International Baccalaureate:

Extended Essay

Investigation of the effects of the relation between the line lengths of pendulums on resonance

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Abstract

This essay studies resonance in a simple harmonic motion with a pendulum, in order to answer the question: "How does the relation between the line lengths of pendulums affect resonance?" This experiment attempts to answer this question in two different measurements: First comparing the time it takes for the resonant to get in resonance with the resonator in different line lengths of pendulums; secondly comparing the amplitude of those pendulums when they get into a resonance. A fishing line, a weight (20g), small spheres made by paper, and a strong LED light is used in this experiment. The weights are attached to a long fishing line and they are pulled and released from a point in order for the weight to do a simple harmonic motion. Then the motion of the paper spheres -which are attached to the same fishing line- is observed in order to get data of their amplitude and time to get in resonance. The line length of the pendulums made by paper spheres differ, so the intended question of "relation between line lengths" can be answered. Line lengths similar to the actual line length of the weight are expected to resonate more and have a greater amplitude. Overall, this experiment has shown that, as the value ($|length\ of\ resonator - length\ of\ resonant|$) gets closer to 0, the amplitude gets greater and the time it takes for the resonant to get in resonance decreases. The study is merely an attempt to better understand the basics of a resonance and the factors that change it. A continuation of this experiment could possibly lead to the study of architecture in order to make buildings, or bridges more resistant to earthquakes and strong winds.

Abstract Word Count: 283

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Introduction and Presentation

1. Scope of Work

The motion of a pendulum in a simple harmonic motion seems relatively simple yet it becomes extremely complex when resonance and damping are taken into count. Because most high-school physics textbooks barely cover a detailed explanation of resonance and the variables that affect it. It came to mind that investigating and experimenting such topic might prove interesting.

This essay is an attempt to study the complex thus interesting topic of resonance and further experimenting it by changing the variables and measuring its consequences. There are a great deal of variables affecting resonance even including the air friction. But in this experiment, the aim is to keep variables constant other than the line lengths of the pendulums so that examining the changes with the varying line lengths will be more accurate. The ratio between the line lengths of the resonator and the resonant has an impact on resonance that cannot be ignored. The first objective is to measure the changing amplitude of the resonant according to its line length, and perhaps derive an approximate equation showing the relation of line length similarity and resonance magnitude. The second objective is to do same with the variable of time -the time a resonant needs to get in resonance with the resonator-, to investigate if there is a relation between the line lengths of pendulums and their time to get in resonance. If I am lucky enough, at the end of this experiment I will have a greater knowledge in resonance and the relation between its variables.

2. Explanation of Simple Harmonic Motion

Simple harmonic motion is a form of motion of a body, where the body has an acceleration always directed to a certain equilibrium point and the acceleration is proportional to the displacement from this equilibrium point. Its mechanics are simply based on the "Second Law of Motion";

$$\vec{F} = m\vec{a}$$

3. Explanation of Resonance

In physics, resonance is the tendency of a system to oscillate with greater amplitude at some frequencies than at others[1]. Ordinarily, there is a resonator and the resonant in the phenomenon of resonance. The resonator, which can be understood from its name, is the body that have the greater energy or amplitude and it sort of shares its energy with the resonant in order to create a motion in which they oscillate together. Resonance occurs when a system is able to store and transfer energy between the bodies. In my case the longest fishing line, the fishing line that both the weight and the paper ball are attached to, is the path that the weight and the paper ball shares energy. Pushing a person in a swing and tidal resonance in order to form bigger waves can be simple yet great examples for resonance.

There are plenty of types in resonance such as: mechanical resonance, acoustic resonance, electrical resonance, optical resonance etc. My experiment includes mechanical resonance

[1] Katsuhiko Ogata (2005). *System Dynamics* (4th ed.). University of Minnesota. p. 617.

and the resonator is the 20g weight and the resonant is the paper ball. Energy is transferred from the oscillating weight to the inert paper ball.

Speaking of resonance in simple pendulum; resonance is highly affected from the relation between the line lengths of the pendulums. My research is in order to find out the particulars of this relationship.

4. Explanation of Damping

Damping is an influence within or upon an oscillatory system that has the effect of reducing, restricting or preventing its oscillations[2].In simple pendulum the amplitude of the pendulum is decreased by time because of this phenomenon, damping.

To reduce the influence of damping in my investigation I noted the amplitude of the paper ball on its first swing. Otherwise the result would be incorrect because I would have measured much smaller amplitudes

[2] dampening - Collins English Dictionary - Complete & Unabridged 10th Edition

Explanation of the experiment

1. Preparing the Experiment

1) First a fishing line is stretched between two walls. It has been tried to be tight as possible because a loose line will result in errors in data collection.
2) After stretching the fishing line, a weight is hanged nearly in the middle with another fishing line of 15cm's. (Which then will be replaced with 20cm and 25cm lines in order to further investigate.)
3) Approximately 5cm's next to weight a paper ball is hanged with 10cm's, 15cm's, 20cm's, 25cm's, and 30cm's of fishing lines respectively. Because the line length is an independent variable in this investigation, the line length of the paper balls' line has frequently changed.
4) A strong LED light is hanged it from the ceiling, very close to the fishing line because a clear shadow of the paper balls is needed.
5) Lastly in order to calculate the amplitude of paper balls a length criterion on the floor is needed. The fishing line's shadow is selected as the origin and marked the floor with a board marker using a tape measure 50cm's each way. I drew big lines in every 5cm's and a little lines in other centimes. Also I've put a dot between every line. So my smallest unit was 5mm's.

2. Variables

Control Variables:

- *The weight of the resonator;*

I ensured that this value is fixed by using the same 20g weight that I borrowed from the laboratory of my school.

- *The weight of the resonant;*

I ensured that this value is controlled by using the same paper ball in all trials.

- *The distance between the resonant and the resonator;*

I marked the points that I hang the weight and the paper ball and hanged them from the same point every time.

- *Amplitude of the weight;*

I used a wooden stick to adjust and control the amplitude the weight. I put the wooden stick on the floor, on which I put a measurement system, and pulled the weight until it touched the wooden stick. By this way I could keep the amplitude constant between trials.

Independent Variables:

- *The length of the weights' line;*

I cut the fishing line precisely with a tape measure according to the requirements of the experiment at that time. I repeated the trials with the same length of the weights' line consecutively to make sure it is strictly constant between trials.

- *The length of the paper balls' line;*

I cut the fishing line precisely with a tape measure according to the requirements of the experiment at that time. I cut 10, 15, 20, 25, and 30 cm's of fishing lines at the beginning and attached them with a plastic tape. Because the length of the paper balls' line changes after every 5 trial. In order to keep things smooth I used plastic tape.

Dependent Variables:

- *Amplitude of the paper ball;*

Amplitude of the paper ball is dependent to amplitude of the weight and also the relation between the lengths of the lines of paper ball and the weight.

- *The time to get in resonance;*

There is a connection between the time that paper ball needs to get in resonance with the weight and the ratio of the lengths of their lines.

Basic Drawing of the Experiment

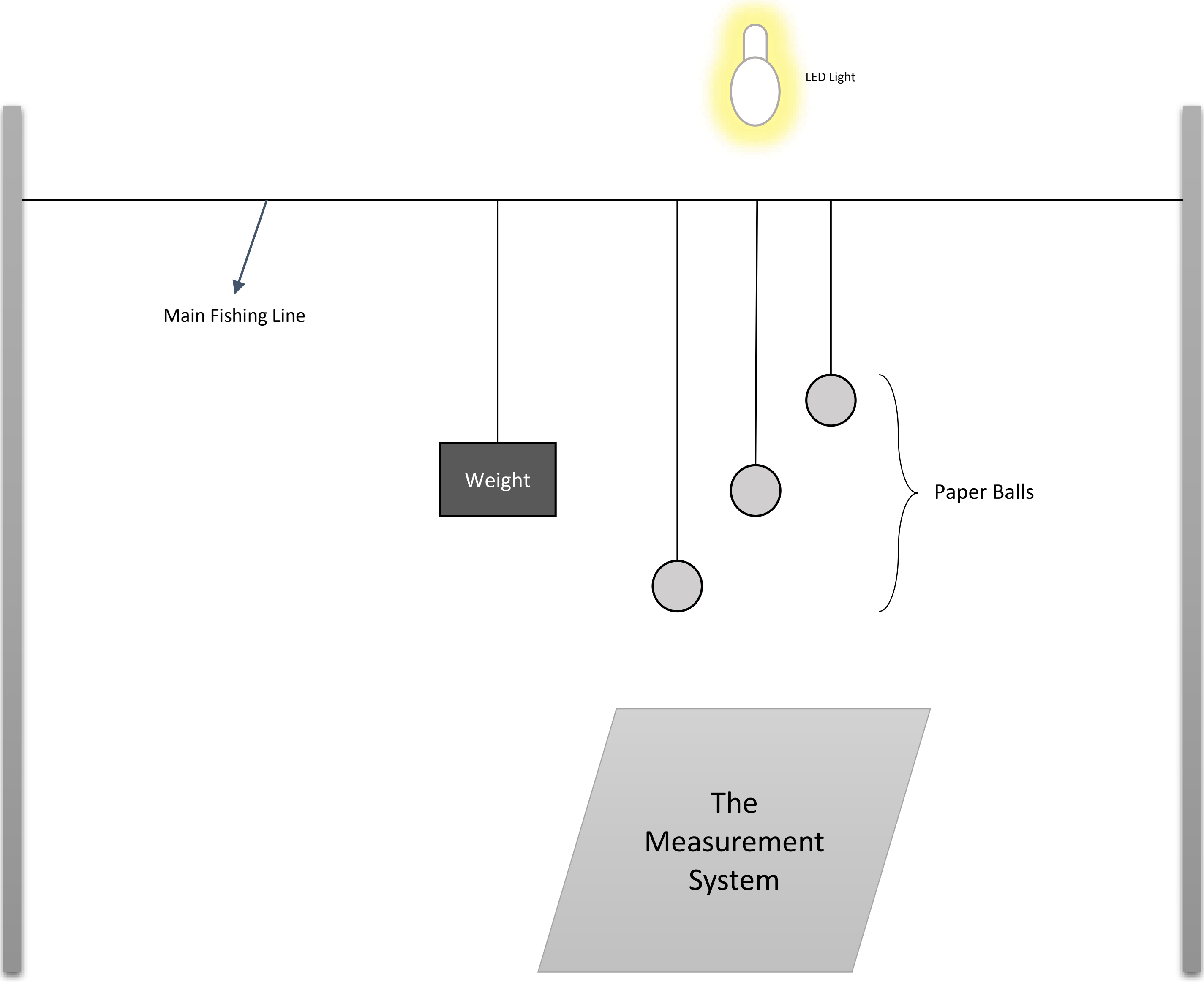


Figure 1: Shows the experiment in a simplified drawing. (*3 balls are just for illustration, there was only one ball at a time during experimentation.)

- LED is just on top of the paper ball because the shadow of it, is highly necessary in order to calculate data
- Because the experiment requires a clear shadow of the paper balls, I only repeated the experiment at night and by only the light of the LED on top of the fishing line

The Measurement System in the Floor

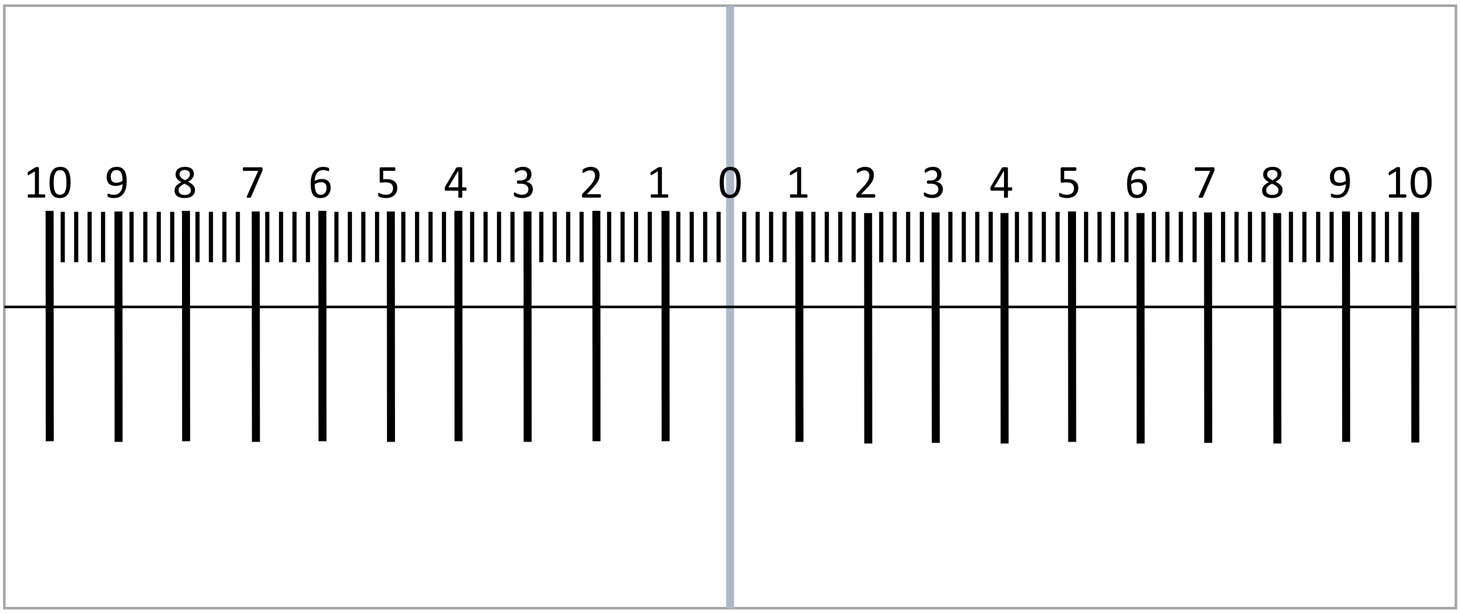


Figure 2: Shows the measurement system on the floor. (*Every number represents 5 centimeters in the grid.)

Adjusting the Amplitude of the Weight

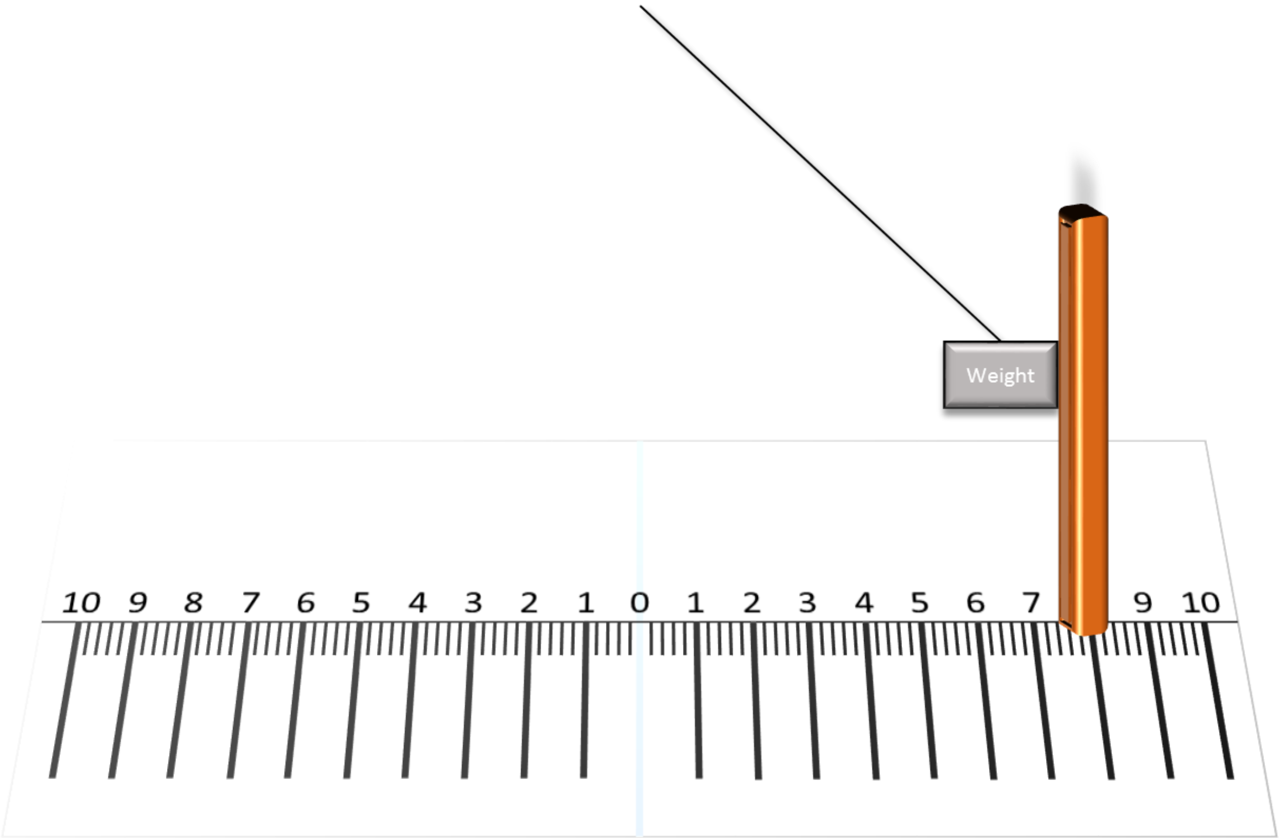


Figure 3: Shows the adjustment of the amplitude of the weight.

Data Collection Method

I have used the basic equation in similar triangles to measure the amplitude of the resonant, paper balls. The light, the paper ball and the floor forms triangles that are similar and has a simple ratio of similarity. With the help of simple geometry I converted the value that I observed from the measurement system in the floor to the amplitude of the paper ball.

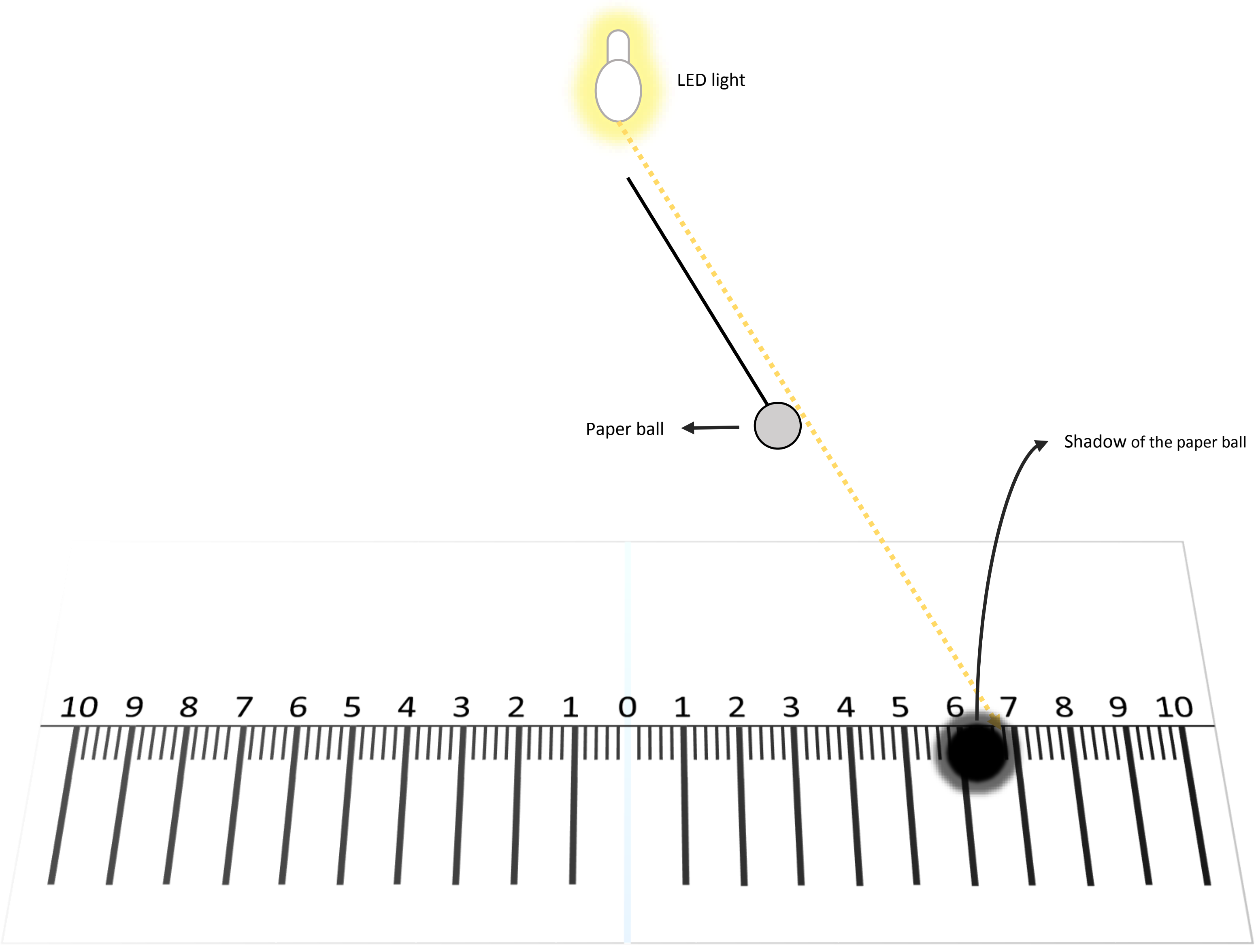
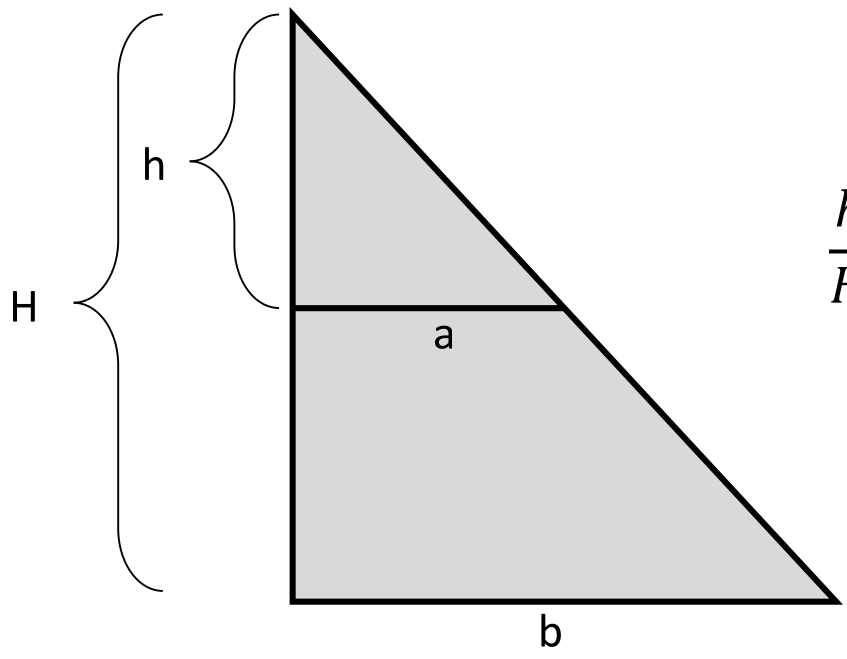


Figure 4: Shows the observation of the amplitude of the shadow of the paper ball.

Triangle Similarity



$$\frac{h}{H} = \frac{a}{b}$$

Figure 5: Shows the way to calculate the actual amplitude of the paper ball

- h: the distance between the light and the paper ball which differs because the length of the paper balls' line changes.
- H: the height of the light from the floor, which is 100 cm's and it is unchanging.
- b: the length I observe from the measurement system
- a: the actual amplitude of the paper ball

- For example while the line length of the paper ball is 20cm's:
  - The distance between the light and the fishing line is 20cm's. So the total distance between the light and the paper ball is 40cm's.
  - So h is 40cm's; and the H is 100 cm's.
  - I have observed 'b' as 26,25cm's;

$\frac{40}{100} = \frac{a}{26,25}$ → a is 10,50 cm's.

Experimental Data Collection for Amplitude

Raw Data Tables

Line Length of the Weight ±0.1cm	Line Length of the Paper Ball ±0.1cm	Amplitude of the Weight ±0.1cm	Trial Count	Amplitude of the Paper Ball ±0.25cm
15	10	12	Trial 1	10.70
			Trial 2	10.60
			Trial 3	10.50
			Trial 4	10.40
			Trial 5	10.80
	15		Trial 1	11.60
			Trial 2	11.80
			Trial 3	11.70
			Trial 4	11.70
			Trial 5	11.90
	20		Trial 1	10.50
			Trial 2	10.30
			Trial 3	10.60
			Trial 4	10.40
			Trial 5	10.40
	25		Trial 1	9.50
			Trial 2	9.70
			Trial 3	9.50
			Trial 4	9.60
			Trial 5	9.10
	30		Trial 1	8.30
			Trial 2	7.90
			Trial 3	8.50
			Trial 4	9.10
			Trial 5	8.20

Table 1: Shows the data of amplitude when the line length of the weight is 15 cm’s.

Line Length of the Weight ±0.1cm	***Line Length of the Paper Ball ±0.1cm***	***Amplitude of the Weight ±0.1cm***	***Trial Count***	***Amplitude of the Paper Ball ±0.25cm***
20	***10***	***12***	***Trial 1***	***9.30***
			Trial 2	***9.10***
			Trial 3	***9.40***
			Trial 4	***9.50***
			Trial 5	***9.70***
	15		***Trial 1***	***10.60***
			Trial 2	***10.30***
			Trial 3	***10.40***
			Trial 4	***10.50***
			Trial 5	***10.30***
	20		***Trial 1***	***11.80***
			Trial 2	***11.70***
			Trial 3	***11.70***
			Trial 4	***11.90***
			Trial 5	***11.80***
	25		***Trial 1***	***10.30***
			Trial 2	***10.40***
			Trial 3	***10.10***
			Trial 4	***10.50***
			Trial 5	***10.30***
	30		***Trial 1***	***9.20***
			Trial 2	***9.20***
			Trial 3	***9.30***
			Trial 4	***9.50***
			Trial 5	***9.30***

Table 2: Shows the data of amplitude when the line length of the weight is 20 cm’s.

Line Length of the Weight ±0.1cm	*Line Length of the Paper Ball ±0.1cm*	*Amplitude of the Weight ±0.1cm*	*Trial Count*	*Amplitude of the Paper Ball ±0.25cm*
25	10	12	Trial 1	8.50
			Trial 2	8.30
			Trial 3	8.40
			Trial 4	8.50
			Trial 5	8.20
	15		Trial 1	9.50
			Trial 2	9.20
			Trial 3	9.40
			Trial 4	9.30
			Trial 5	9.40
	20		Trial 1	10.50
			Trial 2	10.40
			Trial 3	10.10
			Trial 4	10.60
			Trial 5	10.40
	25		Trial 1	11.80
			Trial 2	11.60
			Trial 3	11.80
			Trial 4	11.70
			Trial 5	11.90
	30		Trial 1	10.00
			Trial 2	10.30
			Trial 3	10.10
			Trial 4	10.30
			Trial 5	10.40

Table 3: Shows the data of amplitude when the line length of the weight is 25 cm's.

Data Analysis for Amplitude

Line Length of the Weight ±0.1cm	*Line Length of the Paper Ball ±0.1cm*	*Amplitude of the Weight ±0.1cm*	*Average of the Amplitude ±0.25cm*	*Standard Deviation*
15	***10***	***12***	***10,60***	***0,141***
	15		***11,74***	***0,102***
	20		***10,44***	***0,102***
	25		***9,48***	***0,204***
	30		***8,40***	***0,400***

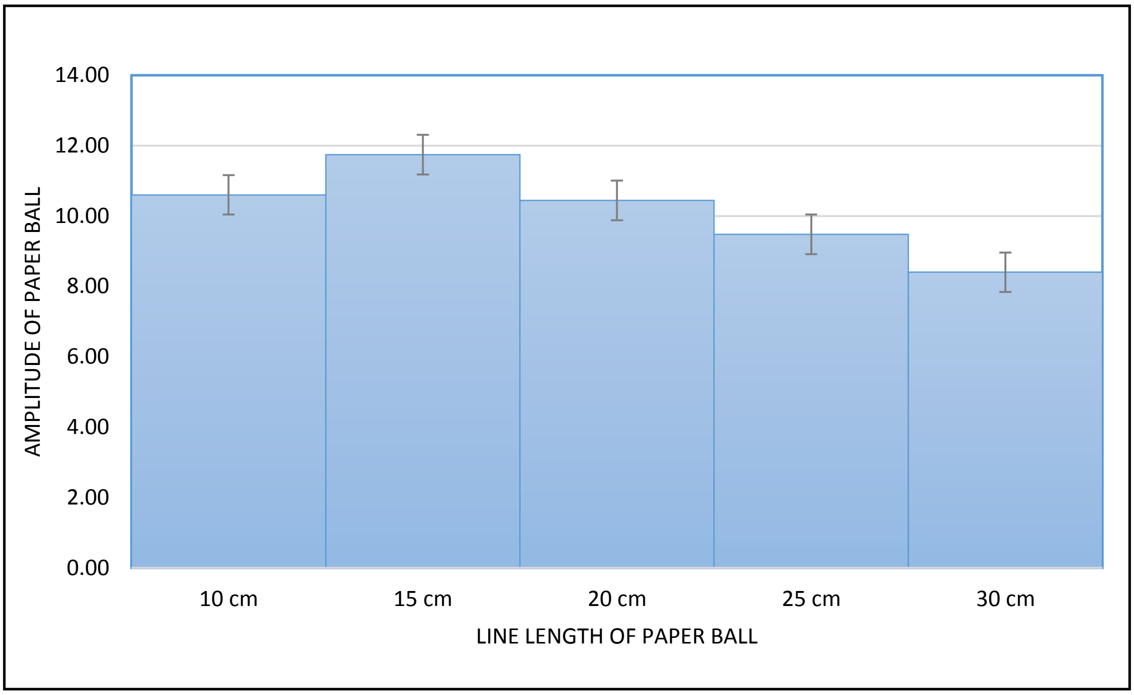
Table 4: Shows statistical summary of the data of amplitude when the line length of the weight is 15 cm's.

Line Length of the Weight ±0.1cm	*Line Length of the Paper Ball ±0.1cm*	*Amplitude of the Weight ±0.1cm*	*Average of the Amplitude ±0.25cm*	*Standard Deviation*
15	***10***	***12***	***9,40***	***0,200***
	15		***10,42***	***0,117***
	20		***11,78***	***0,075***
	25		***10,32***	***0,133***
	30		***9,30***	***0,110***

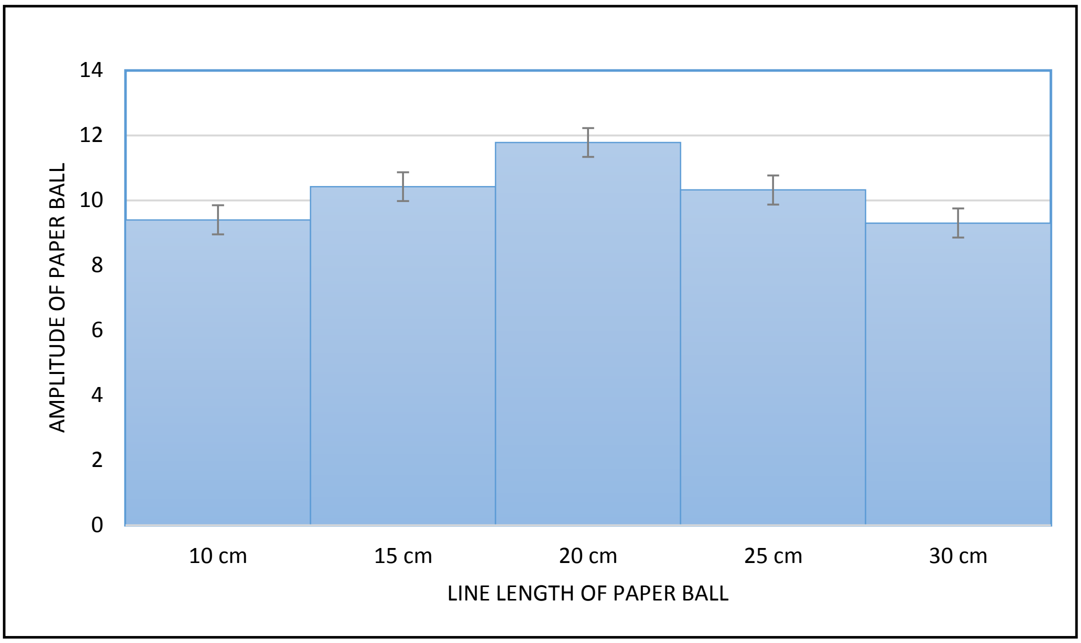
Table 5: Shows statistical summary of the data of amplitude when the line length of the weight is 20 cm's.

Line Length of the Weight ±0.1cm	*Line Length of the Paper Ball ±0.1cm*	*Amplitude of the Weight ±0.1cm*	*Average of the Amplitude ±0.25cm*	*Standard Deviation*
15	*10*	*12*	*8,38*	*0,117*
	15		*9,36*	*0,102*
	20		*10,4*	*0,167*
	25		*11,76*	*0,102*
	30		*10,22*	*0,147*

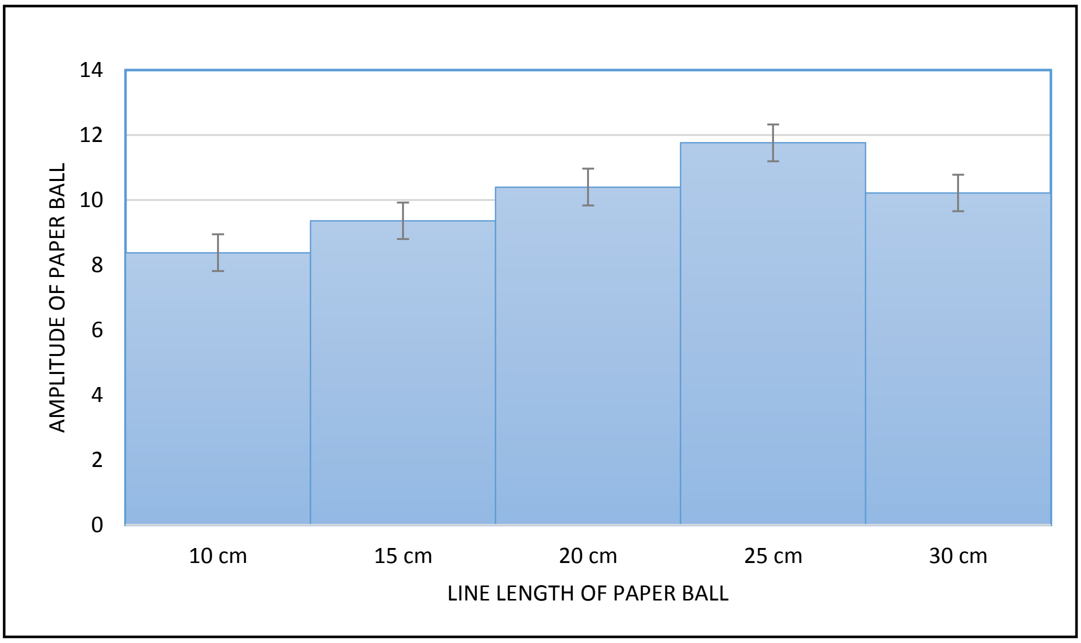
Table 6: Shows statistical summary of the data of amplitude when the line length of the weight is 25 cm's.



Graph 1: Shows the amplitude of the paper ball when the line length of the weight is 15 cm's.



Graph 2: Shows the amplitude of the paper ball when the line length of the weight is 20 cm's.



Graph 3: Shows the amplitude of the paper ball when the line length of the weight is 25 cm's.

Interpretation of the Graphs and the Data of Amplitude

In *Table 1*, as expected, the amplitude of the paper ball, that's length is 15cm's, is the greatest and it is very close to the amplitude of the weight. Also the amplitudes of the paper balls with the line length of 10 and 20 cm's are very close to each other. This gets to mind that the absolute value of difference between the line lengths is the variable that goes into count for resonance magnitude calculation.

In *Table 2*, paper ball with the line length of 20cm's has the greatest amplitude again as anticipated. 15 & 25 cm's and 10 & 30 cm's have very similar amplitudes when compared to each other. As in *Graph 2,* the values equally decrease from the middle value to the right and left. This again leads me to an idea that absolute difference between the lengths of lines is a highly important factor in resonance.

Same idea carries over in *Table 3*. As can be seen in *Graph 3,* the amplitude of the paper ball with the line length of 25 cm's is the greatest and the amplitudes of the paper balls with the line length of 20 cm's and 30 cm's are very similar. If the data is correct it assures that the absolute difference between lines' lengths determine the resonance magnitude.

Standard deviation of the data is mostly under (<0.200) so the data is pretty accurate and shows that it can be trusted. Proceeding from here, saying that the magnitude of resonance is directly related with the ratio between the lengths of the lines that are resonating, would be valid.

Experimental Data Collection for Time

The experiment to find the time it takes for the resonant –paper ball- to get in resonance with the resonator –weight- is only repeated while the line length of the weight is 20 cm's. As the previous data shows, while the line length of the weight is 20 cm's, the data is perfectly distributed. So it is decided that doing the experiment for time while the line length of the weight is 20 cm's would be highly accurate.

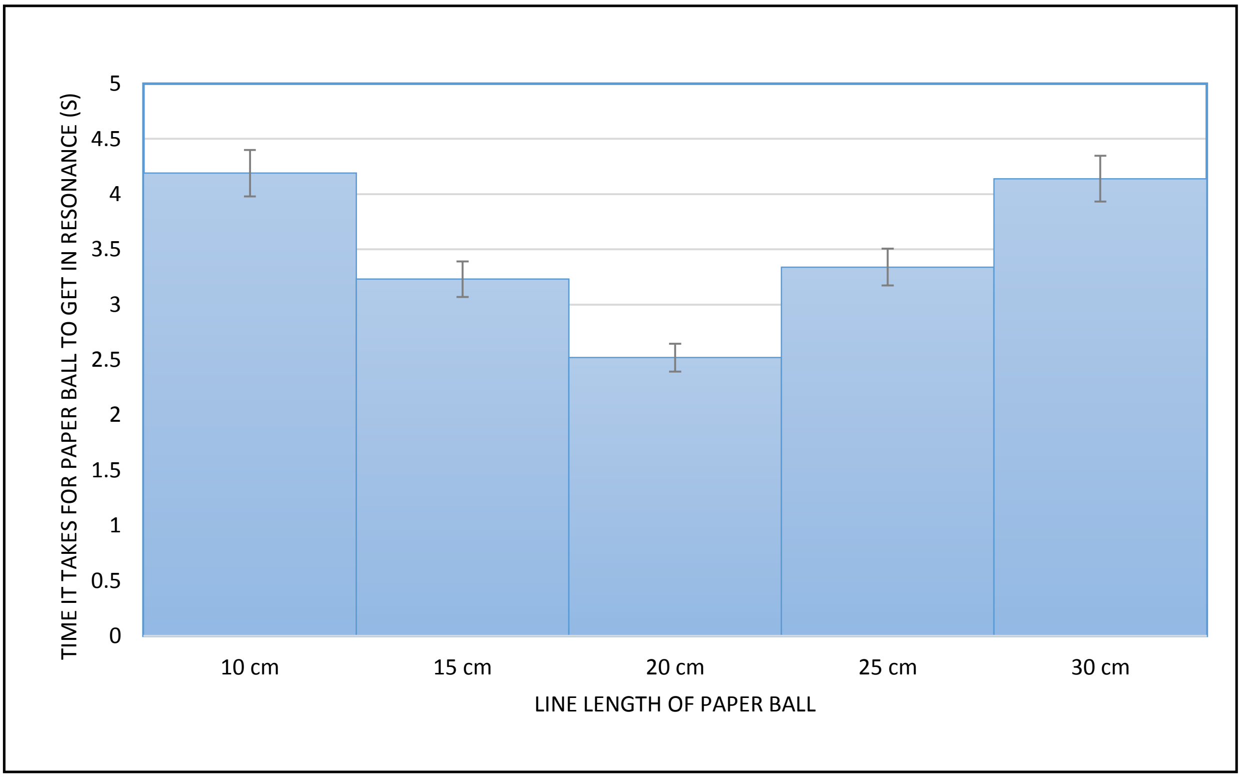
Length of the Weight ±0.1cm	Length of the Paper Ball ±0.1cm	Amplitude of the Weight ±0.1cm	Trial Count	Time it Takes for Paper Ball to Get in Resonance ±0.01s
20	10	12	Trial 1	4.15
			Trial 2	4.25
			Trial 3	4.21
			Trial 4	4.38
			Trial 5	3.98
	15		Trial 1	3.35
			Trial 2	3.17
			Trial 3	3.32
			Trial 4	3.20
			Trial 5	3.12
	20		Trial 1	2.43
			Trial 2	2.64
			Trial 3	2.47
			Trial 4	2.54
			Trial 5	2.52
	25		Trial 1	3.42
			Trial 2	3.26
			Trial 3	3.27
			Trial 4	3.51
			Trial 5	3.24
	30		Trial 1	3.87
			Trial 2	4.35
			Trial 3	4.26
			Trial 4	3.97
			Trial 5	4.23

Table 7: Shows the data of time when the line length of the weight is 20 cm's.

Data Analysis for Time

Length of the Weight ±0.1cm	*Length of the Paper Ball ±0.1cm*	*Amplitude of the Weight ±0.1cm*	*Average of the Time ±0.01s*	*Standard Deviation*
20	***10***	***12***	***4.19***	***0.131***
	15		***3.23***	***0.088***
	20		***2.52***	***0.062***
	25		***3.34***	***0.106***
	30		***4.14***	***0.183***

Table 8: Shows statistical summary of the data of time when the line length of the weight is 20 cm's.



Graph 4: Shows the time it takes for paper ball to get in resonance when the amplitude of the weight is 20 cm's.

Interpretation of the Graphs and the Data of Time

The data of time shows a similar attitude as the data of amplitude. The values are very close while the absolute difference of line lengths of the weight and the paper ball are same. As can be seen in *Graph 4,* the time it takes for paper ball to get in resonance is minimum when the line length of the paper ball is 20cm's and the time increases as the line length of the paper ball changes from 20 cm's. The time for 15 & 25 cm's and 10 & 30 cm's are very similar to each other just like in the amplitude experiment.

The standard deviation of the time data is always under (<0.200) and it shows that the data is accurate and precise once again. So it is reliable and valid. So it can be said that, as the line lengths of the pendulums get closer the time decreases as shown in *Table 8.* The data having such a low standard deviation shows that, "The time it takes for a resonant to get in resonance with the resonator is decreased by the difference of their line lengths in a simple pendulum", can be regarded as a fact.

Conclusion and Evaluation

In physics it is a known fact that two waves with similar frequencies get in resonance more than two with totally different frequencies. In this investigation it is shown that this phenomenon is also valid for resonance in simple pendulum, the line length of the pendulum being the frequency.

As the line lengths of the pendulums get closer, they tend to oscillate greater. The absolute difference between the line lengths directly affects the magnitude of resonance.

$$|l_1 - l_2|$$

Where l_1 being the line length of the weight and l_2 being the line length of the paper ball. I think this value is strongly related with the resonance magnitude of two bodies. Because all the data showed that the absolute difference of the line length is important. Whether the line length of the paper ball is greater or smaller doesn’t interests the magnitude of resonance.

Another outcome of this investigation is finding the relation of the line lengths of the pendulums and the time it takes for them to get in resonance. It is shown that just like amplitude, the time is directly related with the value of $|l_1 - l_2|$. As it gets closer to 0 the time it takes for the resonant to get in resonance with the resonate decreases.

If I were to repeat this experiment I would have done it in a much smaller room with a much stronger light. Because the shadow was not very clear and gathering data was unimaginably hard.

Further improvement of this investigation can lead to new construction techniques where as they are less affected by wind and earthquakes.

Word Count: 2974

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