the holes should be kept constant to obtain more reliable results. To do this the pasco air supplier will work at the maximum power level throughout the experiment.
7) Releasing point: Releasing point must be kept same since the gravitational acceleration changes the speed according to the time of the motion. The glider will be released from the point 12.5 cm to make sure the accelerating force applies the same in each trial.

2.4. Equipment

1) Air Supply, PASCO SF-9216[2]
2) 2.0 m Air Track, PASCO SF-9214[3].
3) Glider (Total length 12.5 cm, flag length 1.0 cm) [3].
4) Photogate timers, PASCO ME9215 A, ME9204 B.
5) Wooden supports (10.0x5.0x5.0cm)
6) Ruler, 30.0±0.1cm

Figure 2: Photogate timers, PASCO ME9215 A, ME9204 B

Figure 3: Glider (Total length 12.5 cm, flag length 1.0 cm)

2.5. Method

2.5.1. Method for Part I

1) The air track is set up as shown in Fig. 2, elevating one end of the track with at least 15 cm support.
2) A point, x_1, is chosen near the center of the track, and its position is marked on the track.
3) A starting point for the glider, x_0 , is chosen near the upper end of the track, such that the distance from x_1 to x_0 is 90.0 cm, the position is marked on the track.
4) The photogate timers are placed at points equidistant (40.0 cm) from x_1, as shown in Fig. 2.
5) The photogate timer mode is adjusted to the “pulse” mode, and the time reading is zeroed.
6) Air supply is turned on to its maximum power, and then holding the glider steady for a few seconds, it is released from the point x_1. This step is repeated for 4 times and the time readings are recorded as Trial 1, 2, 3, and 4 in Table 1.
7) After that, each photogate timer is moved 2.5 cm towards the point x_0 , so that the displacement is 5.0 cm smaller than the previous and the point x_0 is kept at the middle. The time is measured for four times again. This is done until there is 5 cm between photogate times.

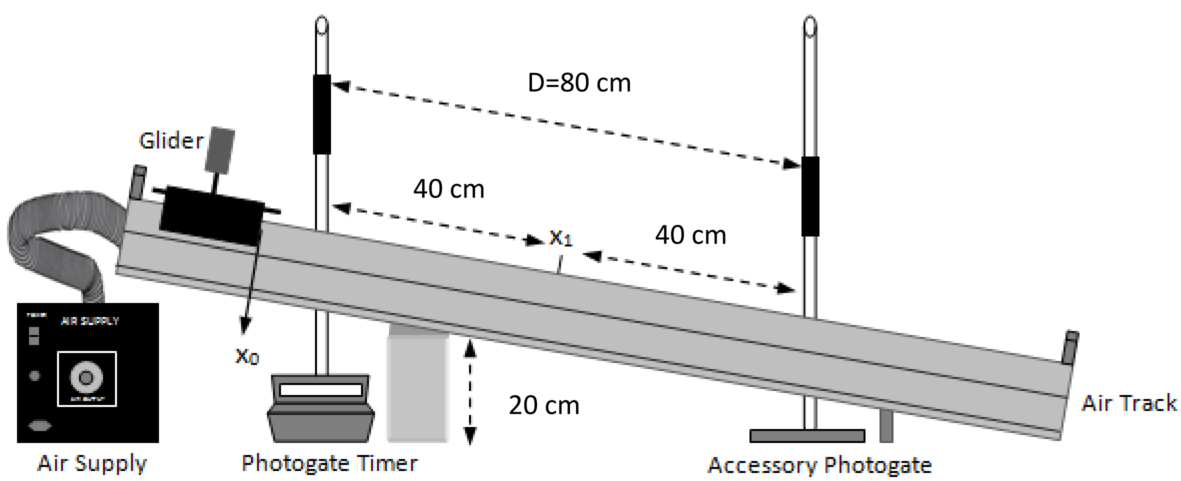


Figure 2. Experimental setup for part I

2.5.2. Method for Part II

1) The air track is set up as shown in Fig. 3, elevating one end of the track with at least 15 cm support.
2) A point, x_1, is chosen near the center of the track, and its position is marked on the track.
3) A starting point for the glider, x_0 , is chosen near the upper end of the track, such that the distance from x_1 to x_0 is 90.0 cm, the position is marked on the track.
4) The first photogate timer is placed at the point between x_1 and x_0, 50.0 cm to x_1, and 40.0 cm to x_0. This position is fixed throughout the experiment.
5) The second photogate timer is placed 5.0 cm away from the first one towards the point x_0.
6) The photogate timer mode is adjusted to the "pulse" mode, and the time reading is zeroed.
7) Air supply is turned on to its maximum power, and then holding the glider steady for a few seconds, it is released from the point x_1. This step is repeated for 4 times and the time readings are recorded as Trial 1, 2, 3, and 4 in Table 3.
8) After that, move the second photogate timer 5.0 cm towards the point x_0 , so that the displacement is 5.0 cm smaller than the previous and the position of the first photogate timer is fixed. The time is measured for four times again. This is done until there is 80 cm between photogate times.

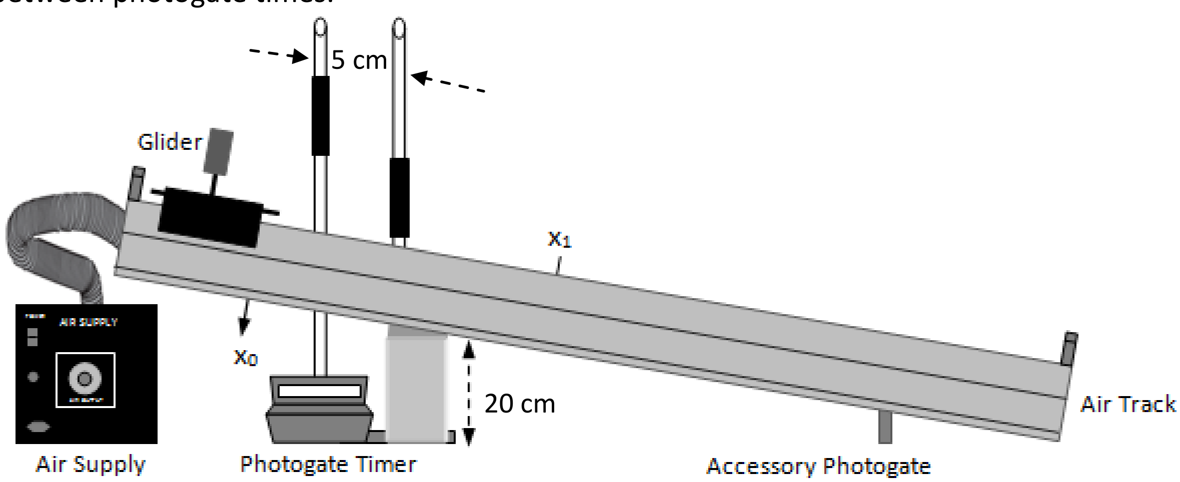


Figure 3: Experimental setup for part II

3. Data Collection and Processing

3.1. Part I

Distance between two photogate timers (cm) (± 0.2cm)	Time of the travel between two photogate timers (s) (± 5%)			
	Trial 1	***Trial 2***	***Trial 3***	***Trial 4***
80.0	0.482	0.483	0.483	0.481
75.0	0.452	0.452	0.452	0.453
70.0	0.423	0.423	0.423	0.422
65.0	0.393	0.393	0.393	0.393
60.0	0.356	0.356	0.356	0.356
55.0	0.328	0.328	0.328	0.328
50.0	0.298	0.299	0.298	0.299
45.0	0.272	0.272	0.272	0.272
40.0	0.244	0.244	0.244	0.244
35.0	0.212	0.212	0.212	0.213
30.0	0.185	0.185	0.185	0.185
25.0	0.155	0.155	0.155	0.155
20.0	0.123	0.125	0.123	0.122
15.0	0.086	0.086	0.086	0.086
10.0	0.061	0.061	0.061	0.061
5.0	0.032	0.032	0.032	0.032

Table 1: Raw Data Table for Part I containing the recorded data for distance between the two photogate timers and time it takes for the glider to travel this distance.

3.1.1. Calculations for 80 cm

Mean of time values:

$$t = \frac{t_1 + t_2 + t_3 + t_4}{4} = \frac{0.482 + 0.483 + 0.483 + 0.481}{4} = 0.482\ s$$

Average velocity for 80.0 cm displacement:

$$v_{avg} = \frac{80.0}{0.482} = 165.975\ cm/s$$

Uncertainty Calculation:

Uncertainty for the time measurement is given as 5% [4]

$$0.482 \times \frac{5}{100} = 0.024 \text{ s}$$

$$0.483 \times \frac{5}{100} = 0.024 \text{ s}$$

$$0.483 \times \frac{5}{100} = 0.024 \text{ s}$$

$$0.481 \times \frac{5}{100} = 0.024 \text{ s}$$

Mean of uncertainties:

$$\Delta t = \frac{\Delta t_1 + \Delta t_2 + \Delta t_3 + \Delta t_4}{4} = \frac{0.024 + 0.024 + 0.024 + 0.024}{4} = 0.024\ s$$

Uncertainty for the random error of the distance between the two photogate timers is 0.2 cm

0.2/80.0= 0.025 s

0.025 x 100= 0.25%

0.024/0.482= 0.050 s

0.050x 100 = 5%

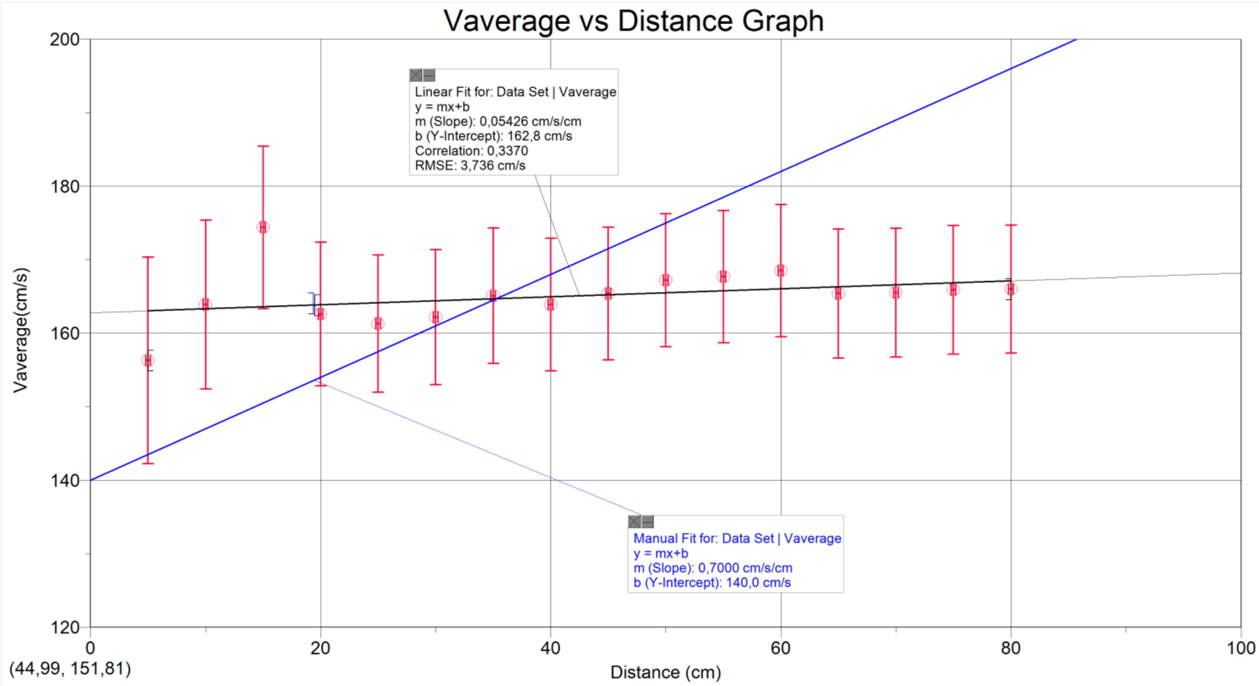
0.025+5.0=5.25%

$V_{average}$= 165.9 ± 5.25% cm/s

=165.9 ± 8.7 cm/s

Distance between two photogates (± 0.2cm)	**Mean of the time (s)**	**Uncertainty of the mean of the time values (s)**	**$V_{average}$ (cm/s)**	**Uncertainty of the $V_{average}$ (cm/s)**
80.0	0.482	0.024	166.0	8.7
75.0	0.452	0.023	165.9	8.7
70.0	0.423	0.021	165.5	8.7
65.0	0.393	0.020	165.4	8.8
60.0	0.356	0.018	168.5	9.0
55.0	0.328	0.016	167.7	9.0
50.0	0.299	0.015	167.2	9.0
45.0	0.272	0.014	165.4	9.0
40.0	0.244	0.012	163.9	9.0
35.0	0.212	0.011	165.1	9.2
30.0	0.185	0.009	162.2	9.2
25.0	0.155	0.008	161.3	9.4
20.0	0.123	0.006	162.6	9.8
15.0	0.086	0.004	174.4	11.0
10.0	0.061	0.003	163.9	11.5
5.0	0.032	0.002	156.3	14.1

Table 2: Processed Data table for Part I containing the calculated average velocities for the displacements given in the table 1 and their calculated uncertainties.



Graph 1: $V_{average}$ vs. Distance graph made by the software called Logger pro

Y intercept of the linear fit line: 162.8 cm/s

Y intercept of the worst line : 140.0 cm/s

Uncertainty of the result is 162.8 cm/s -140.0 cm/s= 22.8 cm/s

Final result is 162.8 ± 22.8 cm/s

Distance between two photogates (± 0.1cm)	Time of the motion between two photogates (s) (± 5%)			
	Trial 1	Trial 2	Trial 3	Trial 4
80.0	0.482	0.482	0.482	0.482
75.0	0.457	0.457	0.456	0.457
70.0	0.431	0.431	0.431	0.431
65.0	0.405	0.405	0.405	0.405
60.0	0.380	0.380	0.380	0.380
55.0	0.348	0.349	0.349	0.349
50.0	0.324	0.324	0.324	0.324
45.0	0.297	0.297	0.296	0.296
40.0	0.269	0.269	0.269	0.269
35.0	0.238	0.238	0.238	0.238
30.0	0.212	0.212	0.212	0.212
25.0	0.181	0.181	0.181	0.181
20.0	0.146	0.146	0.146	0.146
15.0	0.114	0.114	0.114	0.114
10.0	0.079	0.079	0.079	0.079
5.0	0.041	0.041	0.041	0.041

Table 3: Raw Data Table for Part 2 containing the recorded data for distance between the two photogate timers and time it takes for the glider to travel this distance.

Calculations for 80 cm :

Mean of time : $\frac{t1+t2+t3+t4}{4}$

$\frac{0.482+0.482+0.482+0.482}{4}$=0.482 s

Uncertainty Calculation:

Uncertainty for the pasco photogates in the Figure 2 is 5%[4]

0.482x(5/100)=0.024 s

0.483x(5/100)= 0.024 s

0.483x(5/100)=0.024 s

0.481x(5/100) =0.024 s

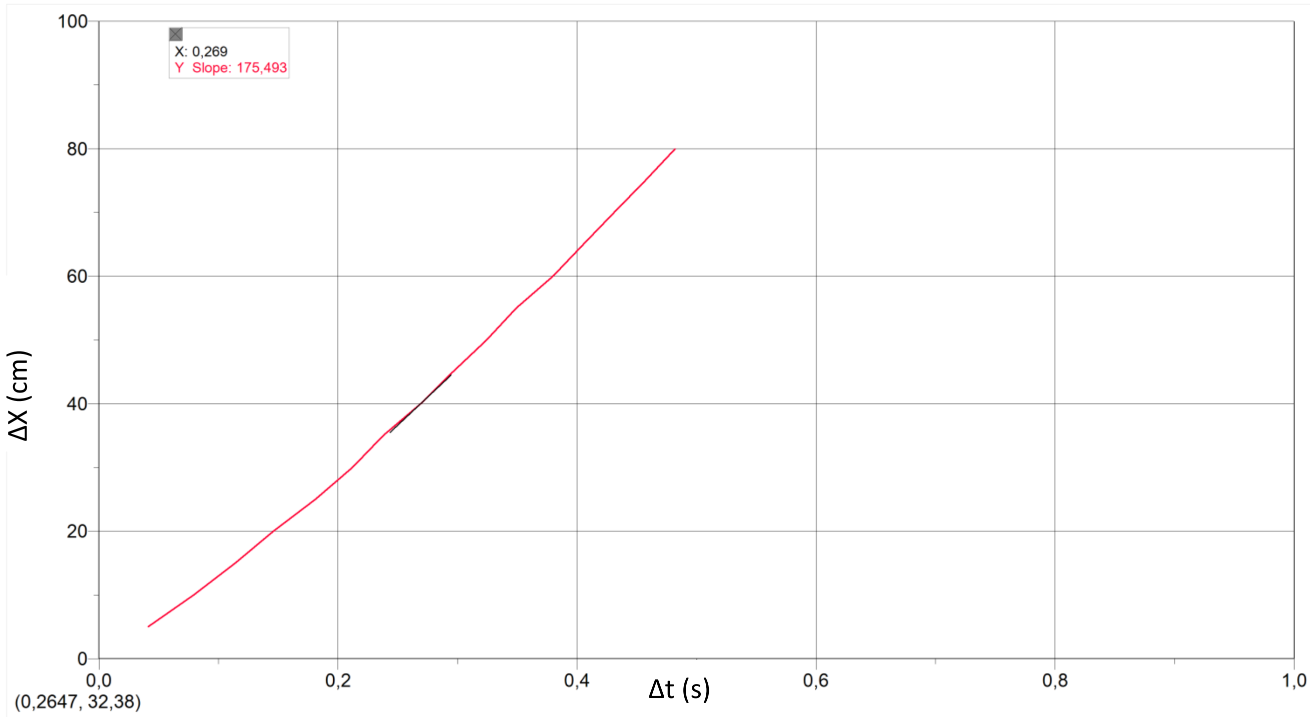
Mean of the uncertainties =$\frac{x1+x2+x3+x4}{4}$

=$\frac{0.024+0.024+0.024+0.024}{4}$

=±0.024 s

Distance between two photogates (± 0.2cm)	**Mean of the time(s)**	**Uncertainty of the mean of the time values(s)**
80.0	0.482	0.024
75.0	0.457	0.023
70.0	0.431	0.022
65.0	0.405	0.020
60.0	0.380	0.019
55.0	0.349	0.017
50.0	0.324	0.016
45.0	0.296	0.015
40.0	0.269	0.013
35.0	0.238	0.012
30.0	0.212	0.011
25.0	0.181	0.009
20.0	0.146	0.007
15.0	0.114	0.006
10.0	0.079	0.004
5.0	0.041	0.002

Table 4: Processed data table for part 2 including the average time values recorded during the procedure of part 2.



Graph 2: X vs. T graph made by the software called Logger pro

Slope of the tangent line at the 40 cm is 175.493 cm/s

4. Conclusion and Evaluation

This study focuses on one dimensional motion of an object moving along an inclined plane under the influence of gravitational force. The purpose is to estimate the instantaneous velocity of the object at the middle point of its travelling path. Instantaneous velocity is the velocity of an object at any instant, which is obtained from the average velocity by taking the time interval zero. This is the mathematical definition of the derivative of position of an object with respect to time. The instantaneous velocity of the object at the middle point of it path is estimated by two different methods based on the theoretical definitions. In the first method, average velocities measured for varying time intervals are collected and the instantaneous velocity is estimated by shrinking the time interval closer and closer to zero. In the second method, position of the object with respect to time is measured and the instantaneous velocity is estimated by finding the slope of the tangent line at time when the object is at the middle. The latter is the usual method to find the instantaneous velocity of an object at any instant. The results obtained from the two methods are compared to test the reliability of the former method. It is found that the instantaneous velocity is well estimated by shrinking the time interval measured for a series of average velocity values, as it is compatible with the one found by the conventional way.

The purpose of this study is to estimate the instantaneous velocity of an object moving under the influence of the gravity along an inclined plane, at the middle point of its travelling path. There are several ways to measure the instantaneous velocity of an object, however in this study the instantaneous velocity is estimated via a series of average velocity measurements performed for a particular point to be fixed at the middle. The idea behind this method is associated with the definition of instantaneous velocity. Instantaneous velocity is defined as the rate at which position is changing with time at a given instant. This definition is originated from the definition of average velocity in the limit as the associated time interval goes to zero. Starting from this mathematical definition, one can consider that, among a series average velocity measurements, the closest approximation to the instantaneous velocity at the middle point is the one with the smallest time interval. This smallest time interval corresponds to the smallest displacement. That is why the graph of average velocity vs. displacement is desired to be obtained. In this graph the vertical intersection point must give the instantaneous velocity at the middle point.

In the first part of the experimental procedure, Part I, the point of interest is determined and time is measured for 16 different displacement values for 4 times. For each displacement value, the point of interest is fixed at the middle, so that in each step the photogate timers are moved closer and closer with the same amount of distance. For each displacement value, mean time is calculated for 4 trials, and the average velocity is calculated by dividing displacement to time. Finally, a data set of average velocity vs. displacement is obtained with corresponding errors. This data set is used to obtain an average velocity vs. displacement graph by using the software called "Logger Pro". An automatic linear fit and a manual worst line are obtained. The instantaneous velocity is found by recording the vertical interception of the best line and the corresponding error by the difference of the vertical interceptions of the two lines. The result is as follows:

$v \pm \Delta v =$162.8 ± 22.8 cm/s

In the second part of the experimental procedure, the instantaneous velocity at the same point is found by using the x-t graph. The time readings for varying positions along the travelling path are recorded, and the position time graph is obtained. The instantaneous velocity is found by the slope of the tangent line at time corresponding to the position of the point of interest, and the result is found to be:

175.493 cm/s

This value is in the error range of the result from the first part of the experiment. Therefore, one can conclude that the instantaneous velocity is well estimated by the method in Part I. The results of both methods are compatible, so the method in Part I is a reasonable method to estimate the instantaneous velocity from a data set of average velocities.

In the experiment, the time readings include 5% error. This is the main contribution to the error found for the estimated instantaneous value. This error can be minimized by using an updated model photogate timer. In these days, latest versions of photogate timers read time with an error margin of 0.05%. Still, in comparison with the usage of stopwatch manually, these reading are a lot more accurate and precise.

In the experiment, the friction between the glider and the track where the glider is sliding on is eliminated by using an air supply which blows air from the holes in the air track. This provides more reliable readings since the friction effects are ignored in the calculations.

The random errors from the length measurements are 0.2 cm due to the milimetric rulers. This is due to the observer's limitations in the eyesight. Since the track is inclined, the desired position of the photogate timers could only be arranged within these limitations.

The heads of the photogate timers are movable; therefore there must be some additional disarrangement in constructing the setup in each step. In other words, the heads of the timers should be parallel to each other and perpendicular to the track, however; during the experimental procedure, while changing the positions of the timers, there might be some differences in parallel and perpendicular locations of the equipment.

Bibliography

1) Fundamentals of Physics, Halliday &Resnick, 9th edition
2) http://www.pasco.com/prodCatalog/SF/SF-9216_variable-output-air-supply/
3) http://www.pasco.com/prodCatalog/SF/SF-9214_glider-air-track/
4) Atılım University Lab Manual for Physics 101