MATHEMATICS EXTENDED ESSAY

"How Restaurants Use Linear Programming For Menu Planning?"

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ABSTRACT

Most of the people besides some engineers and mathematics professors; knows and uses mathematics only when they are receiving education or going to school. They do not know the areas of uses of mathematics in real life and how does it simplifies works and helps people. However; mathematics is indispensable in a lot of areas and it is needed for jobs in order to be planned and arranged for the highest profits.

Linear programming is a mathematical technique that is used in computer modeling (simulation) to find the best possible solution in allocating limited resources (energy, machines, materials, money, personnel, space, time, etc.) to achieve maximum profit or minimum cost. In this study I am going to use linear programming in an area of work in order to show that how could it be used and how does it simplifies works and calculations of profit.

Word Count: 146

RESEARCH QUESTION

How does restaurants use linear programming for menu planning?

INTRODUCTION

In this study I will write about linear programming, how restaurants use linear programming for menu planning, a restaurant menu problem and how it is solved. I will explain all the calculations to solve the problem in this homework.

Form of the linear programming is what I learned first. There is four main parts to apply linear programming to a problem. First three parts are a linear function to be maximized, problem constraints and nonnegative variables. These three parts are written with looking at the numbers in the problem. Most important one is the function to be maximized; therefore I can decide which function will be worked on. Problem constraints are changing with how many constraints I want to put. The nonnegative variables are included to the general parts and don't change the solution. Fourth part is the feasibility region graph. I can see the maximum profit with examining the graph.

Secondly there is a restaurant menu problem, which I wrote to solve. In this problem two friends decide to open a restaurant and thinks about maximum profit per day. Therefore two friends use linear programing and feasibility graph to see their decisions. After the first graph, they see that their profit per day is 376 TL. After that they think about increasing the profit. They hire someone to increase cooking hours of the restaurant and use linear programming again to see what happened. Then they see that their profit increased to 524 TL. Therefore they think that they do very good with hiring someone to the restaurant. Then they start thinking about increasing the profit per day again. They look to the graph and decide to make an advertisement to increase the sales of pitas. They pay 900 TL for advertisement for a month and again use linear programming. Then they look to the feasibility graph and see their profit per day is decreased to 452 TL. Then they decided to hire another person and saw that their profit per day becomes 508 TL. After that they decide to stop the advertisement and stop hiring the second person for increasing cooking hours their profit become 524 TL per day again.

In the end of this study, there is conclusion and bibliography part. In the conclusion part I write about what I learned from this homework. In the bibliography part I write the internet cites which I read for this homework.

LINEAR PROGRAMMING

Linear programming is a mathematical technique used in computer modeling (simulation) to find the best possible solution in allocating limited resources (energy, machines, materials, money, personnel, space, time, etc.) to achieve maximum profit or minimum cost. However, it is applicable only where all relationships are linear and can accommodate only a limited class of cost functions. For problems involving more complex cost functions, another technique called “mixed integer modeling” is employed. It is developed by the Russian economist Leonid Kantorovich and the US economist C. Koopmans, on the basis of the work of Russian mathematician Andrei Nikolayevich Kolmogorov.

Also linear programming is the process of taking various linear inequalities relating to some situations, and finding the “best” value obtainable under those conditions. A typical example would be taking the limitations of materials and labor, and then determining the “best” production levels for maximal profits under those conditions.

HOW RESTAURANTS USE LINEAR PROGRAMMING FOR MENU PLANNING?

Restaurants use linear programming for menu planning. It uses basic algebra to optimize meal production and thereby increase restaurant profits. Linear algebra reflects a direct relationship between an increase or decrease in food resources, and an increase or decrease in meal production. For example, if the kitchen has only half its needed supply of cream base, then it can only prepare half its normal amount of cream soups. Additionally, management can determine the cost of preparing different menu items to decide how many of each menu item to prepare for optimal profit.

Also restaurants use linear programming for very complex mathematical calculations. For this calculations the maximum table which restaurants can serve meal, cooking hours of the cooks, the price of the ingredients of the meals, cooking hour of the one meal, how many customer is expected each day, advertisement, how many meals to be served and other estimations are very important. Generally the aim of the restaurants is maximizing the profit of the restaurant and all the things for linear programing are shaping the constraints of the problem. If the constraints are very high then the problem becomes more complicated.

FORM OF LINEAR PROGRAMMING

The standard form of the linear programming is the usual and the most intuitive form of describing a linear programming problem. It consists of main four parts.

1- A linear function to be maximized

$f(x_1, x_2) = c_1x_1 + c_2x_2$

2- Problem constraints

$a_1x_1 + a_2x_2 \leq b_1$

$a_3x_1 + a_4x_2 \leq b_2$

$a_5x_1 + a_6x_2 \leq b_3$

3- Nonnegative variables

$x_1 \geq 0$

$x_2 \geq 0$

4- The feasibility region graph

When you test the corner points of the intersection of the graph of the previous three parts, you should obtain the maximum or minimum value of the problem.

RESTAURANT MENU PROBLEM

Two friends decided to open a restaurant which sells meals from the Black Sea in Ankara. One of them was a cook and the other one was a business administrator. Although there were other restaurants which sell meals from the Black Sea region, their plan was preparing a good restaurant menu and be successful in their restaurants. So they decided to rent a place for their restaurants and started about thinking the meals they will serve.

Because they don’t have a lot of money for investigation, firstly they decided to serve just two meals to the customers, one was a meal of anchovy and other was a pita. The cook thought about the plan and he decided to obey the plan because by this way they was able to save money for other menu meals and be better about the anchovy meal and the pita. And also the cook decided to serve desserts, soup and salad with the meals.

The next thing they considered was how much they will spend for shopping and how many hours will it take to prepare meals. Therefore, they tried to guess how many meals they will sell each night. They guessed that they will sell maximum 50 meals each night. Each anchovy meal requires 20 minutes to prepare and each pita requires 40 minutes to prepare.

The cook can work 12 hours per day. They also guessed that they will sell at least 2 anchovy meal when they sell one pita because they have a restaurant which sells the Black Sea region meals. Also they decided that at least 1 of their 9 customers will anyway order pitas. The profit from each anchovy meal will be 12 TL and the profit from each pita will be 10 TL.

The question is which condition gives their maximum profit and what happens if they decide to chance some of their guesses.

RESTAURANT MENU PROBLEM SOLUTION

The variables:

x_1: number of anchovy meal

x_2: number of pita

1. A linear function to be maximized:

$z = 12 x_1 + 10 x_2$

z: total profit per day (TL)

$12 x_1$: profit from anchovy meal per day

$10 x_2$: profit from a pita per day

2. Problem constraints

$x_1 + x_2 \leq 50$ (the maximum meals guessed each day)

$20 x_1 + 40 x_2 \leq 720$ (minutes needed to prepare meals, 12*60 = 720)

$x_1 - 2 x_2 \geq 0$ (they will sell at least 2 anchovy meal when they sell one pita)

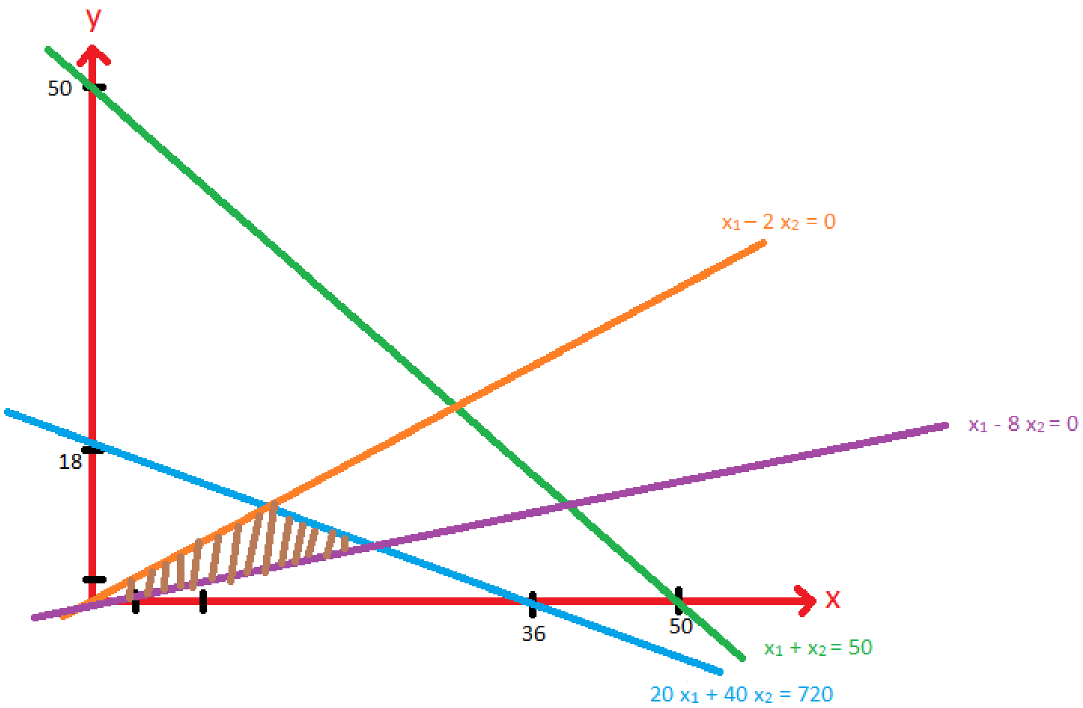
x_1 - 8 $x_2 \leq 0$ (at least 1 of their 9 customers will anyway order pitas)

3. Nonnegative variables

$x_1 \geq 0$

$x_2 \geq 0$

4. The feasibility region graph



In this step we should check the corner points of the shaded area. There are two important corner points in this shaded area. The points are the intersection of x_1 - 8 $x_2 \leq$ 0 and 20 x_1 + 40 $x_2 \leq 720$ equations and the intersection of $x_1 - 2 x_2 \geq 0$ and 20 x_1 + 40 $x_2 \leq 720$ equations.

To find the first point we should solve the first equations.

x_1 - 8 $x_2 = 0$ and 20 x_1 + 40 x_2 = 720

$-20\ x_1 + 160\ x_2 = 0$

$20\ x_1 + 40\ x_2 = 720$

$200\ x_2 = 720$, $x_2 = 3.6$, $x_1 = 28.8$

We can’t have 3.6 pitas and 28.8 anchovy meals. So we should look at the graph and take 28 anchovy meals. When we look at first equation $x_2 = 3.5$. We can’t have 3.5 pitas therefore we should check 4 pitas which is good for $x_1 - 8\ x_2 \leq 0$. And also it is okay for the second equation $20\ x_1 + 40\ x_2 \leq 720$ because 20*28 + 40*4 = 720. Therefore our first check point is (28, 4).

To find the second point we should solve the second equations.

$x_1 – 2\ x_2 = 0$ and $20\ x_1 + 40\ x_2 = 720$

$20\ x_1 - 40\ x_2 = 0$

$20\ x_1 + 40\ x_2 = 720$

$40\ x_1 = 720$, $x_1 = 18$, $x_2 = 9$

Therefore our second point is (18, 9).

Now we should calculate the maximum profit for two points. Bigger one will be our maximum profit point.

$z = 12\ x_1 + 10\ x_2$

z = 12 * 28 + 10 * 4 = 376 TL (for (28, 4) point)

z = 12 * 18 + 10 * 9 = 306 TL (for (18, 9) point)

Therefore for this problem our maximum profit is **376 TL** per day. For this problem 28 anchovy meals and 4 pitas should be sold.

Now check if all the conditions are satisfied.

$x_1 + x_2 \leq 50$

$28 + 4 = 32 < 50$ √

$20\,x_1 + 40\,x_2 \leq 720$

$20 * 28 + 40 * 4 = 720 \leq 720$ √

$x_1 - 2\,x_2 \geq 0$

$28 - 2 * 4 = 20 > 0$ √

$x_1 - 8\,x_2 \leq 0$

$28 - 8 * 4 = -4 < 0$ √

Now two friends found 376 TL very low and started to think a solution to the problem. When they looked to the graph they saw that $20\,x_1 + 40\,x_2 = 720$ equation is the problem. Therefore they decide to increase cooking hours by hiring someone to help the cook. They hired someone and decided to pay her 40 TL per day. By this cooking hours are increased from 12 hours to 18 hours. Finally, they started to draw the graph again to see what happens to the graph.

The variables:

x_1: number of anchovy meal

x_2: number of pita

1. A linear function to be maximized:

$z = 12\ x_1 + 10\ x_2$

z: total profit per day (TL)

$12\ x_1$: profit from anchovy meal per day

$10\ x_2$: profit from a pita per day

2. Problem constraints

$x_1 + x_2 \leq 50$ (the maximum meals guessed each day)

$20\ x_1 + 40\ x_2 \leq 1080$ (minutes needed to prepare meals, 18*60 = 1080)

$x_1 - 2\ x_2 \geq 0$ (they will sell at least 2 anchovy meal when they sell one pita)

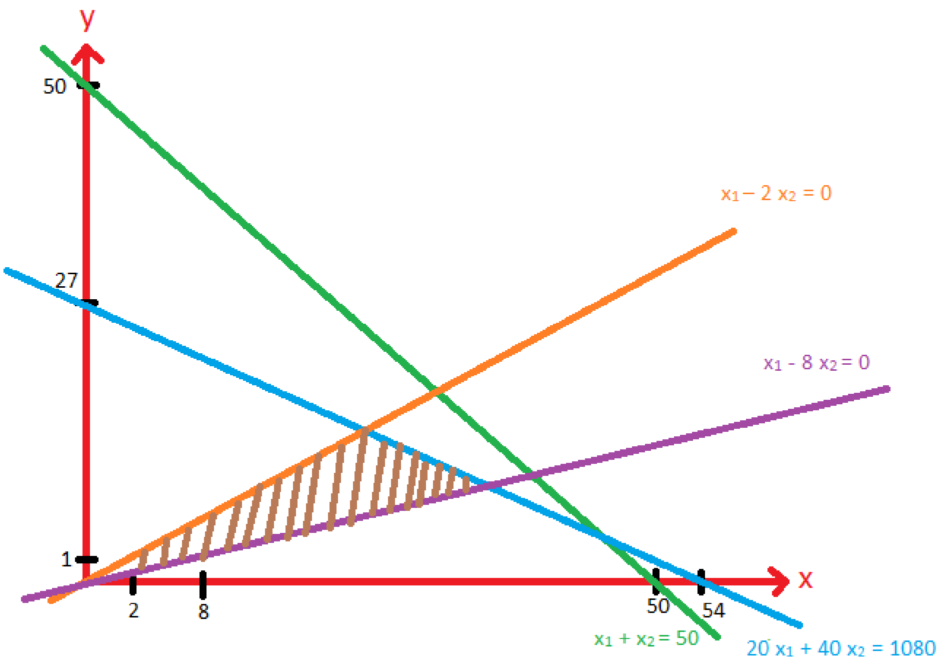
$x_1 - 8\ x_2 \leq 0$ (at least 1 of their 9 customers will anyway order pitas)

3. Nonnegative variables

$x_1 \geq 0$

$x_2 \geq 0$

4. The feasibility region graph



In this step which should again check the corner points of the shaded area. There are two important corner points in this shaded area. The points are the intersection of $x_1 - 8\ x_2 \leq 0$ and $20\ x_1 + 40\ x_2 \leq 1080$ equations and the intersection of $x_1 - 2\ x_2 \geq 0$ and $20\ x_1 + 40\ x_2 \leq 1080$ equations.

To find the first point we should solve the first equations.

$x_1 - 8\ x_2 = 0$ and $20\ x_1 + 40\ x_2 = 1080$

$-20\ x_1 + 160\ x_2 = 0$

$20\ x_1 + 40\ x_2 = 1080$

$200\ x_2 = 1080$, $x_2 = 5.4$, $x_1 = 43.2$

We can't have 5.4 pitas and 43.2 anchovy meals. So we should look at the graph and take 43 anchovy meals. When we look at first equation $x_2 = 5.375$. We can't have 5.375 pitas therefore we should check 6 pitas which is good for $x_1 - 8\ x_2 \leq 0$. But it is not okay for the second equation $20\ x_1 + 40\ x_2 \leq 1080$ because 20*43 + 40*6 = 1100 > 1080. Now we should try (42, 6) point. 20*42 + 40*6 = 1080. Therefore our first check point is (42, 6).

To find the second point we should solve the second equations.

$x_1 - 2\ x_2 = 0$ and $20\ x_1 + 40\ x_2 = 1080$

$20\ x_1 - 40\ x_2 = 0$

$20\ x_1 + 40\ x_2 = 1080$

$40\ x_1 = 1080$, $x_1 = 27$, $x_2 = 13.5$ (we can't have 13.5 pitas therefore we should take 13 pitas)

Therefore our second point is (27, 13).

Now we should calculate the maximum profit for two points. Bigger one will be our maximum profit point.

$z = 12\ x_1 + 10\ x_2$

z = 12 * 42 + 10 * 6 = 564 TL (for (42, 6) point)

z = 12 * 27 + 10 * 13 = 454 TL (for (27, 13) point)

Therefore for this problem our maximum profit is 564 – 40 (hiring money) = **524 TL** per day. For this problem 42 anchovy meals and 6 pitas should be sold.

Now check if all the conditions are satisfied.

$x_1 + x_2 \leq 50$

42 + 6 = 48 < 50 √

$20\ x_1 + 40\ x_2 \leq 1080$

$20 * 42 + 40 * 6 = 1080 \leq 1080$ √

$x_1 – 2\ x_2 \geq 0$

$42 – 2 * 6 = 30 > 0$ √

$x_1 - 8\ x_2 \leq 0$

$42 – 8 * 6 = -6 < 0$ √

Now two friends thought about increasing 524 TL started to think a solution to the problem. When they looked to the graph they saw that $x_1 - 8\ x_2 \leq 0$ (at least 1 of their 9 customers will anyway order pitas) equation is the problem. Therefore they decide to make an advertisement to increase the sales of pita. They give 900 TL for a month for the advertisement of the pita and realized that at least 1 of 5 customers will order pitas. Finally, they started to draw the graph again to see what happens to the graph.

The variables:

x_1: number of anchovy meal

x_2: number of pita

1. A linear function to be maximized:

$z = 12\ x_1 + 10\ x_2$

z: total profit per day (TL)

$12\ x_1$: profit from anchovy meal per day

$10\ x_2$: profit from a pita per day

2. Problem constraints

$x_1 + x_2 \leq 50$ (the maximum meals guessed each day)

$20\ x_1 + 40\ x_2 \leq 1080$ (minutes needed to prepare meals, 18*60 = 1080)

$x_1 – 2\ x_2 \geq 0$ (they will sell at least 2 anchovy meal when they sell one pita)

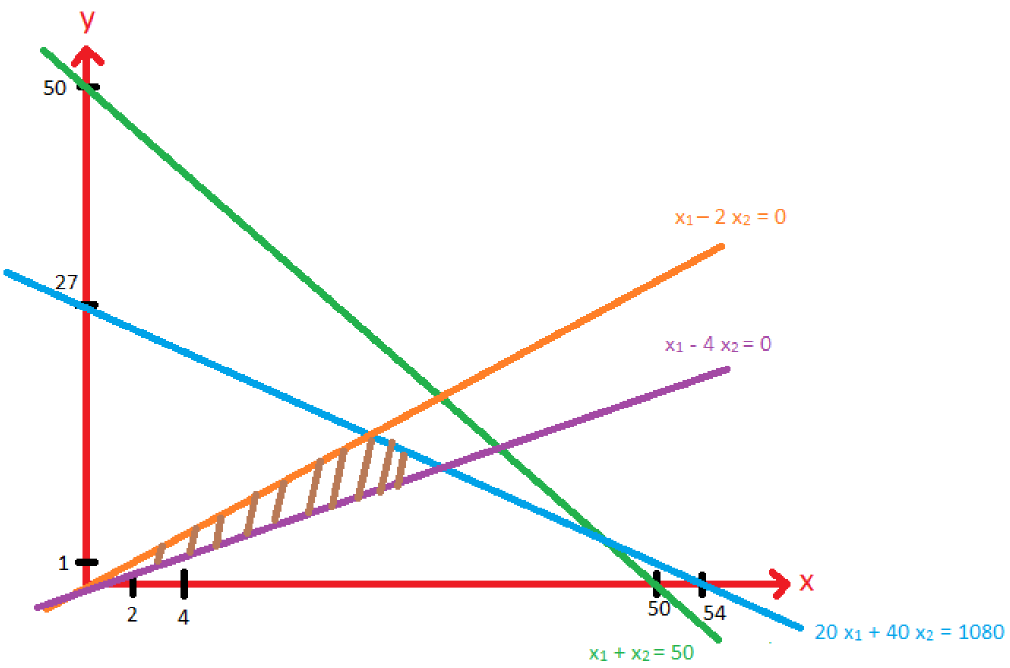
$x_1 - 4\ x_2 \leq 0$ (at least 1 of their 5 customers will anyway order pitas)

3. Nonnegative variables

$x_1 \geq 0$

$x_2 \geq 0$

4. The feasibility region graph



In this step which should again check the corner points of the shaded area. There are two important corner points in this shaded area. The points are the intersection of $x_1 - 4x_2 \leq 0$ and $20x_1 + 40x_2 \leq 1080$ equations and the intersection of $x_1 - 2x_2 \geq 0$ and $20x_1 + 40x_2 \leq 1080$ equations.

To find the first point we should solve the first equations.

$x_1 - 4x_2 = 0$ and $20x_1 + 40x_2 = 1080$

$-20x_1 + 80x_2 = 0$

$20x_1 + 40x_2 = 1080$

$120x_2 = 1080$, $x_2 = 9$, $x_1 = 36$

Therefore our first check point is (36, 9).

To find the second point we should solve the second equations.

$x_1 – 2\ x_2 = 0$ and $20\ x_1 + 40\ x_2 = 1080$

$20\ x_1 - 40\ x_2 = 0$

$20\ x_1 + 40\ x_2 = 1080$

$40\ x_1 = 1080$, $x_1 = 27$, $x_2 = 13.5$ (we can't have 13.5 pitas therefore we should take 13 pitas)

Therefore our second point is (27, 13).

Now we should calculate the maximum profit for two points. Bigger one will be our maximum profit point.

$z = 12\ x_1 + 10\ x_2$

$z = 12 * 36 + 10 * 9 = 522$ TL (for (36, 9) point)

$z = 12 * 27 + 10 * 13 = 454$ TL (for (27,13) point)

Therefore for this problem our maximum profit is 522 – 40 (hiring money) – 900/30 = **452 TL** per day. For this problem 36 anchovy meals and 9 pitas should be sold.

Now check if all the conditions are satisfied.

$x_1 + x_2 \leq 50$

$36 + 9 = 45 < 50$ √

$20\ x_1 + 40\ x_2 \leq 1080$

$20 * 36 + 40 * 9 = 1080 \leq 1080$ √

$x_1 – 2\ x_2 \geq 0$

$36 – 2 * 9 = 18 > 0$ √

$x_1 - 4\ x_2 \leq 0$

$36 - 4 * 9 = 0 = 0$ √

But for the advertisement they saw that it doesn't increase the profit. So they decided quit advertisement on pitas.

Now two friends found started to think a solution to the problem to increase the 524 TL per day and also they decided to quit advertisement. When they looked to the graph they saw that $20\ x_1 + 40\ x_2 = 1080$ equation is again the problem. Therefore they decide to increase cooking hours by hiring someone again to help the cook. They hired someone and decided to pay her 40 TL per day. So now there are two persons who are hired to increase the cooking hours and they spend 80 TL now for hired persons. By this cooking hours are increased from 18 hours to 24 hours. Finally, they started to draw the graph again to see what happens to the graph.

The variables:

x_1: number of anchovy meal

x_2: number of pita

1. A linear function to be maximized:

$z = 12\ x_1 + 10\ x_2$

z: total profit per day (TL)

$12\ x_1$: profit from anchovy meal per day

$10\ x_2$: profit from a pita per day

2. Problem constraints

$x_1 + x_2 \leq 50$ (the maximum meals guessed each day)

$20\ x_1 + 40\ x_2 \leq 1440$ (minutes needed to prepare meals, 24*60 = 1440)

$x_1 - 2\ x_2 \geq 0$ (they will sell at least 2 anchovy meal when they sell one pita)

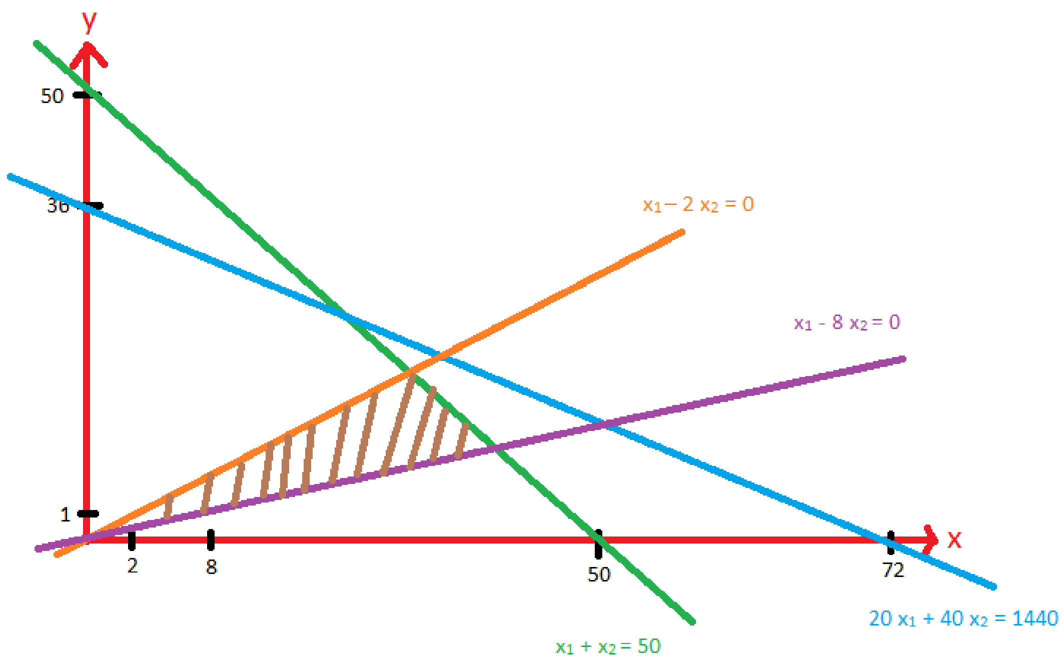
$x_1 - 8\ x_2 \leq 0$ (at least 1 of their 9 customers will anyway order pitas)

3. Nonnegative variables

$x_1 \geq 0$

$x_2 \geq 0$

4. The feasibility region graph



In this step which should again check the corner points of the shaded area. There are two important corner points in this shaded area. The points are the intersection of x_1

$- 8 x_2 \leq 0$ and $x_1 + x_2 \leq 50$ equations and the intersection of $x_1 – 2 x_2 \geq 0$ and $x_1 + x_2 \leq 50$ equations.

To find the first point we should solve the first equations.

$x_1 - 8 x_2 = 0$ and $x_1 + x_2 = 50$

$- x_1 + 8 x_2 = 0$

$x_1 + x_2 = 50$

$9 x_2 = 50$, $x_2 = 5.55$, $x_1 = 44.45$

We can't have 5.55 pitas and 44.45 anchovy meals. So we should look at the graph and take 44 anchovy meals. When we look at first equation $x_2 = 5.5$. We can't have 5.5 pitas therefore we should check 6 pitas which is good for $x_1 - 8 x_2 \leq 0$. And also it is okay for the second equation $x_1 + x_2 \leq 50$ because $44+6 = 50 \leq 50$. Therefore our first check point is (44, 6).

To find the second point we should solve the second equations.

$x_1 – 2 x_2 = 0$ and $x_1 + x_2 = 50$

$- x_1 + 2 x_2 = 0$

$x_1 + x_2 = 50$

$3 x_2 = 50$, $x_2 = 16.66$, $x_1 = 33.34$ (we can't have 16.66 pitas therefore we should take 16 pitas) Therefore our second point is (32, 16).

Now we should calculate the maximum profit for two points. Bigger one will be our maximum profit point.

$z = 12\ x_1 + 10\ x_2$

$z = 12 * 44 + 10 * 6 = 588$ TL (for (44, 6) point)

$z = 12 * 32 + 10 * 16 = 544$ TL (for (27, 13) point)

Therefore for this problem our maximum profit is 588 – 80 (hiring money) = **508 TL** per day. For this problem 44 anchovy meals and 6 pitas should be sold.

Now check if all the conditions are satisfied.

$x_1 + x_2 \leq 50$

$44 + 6 = 50 \leq 50$ √

$20\ x_1 + 40\ x_2 \leq 1440$

$20 * 44 + 40 * 6 = 1120 \leq 1440$ √

$x_1 – 2\ x_2 \geq 0$

$44 – 2 * 6 = 34 > 0$ √

$x_1 - 8\ x_2 \leq 0$

$44 – 8 * 6 = -4 < 0$ √

But for the hiring another person they saw that it doesn’t increase the profit. So they decided quit hiring second person.

CONCLUSION

In this homework I learned about linear programming and to apply it for the restaurant menus. For applying it to restaurant menus I made a lot of researches. Firstly I learned how linear programming is created in the history. I learned about what is linear programming, how a linear programming problem can be written. Also I learned in which areas linear programming are used. Then I choose restaurant menu problem from this areas. After that I decided to write a history of two friends who open a restaurant. Then I thought about what are their thoughts on the restaurant. I learned to write constraint and tried to guess what two friends would think about their restaurants. I learned about drawing feasibility graphs and calculating the maximum profit from this graphs. Then I thought about what they change about their restaurant and how this change will affect the profit.

This homework was very useful for me. I spent a lot of time to understand the linear programming. I see that linear programming can be done for a lot of situations in the life. Linear programming is used enormously in airlines, manufacturing industry, diet problems, restaurant menu problems, company problems and etc. It is simple and very successful. In the future I am planning to use what I learned for this kind of problems. Also I have learned how this kind of homework is helpful for a student. Therefore I will be more enthusiastic about taking homework.

Word Count: 3853

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