TED Ankara College Foundation

Private High School

International Baccalaureate

Extended Essay

Investigating the effect of the throwing arm length on the horizontal distance travelled by an object that is thrown from a catapult

Physics HL

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Abstract

This essay investigates the effect of the throwing arm length on the horizontal distance travelled by an object that is thrown from a catapult. This investigation attempts to answer the following question: “How does the length of the throwing arm affect the horizontal distance travelled by an object that is thrown from a catapult?”. To answer this research question, an experiment was designed and performed. A wooden catapult and five wooden throwing arms with different lengths were constructed. The same object was thrown for five times with each throwing arm and the horizontal distance taken by the object with each throw was measured. The mean horizontal distance the object had travelled was greater when thrown with the longer arms, and the differences between the means were significant ($p<0.001$). This study provides a better understanding of catapults and the principles that it works with.

(Word count: 143 words)

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Introduction and Presentation

Scope of work

In short definition, catapults are devices which hurl objects. They cause projectile motion in which an object is thrown obliquely near the earth's surface and moves along a curved path. Many students are familiar with terms projectile motion, distance, law of conservation of energy, and gravitational force. In this study, these terms and their equations are used for analysing the motions of catapult and finding an answer to the mentioned research question.

Background Information

Catapult

Catapult is a mechanical device used to throw or hurl a projectile a great distance by sudden release of object. Being used since ancient times, catapult has been proven to be one of the most effective mechanisms during warfare.[1] Catapults have been around for centuries and they are used for many different purposes like;

- Fighting a war
- Launching aircrafts
- Surround a town or fortress
- Throwing bombs
- Batting cages
- Tennis courts

There are many types of catapults. In modern times, the word catapult can be used to describe any machine that hurls a projectile. This can include a slingshot

[1] Mancınıklar ve Büyük Ok Atarlar. In: Sezgin F, ed. *İslam'da Bilim ve Teknik (Cilt V).* Second Edition. İstanbul, Turkey: İstanbul Büyükşehir Belediyesi Kültür A. Ş. Yayınları; 2008: 106-119.

used to hurl pebbles, a machine that launches airplanes off aircraft carriers, and of course, the ancient weapons of destruction.

Figure 1. An ancient catapult

One of the most successful weapons that can attack an enemy from a great distance is the catapult. It provides a controlled energy source and a suitable angle for the launch. With a catapult, objects can be thrown to long distances with amazing accuracy.

Even though there are different kinds of catapults, there are some main parts present nearly in all types. These are the main parts that a basic catapult consists of;

- Arm (lever)
- Base
- Tension source
- Basket
- Object being thrown (projectile)

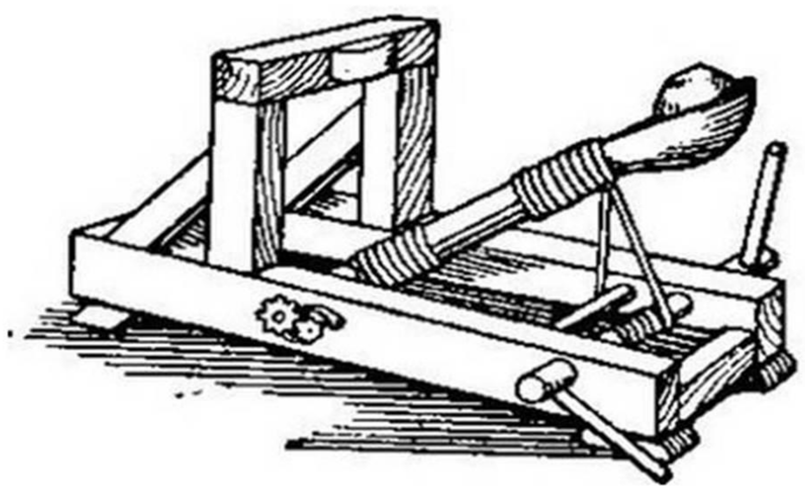


Figure 2. A simple catapult

A catapult's throwing arm has a tail which can be set to different lengths to minimize or maximize the throwing distance. With an experiment the relationship between the length of the throwing arm and the horizontal distance travelled by an object can be found out.

Projectile Motion

An object launched into space without motive power of its own, which travels freely under the action of gravity and air resistance alone, is called a projectile. The complex motion of the projectile contains two simple motions, constant horizontal velocity and uniformly accelerated vertical motion. Horizontal and vertical components are independent of each other.[2]

If an object is given a velocity v at an angle θ with the horizontal, it follows a parabolic path (Figure 3). If there were no gravity, this projectile would move along the line v*t*. In time *t*, the effect of gravity is to bring the projectile a distance $\frac{1}{2}gt^2$ below this line. It is assumed that the air friction is negligible, the horizontal component of the velocity remains constant, and the vertical motion has an uniform acceleration g.

[2] Velocity and Acceleration. In: Weber RL, Manning KV, White MW, Weygand GA, eds. *College Physics*. Fifth Edition. New York, NY: McGraw-Hill; 1974: 41-60.

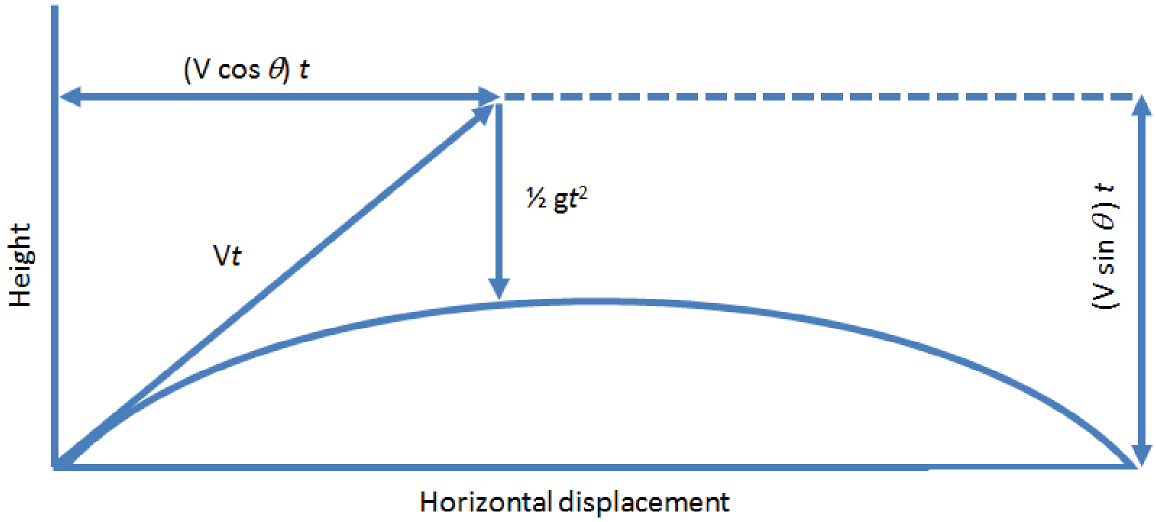


Figure 3. Projectile motion

If the velocity of projectile v makes an angle θ *with the horizontal, the horizontal component is* $v_x = v\ x\ cos\theta$*, the vertical component is* $v_y = v\ x\ sin\theta$*. The horizontal velocity remains constant, but the vertical component in the negative (downward) direction along the y axis changes. The vertical component of velocity at any time t is* $v_y = v\ x\ sin\theta - gt_r$*. The projectile rises until the vertical component of the velocity becomes zero.*[3] At this instant, the height is maximum, and the time of rise t_r is given by

$$0 = v\ x\ sin\theta - gt_r$$

$$t_r = \frac{v\ x\ sin\theta}{g}$$

The total time that projectile is in the air is the sum of time of rise t_r and the time of fall t_f. Throughout that time of flight the horizontal component of the velocity remains constant at $v\ x\ cos\ \theta$. The horizontal range of the projectile is therefore $v\ x\ cos\theta\ x\ (t_r + t_f)$. If the projectile falls to the same height from which it was projected, the time of fall is same as the time of rise, and the time of flight is given by

$$t_r + t_f = \frac{2v\ x\ sin\theta}{g}$$

[3] Translational Motion. In: Smith AW, Cooper JN, eds. *Elements of Physics*. Eighth Edition. New York, NY: McGraw-Hill; 1972: 49-63.

Planning A

Aim

The aim of this experiment is to investigate the effect of throwing arm length on the horizontal distance travelled by an object that is thrown from a catapult.

Research Question

How does the length of the throwing arm affect the horizontal distance travelled by an object that is thrown from a catapult?

Hypothesis

Increasing length of the throwing arm, also increases the horizontal distance travelled by the thrown object.

Key Variables

Independent Variable: Length of the throwing arm

Dependent variable: Horizontal distance travelled by the object

Controlled Variables:

- Dimensions of the object
- Type of the object (a spherical stone)
- Mass of the object (6 grams)
- Catapult
- Initial object orientation

Constant Variables:

- Temperature
- Pressure
- Gravitational acceleration

Planning B

Materials [4]

- 3 Pieces of wood 30cm long
- 8 Pieces of wood 15cm long (3.5cm*2cm*15cm)
- Screws
- Elastic rope or plastic band
- 2 Hooks
- Glue
- 1 Metal bar 15cm long

Figure 4. Materials used for building up the catapult

[4] www.stormthecastle.com/catapult/how-to-build-a-catapult.htm

Building Up the Catapult

- Cut 2 of 15cm long wood pieces from their ends with 45 degree angles.

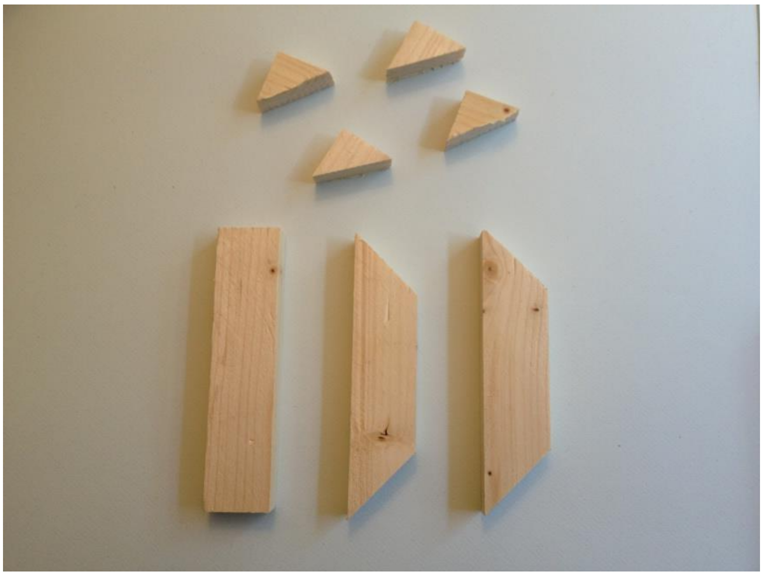


Figure 5. Cutting the ends of wooden pieces

- Use wood adhesive to glue one 15cm long wood to one of the wood pieces that were cut and then drive a nail in wood pieces to strengthen the structure.
- This time use wood adhesive to glue this structure to one of the 30cm long woods and then use 2 screws to firm the composition.

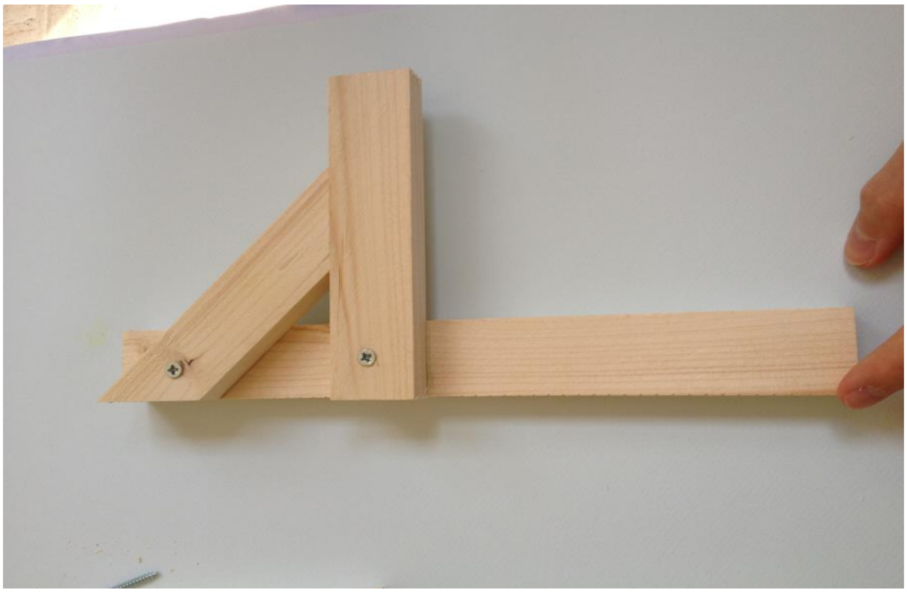


Figure 6. Using glue and screws to firm the composition

- Build other side of catapult as it would be mirror image of the first one.

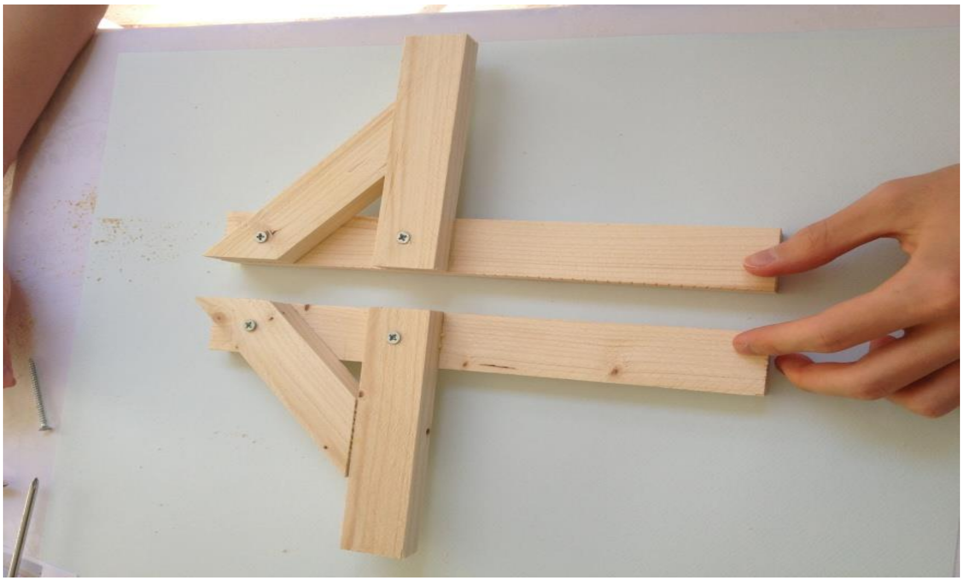


Figure 7. Building the other side of the catapult

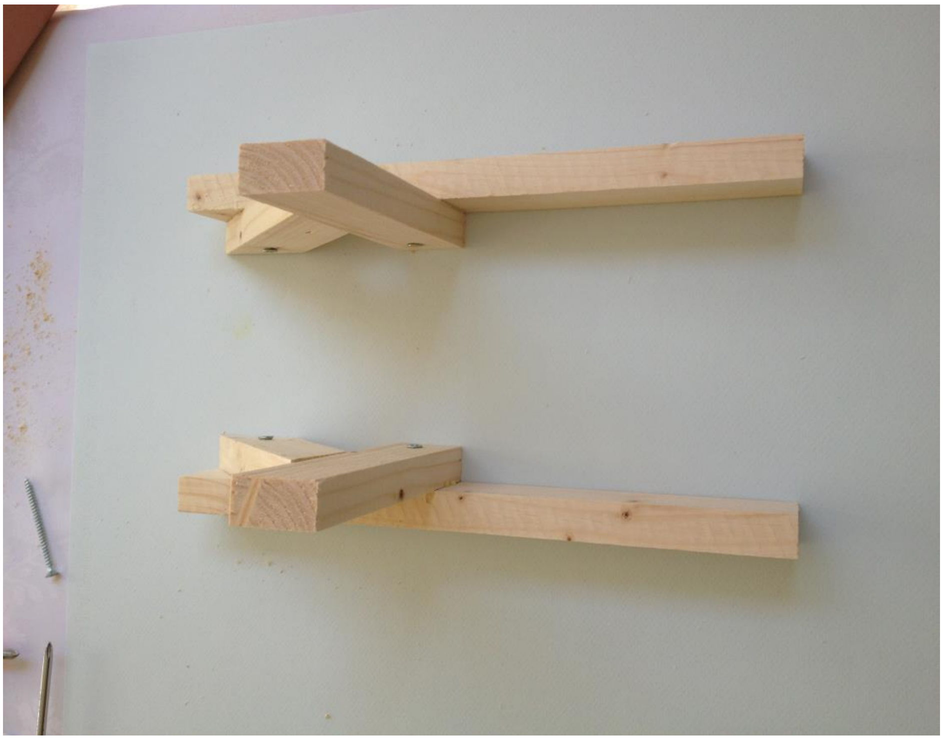


Figure 8. Aligning the sides parallel to each other

- Drill holes in 15cm long vertically oriented parallel wood pieces so that a metal bar can pass through them. But while drilling these holes, drill one of the woods from side to side, since the metal bar must come out of the catapult each time we are going to change throwing arms later in the experiment.
- Connect two sides of catapult together using two of the 15cm long wood pieces.

Figure 9. Connecting the sides of the catapult (top view)

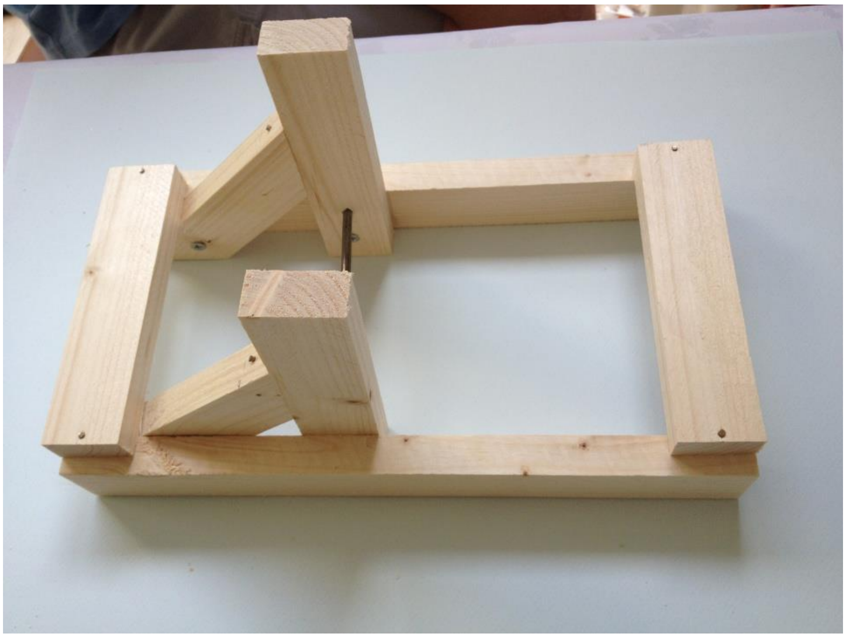


Figure 10. Connecting the sides of the catapult (side view)

- Connect top of two 15cm long vertical wood pieces by using a 15cm long wood piece.

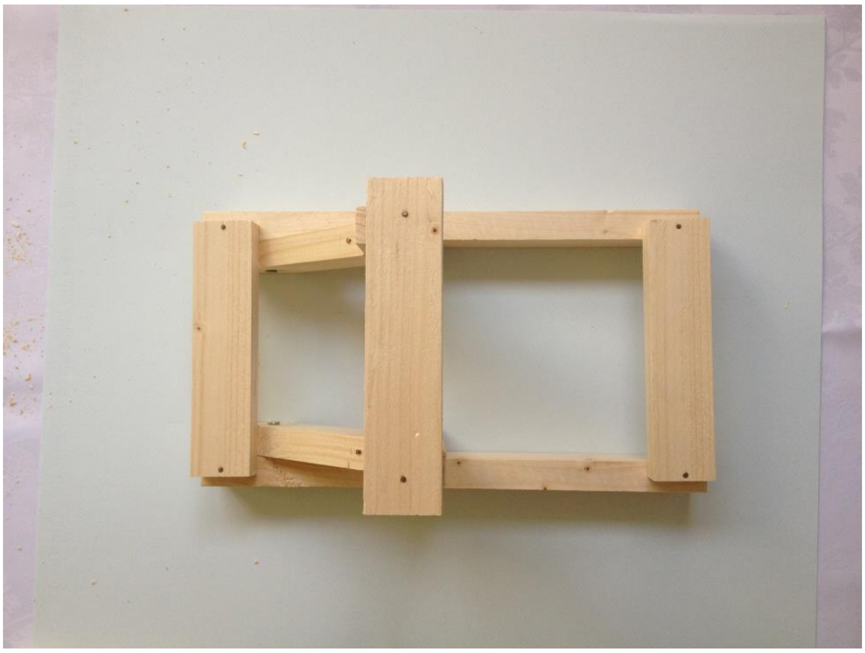


Figure 11. Connecting the top with a wooden piece

- Drill a hole in 25cm long wood piece, this piece is going to be the first throwing arm.
- Put a hook on the other end of wood as it will be 6cm away from the end.
- Nail a basket between the hook and the end as it will be 1cm away from the end.

Figure 12. Placing hook and basket at the end (top view)

Figure 13. Placing hook and basket at the end (side view)

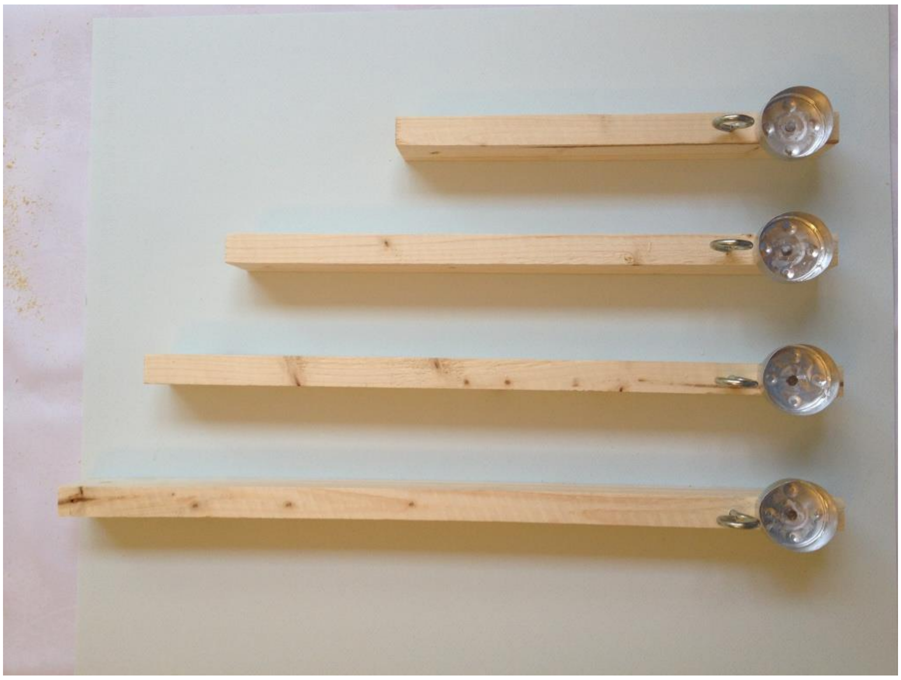


Figure 14. Placing hooks and baskets on the remaining throwing arms

- Push metal bar through catapult arm as it would be between 2 upright woods.
- Put the other hook in front of the catapult.

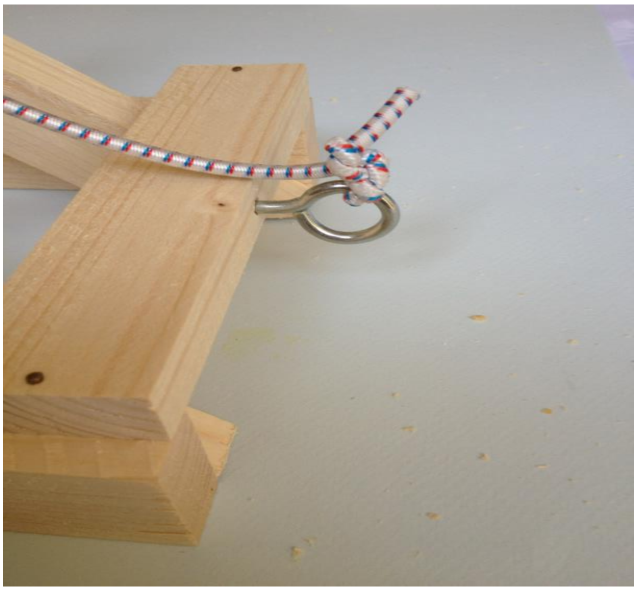


Figure 15. Placing a hook at the front of catapult and tying an elastic rope between the hooks

- Tie an elastic rope from the hook in the front to the hook on the catapult arm.
- Catapult is ready to fire, press down on throwing arm to hurl an object.
- Any solid object that could fit in the basket can be used in this throwing experiment. For this study, a spherical 6gr stone is used.
- Make 5 trials for 25cm long throwing arm.
- Repeat the same steps done for the 25cm long wood piece for each 30cm, 35cm, 40cm, and 45cm long pieces.

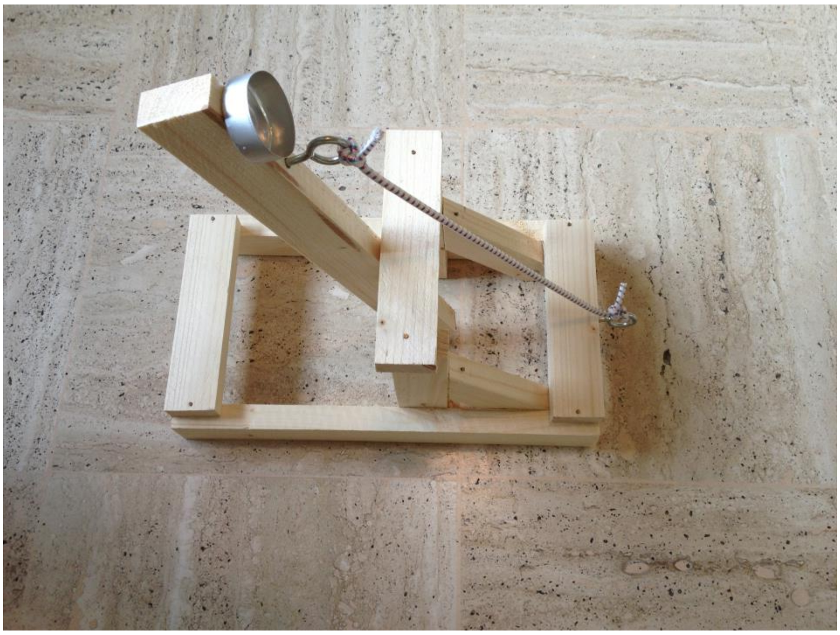


Figure 16. Catapult (side view)

Figure 17. Catapult (front view)

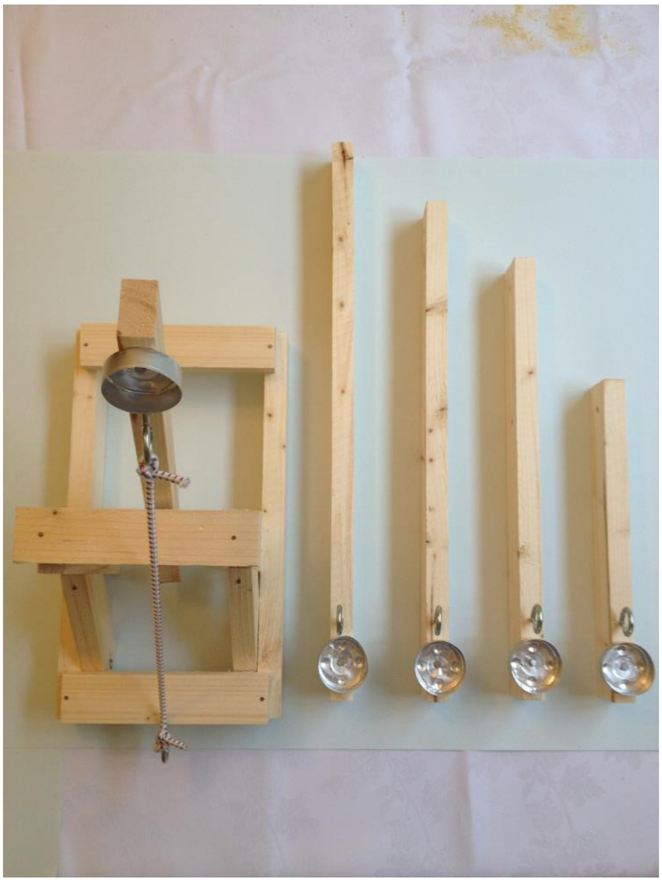


Figure 18. Catapult and the throwing arms

Methods and Data Collection

To investigate the effect of throwing arm lenght on the horizontal distance taken by an object thrown from a catapult, the same object was thrown with throwing arms having different lengths. All other factors such as the tension of elastic rope and the force applied for streching the throwing arm before each throw were kept constant in order to make sure that “the length of the throwing arm” was the only factor being changed during the whole experiment.

A spherical stone with the weight of 6 grams was thrown with each throwing arm (arm lengths were 25cm, 30cm, 35cm, 40cm, and 45cm). With each throwing arm, five consecutive throws were executed, and the horizontal distance the stone had taken from the catapult was measured. Uncertainty was also provided along with the five distance measurements for each throwing arm.

During data analysis, since there were only five measurements for the each throwing arm, it was assumed that the data was unevenly distributed. Therefore, the **Kruskal-Wallis test**, which is more suitable for non-parametric comparisons between more than 2 groups with uneven distributions, was chosen to compare the means.[5,6] As the result of test, *p* value less than 0.05 was considered to be significant.

[5] Kruskal WH, Wallis WA. Use of ranks in one criterion variance analysis. Journal of American Statistical Association 1952; 47: 583-621.

[6] Mann HB, Whitney DR. On a test whether one of two random variables is stochastically larger than other. Annals of Mathematical Statistics 1947; 18: 50-56.

Results and Data Processing

Row Data Chart

Length of Throwing Arm ±0.1 cm	Trial	Mass of Projectile ±0.001 g	Horizontal Distance Travelled by Projectile ±0.1 cm
25.0	1.	6.000	**269.0**
	2.	6.000	**281.0**
	3.	6.000	**262.0**
	4.	6.000	**290.0**
	5.	6.000	**276.0**
30.0	1.	6.000	**308.0**
	2.	6.000	**297.0**
	3.	6.000	**315.0**
	4.	6.000	**304.0**
	5.	6.000	**323.0**
35.0	1.	6.000	**347.0**
	2.	6.000	**362.0**
	3.	6.000	**354.0**
	4.	6.000	**336.0**
	5.	6.000	**343.0**
40.0	1.	6.000	**395.0**
	2.	6.000	**368.0**
	3.	6.000	**379.0**
	4.	6.000	**371.0**
	5.	6.000	**388.0**
45.0	1.	6.000	**424.0**
	2.	6.000	**411.0**
	3.	6.000	**430.0**
	4.	6.000	**417.0**
	5.	6.000	**405.0**

Table 1. This chart shows all obtained distance values for each throwing arm length. First column shows throwing arm lengths while last column shows horizontal distances travelled by the projectile. For each throwing arm length, five trials were made and recorded.

In this experiment, horizontal distances travelled by the object were measured by a ruler, and since the smallest division on the ruler is 1 mm, uncertainty of these values are ± 0.1cm.

In order to plot a graph showing the relationship between throwing arm length and horizontal distance travelled by the projectile, we need to calculate average distance value for each throwing arm.

Average Distance Calculation for the First Throwing Arm

First throwing arm length: 25.0 cm ± 0.1

First distance value: 269.0 cm ± 0.1

Second distance value: 281.0 cm ± 0.1

Third distance value: 262.0 cm ± 0.1

Fourth distance value: 290.0 cm ± 0.1

Fifth distance value: 276.0 cm ± 0.1

Average distance = $\frac{269 + 281 + 262 + 290 + 276}{5}$

= 276.0 cm ± 0.1

Average Distance Calculation for the Second Throwing Arm

Second throwing arm length: 30.0 cm ± 0.1

First distance value: 308.0 cm ± 0.1

Second distance value: 297.0 cm ± 0.1

Third distance value: 315.0 cm ± 0.1

Fourth distance value: 304.0 cm ± 0.1

Fifth distance value: 323.0 cm ± 0.1

Average distance = $\dfrac{308 + 297 + 315 + 304 + 323}{5}$

= 309.0 cm ± 0.1

Average Distance Calculation for the Third Throwing Arm

Third throwing arm length: 35.0 cm ± 0.1

First distance value: 347.0 cm ± 0.1

Second distance value: 362.0 cm ± 0.1

Third distance value: 354.0 cm ± 0.1

Fourth distance value: 336.0 cm ± 0.1

Fifth distance value: 343.0 cm ± 0.1

Average distance = $\dfrac{347 + 362 + 354 + 336 + 343}{5}$

= 348.0 cm ± 0.1

Average Distance Calculation for the Fourth Throwing Arm

Fourth throwing arm length: 40.0 cm ± 0.1

First distance value: 395.0 cm ± 0.1

Second distance value: 368.0 cm ± 0.1

Third distance value: 379.0 cm ± 0.1

Fourth distance value: 371.0 cm ± 0.1

Fifth distance value: 388.0 cm ± 0.1

Average distance = $\dfrac{395 + 368 + 379 + 371 + 388}{5}$

= 380.0 cm ± 0.1

Average Distance Calculation for Fifth Throwing Arm

Fifth throwing arm length: 45.0 cm ± 0.1

First distance value: 424.0 cm ± 0.1

Second distance value: 411.0 cm ± 0.1

Third distance value: 430.0 cm ± 0.1

Fourth distance value: 417.0 cm ± 0.1

Fifth distance value: 405.0 cm ± 0.1

Average distance = $$\frac{424.0 + 411.0 + 430.0 + 417.0 + 405.0}{5}$$ = 417.0 cm ± 0.1

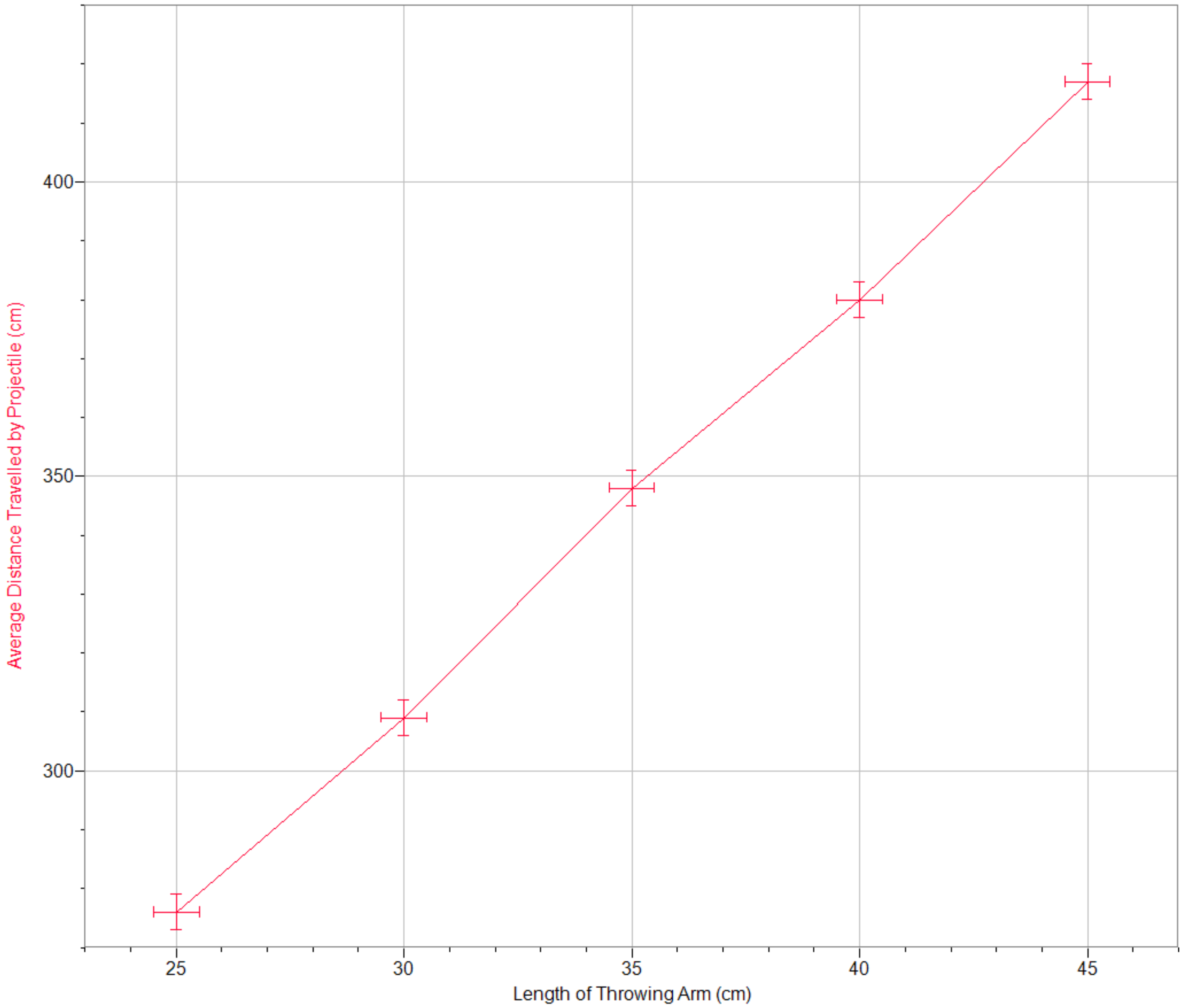
Length of Throwing Arm ± 0.1 cm	Average Distance Travelled by the Projectile ± 0.1 cm

As shown above, average distance values were calculated for each throwing arm. All obtained averages are provided in the following chart.

25.0 cm	**276.0 cm**
30.0 cm	**309.0 cm**
35.0 cm	**348.0 cm**
40.0 cm	**380.0 cm**
45.0 cm	**417.0 cm**

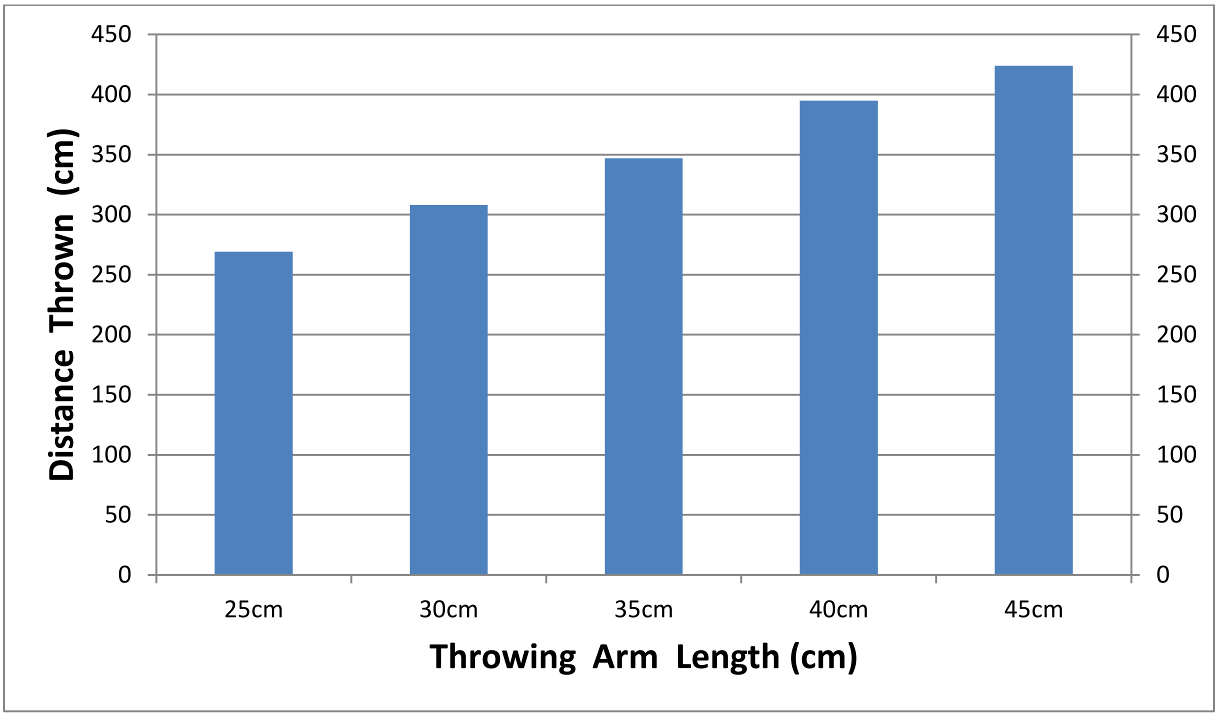
Table 2. This chart shows all throwing arm lengths and average distance values. Each data on the chart is presented with its uncertainty and unit. All calculated average distance values are written next to their corresponding throwing arm lengths.

The graph given below shows the relationship between throwing arm length and average distance taken. As it can be seen from the graph, length of throwing arm is directly proportional to the average distance travelled by the projectile.

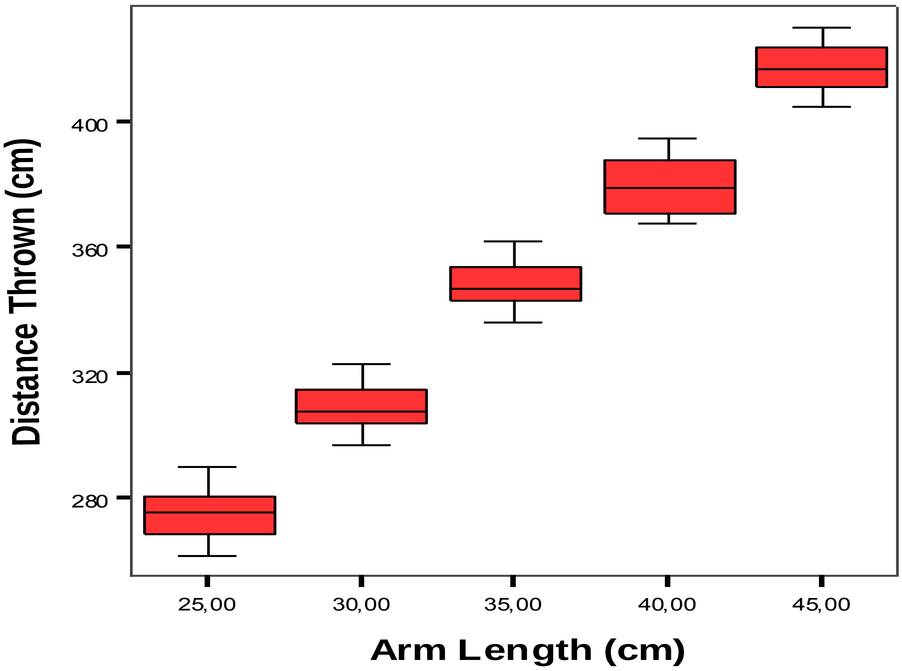


Graph 1. This graph shows the relationship between throwing arm length and average distance taken. As length of throwing arm increases, average distance also increases. In graph, horizontal lines represent uncertainty of throwing arm length while vertical lines represent uncertainty of average distance travelled by the object. Each data is presented with its unit.

The data distribution and mean/median values are provided as bar and box-plot in Graphs 2 & 3. A trend towards increase in distance travelled with the increase in throwing arm length was noted in these graphs.



Graph 2. Bar graph showing the mean distances travelled for each throwing arm.



Graph 3. Box-plot graph showing the median distances travelled, along with the interquartile ranges, for each throwing arm.

In accordance with these graphs, the average distances the stone had travelled was greater when thrown with the longer arms, and the differences between the averages were significant (Kruskal Wallis test, $p<0.001$).

Conclusion and Evaluation

This experiment basically tests whether an increase in the throwing arm length of the catapult actually increases the horizontal distance taken by the projectile. Five throws were executed by each throwing arm to obtain an average value for that specific arm.

The results of the experiment are depicted in Graph 1. In this graph, throwing arm length was represented in the x axis and average horizontal distance thrown in the y axis. The results support the hypothesis that an increase in throwing arm length also increases the horizontal distance travelled by a thrown object. Accordingly, the throwing arm length was directly proportional to the horizontal distance travelled (Graph 1).

The object thrown was a spheric stone weighing 6 grams. The same stone was used through the experiment. The stone chosen for the experiment was free of surface irregularities since the three dimensional configuration of the object could alter the parabolic path taken through the air.

If the lines on a graph pass through the origin or very close to it, this means that there are no systematic errors in the experiment or if there is any, it is so small that it can be neglected. The line on our graph passes very close to the origin meaning that there was no or negligible systematic error to cause a significant difference in our values. Our error bars are reasonable because they are not too big, thus, there were not many deviations in the experiment. In other words, our small error bars indicate that our data and graph are accurate and realistic.

As in all experiments, this study is not free of errors, weaknesses and things to be improved next time. The limitations of this experiment could be summarized as follows:

First, air friction was considered to be negligible in this experiment, but we know that it actually exits. However, air resistance did not seem to cause any significant difference on values, because the object (stone, 6 grams) we used was too small to come across with substantial air resistance, and the time the object spent up in the air was just a few seconds. Since all throws were executed under the same air conditions, air friction was not a changing variable in this experiment. The same thing applies to gravitational force which was virtually constant through all throws.

Second, the flexibility of the rope might have caused problem in experiment, because with each increase in throwing arm length, the rope streched more and might have lost its flexibility.

Finally, in this experiment, we made five trials for each throwing arm length but as in all experiments, it would have been better to make more trials. The more it is repeated the less possibility of random errors exist, thus this contributes to the accuracy of the experiment. Another thing which helps the experiment to be more accurate is to use a wide range of variables. If we had performed the experiment with different weight values, we could have obtained even a wider perspective.

In the experiment, we kept some variables constant to make sure that there will not be any differences caused by them. For example, we used the same stone and the same catapult for all trials. We also did all of our measurement with the same ruler. These controlled variables were kept constant throughout the experiment. Since the length of the elastic rope along with its tension could be confounding factors, outmost attention was paid for aligning and tying the rope in a similar fashion for each throwing arm. Moreover, same amount of streching force was applied during each hurling attempt as possible. Despite all efforts, there were throws unexpectedly short or long. This variation could be attributed to the unintentional tension changes in the elastic rope during the experiment.

As expected, “the horizontal distance taken by the projectile correlated with the length of the throwing arm”. The average distance taken by the projectile, which was a stone in this experiment, significantly increased by the increase in throwing arm lengths (Kruskal-Wallis test, $p<0.001$).

$$mgh + \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

Law of conservation of energy can be used to explain the result of this experiment. The formula mgh represents the potential energy of the object and it is different for each throwing arm length. Even though mass of stone (m) and gravitational acceleration (g) are kept constant, the height between ground and stone (h) increases due to the increase in throwing arm length. Therefore, as throwing arm becomes longer, stone moves upward and potential energy increases.

The formula $\frac{1}{2}kx^2$ represents the elastic potential energy stored in the elastic rope. Similar to mgh, it is also correlated with the throwing arm length. Even though type of elastic rope (k) is kept constant, the distance elastic rope stretches from the equilibrium position (x) increases, hence the distance the rope has to cover increases.

Both potential energy (mgh) and elastic potential energy ($\frac{1}{2}kx^2$) is directly proportional to throwing arm length. When catapult is fired, all this stored energy is released and converts into kinetic energy ($\frac{1}{2}mv^2$). Since sum of potential energy and elastic potential energy is greater in longer throwing arms, this means their projectiles would have greater kinetic energies. Therefore, with their greater kinetic energies, they can travel farther than other projectiles thrown by shorter throwing arms.

When we consider that the only changing variable is the throwing arm length, the following explanation is plausible. The rotational movement (torsion) of each throwing arm changes according to the throwing arm lengths, and as a result, the vector the projectile takes during hurling (striking) varies. Since the horizontal distance taken by the projectile is expressed with the formula [$V \cos\theta\ (t_r + t_f)$], the angle of the vector could be the major determinant in our experiment. When the hurling (striking) angle is smaller the cosine value is greater, and the horizontal distance taken by the projectile is longer. Therefore we may finally assume that the increase in throwing arm length is associated with a decrease in hurling (striking) angle, which eventually results in a longer horizontal distance taken by the projectile.

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- www.stormthecastle.com/catapult/how-to-build-a-catapult.htm

Appendices

Five consecutive throws were executed, and the horizontal distance the stone had taken from the catapult was measured and recorded. A mean distance was calculated and expressed as mean ± SE for all throwing arms measuring 25cm, 30cm, 35cm, 40cm, and 45cm. The distance measurements for each throwing arm are presented in Table 1 Supplement.

Arm length ±0.1cm	Throw #1 ±0.1cm	Throw #2 ±0.1cm	Throw #3 ±0.1cm	Throw #4 ±0.1cm	Throw #5 ±0.1cm	Mean±SE (cm)
25.0	269.0	281.0	262.0	290.0	276.0	275±4,823
30.0	308.0	297.0	315.0	304.0	323.0	309±4,479
35.0	347.0	362.0	354.0	336.0	343.0	348±4,479
40.0	395.0	368.0	379.0	371.0	388.0	380±5,073
45.0	424.0	411.0	430.0	417.0	405.0	417±4,456

Table 1 Supplement. Horizontal distances taken by the projectile according to throwing arm lengths. SE: Standard Error

Along with their standard errors, the mean distances travelled by the 25cm, 30cm, 35cm, 40cm, and 45cm throwing arms were 275±4,823cm, 309±4,479cm, 348±4,479cm, 380±5,073cm, and 417±4,456cm, respectively.