T.E.D ANKARA COLLEGE
FOUNDATION HIGH SCHOOL

Investigating the electrical conductivity of pencil lines written on a sheet

Physics SL Extended Essay

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A. ABSTRACT

Conductance of electricity is used today in many technological devices by billions of people and it is the most widespread energy sources throughout the world. Electrical conductance of materials depends on some subjects such as electrical conductivity and geometry of objects. The measure of a material of being an insulator or a conductor is determined by electrical conductivity. It is the inverse of electrical resistance. The resistance of any material also is determined by its geometry and its own specific electric resistance (in other words electrical resistivity).

This investigation generally inquires the effect of geometry of material to its electrical resistance. To investigate the effect of geometry of any conductor on its electrical conductance graphite lines written on sheet is used. Graphite is a common electrical conductor which is used as core of pencils and in many industrial areas. Its conductivity is explained by delocalized π bonds (pi bonds) within its molecular structure. In this investigation to research electrical conductivity of graphite pencil lines on a sheet, firstly resistance of them are measured and by calculating electrical resistivity of graphite lines, the effect of geometry of material is observed.

As a result it is found that the resistances of graphite lines drawn on a sheet are different from each other and they differ according to thickness of the lead too (not the element of lead, means the top of the graphite pencil, the writing part of it). Only thing that prevent electrical conductance is the blanks in lines that cannot be seen with naked eye. Besides it is found that the resistivity of material does not change because of the shape of it and most lines can still conduct electricity.

(word count: 279)

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1. INTRODUCTION

Life without electricity cannot be thought today. At any little power outage daily works nearly come to stop. The reason of that is people's will of controlling their life mostly with electronic devices. These devices make our life completely easy, comfortable and actually provide high life standards because of that it's hard to reject using these stuffs after getting used to it. People know these devices are completely part of their life and they are addicted to them at some point. Nevertheless they generally don't attempt to get rid of them, instead of it they are trying to learn more about them. They participate in some courses, reading articles and try to discover more on them.

What's more is that today electronic is not a subject that is only known by adults. For instance 7 year old child can have basic knowledge about electronic objects; she or he can repair little errors in a computer. Also children can use them even they don't know how to read and write. Besides electronic devices have a large part in housework too. Furthermore people need light as an energy source, even they are going out at night. Therefore everyone should know about nature of electricity, how to use it and with which instruments it can be beneficial. As a result it is valid to identify conductors, insulators and at what range and how they can be used in daily life. My reason of choosing this topic is to show how can a classic daily life instrument became a useful electronic device.

It is important to mention that history of pencil is old as writing itself. Its history goes to the ancient Rome; there scribes had used *stylus* (name of the pencil in Rome) for writing on papyruses. However styluses and modern pencils have one specific difference and that is its core. In Rome pencils were made from lead which has a slightly readable mark but today we use pencils made by several materials such as graphite, charcoal, crayon and grease. (1) This essay includes only study on conductivity of graphite pencils. Graphite pencils are also called lead pencils in daily life usage. However lead does not mean an element Pb, all lead pencils that are used normally are made by graphite. Lead is used for black or dark grey part at the top of the pencil.

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[1] http://www.pencils.com/history.html

2. BACKGROUND INFORMATION

2.1. Electrical Conduction

Electrical conduction is the movement of electrically charged particles through a medium that allows passage of charged particles freely. (2) Free movement of charged particles is caused by electric field around it. There is another term that is close to electrical conduction; it is called conductance. It is important to mention that conductance and conduction are different in definition. Both of them are related with movement of charged particles though an object. However conductance is the measure of the conduction and it is the reciprocal of the resistance.

$$G = \frac{1}{R}$$

G: Conductance (siemens)

R: Resistance (ohm)

Ability of materials to conduct electricity is explained by "Energy Band Theory". According to this theory around one specific atom there are energy levels which are filled by electrons. The outermost shell of an atom is called valence shell. Valence shell is the band having the highest energy. Valence shell can be completely filled or partially filled. However above the valence shell there is another band called conduction band in which valence electrons can freely move. Electrons locate in the conduction band after leaving valence shell. This conduction band can be partially filled or be empty. (3)

Insulators have large gaps between two consecutive bands so the passage of electrons is nearly impossible. In other word there are no free moving electrons within the molecular structure of them. However in semiconductors and conductors distance between two energy levels is shorter so an electron can overlap to the conduction band.

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[2] http://www.sciencedaily.com/articles/e/electrical_conduction.htm

[3] http://www.blurtit.com/q920136.html

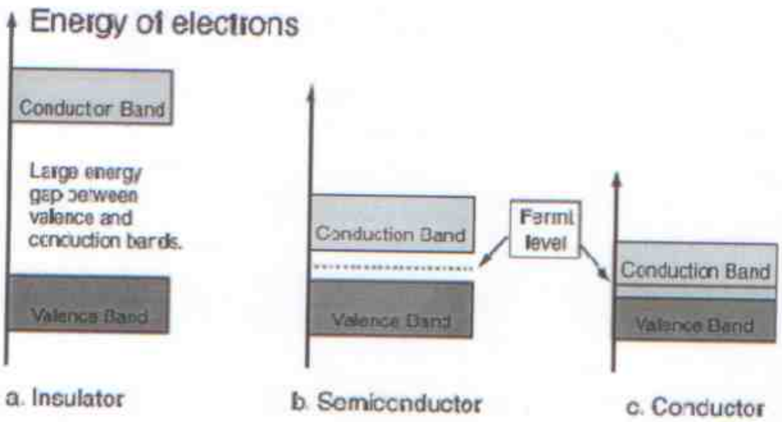

Figure 1: Energy band levels of insulators, semiconductors and conductors and gabs between them [4]

All materials resist against the passage of charged particles in them at different ranges. It is called electrical resistance and it gives the ratio of length of material to its area and also its own specific electric resistance.

$$R = \frac{\rho \times l}{A}$$

R: resistance of material (ohm)

l: length of material (m)

A: cross sectional area of material (m^2)

ρ :specific electrical resistance (resistivity) (ohm.m)

Resistivity is the specific electrical resistance for each material. It can be calculated by measuring resistance of a material when its length and cross sectional area is known. Its unit is ohm.meter (Ω.m) and is shown with ρ (rho).

$$\rho = \frac{A \times R}{l}$$

For instance at room temperature resistivity of pure metals varies from 1.5×10^{-8} Ω.m (silver) to 135×10^{-8} Ω.m (manganese). On the other hand resistivity of insulators is very high compared with conductors. Their resistivity ranges between 10^8 to 10^{16} Ω.m. [5]

[4] http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/band.html

[5] http://www.answers.com/topic/electrical-resistivity

2.2. Graphite

The word graphite comes from the German word "gráphein" means "to write". Graphite is also called black lead or plumbago. Graphite is soft, black and slippery material and it is composed of covalently bonded carbon atoms, it has two dimensional shape and has a layered structure. Because of this layered structure, it can be used in pencils as core material. The written part of the pencils is made by graphite. Its layers slides down each other easily, thus it leave black, readable mark on paper. Graphite is used in lead pencils, making electrodes, arc lights, crucibles and rocket nozzles. Because of its stability under normal conditions, high thermal resistance and lubricity, it can be used for variety of industrial applications.

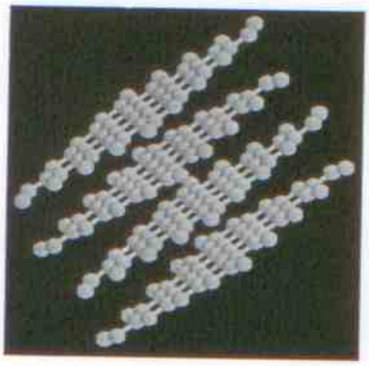

Figure 2: Layered structure of graphite molecule [6]

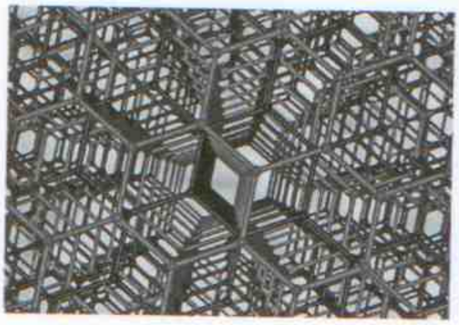

Figure 3: Molecular view of graphite molecules [7]

In covalent bonding atoms of elements make bonds by sharing electrons to complete their last energy levels to octet (eight electrons in the last energy level of an atom) or doublet (two electrons in the last energy level of an atom). If there is not enough atom around one specific atom, this atom makes pi (π) bonds with other atoms around it until completing its electrons to eight at the last energy level. It is not possible to predict that π bond is between which of the atoms because it always changes its locations. In other word there are always moving electrons within the molecule because of that this π bond is also called delocalized π bond. By delocalized π bonds, free moving electrons, conductance of electricity become possible.

Graphite molecules are composed of too many carbon atoms and it forms a giant molecule. Carbon has 4 bond capacities due to having 4 valence electrons. It should normally make 4 covalent bonds to complete its octet. However in graphite molecule, around one carbon atom

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[6] http://staff.jccc.net/PDECELL/biochemistry/carbon.html

[7] http://nanotech_now.com/Art_Gallery/antonio-siber.htm

there are only three other carbon atoms. Thus the last bond is made with one of same atoms around the specific carbon atom. This means that between two carbon atoms there is delocalized π bond. This delocalized π bond let electrons move freely within the molecule. Thus graphite is good electrical conductor. It has very low resistivity (ranges 9 to 40 $\mu\Omega m$), which also support the fact that good conductivity of electricity. Resisivity of graphite changes due to locations where it is mined.

2.3. Pencils

Pencils are also called lead pencils but they have no relation with the element lead actually. This ambiguity is coming from the ancient name of the graphite. When it is first found, because of its black color, people called it blacklead. Actually pencils are made by graphite but beside graphite lots of chemical are used such as wax and clay. In other words pencils that we are using today are chemical combination of graphite and some other chemical substances. These substances are added to increase hardness of it because when only graphite was used for writing, it has seen that it is too soft and easy to break. Other reason of adding clay to the graphite mixture is to give different colors to pencils.

There are 2 common ways of pencil production. First type of production is called extrusion method. In this type graphite and wax are combined with each other and forced to form a string. Then it is cut into pieces according to requirements. Second type of production is made by a machine called billet press. Graphite is combined with clay, this mixture is sent to the machine and cylinder like structure called billet is taken. Lastly, after some different steps core of the pencil is ready to slide down into different sizes. In both type of manufacturing core of the pencils is covered with wood to provide withstand.

Besides, pencils can be used mechanically in daily life. In mechanic pencils, the radius of pencil is constant and after writing, people can have lines with same thickness and better view. Mechanical pencils can have 1.1 mm, 0.9 mm, 0.7 mm, 0.5 mm or 0.3 mm diameters. Diameters of mechanical pencils vary according to their usage. For example 2.0 mm pencils are used for artworks or 0.3 mm pencils are generally used by architects for drawing project. The advantage of them is; their thickness does not change after writing. Thus in this experiment pencils with 0.9 mm, 0.7 mm, 0.5 mm diameters are used to make sure that ever lines have the same width on paper.

Pencils are classified according to their softness and blackness. Softness and color is caused by the other materials and graphite percentage in the pencil. However there are no strict rules for regulation of making pencils. Manufactures determine their own standards and produce their pencils according to these. Nevertheless there are not quite differences between their standards. Lead pencils generally have the colors as showed in the figure 7. From left part of the scale to the right part, blackness and softness of pencil mark increase.

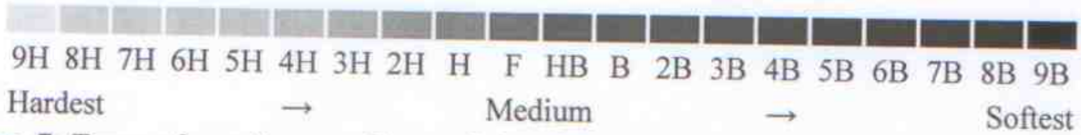


Figure 7: Types of pencils according to their softness and blackness. H is stand for hardness, B is stand for blackness.

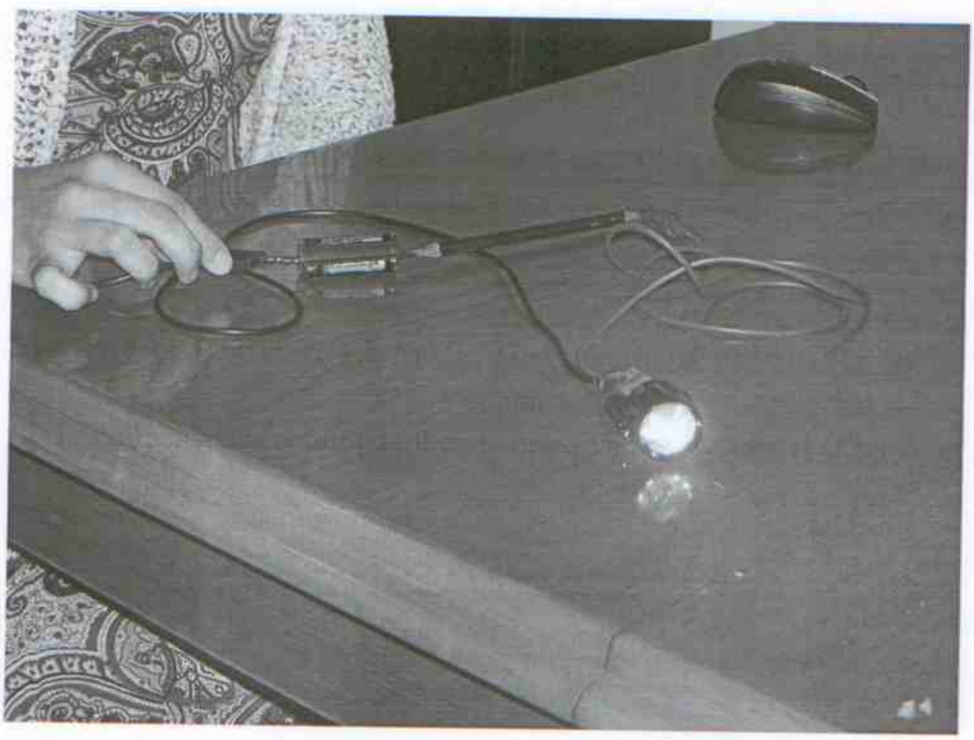


Figure 8: Shows that when a graphite pencil is used as a wire, lamp gives light, by the voltage generated from battery.

In figure 8, to light the pocket lamb 1.2 V rechargeable battery, two wire with crocodile clips and HB graphite pencil is used. Both ends of the pencil are sharpened and cleaned from sawdust around it. One end of the pencil is bind to the battery and the other is connected to the wire. Second end of the battery and light are connected with black wire. In this figure, pencil functions as if it is the part of the wire and allows electric current pass through itself. If the graphite pencil can conduct electricity, then graphite pencil lines, written on a sheet also conduct electricity. Length and thickness of the line can not affect the fact that graphite is a conductive material. These properties only change resistance of lines and conductance of them.

In this experiment 2B graphite pencils with 0.9 cm, 0.7 cm and 0.5 cm diameter are used in order to control width of lines. Then their resistance is measured and by these variables each of the lines' resistivity is calculated. Consequently, experiment of this study will focus on the research question: "How the self conductance of pencil lines on paper change due to the length of lines." According to information that I gathered before starting my experiment, specific electric resistance of any material does not change by its length or area, its shape only affects electron carrying capacity.

3. PLANNING

3.1. Aim

Graphite is an important thermal and electrical conductor and these properties provide it wide range of usage in industrial areas. Besides it has one beneficial role in daily life too; it is the core material of a very popular and useful object that we use during whole our life. Graphite pencils are very common objects and they are also good conductors, which we can meet in everyday life. Aim of this experiment is to investigate the conductivity of pencil lines written on a sheet by using the relation between geometry, resistance and resistivity. Also it is important to conclude a result that explains a relation between the geometry of graphite pencil lines and their conductivity.

3.2. Research Question

How the conductivity of pencil lines written on a sheet are affected due to change in their length and width?

3.3. Hypothesis

In daily life we always use some kinds of pencils like charcoal pencils, graphite pencils... However we are not aware of other possible usages of these common daily life instruments. Actually by carrying any graphite pencil, we also carry a powerful electric conductor too. Everyday while we are drawing or writing something, actually we are making small electrical wires too. It can be said that around all of our notebooks, papers and everywhere that we write with graphite pencil there are millions of thin cables.

These graphite cables are very thin and most probably the have very high resistance to conduct electricity. However theoretically it ca be said that if the pencil itself can conduct electricity, then lines of it can conduct too, no matter whether it is long or thin.

It can be hypothesized that when the lengths of pencil lines get longer and thinner; the electrical conductivity increase, but lines conducts electricity at every size because of the constant resistivity.

3.4. Key Variables

3.4.1. Independent Variables

- Length of pencil lines
- Width of pencil lines

3.4.2. Dependent Variable

- Resistance of pencil lines

3.4.3. Controlled Variables

- Type of pencil (2B)
- Type of the sheet (A4, 80 $g.m^{-2}$ – PaperOne)
- Room Temperature (25.0 ±0.5°C)
- Length of pencil lines for every 30 trial of same the length (5.0 cm, 10.0 cm and 20.0 cm)
- Diameter of the pencils for every line group (one line group consists of both 5.0 cm, 10.0 cm and 20.0 cm long lines and pencils with 0.5 mm, 0.7 mm and 0.9 mm diameter is used for every 90 lines)
- Angle between pencil and sheet during drawing lines ($\cong$ 90°)

3.5. Materials

- Digital multimeter (±0.01MΩ)

- 6 sheets (A4, 80 $g.m^{-2}$ – PaperOne)
- Lead pencil with 0.5 mm diameter (2B, Ultra polymer - TOMBOW)
- Lead pencil with 0.7 mm diameter (2B, Ultra polymer - TOMBOW)
- Lead pencil with 0.9 mm diameter (2B, Mikro)
- 3 Mechanical pencils that can be used with lead pencils with 0.5 mm, 0.7 mm and 0.9 mm diameter
- 30.0 cm ruler (±0.5mm)
- Micrometer (±0.005mm)

3.6. Method

Part 1: Pencil lines with 0.5 mm diameter

1. Measure length of graphite in mechanical pencil with micrometer and record it.
2. Draw 30, 200.0 mm long lines on sheet with same pencil by using ruler. Between lines there should be approximately 1.0 cm long space. In order to measure their resistance with digital multimeter easily, left spaces on the side of the paper too. Be careful about endings of lines; apply same force on the end of lines like it has done on the middle of lines. Second important point to draw lines is the angle between pencil and sheet. To make same color and thickness of graphite mark, angle of pencil should be 90°. Use pencil perpendicular to ruler and sheet as possible as it can be. During drawing lines, written part of the graphite should not be broken. If it is damaged, it must be started from the first step again because length of pencil used is going to be calculated in the further steps.

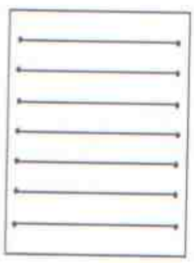


Figure 9: Shows thirty 200.0 mm long lines drawn on a sheet

3. To prevent blanks occur in lines and to obtain continuous line, go through every lines 4 times. Otherwise there can be spaces in graphite lines and continuity of electric

current passage is disturbed. Thus resistance of lines can not be measured because it would like measuring resistance of two different lines.

4. After drawing 30, 200.0 mm long lines, measure last length of the graphite in mechanical pencil with micrometer again in order to calculate volume of the graphite used later.
5. Draw another 30, 100.0 mm lines on second sheet. Between every line leave approximately 1.0 cm space. Go through 4 times on the every line and be careful with the angle of pencil and thickness of lines on sheet.
6. After drawing lines, measure the length of left graphite with micrometer and record it. Again it is important not to break graphite of mechanical pencil. If it is broken measure the length of graphite again and draw 30, 100.0 mm long lines again..
7. Draw 30, 50.0 mm long lines on another sheet by using ruler. Be careful about applying same force on the endings and middle of lines. Also the angle between pencil and sheet should be about 90° and go through every lines 4 times.
8. Measure the length of left graphite and record it. Try not to break graphite. If it is broken, start from the 6th step and draw 30, 50.0 mm long lines again.

> *Note: There should be 90 lines that are drawn with pencil having 0.5 mm diameter. 30 of them are 200.0 mm long, 30 are 100.0 mm long and last 30 lines are 50.0 mm long. There should be 2 graphite pencil lengths for every thirty lines; 3 of them are for beginning of thirty lines and 3 of them are for endings of thirty lines.*

9. Make the calibration of the multimeter and test it on a bare sheet. Control the surface and sheet that you are going to work on whether they have resistance or not. If they have resistance change the place that you work on. If they do not have resistance, start measuring resistance of lines as shown in the figure 10.

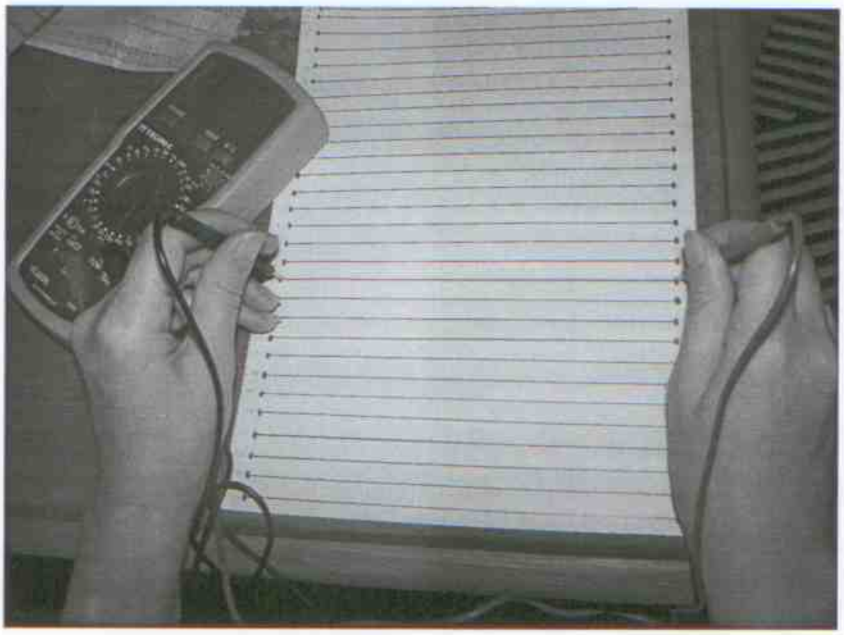


Figure 10: Shows measuring resistance of 200.0 mm pencil lines made with graphite pencil having 0.5 mm diameter by multimeter

Note: If it is hard to measure resistance of lines by touching electrodes to the endings of lines, small spots can be drawn behind lines. While drawing spots, be careful not to shorten or lengthen lines. Spots must be on the end of the lines and should not disturb real lengths of lines. While their resistances are measured, electrodes should be touched between the end of lines and beginning of spots. It is important that spots should not have any affect on resistance of lines. Thus it is advised that not to draw spots, instead of it with proper pressure with graphite pencil while drawing lines would be work too.

Part 2: Pencil lines with 0.7 mm diameter

1. Measure length of graphite in 0.7 mm mechanical pencil with micrometer and record it.
2. Draw 30, 200.0 mm long lines with same pencil and be careful about thickness of lines and angle of pencil. Go through 4 times on every line. If graphite is broken, measure the new length with micrometer and draw lines again.
3. Measure the last length of graphite with micrometer.
4. Draw 30, 100.0 mm long graphite lines and go through on lines 4 times.
5. Measure length of graphite in the mechanical pencil with micrometer.
6. Draw 30, 50.0 mm lines on sheet like the same way with other lines.

7. Measure length of graphite left after drawing.
8. Measure resistances of all 90 lines written by graphite pencil with 0.7 mm diameter and record them.

Part 3: Pencil lines with 0.9 mm diameter

1. Draw thirty 200.0 mm long, thirty 100.0 mm long and thirty 50.0 mm long lines on sheets with graphite pencil with 0.9 mm diameter. Measure lengths of graphite in the mechanical pencil before and after drawing every thirty lines. If the graphite is broken, start drawing from one group of line from the beginning. Be careful about thickness of lines and angle of pencil. Go through each line 4 times.
2. Measure resistances of lines and record them.

Note: At the end of the data collection there should be 270 lines and resistance value, 90 of these lines will be drawn with 0.5 mm diameter pencil, 90 will be drawn with pencil having 0.7 mm diameter and last 90 will be drawn with 0.9 mm diameter pencil.

4. DATA COLLECTION

Length of the graphite in the mechanical pencil is measured before and after drawing each 30 lines. These lengths are recorded in mm and placed in tables according to diameter of used pencil and length of lines. Then resistances of each 90 trial of one pencil with specific diameter are measured and recorded in MΩ in the tables. Data Collection is shown in the 7.1. Appendix (page: 17)

5. DATA PROCESSING AND PRESENTATION

In data processing all units are changed into the SI system. After that average cross-sectional area of lines are calculated in order to find the resistivity of graphite. Average cross-sectional area is found by using the volume of the pencil used while drawing lines. The calculation process is shown the tables in 7.2. Appendix (page: 26). Lastly the resistivity of each line is calculated by using the formula $\rho = \frac{R \times A}{l}$ and calculations are shown in detail in 7.3 Appendix (page:33).

6. CONCLUSION

Aim of this experiment was to determine whether the length and width of the graphite pencil lines on paper affect the conductance of graphite. To find out the changes in the conductance of lines three type of pencils are used and for each type of pencil three different lengths are drawn. Their resistances are measured with multi-meter to show that they have proper resistance value to permit electron passage from one side to the end. To show that lines are still conductive and to show that there is no change in the chemical properties of graphite their resistivity (which means specific electrical resistance) is calculated by the formula $\rho = R \times \frac{A}{l}$ [p=resistvity (ohm$\times$meter: Ωm), R=Resistance (ohm: Ω), A=Cross sectional area (m^2), l: length of line (m)].

Resistivity of graphite with 0.5 mm diameter = 0.0032 ± 14.0 % Ωm

200.0 mm long lines= 0.0029 ± 2.4 % Ω.m

100.0 mm long lines= 0.0027 ± 4.0 % Ωm

50.0 mm long lines= 0.0040 ± 7.0 % Ωm

Resistivity of graphite with 0.7 mm diameter = 0.0090 ± 18.0 % Ωm

200.0 mm long lines= 0.0063 ± 2.4 % Ωm

100.0 mm long lines= 0.013± 5.0 % Ωm

50.0 mm long lines= 0.0077± 10.0 % Ωm

Resistivity of graphite with 0.9 mm diameter = 0.0073 ± 14.0 %Ωm

200.0 mm long lines= 0.010 ± 2.4 % Ωm

100.0 mm long lines= 0.0083 ± 5.0 % Ωm

50.0 mm long lines= 0.0037 ± 10.0 % Ωm

During the experiment it is seen that length of lines are proportional with resistance of it [$(R \propto l)$: R=Resistance, l=length of line]. Secondly when the width of lines is increased it is seen that the resistance of lines decrease [$\left(R \propto \frac{1}{A} \right)$: R=Resistance, A=Cross sectional area]. Resistivities which should be same for every shape of specific material are calculated different from each other because of working on thin and very small sized lines. Nevertheless all lines conserved their conductance property. Thus it can be stated that the hypothesis of the experiment is true.

Evaluation

Spaces within the graphite lines: There are some spaces in the graphite lines which prevent electron passages from one side to another. These spaces are caused by not applying enough force on sheet while drawing lines. (For example; Trial 3 in Test 1-200.0mm) These lines are calculated as if their resistances are zero. However it can not be said that graphite lines don't conduct electricity. To prevent this error source it can be passed on these lines more than four times.

Maximum resistance measured by multimeter: The multimeter used in this experiment can measure maximum 20.0 MΩ. However resistances of some lines are more than 20.0 MΩ since the by multimeter used in this experiment resistances of these lines can not be shown. They are ignored in the calculation and taken as if they are zero. To prevent this error, another multimeter may be used that higher resistances can be measured with.

Oxidation of graphite lines: In this experiment, before measuring the resistances of lines all lines are drawn and make wait for some time in contact with air so that oxygen too. This cause oxidation of graphite lines at small levels. This affects the measured resistances of lines. Prevent this error source lines should make wait under a condition that avoids the contact of graphite with oxygen. Secondly if the resistances of lines are measured right after they are drawn this error source can be prevented too.

7. APPENDIX

7.1 DATA COLLECTION

7.1.1 Test 1

Diameter of graphite pencil used: 0.5 mm
Softness & Blackness of graphite pencil: 2B
Room Temperature: 25.0 ±0.5°C

	Length of line (mm) (±0.5)	Length of pencil before it is used(h_i) (mm) (±0.05)	Length of pencil after it is used(h_f) (mm) (±0.05)
200mm pencil line	200.0	54.50	49.00
100mm pencil line	100.0	58.00	54.50
50mm pencil line	50.0	60.00	58.00

Table 1.1: Size of pencil with 0.5mm diameter before and after drawing 200.0mm, 100.0mm and 50.0mm long lines

Trial	Line Resistance (MΩ) (± 0.01)
1	4.01
2	2.36
3	-
4	-
5	6.71
6	-
7	-
8	-
9	-
10	-
11	4.72
12	-
13	-
14	-
15	7.60
16	-
17	4.50
18	10.10
19	4.80
20	-
21	12.36
22	-
23	6.53
24	-
25	7.05
26	7.92
27	3.21
28	4.95
29	3.74
30	7.55

Table 1.2: Measured resistance of 200.0 mm long lines written with 0.5mm pencil on sheet.

Trial	Line Resistance (MΩ) (± 0.01)
1	0.42
2	0.40
3	0.39
4	0.46
5	0.77
6	1.08
7	0.70
8	1.61
9	0.93
10	6.92
11	1.30
12	1.70
13	1.45
14	0.62
15	0.77
16	0.79
17	0.70
18	0.63
19	0.76
20	1.46
21	1.93
22	1.30
23	1.31
24	1.19
25	1.13
26	0.67
27	0.53
28	0.82
29	1.71
30	0.75

Table 1.3: Measured resistance of 100.0 mm long lines written with 0.5mm pencil on sheet.

Trial	Line Resistance (MΩ) (± 0.01)
1	-
2	3.52
3	0.60
4	1.15
5	1.40
6	1.27
7	1.87
8	0.55
9	1.37
10	1.86
11	1.51
12	0.79
13	0.68
14	0.54
15	1.03
16	0.51
17	0.72
18	0.55
19	0.12
20	0.26
21	0.16
22	0.21
23	0.26
24	0.30
25	0.13
26	0.24
27	0.27
28	0.31
29	0.48
30	0.18

Table 1.4: Measured resistance of 50.0 mm long lines written with 0.5mm pencil on sheet

7.1.2. Test 2

Diameter of graphite pencil used: 0.7 mm
Softness & Blackness of graphite pencil: 2B
Room Temperature: 25.0 ±0.5°C

	Length (mm) (±0.5)	Length of pencil before it is used(h_i) (mm) (±0.05)	Length of pencil after it is used(h_f) (mm) (±0.05)
200mm pencil line	200.0	42.00	37.00
100mm pencil line	100.0	44.50	42.00
50mm pencil line	50.0	45.70	44.50

Table 2.1: Size of pencil with 0.7mm diameter before and after drawing 200.0mm, 100.0mm and 50.0mm long lines

Trial	Line Resistance (MΩ) (± 0.01)
1	4.41
2	5.76
3	1.92
4	1.19
5	4.86
6	3.34
7	2.90
8	2.62
9	2.57
10	3.25
11	4.80
12	7.32
13	3.45
14	3.24
15	2.79
16	7.32
17	11.90
18	7.06
19	7.96
20	-
21	4.73
22	-
23	2.14
24	3.56
25	2.21
26	2.00
27	2.45
28	5.70
29	4.03
30	3.35

Table 2.2: Measured resistance of 200.0 mm long lines written with 0.7mm pencil on sheet.

Trial	Line Resistance (MΩ) (± 0.01)
1	-
2	8.30
3	9.81
4	11.31
5	12.20
6	5.20
7	11.60
8	5.93
9	3.05
10	3.36
11	7.50
12	-
13	2.96
14	4.67
15	2.26
16	1.06
17	1.60
18	3.01
19	2.92
20	3.32
21	2.19
22	2.28
23	1.80
24	3.68
25	1.40
26	3.60
27	1.05
28	1.36
29	0.68
30	1.51

Table 2.3: Measured resistance of 100.0 mm long lines written with 0.7mm pencil on sheet.

Trial	Line Resistance (MΩ) (± 0.01)
1	-
2	1.32
3	3.89
4	1.16
5	1.84
6	3.92
7	2.12
8	2.66
9	4.26
10	1.28
11	1.00
12	1.38
13	1.18
14	1.48
15	0.47
16	0.76
17	1.00
18	1.08
19	0.84
20	0.59
21	0.86
22	0.62
23	0.87
24	0.89
25	0.16
26	0.42
27	0.48
28	0.46
29	0.92
30	1.10

Table 2.4: Measured resistance of 50.0 mm long lines written with 0.7mm pencil on sheet.

7.1.3. Test 3

Diameter of graphite pencil used: 0.9 mm
Softness & Blackness of graphite pencil: 2B
Room Temperature: 25.0 ±0.5°C

	Length of line (mm) (±0.5)	Length of pencil before it is used(h_i) (mm) (±0.05)	Length of pencil after it is used(h_f) (mm) (±0.05)
200mm pencil line	200.0	72.00	67.70
100mm pencil line	100.0	74.30	72.00
50mm pencil line	50.0	75.50	74.30

Table 3.1: Size of pencil with 0.9mm diameter before and after drawing 200.0mm, 100.0mm and 50.0mm long lines

Trial	Line Resistance (MΩ) (± 0.01)
1	2.95
2	5.54
3	-
4	4.45
5	7.85
6	-
7	10.78
8	3.22
9	7.65
10	3.55
11	2.11
12	2.87
13	-
14	7.61
15	7.50
16	15.6
17	-
18	4.30
19	4.65
20	4.65
21	6.50
22	2.55
23	2.23
24	14.86
25	5.01
26	2.20
27	2.35
28	1.16
29	1.78
30	1.81

Table 3.2: Measured resistance of 200.0 mm long lines written with 0.9mm pencil on sheet.

Trial	Line Resistance (MΩ) (± 0.01)
1	0.72
2	0.44
3	1.40
4	0.43
5	1.08
6	0.50
7	0.80
8	0.93
9	1.87
10	3.58
11	3.13
12	2.11
13	1.66
14	0.53
15	1.80
16	1.35
17	1.49
18	1.45
19	0.63
20	1.48
21	5.16
22	0.87
23	1.39
24	1.49
25	0.83
26	1.69
27	6.93
28	2.00
29	1.11
30	1.95

Table 3.3: Measured resistance of 100.0 mm long lines written with 0.9mm pencil on sheet.

Trail	Line Resistance (MΩ) (± 0.01)
1	0.36
2	0.55
3	0.35
4	1.44
5	0.29
6	0.75
7	0.47
8	0.31
9	1.51
10	0.46
11	0.14
12	0.23
13	0.14
14	0.19
15	0.14
16	0.18
17	0.18
18	0.17
19	0.12
20	0.26
21	0.21
22	0.24
23	0.25
24	0.34
25	0.21
26	0.24
27	0.45
28	0.26
29	0.62
30	0.18

Table 3.4: Measured resistance of 50.0 mm long lines written with 0.9mm pencil on sheet.

7.2 DATA PROCESSING AND PRESENTATION

Test 1

Diameter of graphite pencil used (2r): 0.5 mm
Radius of graphite pencil used (r): 0.25 mm

	Length of line (l)		Length of pencil before it is used (h_i)		Length of pencil after it is used (h_f)		Length of pencil used	Volume of pencil used	Average volume of line	Average cross sectional area of pencil lines
	(mm) (±0.5)	(m) (±0.0005)	(mm) (±0.05)	(m) (±0.00005)	(mm) (±0.05)	(m) (±0.00005)	($\Delta h = h_i - h_f$) (m) (±0.0001)	($V = \pi r^2 \Delta h$) (m^3)	$\left(V_{av} = \frac{V}{30}\right)$ (m^3)	$\left(A = \frac{V_{av}}{l}\right)$ (m^2)
200.0 mm pencil line	200.0	0.2000	54.50	0.05450	49.00	0.04900	0.0055	1.1×10^{-9}	3.7×10^{-11}	1.8×10^{-10}
100.0 mm pencil line	100.0	0.1000	58.00	0.05800	54.50	0.05450	0.0035	6.9×10^{-10}	2.3×10^{-11}	2.3×10^{-10}
50.0 mm pencil line	50.0	0.0500	60.00	0.06000	58.00	0.05800	0.0020	3.9×10^{-10}	1.3×10^{-11}	2.6×10^{-10}

Table 4.1.1: Cross sectional area of 0.5 mm pencil lines on paper for 200.0mm, 100.0mm and 50.0mm long lines

Trial	200.0 (mm) (±0.5)			100 mm (mm) (±0.5)			50.0 (mm) (±0.5)		
	Line resistance (R)		Resistivity of graphite	Line resistance (R)		Resistivity of graphite	Line resistance (R)		Resistivity of graphite
	(MΩ) (±0.01)	(Ω) (±10^{-7})	$\left(\rho = \frac{R \times A}{l}\right)$ (Ωm)	(MΩ) (±0.01)	(Ω) (±10^{-7})	$\left(\rho = \frac{R \times A}{l}\right)$ (Ωm)	(MΩ) (±0.01)	(Ω) (±10^{-7})	$\left(\rho = \frac{R \times A}{l}\right)$ (Ωm)
1	4.01	4.01×10^{-6}	0.0036	0.42	0.42×10^{-6}	0.00096	-	-	-
2	2.36	2.36×10^{-6}	0.0021	0.40	0.40×10^{-6}	0.00092	3.52	3.52×10^{-6}	0.018
3	-	-	-	0.39	0.39×10^{-6}	0.00089	0.60	0.6×10^{-6}	0.0031
4	-	-	-	0.46	0.46×10^{-6}	0.0010	1.15	1.15×10^{-6}	0.0060
5	6.71	6.71×10^{-6}	0.0060	0.77	0.77×10^{-6}	0.0018	1.40	1.4×10^{-6}	0.0073
6	-	-	-	1.08	1.08×10^{-6}	0.0025	1.27	1.27×10^{-6}	0.0066
7	-	-	-	0.70	0.70×10^{-6}	0.0016	1.87	1.87×10^{-6}	0.0098
8	-	-	-	1.61	1.61×10^{-6}	0.0037	0.55	0.55×10^{-6}	0.0029
9	-	-	-	0.93	0.93×10^{-6}	0.0021	1.37	1.37×10^{-6}	0.0072
10	-	-	-	6.92	6.92×10^{-6}	0.016	1.86	1.86×10^{-6}	0.0097
11	4.72	4.72×10^{-6}	0.0042	1.30	1.30×10^{-6}	0.0030	1.51	1.51×10^{-6}	0.0079
12	-	-	-	1.70	1.70×10^{-6}	0.0039	0.79	0.79×10^{-6}	0.0041
13	-	-	-	1.45	1.45×10^{-6}	0.0033	0.68	0.68×10^{-6}	0.0036
14	-	-	-	0.62	0.62×10^{-6}	0.0014	0.54	0.54×10^{-6}	0.0028
15	7.60	7.60×10^{-6}	0.0068	0.77	0.77×10^{-6}	0.0018	1.03	1.03×10^{-6}	0.0054
16	-	-	-	0.79	0.79×10^{-6}	0.0018	0.51	0.51×10^{-6}	0.0027
17	4.50	4.50×10^{-6}	0.0040	0.70	0.70×10^{-6}	0.0016	0.72	0.72×10^{-6}	0.0038
18	10.10	10.10×10^{-6}	0.0091	0.63	0.63×10^{-6}	0.0014	0.55	0.55×10^{-6}	0.0029
19	4.80	4.80×10^{-6}	0.0043	0.76	0.76×10^{-6}	0.0017	0.12	0.12×10^{-6}	0.00062
20	-	-	-	1.46	1.46×10^{-6}	0.0034	0.26	0.26×10^{-6}	0.0014
21	12.36	12.36×10^{-6}	0.011	1.93	1.93×10^{-6}	0.0044	0.16	0.16×10^{-6}	0.00083
22	-	-	-	1.30	1.30×10^{-6}	0.0030	0.21	0.21×10^{-6}	0.0011
23	6.53	6.53×10^{-6}	0.0059	1.31	1.31×10^{-6}	0.0030	0.26	0.26×10^{-6}	0.0014
24	-	-	-	1.19	1.19×10^{-6}	0.0027	0.30	0.3×10^{-6}	0.0016
25	7.05	7.05×10^{-6}	0.0063	1.13	1.13×10^{-6}	0.0026	0.13	0.13×10^{-6}	0.00068
26	7.92	7.92×10^{-6}	0.0071	0.67	1.13×10^{-6}	0.0015	0.24	0.24×10^{-6}	0.0012
27	3.21	3.21×10^{-6}	0.0029	0.53	0.53×10^{-6}	0.0012	0.27	0.27×10^{-6}	0.0014
28	4.95	4.95×10^{-6}	0.0044	0.82	0.82×10^{-6}	0.0019	0.31	0.31×10^{-6}	0.0016
29	3.74	3.74×10^{-6}	0.0034	1.71	1.71×10^{-6}	0.0039	0.48	0.48×10^{-6}	0.0025
30	7.55	7.55×10^{-6}	0.0068	0.75	0.75×10^{-6}	0.0017	0.18	0.18×10^{-6}	0.00094

Table 4.1.2: Resistivity of 200.0 mm, 100.0 mm and 50.0 mm long lines drawn with 0.5 mm graphite pencil

Test 2

Diameter of graphite pencil used (2r): 0.7 mm
Radius of graphite pencil used (r): 0.35 mm

	Length of line (l)		Length of pencil before it is used (h_i)		Length of pencil after it is used (h_f)		Length of pencil used	Volume of pencil used	Average volume of	Average cross sectional area of pencil
	(mm) (±0.5)	(m) (±0.0005)	(mm) (±0.05)	(m) (±0.00005)	(mm) (±0.05)	(m) (±0.00005)	($\Delta h = h_i - h_f$) (m) (±0.0001)	($V = \pi r^2 \Delta h$) (m^3)	line $\left(V_{av} = \frac{V}{30}\right)$ (m^3)	lines $\left(A = \frac{V_{av}}{l}\right)$ (m^2)
200.0 mm pencil line	200.0	0.2000	42.00	0.04200	37.00	0.03700	0.0050	1.9×10^{-9}	6.3×10^{-11}	3.2×10^{-10}
100.0 mm pencil line	100.0	0.1000	44.50	0.04450	42.00	0.04250	0.0025	9.6×10^{-10}	3.2×10^{-11}	3.2×10^{-10}
50.0 mm pencil line	50.0	0.0500	45.70	0.04570	44.50	0.04450	0.0012	4.6×10^{-10}	1.5×10^{-11}	3.0×10^{-10}

Table 4.2.1: Cross sectional area of 0.7 mm pencil lines on paper for 200.0mm, 100.0mm and 50.0mm long lines

Trial	200.0 (mm) (±0.5) Line resistance (R) (MΩ) (±0.01)	200.0 (mm) (±0.5) Line resistance (R) (Ω) (±10^{-7})	200.0 (mm) (±0.5) Resistivity of graphite $\left(\rho = \frac{R \times A}{l}\right)$ (Ωm)	100 mm (mm) (±0.5) Line resistance (R) (MΩ) (±0.01)	100 mm (mm) (±0.5) Line resistance (R) (Ω) (±10^{-7})	100 mm (mm) (±0.5) Resistivity of graphite $\left(\rho = \frac{R \times A}{l}\right)$ (Ωm)	50.0 (mm) (±0.5) Line resistance (R) (MΩ) (±0.01)	50.0 (mm) (±0.5) Line resistance (R) (Ω) (±10^{-7})	50.0 (mm) (±0.5) Resistivity of graphite $\left(\rho = \frac{R \times A}{l}\right)$ (Ωm
1	4.41	4.41×10^{-6}	0.0070	-	-	-	-	-	-
2	5.76	5.76×10^{-6}	0.0092	8.30	8.3×10^{-6}	0.026	1.32	1.32×10^{-6}	0.0079
3	1.92	1.92×10^{-6}	0.0031	9.81	9.81×10^{-6}	0.031	3.89	3.89×10^{-6}	0.023
4	1.19	1.19×10^{-6}	0.0019	11.31	11.31×10^{-6}	0.036	1.16	1.16×10^{-6}	0.0070
5	4.86	4.86×10^{-6}	0.0078	12.2	12.2×10^{-6}	0.039	1.84	1.84×10^{-6}	0.011
6	3.34	3.34×10^{-6}	0.0053	5.20	5.2×10^{-6}	0.017	3.92	3.92×10^{-6}	0.024
7	2.90	2.90×10^{-6}	0.0046	11.60	11.6×10^{-6}	0.037	2.12	2.12×10^{-6}	0.013
8	2.62	2.62×10^{-6}	0.0042	5.93	5.93×10^{-6}	0.019	2.66	2.66×10^{-6}	0.016
9	2.57	2.57×10^{-6}	0.0041	3.05	3.05×10^{-6}	0.0098	4.26	4.26×10^{-6}	0.026
10	3.25	3.25×10^{-6}	0.0052	3.36	3.36×10^{-6}	0.011	1.28	1.28×10^{-6}	0.0077
11	4.80	4.80×10^{-6}	0.0077	7.50	7.50×10^{-6}	0.024	1.00	1.00×10^{-6}	0.0060
12	7.32	7.32×10^{-6}	0.012	-	-	-	1.38	1.38×10^{-6}	0.0083
13	3.45	3.45×10^{-6}	0.0055	2.96	2.96×10^{-6}	0.0095	1.18	1.18×10^{-6}	0.0071
14	3.24	3.24×10^{-6}	0.0052	4.67	4.67×10^{-6}	0.015	1.48	1.48×10^{-6}	0.0089
15	2.79	2.79×10^{-6}	0.0045	2.26	2.26×10^{-6}	0.0072	0.47	0.47×10^{-6}	0.0028
16	7.32	7.32×10^{-6}	0.012	1.06	1.06×10^{-6}	0.0034	0.76	0.76×10^{-6}	0.0046
17	11.90	11.90×10^{-6}	0.019	1.60	1.60×10^{-6}	0.0051	1.00	1.00×10^{-6}	0.0060
18	7.06	7.06×10^{-6}	0.011	3.01	3.01×10^{-6}	0.0096	1.08	1.08×10^{-6}	0.0065
19	7.96	7.96×10^{-6}	0.013	2.92	2.92×10^{-6}	0.0093	0.84	0.84×10^{-6}	0.0050
20	-	-	-	3.32	3.32×10^{-6}	0.011	0.59	0.59×10^{-6}	0.0035
21	4.73	4.73×10^{-6}	0.0076	2.19	2.19×10^{-6}	0.0070	0.86	0.86×10^{-6}	0.0052
22	-	-	-	2.28	2.28×10^{-6}	0.0073	0.62	0.62×10^{-6}	0.0037
23	2.14	2.14×10^{-6}	0.0034	1.80	1.80×10^{-6}	0.0058	0.87	0.87×10^{-6}	0.0052
24	3.56	3.56×10^{-6}	0.0057	3.68	3.68×10^{-6}	0.012	0.89	0.89×10^{-6}	0.0053
25	2.21	2.21×10^{-6}	0.0035	1.40	1.40×10^{-6}	0.0045	0.16	0.16×10^{-6}	0.00096
26	2.00	2.00×10^{-6}	0.0032	3.60	3.60×10^{-6}	0.012	0.42	0.42×10^{-6}	0.0025
27	2.45	2.45×10^{-6}	0.0039	1.05	1.05×10^{-6}	0.0034	0.48	0.48×10^{-6}	0.0029
28	5.70	5.70×10^{-6}	0.0091	1.36	1.36×10^{-6}	0.0044	0.46	0.46×10^{-6}	0.0028
29	4.03	4.03×10^{-6}	0.0064	0.68	0.68×10^{-6}	0.0022	0.92	0.92×10^{-6}	0.0055
30	3.35	3.35×10^{-6}	0.0054	1.51	1.51×10^{-6}	0.0048	1.1	1.1×10^{-6}	0.0066

Table 4.2.2: Resistivity of 200.0 mm, 100.0 mm and 50.0 mm long lines drawn with 0.7 mm graphite pencil

Test 3

Diameter of graphite pencil used (2r): 0.9 mm
Radius of graphite pencil used (r): 0.45 mm

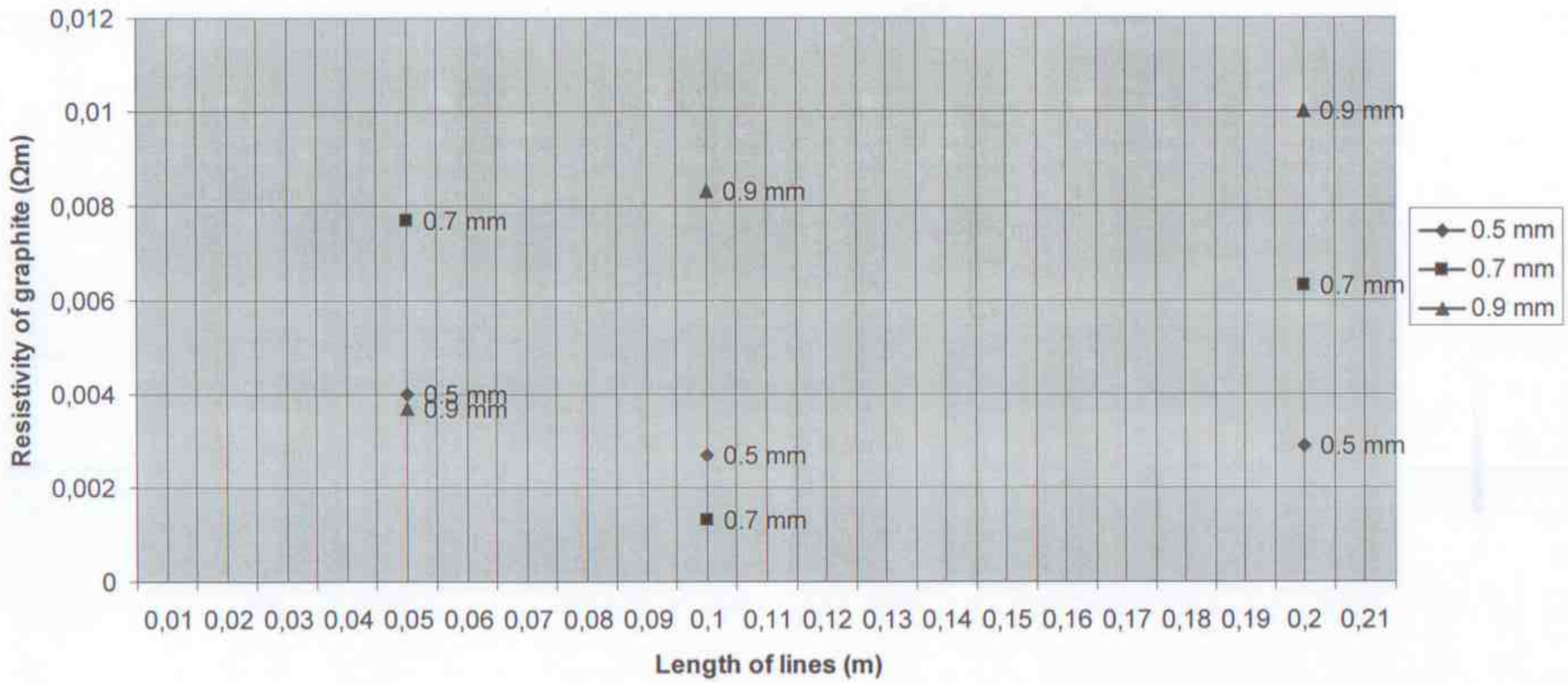
	Length of line (l)		Length of pencil before it is used (h_i)		Length of pencil after it is used (h_f)		Length of pencil used	Volume of pencil used	Average volume of	Average cross sectional area of pencil
	(mm) (±0.5)	(m) (±0.0005)	(mm) (±0.05)	(m) (±0.00005)	(mm) (±0.05)	(m) (±0.00005)	$(\Delta h = h_i - h_f)$ (m) (±0.0001)	$(V = \pi r^2 \Delta h)$ (m^3)	line $\left(V_{av} = \frac{V}{30}\right)$ (m^3)	lines $\left(A = \frac{V_{av}}{l}\right)$ (m^2)
200.0 mm pencil line	200.0	0.2000	72.00	0.07200	67.70	0.06770	0.0043	2.7×10^{-9}	9.0×10^{-11}	4.5×10^{-10}
100.0 mm pencil line	100.0	0.1000	74.30	0.07430	72.00	0.07200	0.0023	1.5×10^{-9}	5.0×10^{-11}	5.0×10^{-10}
50.0 mm pencil line	50.0	0.0500	75.50	0.07550	74.30	0.07430	0.0012	7.6×10^{-10}	2.5×10^{-11}	5.1×10^{-10}

Table 4.3.1: Cross sectional area of 0.9 mm pencil lines on paper for 200.0mm, 100.0mm and 50.0mm long lines

Trial	200.0 (mm) (±0.5)			100.0 (mm) (±0.5)			50.0 (mm) (±0.5)		
	Line resistance (R)		Resistivity of graphite	Line resistance		Resistivity of graphite	Line resistance		Resistivity of graphite
	(MΩ) (±0.01)	(Ω) (±10^{-7})	$\left(\rho = \frac{R \times A}{l}\right)$ (Ωm)	(MΩ) (±0.01)	(Ω) (±10^{-7})	$\left(\rho = \frac{R \times A}{l}\right)$ (Ωm)	(MΩ) (±0.01)	(Ω) (±10^{-7})	$\left(\rho = \frac{R \times A}{l}\right)$ (Ωm)
1	2.95	2.95×10^{-6}	0.0066	0.72	0.72×10^{-6}	0.0036	0.36	0.36×10^{-6}	0.0037
2	5.54	5.54×10^{-6}	0.012	0.44	0.44×10^{-6}	0.0022	0.55	0.55×10^{-6}	0.0056
3	-	-	-	1.40	1.40×10^{-6}	0.0070	0.35	0.35×10^{-6}	0.0036
4	4.45	4.45×10^{-6}	0.010	0.43	0.43×10^{-6}	0.0022	1.44	1.44×10^{-6}	0.015
5	7.85	7.85×10^{-6}	0.017	1.08	1.08×10^{-6}	0.0054	0.29	0.29×10^{-6}	0.0030
6	-	-	-	0.50	0.50×10^{-6}	0.0025	0.75	0.75×10^{-6}	0.0076
7	10.78	10.78×10^{-6}	0.024	0.80	0.80×10^{-6}	0.0040	0.47	0.47×10^{-6}	0.0048
8	3.22	3.22×10^{-6}	0.0072	0.93	0.93×10^{-6}	0.0046	0.31	0.31×10^{-6}	0.0032
9	7.65	7.65×10^{-6}	0.017	1.87	1.87×10^{-6}	0.0094	1.51	1.51×10^{-6}	0.015
10	3.55	3.55×10^{-6}	0.0080	3.58	3.58×10^{-6}	0.018	0.46	0.46×10^{-6}	0.0047
11	2.11	2.11×10^{-6}	0.0047	3.13	3.13×10^{-6}	0.016	0.14	0.14×10^{-6}	0.0014
12	2.87	2.87×10^{-6}	0.0064	2.11	2.11×10^{-6}	0.010	0.23	0.23×10^{-6}	0.0023
13	-	-	-	1.66	1.66×10^{-6}	0.0083	0.14	0.14×10^{-6}	0.0014
14	7.61	7.61×10^{-6}	0.017	0.53	0.53×10^{-6}	0.0026	0.19	0.19×10^{-6}	0.0019
15	7.50	7.50×10^{-6}	0.017	1.80	1.80×10^{-6}	0.0090	0.14	0.14×10^{-6}	0.0014
16	15.60	15.6×10^{-6}	0.035	1.35	1.35×10^{-6}	0.0068	0.18	0.18×10^{-6}	0.0018
17	-	-	-	1.49	1.49×10^{-6}	0.0074	0.18	0.18×10^{-6}	0.0018
18	4.30	4.30×10^{-6}	0.0097	1.45	1.45×10^{-6}	0.0072	0.17	0.17×10^{-6}	0.0017
19	4.65	4.65×10^{-6}	0.010	0.63	0.63×10^{-6}	0.0032	0.12	0.12×10^{-6}	0.0012
20	4.65	4.65×10^{-6}	0.010	1.48	1.48×10^{-6}	0.0074	0.26	0.26×10^{-6}	0.0026
21	6.50	6.50×10^{-6}	0.015	5.16	5.16×10^{-6}	0.026	0.21	0.21×10^{-6}	0.0021
22	2.55	2.55×10^{-6}	0.0057	0.87	0.87×10^{-6}	0.0044	0.24	0.24×10^{-6}	0.0024
23	2.23	2.23×10^{-6}	0.0050	1.39	1.39×10^{-6}	0.0070	0.25	0.25×10^{-6}	0.0026
24	14.86	14.86×10^{-6}	0.033	1.49	1.49×10^{-6}	0.0074	0.34	0.34×10^{-6}	0.0035
25	5.01	5.01×10^{-6}	0.011	0.83	0.83×10^{-6}	0.0042	0.21	0.21×10^{-6}	0.0021
26	2.20	2.20×10^{-6}	0.0050	1.69	1.69×10^{-6}	0.0084	0.24	0.24×10^{-6}	0.0025
27	2.35	2.35×10^{-6}	0.0053	6.93	6.93×10^{-6}	0.035	0.45	0.45×10^{-6}	0.0046
28	1.16	1.16×10^{-6}	0.0026	2.00	2.00×10^{-6}	0.010	0.26	0.26×10^{-6}	0.0027
29	1.78	1.78×10^{-6}	0.0040	1.11	1.11×10^{-6}	0.0056	0.62	0.62×10^{-6}	0.0063
30	1.81	1.81×10^{-6}	0.0041	1.95	1.95×10^{-6}	0.0098	0.18	0.18×10^{-6}	0.0018

Table 4.3.2: Resistivity of 200.0 mm, 100.0 mm and 50.0 mm long lines drawn with 0.9 mm graphite pencil

Graph 1: Grahite resistivity of graphite pencils with 0.5 mm, 0.7 mm and 0.9 mm diameter and length of lines



Diameter of pencil (mm)	0.5			0.7			0.9		
Length of line (m)	0.20	0.10	0.05	0.20	0.10	0.05	0.20	0.10	0.05
Average resistivity (Ωm)	0.0029	0.0027	0.0040	0.0063	0.0013	0.0077	0.0100	0.0083	0.0037

7.3. Calculations Based on Test Results and Error Calculation

7.3.1 Test 1: Graphite pencil with 0.5 mm diameter

Diameter of graphite pencil used (2r): 0.5 mm
Radius of graphite pencil used (r): 0.25×10^{-3} m

Cross-Sectional Area Calculation

200.0 mm pencil lines
Length of line (l) = 0.2000 ± 0.0005 m

$$\text{Percentage error} = \left(\frac{\text{relative error}}{\text{literature value}}\right)\times 100$$
$$= \left(\frac{0.0005}{0.2000}\right)\times 100$$
$$= 0.25\ \%$$
$$\cong 0.2\ \%$$

Length of line (l) = 0.2000 ± 0.2 % m
Initial length of pencil (h_i) = 0.05450 ± 0.00005 m
Final length of pencil (h_f) = 0.04900 ± 0.00005 m
Length of pencil used (Δh) = $h_i - h_f$

$$=(0.05450 \pm 0.00005) - (0.04900 \pm 0.00005) = 0.0055 \pm 0.0001 \text{ m}$$

$$\text{Percentage error} = \left(\frac{0.0001}{0.0055}\right)\times 100$$
$$= 1.818181818\ \%$$
$$\cong 2\ \%$$

Length of pencil used (Δh) = 0.0055 ± 2 %
Volume of pencil used (V) = $\pi r^2 \Delta h$

$$= \pi \times (0.25\times 10^{-3})^2 \times (0.0055 \pm 2\%)$$
$$= 1.08036 \times 10^{-9} \pm 2\ \%\ \text{m}^3$$
$$\cong 1.1\times 10^{-9} \pm 2\ \%\ \text{m}^3$$

$$\text{Average volume of line } (V_{av}) = \frac{V}{30}$$
$$= \frac{1.1\times 10^{-9} \pm 2\ \%}{30}$$
$$= 3.66667\times 10^{-11} \pm 2\ \%\ \text{m}^3$$
$$\cong 3.7\times 10^{-11} \pm 2\ \%\ \text{m}^3$$

$$\text{Average cross sectional area of pencil lines (A)} = \frac{V_{av}}{l}$$
$$= \frac{3.7\times 10^{-11} \pm 2\%}{0.2000 \pm 0.2\%}$$
$$= 1.85\times 10^{-10} \pm 2.2\ \%\ \text{m}^2$$
$$\cong 1.8 \times 10^{-10} \pm 2.2\ \%\ \text{m}^2$$

100.0 mm pencil lines

Length of line (l) = 0.1000 ± 0.0005 m

$$\text{Percentage error} = \left(\frac{\text{relative error}}{\text{literature value}}\right)\times 100$$

$$= \left(\frac{0.0005}{0.1000}\right)\times 100$$

$$= 0.5\ \%$$

Length of line (l) = 0.1000 ± 0.5 % m

Initial length of pencil (h_i) = 0.05800 ± 0.00005 m

Final length of pencil (h_f) = 0.05450 ± 0.00005 m

Length of pencil used (Δh) = h_i - h_f

$$=(0.05800 \pm 0.00005) - (0.05450 \pm 0.00005) = 0.0035 \pm 0.0001 \text{ m}$$

$$\text{Percentage error} = \left(\frac{0.0001}{0.0035}\right)\times 100$$

$$= 2.857142857\ \%$$

$$\cong 3\ \%$$

Length of pencil used (Δh) = 0.0035 ± 3 %

Volume of pencil used (V) = $\pi r^2 \Delta h$

$$= \pi \times (0.25\times 10^{-3})^2 \times (0.0035 \pm 3\%)$$

$$= 6.875 \times 10^{-10} \pm 3\ \%\ \text{m}^3$$

$$\cong 6.9 \times 10^{-10} \pm 3\ \%\ \text{m}^3$$

$$\text{Average volume of line } (V_{av}) = \frac{V}{30}$$

$$= \frac{6.9\times 10^{-10} \pm 3\ \%}{30}$$

$$= 2.3 \times 10^{-11} \pm 3\ \%\ \text{m}^3$$

$$\cong 2.3 \times 10^{-11} \pm 3\ \%\ \text{m}^3$$

$$\text{Average cross sectional area of pencil lines (A)} = \frac{V_{av}}{l}$$

$$= \frac{2.3\times 10^{-11} \pm 3\%}{0.1000 \pm 0.5\%}$$

$$= 2.3 \times 10^{-10} \pm 3.5\ \%\ \text{m}^2$$

50.0 mm pencil lines

Length of line (l) = 0.0500 ± 0.0005 m

$$\text{Percentage error} = \left(\frac{\text{relative error}}{\text{literature value}}\right)\times 100$$

$$= \left(\frac{0.0005}{0.0500}\right)\times 100$$

$$= 1\%$$

Length of line (l) = 0.0500 ± 1 % m

Initial length of pencil (h_i) = 0.06000 ± 0.00005 m

Final length of pencil (h_f) = 0.05800 ± 0.00005 m

Length of pencil used (Δh) = h_i - h_f

$$=(0.06000 \pm 0.00005) - (0.05800 \pm 0.00005) = 0.0020 \pm 0.0001 \text{ m}$$

Percentage error = $\left(\frac{0.0001}{0.0020}\right)\times 100$

$= 5\ \%$

Length of pencil used (Δh) = 0.0020 ± 5 %

Volume of pencil used (V) = $\pi r^2 \Delta h$

$= \pi \times (0.25\times 10^{-3})^2 \times (0.0020 \pm 5\%)$

$= 3.92857\ \times 10^{-10} \pm 5\%\ m^3$

$\cong 3.9\times 10^{-10} \pm 5\%\ m^3$

Average volume of line (V_{av}) = $\frac{V}{30}$

$= \frac{3.9\times 10^{-10} \pm 5\%}{30}$

$= 1.3\times 10^{-11} \pm 5\ \%\ m^3$

Average cross sectional area of pencil lines (A) = $\frac{V_{av}}{l}$

$= \frac{1.3\times 10^{-11} \pm 5\%}{0.0500 \pm 1\%}$

$= 2.6\times 10^{-10} \pm 6\ \%\ m^2$

Resistivity Calculation
200.0 mm pencil lines

Percentage Error of Resistance = $\left(\frac{\text{relative error}}{\text{Resistance}}\right)\times 100$

Trial 1: Percentage Error = $\left(\frac{10^{-7}}{4.01\times 10^{-6}}\right)\times 100 = 2.49377\times 10^{-12}\ \%$

$\cong 2.5\ \times 10^{-12}\ \%$

Trial 2: Percentage Error = $\left(\frac{10^{-7}}{2.36\times 10^{-6}}\right)\times 100 = 4.23729\times 10^{-12}\ \%$

$\cong 4.2\ \times 10^{-12}\ \%$

Trial 5: Percentage Error = $\left(\frac{10^{-7}}{6.71\times 10^{-6}}\right)\times 100 = 1.49031\times 10^{-12}\ \%$

$\cong 1.5\ \times 10^{-12}\ \%$

Trial 11: Percentage Error = $\left(\frac{10^{-7}}{4.72\times 10^{-6}}\right)\times 100 = 2.11864\times 10^{-12}\ \%$

$\cong 2.1\ \times 10^{-12}\ \%$

Trial 15: Percentage Error = $\left(\frac{10^{-7}}{7.60\times 10^{-6}}\right)\times 100 = 1.31579\times 10^{-12}\ \%$

$\cong 1.3\ \times 10^{-12}\ \%$

Trial 17: Percentage Error = $\left(\frac{10^{-7}}{4.50\times 10^{-6}}\right)\times 100 = 2.22222\times 10^{-12}\ \%$

$\cong 2.2\ \times 10^{-12}\%$

Trial 18: Percentage Error $=\left(\frac{10^{-7}}{10.10\times10^{-6}}\right)\times100=9.90099\times10^{-12}\,\%$

$\cong 9.9\times10^{-12}\ \%$

Trial 19: Percentage Error $=\left(\frac{10^{-7}}{4.80\times10^{-6}}\right)\times100=2.08333\times10^{-12}\,\%$

$\cong 2.1\times10^{-12}\ \%$

Trial 21: Percentage Error $=\left(\frac{10^{-7}}{13.36\times10^{-6}}\right)\times100=8.09061\times10^{-13}\,\%$

$\cong 8.1\times10^{-13}\ \%$

Trial 23: Percentage Error $=\left(\frac{10^{-7}}{6.53\times10^{-6}}\right)\times100=1.53139\times10^{-12}\,\%$

$\cong 1.5\times10^{-12}\ \%$

Trial 25: Percentage Error $=\left(\frac{10^{-7}}{7.05\times10^{-6}}\right)\times100=1.41844\times10^{-12}\,\%$

$\cong 1.4\times10^{-12}\ \%$

Trial 26: Percentage Error $=\left(\frac{10^{-7}}{7.92\times10^{-6}}\right)\times100=1.26263\times10^{-12}\,\%$

$\cong 1.3\times10^{-12}\ \%$

Trial 27: Percentage Error $=\left(\frac{10^{-7}}{3.21\times10^{-6}}\right)\times100=3.11526\times10^{-12}\,\%$

$\cong 3.1\times10^{-12}\ \%$

Trial 28: Percentage Error $=\left(\frac{10^{-7}}{4.95\times10^{-6}}\right)\times100=2.0202\times10^{-12}\,\%$

$\cong 2.0\times10^{-12}\ \%$

Trial 29: Percentage Error $=\left(\frac{10^{-7}}{3.74\times10^{-6}}\right)\times100=2.6738\times10^{-12}\,\%$

$\cong 2.7\times10^{-12}\ \%$

Trial 30: Percentage Error $=\left(\frac{10^{-7}}{7.55\times10^{-6}}\right)\times100=1.3245\times10^{-12}\,\%$

$\cong 1.3\times10^{-12}\ \%$

Percentage errors of resistances are too small to be shown in the percentage error calculation. Therefore they are ignored and not calculated in the error calculation.

Resistivity (ρ) $=\frac{R\times A}{l}$

Trial 1: $\rho=\frac{4.01\times10^{-6}\times1.8\times10^{-10}}{0.2000}=3.609\times10^{-3}\,\Omega.\text{m}$

$\cong 3.6\times10^{-3}\pm2.4\ \%\ \Omega.\text{m}$

Trial 2: $\rho=\frac{2.36\times10^{-6}\times1.8\times10^{-10}}{0.2000}=2.124\times10^{-3}\,\Omega.\text{m}$

$\cong 2.1\times10^{-3}\pm2.4\ \%\ \Omega.\text{m}$

Trial 3: -
Trial 4: -

Trial 5: $\rho = \dfrac{6.71\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 6.039\times10^{-3}\,\Omega.m$

$\cong 6.0\times10^{-3} \pm 2.4\,\%\,\Omega.m$

Trial 6: -
Trial 7: -
Trial 8: -
Trial 9: -
Trial 10: -

Trial 11: $\rho = \dfrac{4.72\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 4.248\times10^{-3}\,\Omega.m$

$\cong 4.2\times10^{-3} \pm 2.4\,\%\,\Omega.m$

Trial 12: -
Trial 13: -
Trial 14: -

Trial 15: $\rho = \dfrac{7.60\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 6.840\times10^{-3}\,\Omega.m$

$\cong 3.6\times10^{-3} \pm 2.4\,\%\,\Omega.m$

Trial 16: -

Trial 17: $\rho = \dfrac{4.50\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 4.050\times10^{-3}\,\Omega.m$

$\cong 4.0\times10^{-3} \pm 2.4\,\%\,\Omega.m$

Trial 18: $\rho = \dfrac{10.10\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 9.090\times10^{-3}\,\Omega.m$

$\cong 9.1\times10^{-3} \pm 2.4\,\%\,\Omega.m$

Trial 19: $\rho = \dfrac{4.80\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 4.320\times10^{-3}\,\Omega.m$

$\cong 4.3\times10^{-3} \pm 2.4\,\%\,\Omega.m$

Trial 20: -

Trial 21: $\rho = \dfrac{12.36\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 1.112\times10^{-2}\,\Omega.m$

$\cong 1.1\times10^{-2} \pm 2.4\,\%\,\Omega.m$

Trial 22: -

Trial 23: $\rho = \dfrac{6.53\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 5.877\times10^{-3}\,\Omega.m$

$\cong 5.9\times10^{-3} \pm 2.4\,\%\,\Omega.m$

Trial 24: -

Trial 25: $\rho = \dfrac{7.05\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 6.345\times10^{-3}\,\Omega.m$

$\cong 6.3\times10^{-3} \pm 2.4\,\%\,\Omega.m$

Trial 26: $\rho = \dfrac{7.92\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 7.128\times10^{-3}\,\Omega.m$

$\cong 7.1\times10^{-3} \pm 2.4\,\%\,\Omega.m$

Trial 27: $\rho = \dfrac{3.21\times10^{-6}\times1.8\times10^{-10}}{0.2000} = 2.889\times10^{-3}\,\Omega.m$

$$\cong 2.9\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

Trial 28: $\rho=\dfrac{4.95\times10^{-6}\times1.8\times10^{-10}}{0.2000}=4.455\times10^{-3}\,\Omega.\text{m}$

$$\cong 4.4\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

Trial 29: $\rho=\dfrac{3.74\times10^{-6}\times1.8\times10^{-10}}{0.2000}=3.366\times10^{-3}\,\Omega.\text{m}$

$$\cong 3.4\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

Trial 30: $\rho=\dfrac{7.55\times10^{-6}\times1.8\times10^{-10}}{0.2000}=6.795\times10^{-3}\,\Omega.\text{m}$

$$\cong 3.8\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

Average resistivity of 200.0mm long lines ($\rho_{av200.0}$) = $\dfrac{\sum_{k=1}^{30}\rho_k}{30}$

$$=\frac{\rho_1+\rho_2+\rho_3+\ldots+\rho_{30}}{30}$$

$$=0.002933\ \Omega.\text{m}$$

$$\cong 0.0029 \pm 2.4\ \%\ \Omega.\text{m}$$

Relative error = $\left(\dfrac{\text{percentage error}\times\text{resistance}}{100}\right)$

$$=\left(\frac{0.0029\times2.4}{100}\right)=0.0000696$$

$$\cong 0.000070$$

$\rho_{av200.0} = 0.0029 \pm 0.000070\ \Omega.\text{m}$

100.0 mm pencil lines

Trial 1: $\rho=\dfrac{0.42\times10^{-6}\times2.3\times10^{-10}}{0.1000}=9.660\times10^{-4}\,\Omega.\text{m}$

$$\cong 9.7\times10^{-4} \pm 4.0\ \%\Omega.\text{m}$$

Trial 2: $\rho=\dfrac{0.40\times10^{-6}\times2.3\times10^{-10}}{0.1000}=9.200\times10^{-4}\,\Omega.\text{m}$

$$\cong 9.2\times10^{-4} \pm 4.0\ \%\Omega.\text{m}$$

Trial 3: $\rho=\dfrac{0.39\times10^{-6}\times2.3\times10^{-10}}{0.1000}=8.970\times10^{-4}\,\Omega.\text{m}$

$$\cong 9.0\times10^{-4} \pm 4.0\ \%\Omega.\text{m}$$

Trial 4: $\rho=\dfrac{0.46\times10^{-6}\times2.3\times10^{-10}}{0.1000}=1.058\times10^{-3}\,\Omega.\text{m}$

$$\cong 1.0\times10^{-3} \pm 4.0\ \%\Omega.\text{m}$$

Trial 5: $\rho=\dfrac{0.77\times10^{-6}\times2.3\times10^{-10}}{0.1000}=1.771\times10^{-3}\,\Omega.\text{m}$

$$\cong 1.8\times10^{-3} \pm 4.0\ \%\Omega.\text{m}$$

Trial 6: $\rho=\dfrac{1.08\times10^{-6}\times2.3\times10^{-10}}{0.1000}=2.484\times10^{-3}\,\Omega.\text{m}$

$$\cong 2.5\times10^{-3} \pm 4.0\ \%\Omega.\text{m}$$

Trial 7: $\rho = \frac{0.70 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$1.610 \times 10^{-3}\ \Omega.m$

$\cong 1.6 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 8: $\rho = \frac{1.61 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$3.703 \times 10^{-3}\ \Omega.m$

$\cong 3.7 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 9: $\rho = \frac{0.93 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$1.139 \times 10^{-3}\ \Omega.m$

$\cong 1.1 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 10 $\rho = \frac{6.92 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$1.592 \times 10^{-3}\ \Omega.m$

$\cong 1.6 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 11 $\rho = \frac{1.30 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$2.990 \times 10^{-3}\ \Omega.m$

$\cong 3.0 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 12 $\rho = \frac{1.70 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$3.910 \times 10^{-3}\ \Omega.m$

$\cong 3.9 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 13 $\rho = \frac{1.45 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$3.335 \times 10^{-2}\ \Omega.m$

$\cong 3.3 \times 10^{-2} \pm 4.0\ \%\Omega.m$

Trial 14 $\rho = \frac{0.62 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$1.426 \times 10^{-3}\ \Omega.m$

$\cong 1.4 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 15 $\rho = \frac{0.77 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$1.771 \times 10^{-3}\ \Omega.m$

$\cong 1.8 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 16 $\rho = \frac{0.79 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$1.817 \times 10^{-3}\ \Omega.m$

$\cong 1.8 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 17 $\rho = \frac{0.70 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$1.610 \times 10^{-3}\ \Omega.m$

$\cong 1.6 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 18 $\rho = \frac{0.63 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$1.449 \times 10^{-3}\ \Omega.m$

$\cong 1.4 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 19 $\rho = \frac{0.76 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$1.748 \times 10^{-3}\ \Omega.m$

$\cong 1.7 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 20 $\rho = \frac{1.46 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$3.358 \times 10^{-3}\ \Omega.m$

$\cong 3.4 \times 10^{-3} \pm 4.0\ \%\Omega.m$

Trial 21: $\rho = \frac{1.93 \times 10^{-6} \times 2.3 \times 10^{-10}}{0.1000}$ =$4.439 \times 10^{-3}\ \Omega.m$

$$\cong 4.4\times10^{-3} \pm 4.0\ \%\Omega.m$$

Trial 22: $\rho = \dfrac{1.30\times10^{-6}\times2.3\times10^{-10}}{0.1000} = 2.990\times10^{-3}\ \Omega.m$

$$\cong 3.0\times10^{-3} \pm 4.0\ \%\Omega.m$$

Trial 23: $\rho = \dfrac{1.31\times10^{-6}\times2.3\times10^{-10}}{0.1000} = 3.013\times10^{-3}\ \Omega.m$

$$\cong 3.0\times10^{-3} \pm 4.0\ \%\Omega.m$$

Trial 24: $\rho = \dfrac{1.19\times10^{-6}\times2.3\times10^{-10}}{0.1000} = 2.737\times10^{-3}\ \Omega.m$

$$\cong 3.7\times10^{-3} \pm 4.0\ \%\Omega.m$$

Trial 25: $\rho = \dfrac{1.13\times10^{-6}\times2.3\times10^{-10}}{0.1000} = 2.599\times10^{-3}\ \Omega.m$

$$\cong 3.6\times10^{-3} \pm 4.0\ \%\Omega.m$$

Trial 26: $\rho = \dfrac{0.67\times10^{-6}\times2.3\times10^{-10}}{0.1000} = 1.541\times10^{-3}.m$

$$\cong 1.5\times10^{-3} \pm 4.0\ \%\Omega.m$$

Trial 27: $\rho = \dfrac{0.53\times10^{-6}\times2.3\times10^{-10}}{0.1000} = 1.219\times10^{-3}\ \Omega.m$

$$\cong 1.2\times10^{-3} \pm 4.0\ \%\Omega.m$$

Trial 28: $\rho = \dfrac{0.82\times10^{-6}\times2.3\times10^{-10}}{0.1000} = 1.886\times10^{-3}\ \Omega.m$

$$\cong 1.9\times10^{-3} \pm 4.0\ \%\Omega.m$$

Trial 29: $\rho = \dfrac{1.71\times10^{-6}\times2.3\times10^{-10}}{0.1000} = 3.993\times10^{-3}\ \Omega.m$

$$\cong 4.0\times10^{-3} \pm 4.0\ \%\Omega.m$$

Trial 30: $\rho = \dfrac{0.75\times10^{-6}\times2.3\times10^{-10}}{0.1000} = 1.725\times10^{-3}\ \Omega.m$

$$\cong 1.7\times10^{-3} \pm 4.0\ \%\Omega.m$$

Average resistivity of 100.0mm long lines (ρ_{av}) = $\dfrac{\sum_{k=1}^{30}\rho_k}{30}$

$$= \frac{\rho_1 + \rho_2 + \rho_3 + \ldots + \rho_{30}}{30}$$

$$= 0.0027 \pm 4.0\ \%\ \Omega.m$$

Relative error = $\left(\dfrac{\text{percentage error} \times \text{resistance}}{100}\right)$

$$= \left(\frac{0.0027\times4.0}{100}\right) = 0.000108$$

$$\cong 0.00011$$

$\rho_{av100.0} = 0.0027 \pm 0.00011\ \Omega.m$

50.0 mm pencil lines

Trial 1: -

Trial 2: $\rho = \frac{3.52\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =1.830×10^{-2} Ω.m

$\cong 1.8\times10^{-2} \pm 7.0$ %Ω.m

Trial 3: $\rho = \frac{0.60\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =3.120×10^{-3} Ω.m

$\cong 3.1\times10^{-3} \pm 7.0$ %Ω.m

Trial 4: $\rho = \frac{1.15\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =5.980×10^{-3} Ω.m

$\cong 6.0\times10^{-3} \pm 7.0$ %Ω.m

Trial 5: $\rho = \frac{1.40\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =7.280×10^{-3} Ω.m

$\cong 7.3\times10^{-3} \pm 7.0$ %Ω.m

Trial 6: $\rho = \frac{1.27\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =6.604×10^{-3} Ω.m

$\cong 6.6\times10^{-3} \pm 7.0$ %Ω.m

Trial 7: $\rho = \frac{1.87\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =9.724×10^{-3} Ω.m

$\cong 9.7\times10^{-3} \pm 7.0$ %Ω.m

Trial 8: $\rho = \frac{0.55\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =2.860×10^{-3} Ω.m

$\cong 2.9\times10^{-3} \pm 7.0$ %Ω.m

Trial 9: $\rho = \frac{1.37\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =7.124×10^{-3} Ω.m

$\cong 7.1\times10^{-3} \pm 7.0$ %Ω.m

Trial 10: $\rho = \frac{1.86\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =9.672×10^{-3} Ω.m

$\cong 9.7\times10^{-3} \pm 7.0$ %Ω.m

Trial 11: $\rho = \frac{1.51\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =7.852×10^{-3} Ω.m

$\cong 7.8\times10^{-3} \pm 7.0$ %Ω.m

Trial 12: $\rho = \frac{0.79\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =4.108×10^{-3} Ω.m

$\cong 4.1\times10^{-3} \pm 7.0$ %Ω.m

Trial 13: $\rho = \frac{0.68\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =3.536×10^{-3} Ω.m

$\cong 3.5\times10^{-3} \pm 7.0$ %Ω.m

Trial 14: $\rho = \frac{0.54\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =2.808×10^{-3} Ω.m

$\cong 2.8\times10^{-3} \pm 7.0$ %Ω.m

Trial 15: $\rho = \frac{1.03\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ =5.356×10^{-3} Ω.m

$\cong 5.4\times10^{-3} \pm 7.0$ %Ω.m

Trial 16: $\rho = \dfrac{0.51\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=2.652\times10^{-3}\,\Omega.m$

$\cong 2.6\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 17: $\rho = \dfrac{0.72\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=3.774\times10^{-3}\,\Omega.m$

$\cong 3.8\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 18: $\rho = \dfrac{0.55\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=2.860\times10^{-3}\,\Omega.m$

$\cong 2.9\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 19: $\rho = \dfrac{0.12\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=6.240\times10^{-3}\,\Omega.m$

$\cong 6.2\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 20: $\rho = \dfrac{0.26\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=1.352\times10^{-3}\,\Omega.m$

$\cong 1.4\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 21: $\rho = \dfrac{0.16\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=8.320\times10^{-3}\,\Omega.m$

$\cong 8.3\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 22: $\rho = \dfrac{0.21\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=1.092\times10^{-3}\,\Omega.m$

$\cong 1.1\times10^{-3}\pm7.0\%\Omega.m$

Trial 23: $\rho = \dfrac{0.26\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=1.352\times10^{-3}\,\Omega.m$

$\cong 1.4\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 24: $\rho = \dfrac{0.30\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=1.560\times10^{-3}\,\Omega.m$

$\cong 1.6\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 25: $\rho = \dfrac{0.13\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=6.760\times10^{-3}\,\Omega.m$

$\cong 6.8\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 26: $\rho = \dfrac{0.24\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=1.248\times10^{-3}\,\Omega.m$

$\cong 1.2\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 27: $\rho = \dfrac{0.27\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=1.404\times10^{-3}\,\Omega.m$

$\cong 1.4\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 28: $\rho = \dfrac{0.31\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=1.612\times10^{-3}\,\Omega.m$

$\cong 1.6\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 29: $\rho = \dfrac{0.48\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=2.469\times10^{-3}\,\Omega.m$

$\cong 2.5\times10^{-3}\pm7.0\,\%\Omega.m$

Trial 30: $\rho = \frac{0.18\times10^{-6}\times2.6\times10^{-10}}{0.0500}$ $=9.360\times10^{-3}\,\Omega.\text{m}$

$\cong 9.3\times10^{-3} \pm 7.0\ \%\Omega.\text{m}$

Average resistivity of 50.0mm long lines (ρ_{av}) = $\frac{\sum_{k=1}^{30}\rho_k}{30}$

$= \frac{\rho_1+\rho_2+\rho_3+...+\rho_{30}}{30}$

$=0.003959\ \Omega.\text{m}$

$\cong 0.0040 \pm 7.0\ \%\ \Omega.\text{m}$

Relative error = $\left(\frac{\text{percentage error}\times\text{resistivity}}{100}\right)$

$=\left(\frac{0.0040\times7.0}{100}\right)= 0.00028$

$\rho_{av50.0} = 0.0040 \pm 0.00028\ \Omega.\text{m}$

Average resistivity of graphite with 0.5 mm diameter ($\rho_{av0.5}$) = $\frac{\rho_{av200.0}+\rho_{av100.0}+\rho_{av50.0}}{3}$

$=0.0032\ \Omega.\text{m}$

relative error of $\rho_{av0.5}$ = 0.000070+0.00011+0.00028 = 0.00046

$\rho_{av0.5}$ with relative error = $0.0032 \pm 0.00046\ \Omega.\text{m}$

percentage error = $\left(\frac{0.00046}{0.0032}\right)\times100$ = 14.375 %

$\cong 14.0\ \%$

$\rho_{av0.5}$ with percentage error = $0.0032 \pm 14.0\ \%\ \Omega.\text{m}$

7.3.2. Test 2: Graphite pencil with 0.7 mm diameter

Cross-Sectional Area Calculation

Diameter of graphite pencil used (2r): 0.7 mm
Radius of graphite pencil used (r): 0.35×10^{-3} m

200.0 mm pencil lines

Length of line (l) = 0.2000 ± 0.0005 m

Percentage error = $\left(\frac{\text{relative error}}{\text{literature value}}\right)\times100$

$=\left(\frac{0.0005}{0.2000}\right)\times100$

$= 0.25\ \%$

$\cong 0.2\ \%$

Length of line (l) = $0.2000 \pm 0.2\ \%$ m

Initial length of pencil (h_i) = 0.04200 ± 0.00005 m
Final length of pencil (h_f) = 0.03700 ± 0.00005 m
Length of pencil used (Δh) = h_i - h_f

$$=(0.04200 \pm 0.00005) - (0.03700 \pm 0.00005) = 0.0050 \pm 0.0001 \text{ m}$$

$$\text{Percentage error} = \left(\frac{0.0001}{0.0050}\right) \times 100$$

$$= 2\ \%$$

Length of pencil used (Δh) = 0.0050 ± 2 %

Volume of pencil used (V) = $\pi r^2 \Delta h$

$$= \pi \times (0.35 \times 10^{-3})^2 \times (0.0050 \pm 2\%)$$
$$= 1.925 \times 10^{-9} \pm 2\ \%\ \text{m}^3$$
$$\cong 1.9 \times 10^{-9} \pm 2\ \%\ \text{m}^3$$

$$\text{Average volume of line } (V_{av}) = \frac{V}{30}$$

$$= \frac{1.9 \times 10^{-9} \pm 2\ \%}{30}$$
$$= 6.33333 \times 10^{-11} \pm 2\ \%\ \text{m}^3$$
$$\cong 6.3 \times 10^{-11} \pm 2\ \%\ \text{m}^3$$

$$\text{Average cross sectional area of pencil lines (A)} = \frac{V_{av}}{l}$$

$$= \frac{6.3 \times 10^{-11} \pm 2\%}{0.2000 \pm 0.2\%}$$
$$= 3.15 \times 10^{-10} \pm 2.2\ \%\ \text{m}^2$$
$$\cong 3.2 \times 10^{-10} \pm 2.2\ \%\ \text{m}^2$$

100.0 mm pencil lines

Length of line (l) = 0.1000 ± 0.0005 m

$$\text{Percentage error} = \left(\frac{\text{relative error}}{\text{literature value}}\right) \times 100$$

$$= \left(\frac{0.0005}{0.1000}\right) \times 100$$

$$= 0.5\ \%$$

Length of line (l) = 0.1000 ± 0.5 % m
Initial length of pencil (h_i) = 0.04450 ± 0.00005 m
Final length of pencil (h_f) = 0.04250 ± 0.00005 m
Length of pencil used (Δh) = h_i - h_f

$$=(0.04450 \pm 0.00005) - (0.04250 \pm 0.00005) = 0.0025 \pm 0.0001 \text{ m}$$

$$\text{Percentage error} = \left(\frac{0.0001}{0.0025}\right) \times 100$$

$$=4\ \%$$

Length of pencil used (Δh) = 0.0025 ± 4 %

Volume of pencil used (V) = $\pi r^2 \Delta h$

$$= \pi \times (0.35 \times 10^{-3})^2 \times (0.0025 \pm 4\%)$$
$$= 9.625 \times 10^{-10} \pm 4\ \%\ \text{m}^3$$
$$\cong 9.6 \times 10^{-10} \pm 4\ \%\ \text{m}^3$$

Average volume of line (V_{av}) $= \dfrac{V}{30}$

$$= \frac{9.6\times10^{-10}\pm 4\ \%}{30}$$
$$= 3.2\times10^{-11}\pm 4\ \%\ m^3$$

Average cross sectional area of pencil lines (A) $= \dfrac{V_{av}}{l}$

$$= \frac{3.2\times10^{-11}\pm 4\%}{0.1000\pm 0.5\%}$$
$$= 3.2\times10^{-10}\pm 4.5\ \%\ m^2$$

50.0 mm pencil lines

Length of line (l) = 0.0500 ± 0.0005 m

$$\text{Percentage error} = \left(\frac{\text{relative error}}{\text{literature value}}\right)\times100$$
$$= \left(\frac{0.0005}{0.0500}\right)\times100$$
$$= 1\%$$

Length of line (l) = 0.0500 ± 1 % m

Initial length of pencil (h_i) = 0.04570 ± 0.00005 m

Final length of pencil (h_f) = 0.04450 ± 0.00005 m

Length of pencil used (Δh) = h_i - h_f

$$=(0.04570 \pm 0.00005) - (0.04450\pm 0.00005) = 0.0012 \pm 0.0001\ m$$

$$\text{Percentage error} = \left(\frac{0.0001}{0.0020}\right)\times100$$
$$= 8.33333\ \%$$
$$\cong 8\ \%$$

Length of pencil used (Δh) = 0.0012 ± 8 %

Volume of pencil used (V) = $\pi r^2\Delta h$

$$= \pi\times(0.35\times10^{-3})^2\times(0.0012\pm8\%)$$
$$= 4.62\times10^{-10}\pm 8\%\ m^3$$
$$\cong 4.6\times10^{-10}\pm 8\%\ m^3$$

Average volume of line (V_{av}) $= \dfrac{V}{30}$

$$= \frac{4.6\times10^{-10}\pm 8\%}{30}$$
$$= 1.53333\times10^{-11}\pm 8\ \%\ m^3$$
$$\cong 1.5\times10^{-11}\pm 8\ \%\ m^3$$

Average cross sectional area of pencil lines (A) $= \dfrac{V_{av}}{l}$

$$= \frac{1.5\times10^{-11}\pm8\%}{0.0500\pm1\%}$$
$$= 3.0\times10^{-10}\pm 9\ \%\ m^2$$

Resistivity Calculation
200.0 mm pencil lines

Trial 1: $\rho = \dfrac{4.41\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 7.056\times10^{-3}\,\Omega.\text{m}$
$\cong 7.0\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 2: $\rho = \dfrac{5.76\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 9.216\times10^{-3}\,\Omega.\text{m}$
$\cong 9.2\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 3: $\rho = \dfrac{1.92\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 3.072\times10^{-3}\,\Omega.\text{m}$
$\cong 3.1\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 4: $\rho = \dfrac{1.19\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 1.904\times10^{-3}\,\Omega.\text{m}$
$\cong 1.9\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 5: $\rho = \dfrac{4.86\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 7.776\times10^{-3}\,\Omega.\text{m}$
$\cong 7.8\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 6: $\rho = \dfrac{3.34\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 4.334\times10^{-3}\,\Omega.\text{m}$
$\cong 4.3\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 7: $\rho = \dfrac{2.90\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 4.640\times10^{-3}\,\Omega.\text{m}$
$\cong 4.6\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 8: $\rho = \dfrac{2.62\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 4.192\times10^{-3}\,\Omega.\text{m}$
$\cong 4.2\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 9: $\rho = \dfrac{2.57\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 4.112\times10^{-3}\,\Omega.\text{m}$
$\cong 4.1\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 10: $\rho = \dfrac{3.25\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 5.200\times10^{-3}\,\Omega.\text{m}$
$\cong 5.2\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 11: $\rho = \dfrac{4.80\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 7.680\times10^{-3}\,\Omega.\text{m}$
$\cong 7.7\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 12: $\rho = \dfrac{7.32\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 1.171\times10^{-3}\,\Omega.\text{m}$
$\cong 1.2\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 13: $\rho = \dfrac{3.45\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 5.520\times10^{-3}\,\Omega.\text{m}$
$\cong 5.5\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 14: $\rho = \dfrac{3.24\times10^{-6}\times3.2\times10^{-10}}{0.2000} = 5.184\times10^{-3}\,\Omega.\text{m}$
$\cong 5.2\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 15: $\rho = \dfrac{2.79\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=4.464\times10^{-3}\,\Omega.\text{m}$

$\cong 4.5\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 16: $\rho = \dfrac{7.32\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=1.171\times10^{-2}\,\Omega.\text{m}$

$\cong 1.2\times10^{-2} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 17: $\rho = \dfrac{11.90\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=1.904\times10^{-2}\,\Omega.\text{m}$

$\cong 1.9\times10^{-2} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 18: $\rho = \dfrac{7.06\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=1.130\times10^{-2}\,\Omega.\text{m}$

$\cong 1.1\times10^{-2} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 19: $\rho = \dfrac{7.96\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=1.274\times10^{-2}\,\Omega.\text{m}$

$\cong 1.3\times10^{-2} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 20: -

Trial 21: $\rho = \dfrac{4.73\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=7.568\times10^{-3}\,\Omega.\text{m}$

$\cong 7.6\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 22: -

Trial 23: $\rho = \dfrac{2.14\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=3.424\times10^{-3}\,\Omega.\text{m}$

$\cong 3.4\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 24: $\rho = \dfrac{3.56\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=5.696\times10^{-3}\,\Omega.\text{m}$

$\cong 5.7\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 25: $\rho = \dfrac{2.21\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=3.536\times10^{-3}\,\Omega.\text{m}$

$\cong 3.5\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 26: $\rho = \dfrac{2.00\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=3.200\times10^{-3}\,\Omega.\text{m}$

$\cong 3.2\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 27: $\rho = \dfrac{2.45\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=3.920\times10^{-3}\,\Omega.\text{m}$

$\cong 3.9\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 28: $\rho = \dfrac{5.70\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=9.120\times10^{-3}\,\Omega.\text{m}$

$\cong 9.1\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 29: $\rho = \dfrac{4.03\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=6.448\times10^{-3}\,\Omega.\text{m}$

$\cong 6.4\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Trial 30: $\rho = \dfrac{3.35\times10^{-6}\times3.2\times10^{-10}}{0.2000}$ $=5.360\times10^{-3}\,\Omega.\text{m}$

$\cong 5.4\times10^{-3} \pm 2.4\,\%\,\Omega.\text{m}$

Average resistivity of 200.0mm long lines (ρ_{av}) = $\frac{\sum_{k=1}^{30}\rho_k}{30}$

$$= \frac{\rho_1+\rho_2+\rho_3+...+\rho_{30}}{30}$$

$=0.006333\ \Omega.m$

$\cong 0.0063 \pm 2.4\ \%\ \Omega.m$

Relative error = $\left(\frac{\text{percentage error} \times \text{resistance}}{100}\right)$

$$=\left(\frac{0.0063\times 2.4}{100}\right) = 0.000151$$

$\cong 0.00015$

$\rho_{av200.0} = 0.0063 \pm 0.00015\ \Omega.m$

100.0 mm pencil lines

Trial 1: -

Trial 2: $\rho = \frac{8.30\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 2.656\times10^{-2}\ \Omega.m$

$\cong 2.6\times10^{-2} \pm 5.0\ \%\ \Omega.m$

Trial 3: $\rho = \frac{9.81\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 3.139\times10^{-2}\ \Omega.m$

$\cong 3.1\times10^{-2} \pm 5.0\ \%\ \Omega.m$

Trial 4: $\rho = \frac{11.31\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 3.619\times10^{-2}\ \Omega.m$

$\cong 3.6\times10^{-2} \pm 5.0\ \%\ \Omega.m$

Trial 5: $\rho = \frac{12.20\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 3.904\times10^{-2}\ \Omega.m$

$\cong 3.9\times10^{-2} \pm 5.0\ \%\ \Omega.m$

Trial 6: $\rho = \frac{5.20\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 1.664\times10^{-2}\ \Omega.m$

$\cong 1.7\times10^{-2} \pm 5.0\ \%\ \Omega.m$

Trial 7: $\rho = \frac{11.60\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 3.712\times10^{-2}\ \Omega.m$

$\cong 3.7\times10^{-2} \pm 5.0\ \%\ \Omega.m$

Trial 8: $\rho = \frac{5.93\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 1.898\times10^{-2}\ \Omega.m$

$\cong 1.9\times10^{-2} \pm 5.0\ \%\ \Omega.m$

Trial 9: $\rho = \frac{3.05\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 9.760\times10^{-3}\ \Omega.m$

$\cong 9.8\times10^{-3} \pm 5.0\ \%\ \Omega.m$

Trial 10: $\rho = \frac{3.36\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 1.075\times10^{-2}\ \Omega.m$

$$\cong 1.1\times 10^{-2} \pm 5.0\ \%\ \Omega.m$$

Trial 11: $\rho = \dfrac{7.50\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 2.400\times 10^{-2}\ \Omega.m$

$$\cong 2.4\times 10^{-2} \pm 5.0\ \%\ \Omega.m$$

Trial 12: -

Trial 13: $\rho = \dfrac{2.96\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 9.472\times 10^{-3}\ \Omega.m$

$$\cong 9.5\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 14: $\rho = \dfrac{4.67\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 1.494\times 10^{-2}\ \Omega.m$

$$\cong 1.5\times 10^{-2} \pm 5.0\ \%\ \Omega.m$$

Trial 15: $\rho = \dfrac{2.26\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 7.232\times 10^{-3}\ \Omega.m$

$$\cong 7.2\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 16: $\rho = \dfrac{1.06\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 3.392\times 10^{-3}\ \Omega.m$

$$\cong 3.4\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 17: $\rho = \dfrac{1.60\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 5.120\times 10^{-3}\ \Omega.m$

$$\cong 5.1\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 18: $\rho = \dfrac{3.01\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 9.632\times 10^{-3}\ \Omega.m$

$$\cong 9.6\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 19: $\rho = \dfrac{2.92\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 9.344\times 10^{-3}\ \Omega.m$

$$\cong 9.3\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 20: $\rho = \dfrac{3.32\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 1.062\times 10^{-2}\ \Omega.m$

$$\cong 1.1\times 10^{-2} \pm 5.0\ \%\ \Omega.m$$

Trial 21: $\rho = \dfrac{2.19\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 7.008\times 10^{-3}\ \Omega.m$

$$\cong 7.0\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 22: $\rho = \dfrac{2.28\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 7.296\times 10^{-3}\ \Omega.m$

$$\cong 7.3\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 23: $\rho = \dfrac{1.80\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 5.760\times 10^{-3}\ \Omega.m$

$$\cong 5.8\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 24: $\rho = \dfrac{3.68\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 1.178\times 10^{-3}\ \Omega.m$

$$\cong 1.2\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

Trial 25: $\rho = \dfrac{1.40\times 10^{-6}\times 3.2\times 10^{-10}}{0.1000} = 4.480\times 10^{-3}\ \Omega.m$

$$\cong 4.5\times 10^{-3} \pm 5.0\ \%\ \Omega.m$$

$$\text{Trial 26: } \rho = \frac{3.60\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 1.152\times10^{-2}\ \Omega.\text{m}$$
$$\cong 1.2\times10^{-2} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 27: } \rho = \frac{1.05\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 3.360\times10^{-3}\ \Omega.\text{m}$$
$$\cong 3.4\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 28: } \rho = \frac{1.36\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 4.352\times10^{-3}\ \Omega.\text{m}$$
$$\cong 4.4\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 29: } \rho = \frac{0.68\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 2.176\times10^{-3}\ \Omega.\text{m}$$
$$\cong 2.2\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 30: } \rho = \frac{1.51\times10^{-6}\times3.2\times10^{-10}}{0.1000} = 4.832\times10^{-3}\ \Omega.\text{m}$$
$$\cong 4.8\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Average resistivity of 100.0mm long lines } (\rho_{av}) = \frac{\sum_{k=1}^{30}\rho_k}{30}$$
$$= \frac{\rho_1+\rho_2+\rho_3+...+\rho_{30}}{30}$$
$$= 0.012667\ \Omega.\text{m}$$
$$\cong 0.0013 \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Relative error} = \left(\frac{\text{percentage error}\times\text{resistance}}{100}\right)$$
$$= \left(\frac{0.0013\times5.0}{100}\right) = 0.00065$$

$\rho_{av100.0} = 0.0013 \pm 0.00065\ \Omega.\text{m}$

50.0 mm pencil lines

Trial 1: -

$$\text{Trial 2: } \rho = \frac{1.32\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 7.920\times10^{-3}\ \Omega.\text{m}$$
$$\cong 7.9\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 3: } \rho = \frac{3.89\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 2.334\times10^{-2}\ \Omega.\text{m}$$
$$\cong 2.3\times10^{-2} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 4: } \rho = \frac{1.16\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 6.960\times10^{-3}\ \Omega.\text{m}$$
$$\cong 7.0\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 5: } \rho = \frac{1.84\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 1.104\times10^{-2}\ \Omega.\text{m}$$
$$\cong 1.1\times10^{-2} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 6: $\rho = \dfrac{3.92\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=2.352\times10^{-2}\,\Omega.\text{m}$

$\cong 2.4\times10^{-2} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 7: $\rho = \dfrac{2.12\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=1.272\times10^{-2}\,\Omega.\text{m}$

$\cong 1.3\times10^{-2} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 8: $\rho = \dfrac{2.66\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=1.596\times10^{-2}\,\Omega.\text{m}$

$\cong 1.6\times10^{-2} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 9: $\rho = \dfrac{4.26\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=2.556\times10^{-2}\,\Omega.\text{m}$

$\cong 2.6\times10^{-2} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 10: $\rho = \dfrac{1.28\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=7.680\times10^{-3}\,\Omega.\text{m}$

$\cong 7.7\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 11: $\rho = \dfrac{1.00\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=6.000\times10^{-3}\,\Omega.\text{m}$

$\cong 6.0\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 12: $\rho = \dfrac{1.38\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=8.280\times10^{-3}\,\Omega.\text{m}$

$\cong 8.3\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 13: $\rho = \dfrac{1.18\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=7.080\times10^{-3}\,\Omega.\text{m}$

$\cong 7.1\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 14: $\rho = \dfrac{1.48\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=9.880\times10^{-3}\,\Omega.\text{m}$

$\cong 9.9\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 15: $\rho = \dfrac{0.47\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=2.820\times10^{-3}\,\Omega.\text{m}$

$\cong 2.8\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 16: $\rho = \dfrac{0.76\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=4.560\times10^{-3}\,\Omega.\text{m}$

$\cong 4.6\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 17: $\rho = \dfrac{1.00\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=6.000\times10^{-3}\,\Omega.\text{m}$

$\cong 6.0\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 18: $\rho = \dfrac{1.08\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=6.480\times10^{-3}\,\Omega.\text{m}$

$\cong 6.5\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 19: $\rho = \dfrac{0.84\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=5.040\times10^{-3}\,\Omega.\text{m}$

$\cong 5.0\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$

Trial 20: $\rho = \dfrac{0.59\times10^{-6}\times3.0\times10^{-10}}{0.0500}$ $=3.540\times10^{-3}\,\Omega.\text{m}$

$$\cong 3.5\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 21: $\rho = \dfrac{0.86\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 5.160\times10^{-3}\ \Omega.\text{m}$

$$\cong 5.2\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 22: $\rho = \dfrac{0.62\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 3.720\times10^{-3}\ \Omega.\text{m}$

$$\cong 3.7\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 23: $\rho = \dfrac{0.87\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 5.220\times10^{-3}\ \Omega.\text{m}$

$$\cong 5.2\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 24: $\rho = \dfrac{0.89\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 5.340\times10^{-3}\ \Omega.\text{m}$

$$\cong 5.3\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 25: $\rho = \dfrac{0.16\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 9.600\times10^{-4}\ \Omega.\text{m}$

$$\cong 9.6\times10^{-4} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 26: $\rho = \dfrac{0.42\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 2.520\times10^{-3}\ \Omega.\text{m}$

$$\cong 2.5\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 27: $\rho = \dfrac{0.48\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 2.880\times10^{-3}\ \Omega.\text{m}$

$$\cong 2.9\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 28: $\rho = \dfrac{0.46\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 2.760\times10^{-3}\ \Omega.\text{m}$

$$\cong 2.8\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 29: $\rho = \dfrac{0.92\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 5.520\times10^{-3}\ \Omega.\text{m}$

$$\cong 5.5\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 30: $\rho = \dfrac{1.10\times10^{-6}\times3.0\times10^{-10}}{0.0500} = 6.600\times10^{-3}\ \Omega.\text{m}$

$$\cong 6.6\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Average resistivity of 50.0mm long lines (ρ_{av}) = $\dfrac{\sum_{k=1}^{30}\rho_k}{30}$

$$= \frac{\rho_1+\rho_2+\rho_3+\ldots+\rho_{30}}{30}$$

$$= 0.007667\ \Omega.\text{m}$$

$$\cong 0.0077 \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Relative error} = \left(\frac{\text{percentage error} \times \text{resistivity}}{100}\right)$$
$$= \left(\frac{0.0077 \times 10.0}{100}\right) = 0.00077$$

$\rho_{av50.0} = 0.0077 \pm 0.00077\ \Omega.\text{m}$

Average resistivity of graphite with 0.7 mm diameter ($\rho_{av0.7}$) = $\frac{\rho_{av200.0} + \rho_{av100.0} + \rho_{av50.0}}{3}$

$= 0.0090\ \Omega.\text{m}$

relative error of $\rho_{av0.7} = 0.00015 + 0.00065 + 0.00077 = 0.001571$

$\cong 0.0016$

$\rho_{av0.7}$ with relative error = $0.0090 \pm 0.0016\ \Omega.\text{m}$

percentage error = $\left(\frac{0.0016}{0.0090}\right) \times 100 = 17.77778\ \%$

$\cong 18.0\ \%$

$\rho_{av0.7}$ with percentage error = $0.0090 \pm 18.0\ \%\ \Omega.\text{m}$

7.3.3. Test 3: Graphite pencil with 0.9 mm diameter

Cross-Sectional Area Calculation

Diameter of graphite pencil used (2r): 0.9 mm
Radius of graphite pencil used (r): 0.45×10^{-3} m

200.0 mm pencil lines

Length of line (l) = 0.2000 ± 0.0005 m

$$\text{Percentage error} = \left(\frac{\text{relative error}}{\text{literature value}}\right) \times 100$$
$$= \left(\frac{0.0005}{0.2000}\right) \times 100$$
$$= 0.25\ \%$$
$$\cong 0.2\ \%$$

Length of line (l) = $0.2000 \pm 0.2\ \%$ m
Initial length of pencil (h_i) = 0.07200 ± 0.00005 m
Final length of pencil (h_f) = 0.06770 ± 0.00005 m
Length of pencil used (Δh) = $h_i - h_f$

$= (0.07200 \pm 0.00005) - (0.06770 \pm 0.00005) = 0.0043 \pm 0.0001$ m

$$\text{Percentage error} = \left(\frac{0.0001}{0.0043}\right) \times 100$$
$$= 2.38095\ \%$$
$$\cong 2\ \%$$

Length of pencil used (Δh) = $0.0043 \pm 2\ \%$

Volume of pencil used (V) = $\pi r^2 \Delta h$

$$= \pi \times (0.45\times10^{-3})^2 \times (0.0043 \pm 2\%)$$
$$= 2.73664\ \times 10^{-9} \pm 2\ \%\, m^3$$
$$\cong 2.7\times 10^{-9} \pm 2\ \%\ m^3$$

Average volume of line (V_{av}) = $\dfrac{V}{30}$

$$= \frac{2.7\times10^{-9} \pm 2\ \%}{30}$$
$$= 9.0\times 10^{-11} \pm 2\ \%\ m^3$$

Average cross sectional area of pencil lines (A) = $\dfrac{V_{av}}{l}$

$$= \frac{9.0\times10^{-11} \pm 2\%}{0.2000 \pm 0.2\%}$$
$$= 4.5\times 10^{-10} \pm 2.2\ \%\ m^2$$

100.0 mm pencil lines

Length of line (l) = 0.1000 ± 0.0005 m

Percentage error = $\left(\dfrac{\text{relative error}}{\text{literature value}}\right)\times 100$

$$= \left(\frac{0.0005}{0.1000}\right)\times 100$$
$$= 0.5\ \%$$

Length of line (l) = 0.1000 ± 0.5 % m

Initial length of pencil (h_i) = 0.07430 ± 0.00005 m

Final length of pencil (h_f) = 0.07200 ± 0.00005 m

Length of pencil used (Δh) = h_i - h_f

=(0.07430 ± 0.00005) – (0.07200± 0.00005) = 0.0023 ± 0.0001 m

Percentage error = $\left(\dfrac{0.0001}{0.0023}\right)\times 100$

$$= 4.347826\ \%$$
$$\cong 4\%$$

Length of pencil used (Δh) = 0.0023 ± 4 %

Volume of pencil used (V) = $\pi r^2 \Delta h$

$$= \pi \times (0.45\times10^{-3})^2 \times (0.0023 \pm 4\%)$$
$$= 1.46379\ \times 10^{-9} \pm 4\ \%\, m^3$$
$$\cong 1.5\times 10^{-9} \pm 4\ \%\ m^3$$

Average volume of line (V_{av}) = $\dfrac{V}{30}$

$$= \frac{1.5\times10^{-9} \pm 4\ \%}{30}$$
$$= 5.0\times 10^{-11} \pm 4\ \%\ m^3$$

Average cross sectional area of pencil lines (A) = $\frac{V_{av}}{l}$

$$= \frac{5.0\times10^{-11} \pm 4\%}{0.1000 \pm 0.5\%}$$
$$= 5.0\times 10^{-10} \pm 4.5\ \%\ m^2$$

50.0 mm pencil lines

Length of line (l) = 0.0500 ± 0.0005 m

$$\text{Percentage error} = \left(\frac{\text{relative error}}{\text{literature value}}\right)\times100$$
$$= \left(\frac{0.0005}{0.0500}\right)\times100$$
$$= 1\%$$

Length of line (l) = 0.0500 ± 1 % m

Initial length of pencil (h_i) = 0.07550 ± 0.00005 m

Final length of pencil (h_f) = 0.07430 ± 0.00005 m

Length of pencil used (Δh) = h_i - h_f

$$=(0.07550 \pm 0.00005) - (0.07430 \pm 0.00005) = 0.0012 \pm 0.0001\ m$$

$$\text{Percentage error} = \left(\frac{0.0001}{0.0020}\right)\times100$$
$$= 8.33333\ \%$$
$$\cong 8\ \%$$

Length of pencil used (Δh) = 0.0012 ± 8 %

Volume of pencil used (V) = $\pi r^2 \Delta h$

$$= \pi\times(0.45\times10^{-3})^2\times(0.0012\pm8\%)$$
$$= 7.63714\times10^{-10} \pm 8\%\ m^3$$
$$\cong 7.6\times10^{-10} \pm 8\%\ m^3$$

Average volume of line (V_{av}) = $\frac{V}{30}$

$$= \frac{7.6\times10^{-10} \pm 8\%}{30}$$
$$= 2.54571\times10^{-11} \pm 8\ \%\ m^3$$
$$\cong 2.5\times10^{-11} \pm 8\ \%\ m^3$$

Average cross sectional area of pencil lines (A) = $\frac{V_{av}}{l}$

$$= \frac{2.5\times10^{-11} \pm 8\%}{0.0500 \pm 1\%}$$
$$= 5.066667\times10^{-10} \pm 9\ \%\ m^2$$
$$\cong 5.1\times10^{-10} \pm 9\ \%\ m^2$$

Resistivity Calculation

200.0 mm pencil lines

$$\text{Trial 1: } \rho = \frac{2.95\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 6.638\times10^{-3}\ \Omega.m$$
$$\cong 6.6\times10^{-3} \pm 2.4\ \%\ \Omega.m$$

$$\text{Trial 2: } \rho = \frac{2.95\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 1.247\times10^{-2}\ \Omega.m$$

$\cong 1.2\times 10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 3: -

Trial 4: $\rho = \dfrac{4.45\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 1.001\times 10^{-2}\ \Omega.\text{m}$

$\cong 1.0\times 10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 5: $\rho = \dfrac{7.85\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 1.766\times 10^{-2}\ \Omega.\text{m}$

$\cong 1.8\times 10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 6: -

Trial 7: $\rho = \dfrac{10.78\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 2.426\times 10^{-2}\ \Omega.\text{m}$

$\cong 2.4\times 10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 8: $\rho = \dfrac{3.22\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 7.245\times 10^{-3}\ \Omega.\text{m}$

$\cong 7.2\times 10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 9: $\rho = \dfrac{7.65\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 1.721\times 10^{-2}\ \Omega.\text{m}$

$\cong 1.7\times 10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 10: $\rho = \dfrac{3.55\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 7.988\times 10^{-3}\ \Omega.\text{m}$

$\cong 8.0\times 10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 11: $\rho = \dfrac{2.11\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 4.748\times 10^{-3}\ \Omega.\text{m}$

$\cong 4.7\times 10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 12: $\rho = \dfrac{2.87\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 6.458\times 10^{-3}\ \Omega.\text{m}$

$\cong 6.4\times 10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 13: -

Trial 14: $\rho = \dfrac{7.61\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 9.675\times 10^{-2}\ \Omega.\text{m}$

$\cong 9.7\times 10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 15: $\rho = \dfrac{7.50\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 1.688\times 10^{-2}\ \Omega.\text{m}$

$\cong 1.7\times 10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 16: $\rho = \dfrac{15.60\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 3.510\times 10^{-2}\ \Omega.\text{m}$

$\cong 3.5\times 10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 17: -

Trial 18: $\rho = \dfrac{4.30\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 9.675\times 10^{-3}\ \Omega.\text{m}$

$\cong 9.7\times 10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$

Trial 19: $\rho = \dfrac{4.65\times 10^{-6}\times 4.5\times 10^{-10}}{0.2000} = 1.046\times 10^{-2}\ \Omega.\text{m}$

$\cong 1.0\times 10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$

$$\text{Trial 20: } \rho = \frac{4.65\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 1.046\times10^{-2}\,\Omega.\text{m}$$
$$\cong 1.0\times10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 21: } \rho = \frac{6.50\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 1.463\times10^{-2}\,\Omega.\text{m}$$
$$\cong 1.5\times10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 22: } \rho = \frac{2.55\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 5.738\times10^{-3}\,\Omega.\text{m}$$
$$\cong 5.7\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 23: } \rho = \frac{2.23\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 5.018\times10^{-3}\,\Omega.\text{m}$$
$$\cong 5.0\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 24: } \rho = \frac{14.86\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 3.344\times10^{-2}\,\Omega.\text{m}$$
$$\cong 3.3\times10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 25: } \rho = \frac{5.01\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 1.127\times10^{-2}\,\Omega.\text{m}$$
$$\cong 1.1\times10^{-2} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 26: } \rho = \frac{2.20\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 4.950\times10^{-3}\,\Omega.\text{m}$$
$$\cong 5.0\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 27: } \rho = \frac{2.35\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 5.288\times10^{-3}\,\Omega.\text{m}$$
$$\cong 5.3\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 28: } \rho = \frac{1.16\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 2.610\times10^{-3}\,\Omega.\text{m}$$
$$\cong 2.6\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 29: } \rho = \frac{1.78\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 4.005\times10^{-3}\,\Omega.\text{m}$$
$$\cong 4.0\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Trial 30: } \rho = \frac{1.81\times10^{-6}\times4.5\times10^{-10}}{0.2000} = 4.073\times10^{-3}\,\Omega.\text{m}$$
$$\cong 4.1\times10^{-3} \pm 2.4\ \%\ \Omega.\text{m}$$

Average resistivity of 200.0mm long lines ($\rho_{av200.0}$) = $\dfrac{\sum_{k=1}^{30} \rho_k}{30}$

$$= \frac{\rho_1 + \rho_2 + \rho_3 + \ldots + \rho_{30}}{30}$$
$$= 0.010 \pm 2.4\ \%\ \Omega.\text{m}$$

$$\text{Relative error} = \left(\frac{\text{percentage error} \times \text{resistance}}{100}\right)$$

$$= \left(\frac{0.010 \times 2.4}{100}\right) = 0.00024$$

$\rho_{av200.0} = 0.010 \pm 0.00024\ \Omega.\text{m}$

100.0 mm pencil lines

$$\text{Trial 1: } \rho = \frac{0.72 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 3.6 \times 10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 2: } \rho = \frac{0.44 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 2.2 \times 10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 3: } \rho = \frac{1.40 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 7.0 \times 10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 4: } \rho = \frac{0.43 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 2.150 \times 10^{-3}\ \Omega.\text{m}$$

$$\cong 2.2 \times 10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 5: } \rho = \frac{1.08 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 5.4 \times 10^{-3}\ \Omega.\text{m}$$

$$\text{Trial 6: } \rho = \frac{0.50 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 2.5 \times 10^{-3}\ \Omega.\text{m}$$

$$\text{Trial 7: } \rho = \frac{0.80 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 4.0 \times 10^{-3}\ \Omega.\text{m}$$

$$\text{Trial 8: } \rho = \frac{0.93 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 4.650 \times 10^{-3}\ \Omega.\text{m}$$

$$\cong 4.6 \times 10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 9: } \rho = \frac{1.87 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 9.350 \times 10^{-3}\ \Omega.\text{m}$$

$$\cong 9.4 \times 10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 10: } \rho = \frac{3.58 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 1.790 \times 10^{-2}\ \Omega.\text{m}$$

$$\cong 1.8 \times 10^{-2} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 11: } \rho = \frac{3.13 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 1.565 \times 10^{-2}\ \Omega.\text{m}$$

$$\cong 1.6 \times 10^{-2} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 12: } \rho = \frac{2.11 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 1.055 \times 10^{-2}\ \Omega.\text{m}$$

$$\cong 1.0 \times 10^{-2} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 13: } \rho = \frac{1.66 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 8.3 \times 10^{-3}\ \Omega.\text{m}$$

$$\text{Trial 14: } \rho = \frac{0.53 \times 10^{-6} \times 5.0 \times 10^{-10}}{0.1000} = 2.650 \times 10^{-3}\ \Omega.\text{m}$$

$$\cong 2.6 \times 10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 15: } \rho = \frac{1.80\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 9.0\times10^{-3}\ \Omega.\text{m}$$

$$\text{Trial 16: } \rho = \frac{1.35\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 6.750\times10^{-3}\ \Omega.\text{m}$$

$$\cong 6.8\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 17: } \rho = \frac{1.49\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 7.450\times10^{-3}\ \Omega.\text{m}$$

$$\cong 7.4\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 18: } \rho = \frac{1.45\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 7.250\times10^{-3}\ \Omega.\text{m}$$

$$\cong 7.2\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 19: } \rho = \frac{0.63\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 3.150\times10^{-3}\ \Omega.\text{m}$$

$$\cong 3.2\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 20: } \rho = \frac{1.48\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 7.4\times10^{-3}\ \Omega.\text{m}$$

$$\text{Trial 21: } \rho = \frac{5.16\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 2.580\times10^{-2}\ \Omega.\text{m}$$

$$\cong 2.6\times10^{-2} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 22: } \rho = \frac{0.87\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 4.350\times10^{-3}\ \Omega.\text{m}$$

$$\cong 4.4\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 23: } \rho = \frac{1.39\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 6.950\times10^{-3}\ \Omega.\text{m}$$

$$\cong 7.0\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 24: } \rho = \frac{1.49\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 7.450\times10^{-3}\ \Omega.\text{m}$$

$$\cong 7.4\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 25: } \rho = \frac{0.83\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 4.150\times10^{-3}\ \Omega.\text{m}$$

$$\cong 4.2\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 26: } \rho = \frac{1.69\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 8.450\times10^{-3}\ \Omega.\text{m}$$

$$\cong 8.4\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 27: } \rho = \frac{6.93\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 3.465\times10^{-2}\ \Omega.\text{m}$$

$$\cong 3.5\times10^{-2} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 28: } \rho = \frac{2.00\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 1.0\times10^{-2}\ \Omega.\text{m}$$

$$\text{Trial 29: } \rho = \frac{11.1\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 5.550\times10^{-3}\ \Omega.\text{m}$$

$$\cong 5.6\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 30: } \rho = \frac{1.95\times10^{-6}\times5.0\times10^{-10}}{0.1000} = 9.750\times10^{-3}\ \Omega.\text{m}$$

$$\cong 9.8\times10^{-3} \pm 5.0\ \%\ \Omega.\text{m}$$

Average resistivity of 100.0mm long lines (ρ_{av}) = $\dfrac{\sum_{k=1}^{30}\rho_k}{30}$

$$= \frac{\rho_1+\rho_2+\rho_3+...+\rho_{30}}{30}$$

$$=0.0083333\ \Omega.\text{m}$$

$$\cong 0.0083 \pm 5.0\ \%\ \Omega.\text{m}$$

$$\text{Relative error} = \left(\frac{\text{percentage error}\times\text{resistance}}{100}\right)$$

$$=\left(\frac{0.0083\times5.0}{100}\right)=0.000415$$

$$\cong 0.00042$$

$$\rho_{av100.0} = 0.0083 \pm 0.00042\ \Omega.\text{m}$$

50.0 mm pencil lines

Trial 1: $\rho = \dfrac{0.36\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 3.672\times10^{-3}\ \Omega.\text{m}$

$$\cong 3.7\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 2: $\rho = \dfrac{0.55\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 5.610\times10^{-3}\ \Omega.\text{m}$

$$\cong 5.6\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 3: $\rho = \dfrac{0.35\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 3.570\times10^{-3}\ \Omega.\text{m}$

$$\cong 3.6\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 4: $\rho = \dfrac{1.44\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.469\times10^{-2}\ \Omega.\text{m}$

$$\cong 1.5\times10^{-2} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 5: $\rho = \dfrac{0.29\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 2.958\times10^{-3}\ \Omega.\text{m}$

$$\cong 3.0\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 6: $\rho = \dfrac{0.75\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 7.650\times10^{-3}\ \Omega.\text{m}$

$$\cong 7.6\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 7: $\rho = \dfrac{0.47\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 4.794\times10^{-3}\ \Omega.\text{m}$

$$\cong 4.8\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 8: $\rho = \dfrac{0.31\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 3.162\times10^{-3}\ \Omega.\text{m}$

$$\cong 3.2\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 9: $\rho = \dfrac{1.51\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.540\times10^{-2}\ \Omega.\text{m}$

$$\cong 1.5\times10^{-2} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 10: } \rho = \frac{0.46\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 4.692\times10^{-3}\ \Omega.\text{m}$$
$$\cong 4.7\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 11: } \rho = \frac{0.14\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.428\times10^{-3}\ \Omega.\text{m}$$
$$\cong 2.4\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 12: } \rho = \frac{0.23\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 2.346\times10^{-3}\ \Omega.\text{m}$$
$$\cong 2.3\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 13: } \rho = \frac{0.14\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.428\times10^{-3}\ \Omega.\text{m}$$
$$\cong 1.4\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 14: } \rho = \frac{0.19\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.938\times10^{-3}\ \Omega.\text{m}$$
$$\cong 1.9\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 15: } \rho = \frac{0.14\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.428\times10^{-3}\ \Omega.\text{m}$$
$$\cong 1.4\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 16: } \rho = \frac{0.18\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.836\times10^{-3}\ \Omega.\text{m}$$
$$\cong 1.8\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 17: } \rho = \frac{0.18\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.836\times10^{-3}\ \Omega.\text{m}$$
$$\cong 1.8\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 18: } \rho = \frac{0.17\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.734\times10^{-3}\ \Omega.\text{m}$$
$$\cong 1.7\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 19: } \rho = \frac{0.12\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.224\times10^{-3}\ \Omega.\text{m}$$
$$\cong 1.2\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 20: } \rho = \frac{0.26\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 2.652\times10^{-3}\ \Omega.\text{m}$$
$$\cong 2.6\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 21: } \rho = \frac{0.21\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.142\times10^{-3}\ \Omega.\text{m}$$
$$\cong 2.4\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 22: } \rho = \frac{0.24\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 2.448\times10^{-3}\ \Omega.\text{m}$$
$$\cong 2.4\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

$$\text{Trial 23: } \rho = \frac{0.25\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 2.550\times10^{-3}\ \Omega.\text{m}$$
$$\cong 2.6\times10^{-3} \pm 10.0\ \%\ \Omega.\text{m}$$

Trial 24: $\rho = \dfrac{0.34\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 3.468\times10^{-3}\,\Omega.\text{m}$

$\cong 3.5\times10^{-3} \pm 10.0\,\%\,\Omega.\text{m}$

Trial 25: $\rho = \dfrac{0.21\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 2.142\times10^{-3}\,\Omega.\text{m}$

$\cong 2.1\times10^{-3} \pm 10.0\,\%\,\Omega.\text{m}$

Trial 26: $\rho = \dfrac{0.24\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 2.448\times10^{-3}\,\Omega.\text{m}$

$\cong 2.4\times10^{-3} \pm 10.0\,\%\,\Omega.\text{m}$

Trial 27: $\rho = \dfrac{0.45\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 4.590\times10^{-3}\,\Omega.\text{m}$

$\cong 4.6\times10^{-3} \pm 10.0\,\%\,\Omega.\text{m}$

Trial 28: $\rho = \dfrac{0.26\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 2.652\times10^{-3}\,\Omega.\text{m}$

$\cong 2.6\times10^{-3} \pm 10.0\,\%\,\Omega.\text{m}$

Trial 29: $\rho = \dfrac{0.62\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 6.324\times10^{-3}\,\Omega.\text{m}$

$\cong 6.3\times10^{-3} \pm 10.0\,\%\,\Omega.\text{m}$

Trial 30: $\rho = \dfrac{0.18\times10^{-6}\times5.1\times10^{-10}}{0.0500} = 1.836\times10^{-3}\,\Omega.\text{m}$

$\cong 1.8\times10^{-3} \pm 10.0\,\%\,\Omega.\text{m}$

Average resistivity of 50.0mm long lines (ρ_{av}) = $\dfrac{\sum_{k=1}^{30}\rho_k}{30}$

$= \dfrac{\rho_1 + \rho_2 + \rho_3 + \ldots + \rho_{30}}{30}$

$= 0.0036667\,\Omega.\text{m}$

$\cong 0.0037 \pm 10.0\,\%\,\Omega.\text{m}$

Relative error = $\left(\dfrac{\text{percentage error} \times \text{resistivity}}{100}\right)$

$= \left(\dfrac{0.0037\times10.0}{100}\right) = 0.00037$

$\rho_{av50.0} = 0.0037 \pm 0.00037\,\Omega.\text{m}$

Average resistivity of graphite with 0.9 mm diameter ($\rho_{av0.9}$) = $\dfrac{\rho_{av200.0} + \rho_{av100.0} + \rho_{av50.0}}{3}$

$= 0.0073333\,\Omega.\text{m}$

$\cong 0.0073\,\Omega.\text{m}$

relative error of $\rho_{av0.7} = 0.00015 + 0.00065 + 0.00077 = 0.00101$

$\cong 0.0010$

$\rho_{av0.9}$ with relative error = 0.0073 ± 0.0010 Ω.m

$$\text{percentage error} = \left(\frac{0.0010}{0.0073}\right)\times 100 = 13.69863\ \%$$

$$\cong 14.0\ \%$$

$\rho_{av0.9}$ with percentage error = 0.0073 ± 14.0 % Ω.m

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