TED ANKARA COLLEGE FOUNDATION PRIVATE HIGH SCHOOL

THE INTERNATIONAL BACCALAUREATE PROGRAMME

MATHEMATICS EXTENDED ESSAY

Mathematics In Music

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Research Question: How mathematics and music are related to each other in the way of traditional and patternist perspective?

Abstract

Links between music and mathematics is always been an important research topic ever. Serious attempts have been made to identify these links. Researches showed that relationships are much more stronger than it has been anticipated. This paper identifies these relationships between music and various fields of mathematics. In addition to these traditional mathematical investigations to the field of music, a new approach is presented: Patternist perspective. This new perspective yields new important insights into music theory. These insights are explained in a detailed way unraveling some interesting patterns in the chords and scales. The patterns in the underlying the chords are explained with the help of an abstract device named chord wheel which clearly helps to visualize the most essential relationships in the chord theory. After that, connections of music with some fields in mathematics, such as fibonacci numbers and set theory, are presented. Finally concluding that music is an abstraction of mathematics.

Word Count: 154 words.

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Introduction

Physics is the music of the existence whereas mathematics is its notes. This is the traditional perception of the existence but there are more than it meets the eye: Patterns. Indeed, everything in life are perceived by patterns so as mathematics. For example, one can perceive the sequence starting with: 1,2,3... as consecutive numbers (…4,5,6…) whereas another can perceive it as Fibonacci-like sequence (…5,8,13…). However while connections between music and fields of mathematics like trigonometry, logarithm, geometry (The geometry of musical chords – Dmitri Tymoczko, Princeton University) are deeply analysed over the years and patterns can be thought the fundamental pieces of everything, music has not been analysed in details through patterns. Based on this research of the literature, not only traditional connections between pure mathematics and music but patterns underlying the music are presented in a detailed way concluding with that music is just an abstraction of pure mathematics.

It is also been known that music is about emotions. It is not quite right to think emotions independent from mathematics though. They are indeed mathematics. In the light of the recent developments in computer science, it is now clear that they; love, anger, hatred, disgust etc... are all mathematical models in human mind. Thus, emotions and mathematics are not opposite concepts. On the other hand, it is also equally wrong to try to perform music with pure mathematics. Emotions which are an abstraction of mathematics and even though we did not aware their model in our mind or how they are operating while feelings are aroused, one needs to know intuitively to use and combine them "correctly" with pure mathematics i.e. theory of chords and scales, for performing "good" music. In conclusion, emotions are mathematical models, one is doing mathematics while performing music and one does not indeed aware of this mathematical process being taken place in between our networks of neurons.

Based on the observations and facts stated above, it can be safe to state that music is nothing but an abstraction of mathematics in every perspective. The research question of this extended essay is; How mathematics and music are related to each other in the way of traditional and patternist perspective?

This paper examines mathematics behind the music starting from the fundamentals of sound continuing with the chord theory which is investigated with the patternist perspective. Finally, it investigates the core connection with some fields of pure mathematics and while some applications of certain fields are surveyed in a detailed manner, a complete overview of the complex patterns in music and connections with mathematics is not intended.

MATHEMATICS IN MUSIC

SOUND BASICS

1.1.Physics of Sound:

Sound is the vibrations which can travel only in the existence of a medium. Vibrations take place between the molecules of the substance, and the vibrations move through the medium in the waves of sound. The sound can move in a medium more easily depending on two properties; the nature of the sound and the nature of the medium. For instance; some frequencies can move more easily through certain mediums than the other frequencies, and some frequencies can travel forth. When the vibrations travel through the medium, particles(the molecules of the medium) hit each other and they come back to their previous positions. Since therefore some parts of the medium become denser and they are called condensations. Less dense parts are called rarefactions.

1.2.Properties of Waves

(İ)Speed of Wave

The speed sound waves is strictly dependent on the medium. In air with 21^{o} C the speed is approximately 344 m/s. Sound waves travel much more distances in solid or liquid than in air. For instance, the diffusion of sound waves is 1.4 km/s in water, 5000 km/s in steel. Here is the formual for the calcuation of the speed:

$$c = 20.05 \times \sqrt{273 + T}$$

c : speed of sound, m/sn

T : temperature, ^{o}C

Example:

Speed of sound in 21^{o} C:

$$c = 20.05 \times \sqrt{273 + 21}$$

$$c = 343.86 \text{ m/s}$$

Speed of sound in 0^{o} C

$$c = 20.05 \times \sqrt{273 + 0}$$

$$c = 331.23 \text{ m/sn}$$

Speed of sound in -273^{o} C namely 0^{o} K absolute zero

$c = 20.05 \times \sqrt{(273 - 273)}$

$c = 0$ m/sn

* Giving us the insight that in a medium with no molecules moving, the sound can not diffuse.

(ii)Frequency

Frequency is the number of vibrations of a wave in a second.(Reference 8) Namely it is 1 / sn which **Hertz** abbreviated as **Hz**. When the number of vibrations increase, i.e. the frequency increases, the sound gets sharper, when the number of vibrations decrease the sound gets lower.

Human ear can hear between 20 Hz – 20,000 Hz(20 kHz); although very few people can hear very high frequencies such as 20 kHz. Roughly, the juvenile human cannot hear above 17 kHz whereas elder ones cannot hear above 15 kHz.

(iii)Wavelength

It is the distance over which the wave's shape repeats. (Reference 8) Mathematically, it can be found simply by dividing the speed of the wave to the frequency of the wave.

$\lambda = c / f$

λ : Wavelength, m

c : speed, m/sn

f : frequency, Hz

Example:

Given the temperature as 21^{o} C hence obtaining the speed as 344 m/sn, the wavelength of a wave with 2 kHz is 17 cm.

$\lambda = c / f$

$\lambda = (344 \text{ m/sn}) / (2000 \text{ Hz}) = 0.17 \text{ m} = 17 \text{ cm}$

(İV)**Period**

Period is the duration between the start one cycle to another cycle of the wave. (Reference 8)Mathematically it can be found by taking the reciprocal of the frequency.

T = 1 / f

T : period, sn

f : frequency, Hz

Example:

A 50 Hz wave completes it's one period in 1 / (50 Hz) namely 0.02 seconds i.e. period of a 50 Hz wave is 0.02 sn.

(V)**Amplitude**

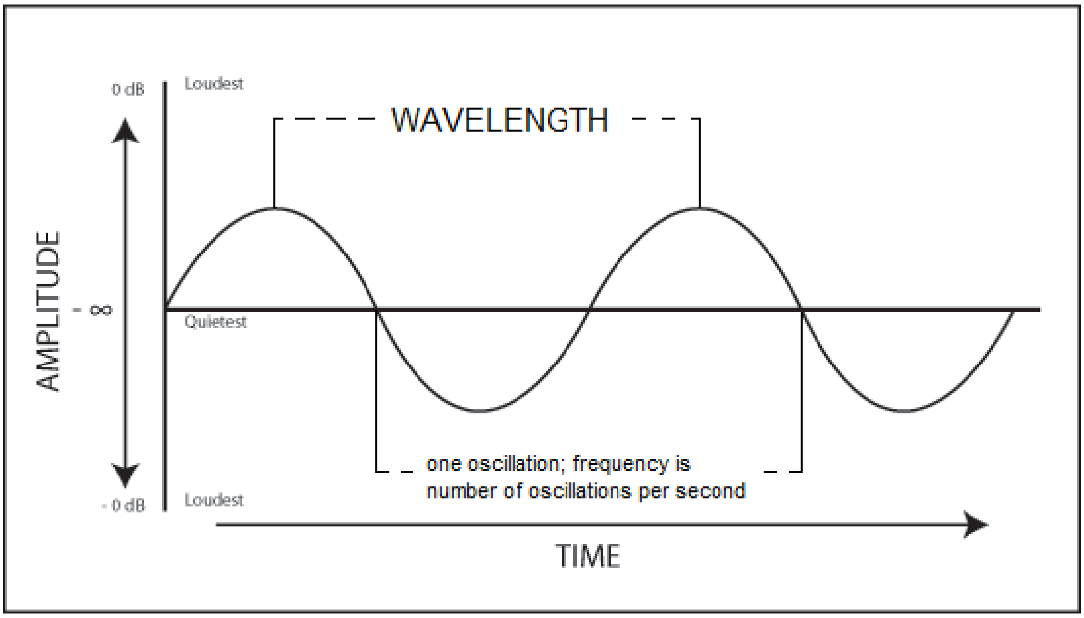

Figure 1 : Properties of the waves. (Reference 13)

1.3.Pitch

Pitch is an auditory sensation in which a listener assigns musical tones to relative positions on a musical scale based primarily on the frequency of vibration. Pitch is closely related to frequency, but the two are not equivalent. Frequency is an objective, scientific concept, whereas pitch is subjective. Sound waves themselves do not have pitch, and their oscillations can be measured to obtain a frequency. It takes a human brain to map the internal quality of pitch. (Reference 12)

1.4.Pitch Frequencies

Here is the table of frequencies of pitches, and pitch ranges of various instruments.

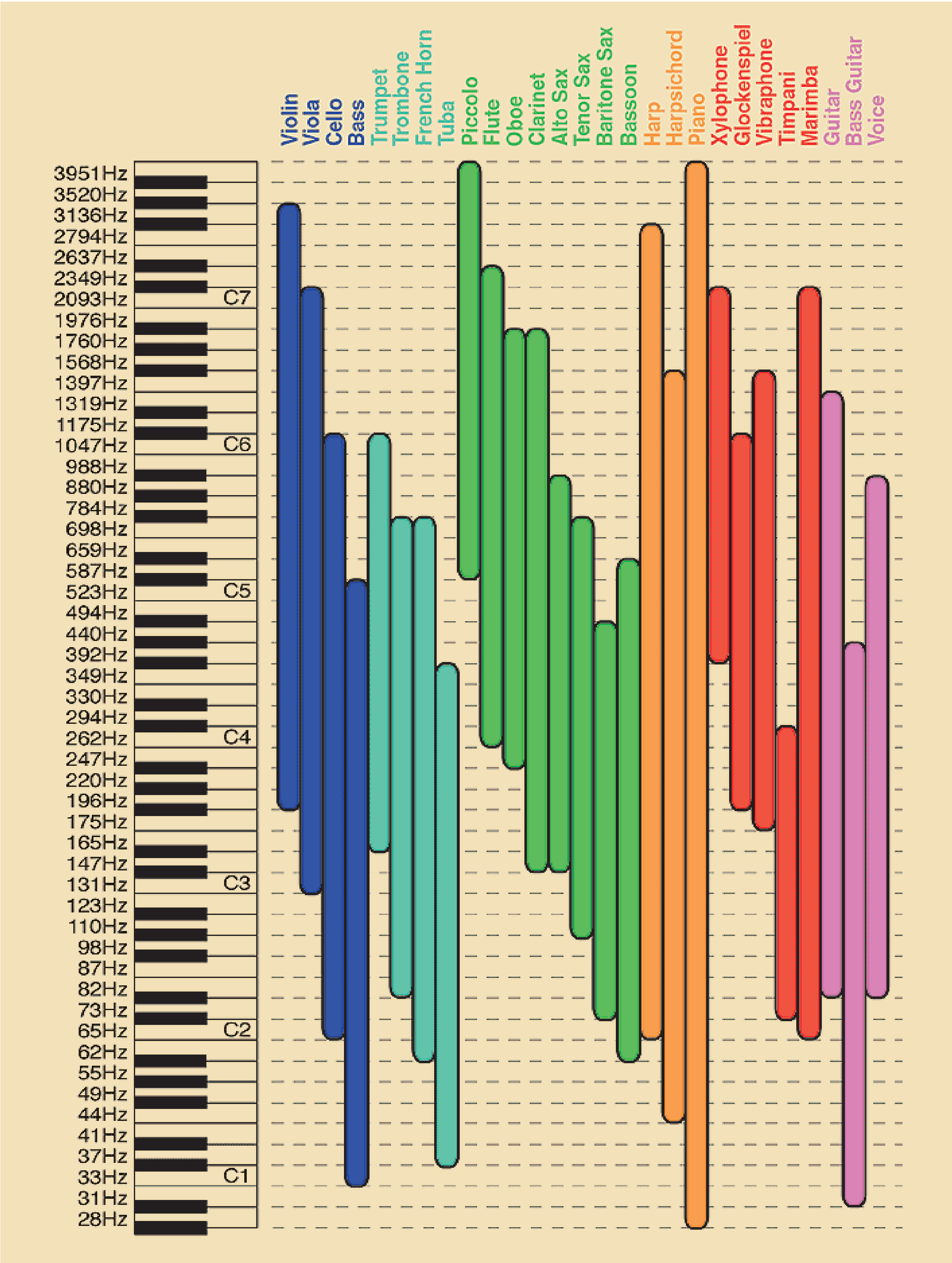

Figure 2 : Piano Key frequencies and Ranges of Various Instruments. (Reference 1)

1.5.Timbre and Fundamental Frequency

The American Standards Association definition 12.9 of timbre describes it as "that attribute of sensation in terms of which a listener can judge that two sounds having the same loudness and pitch are dissimilar", and a note to this definition adds that "timbre depends primarily upon the spectrum of the stimulus, but it also depends upon the waveform, the sound pressure, the frequency location of the spectrum, and the temporal characteristics of the stimulus" (Reference 4).

What this property tells us that, two same pitches with the same volume are totally perceived differently from different instruments. Imagine that a guitar and a piano are played in the same octave and pitch with same volume; since these instruments have different timbres, we can differentiate the instruments by hearing the notes produced.

The lowest frequency produced by any particular instrument is known as the **fundamental frequency.** (Reference 8) The instrument produces partial frequencies with the fundamental frequency named as *partials*. The higher frequencies (above fundamental frequency) produced are named as *upper partials* or *overtones.* Timbre is emerged from the overlapping of the fundamental frequency and the partials. Timbre often referred as the *color of sound.*

Integral multiples of fundamental frequency are the frequencies called as harmonic. As for an example, let's choose A4 note, 440 Hz as fundamental frequency. First harmonic of A4 is itself; second harmonic is (440 x 2) 880 Hz; third harmonic is (440 x 3) 1320 Hz. Octave is referred as 2:1 frequency ratio. One octave above means the twice of the frequency, conversely one octave below is the half of the frequency. (See the table below)

Fundamental Frequency	First Harmonic	Unison
	Second Harmonic	Octave +1
	Third Harmonic	
	Fourth Harmonic	Octave +2

Figure 3 : Generic Fundamental Frequency Table

Applying the table above for the A4 note, 440 Hz:

440 Hz(Fundamental Frequency)	First Harmonic	Unison	A4
880 Hz	Second Harmonic	Octave +1	A5
1320 Hz	Third Harmonic		
1760 Hz	Fourth Harmonic	Octave +2	A6

Figure 4 : Fundamental Frequency Table for A4 note

PATTERNS IN MUSIC THEORY

2.1.CHORDS[1]

(i)What Is Chord?

A sound set composing of minimum three notes, which the relationships between these notes are in 3rds, called a chord. There is a strong relationship between notion of chords and the concept pitch. For example in the scale given below, we can construct a chord according to definition above by combining three notes starting with a random note and jumping to 3rds (either minor or major).

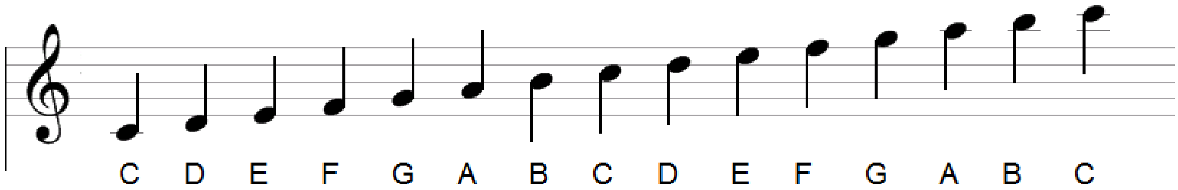

Figure 5 : Natural scale

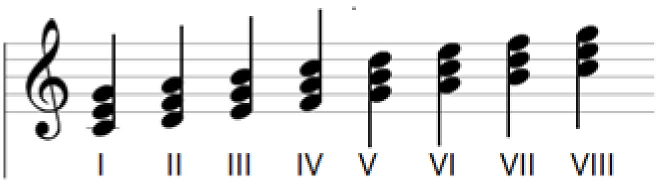

Figure 6 : Harmonized scale of C

I; C – E – G
II; D – F – A
III; E – G – B
IV; F – A – C
V; G – B – D
VI; A – C – E
VII; B – D – F
VIII, OCTAVE ; C – E – G

Hence, we can construct a chord based on any note and we will give the name *TRIAD* for chords consisting of exactly three notes that have interval of 3rd. The base note will be called as *ROOT NOTE* i.e. for the chord **I** it will be C, for **II** it will be D. The chords are constructed according to this root note again for the chord **I**, E will be 3rd of base note C, G will be 3rd of E.

[1] The chords, notes and scales in this chapter are generated using Guitar Pro software.(Reference 6)

The type of triad will be determined by the quality of the 3rd interval; major or minor.

See below fo different types of triads. (Reference 3)

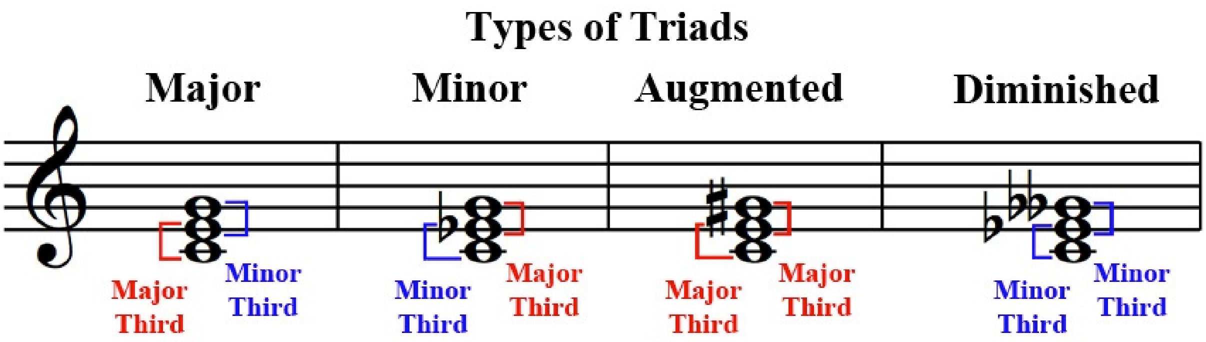

Figure 7 : Triads(Reference 7)

Chords can be constructed with more than 3 notes with 3rd interval relationship; thus becoming *SEVENTH(chord I below)*, *NINTH(chord II)*, *ELEVENTH(chord III)* or *THIRTEENTH(chord IV)* chord. Note that the ninth chord also contains the seventh note, the eleventh contains both the seventh and the ninth and so forth.

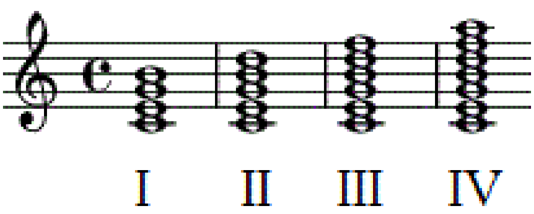

Figure 8 : chords being constructed with more than 3 notes

(ii)Types of Chords

Here types of chord will be introduced very briefly. There are a number of types of chords such as major, minor, dominant and so forth. *(Refer to the appendix for a full list of chord namings)*

The patterns in the chords are based on triads given above. The numbers in the chords indicate the new node added the chord with the respect to its interval relative to root note. For example for the C6 chord, 6 means the 6th of root note which is C, hence giving us the A added in the chord.

(a) Major Chord

This type of chord is based on the major triads and the chords given in figures 9,10 below. (Reference 18)

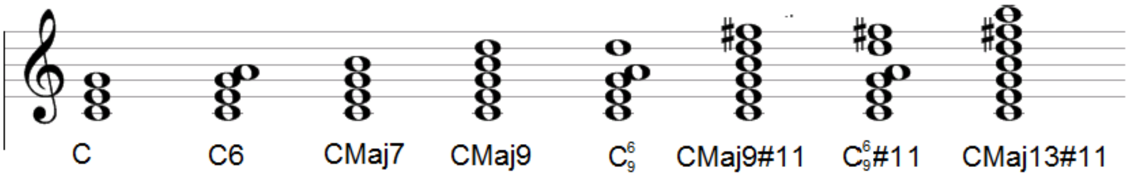

Figure 9 : Major chords of C

A chord can be altered by lowering or raising the 5th of the Chord by a half step; a *flat* 5 (b5) or a *sharp* 5 (#5 or +5). Some Chords like the CMaj7(#5,b5) have to use a flat for the sharp as the "G" would be too confusing with both a flat and a sharp "G".

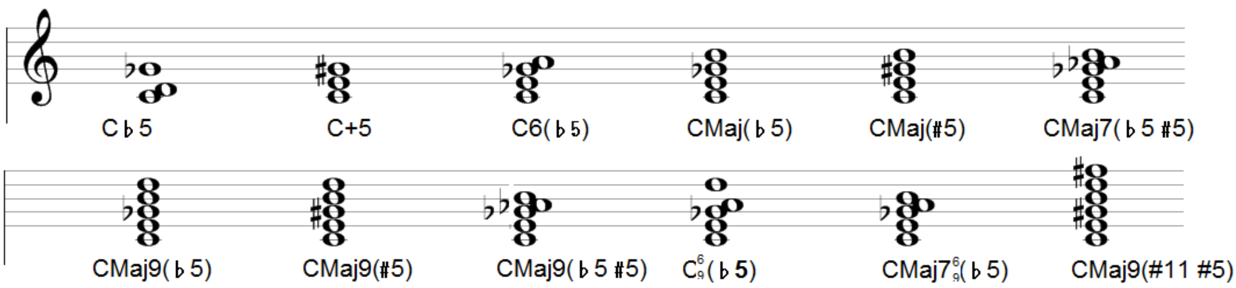

Figure 10 : Altered Major chords of C

(b) Minor Chord

This type of chord is based on the major triads and the chords given in figure 11, 12 below. (Reference 2,3)

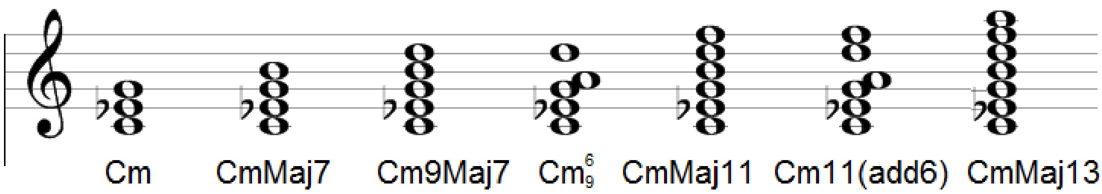

Figure 11 : Minor chords of C

The alteration process can be performed exactly the same way in the construction of Major chord.

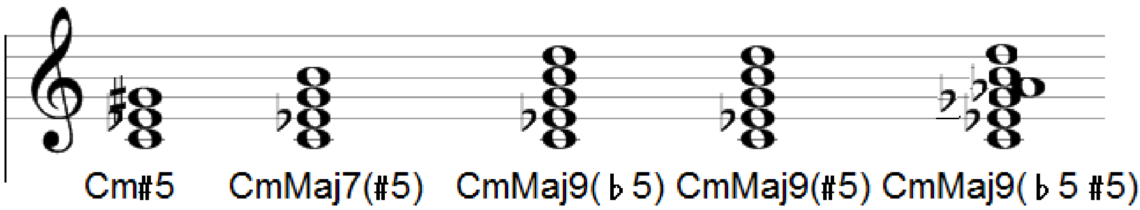

Figure 12 : Altered Minor chords of C

(c) Dominant Chord

This type of chord is based on the major triads and the chords given in figure 13 below. (Reference 2,3)

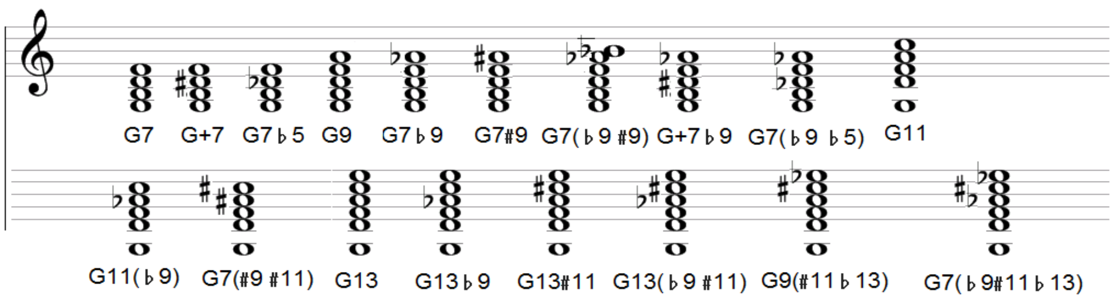

Figure 13 : Dominant chords of C including Alterations

(d) Diminished Chord

This type of chord is based on the major triads and the chords given in figure 14. (Reference 2,3)

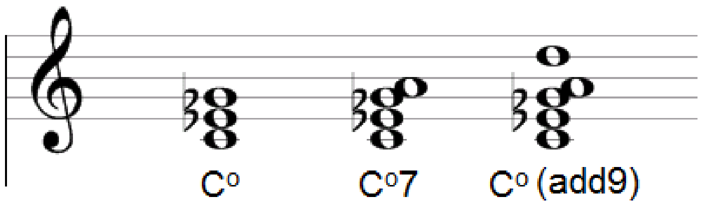

Figure 14 : Diminished chords of C

(e) Other Chord Forms(Reference 18)

A "sus4" Chord (suspended 4th) is basically a triad with an altered 3rd raised to 4th. Example: "C" Triad consists of C-E-G, but when altered to a [Csus4] the "E" is augmented up to the next scale degree (F) and is written as C-F-G.

Another type of Chord used a lot in Rock music is the "Power Chord" (also known by other names) which basically uses only the root and 5th of the triad. It can be notated as [C5] or [C no3].

Other variations on Chords can be written with an "add" notice. For example, [CMaj(add9)] would be a "C" Major Triad with the 9th (D) added on top. Different that the Maj9 Chord as the Major 7th is deliberately left out.

(iii)Harmonized Scales

In any major key there are seven basic chords, known as the *diatonic chords.* (Reference 3) These chords are constructed by using every other note of the major scale. There is one chord for each unique note of the scale.

The chord families of C and Cm is given below. Chord Is represent the *tonic* chord of the family.

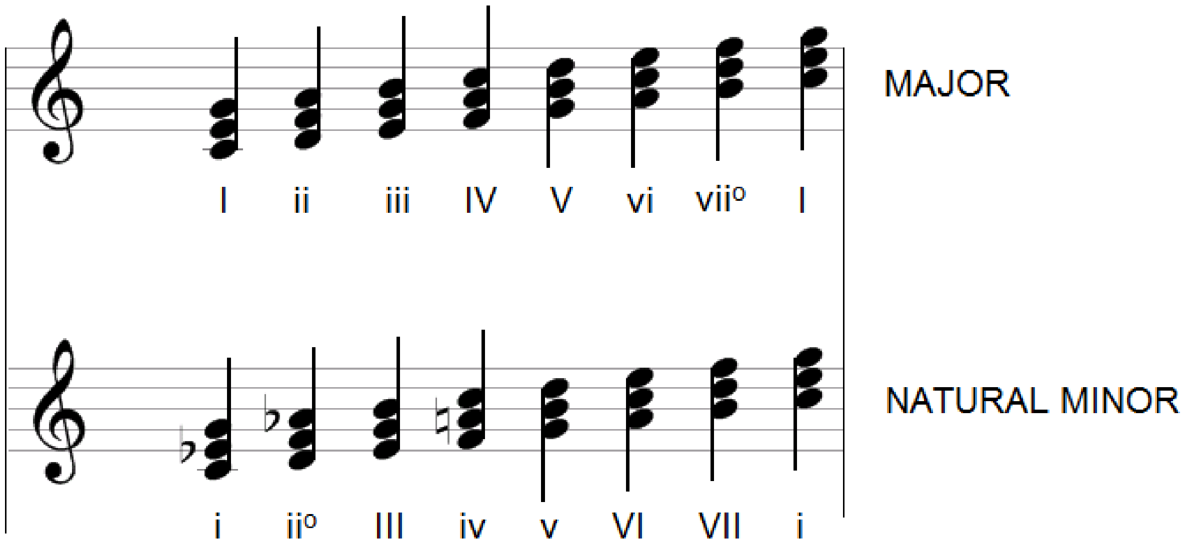

Figure 15 : Harmonized Scale of C and Cm

(a)Major Scale

The roman numerals given indicates the nature of the chord such that;

I, IV, V represents MAJOR chords; C, F, G respectively.

ii, iii, vi represents MINOR chords; Dm, Em, Am respectively.

vii^{o} represents DIMINISHED chord; B^{o} .

(b)Minor Scale

The roman numerals given indicates the nature of the chord such that;

III, VI, VII represents MAJOR chords; Eb, Ab, Bb respectively.

i, iv , v represents MINOR chords; Cm, Fm, Gm respectively.

ii^{o} represents DIMINISHED chord; D^{o} .

(iv)Chord Relationships

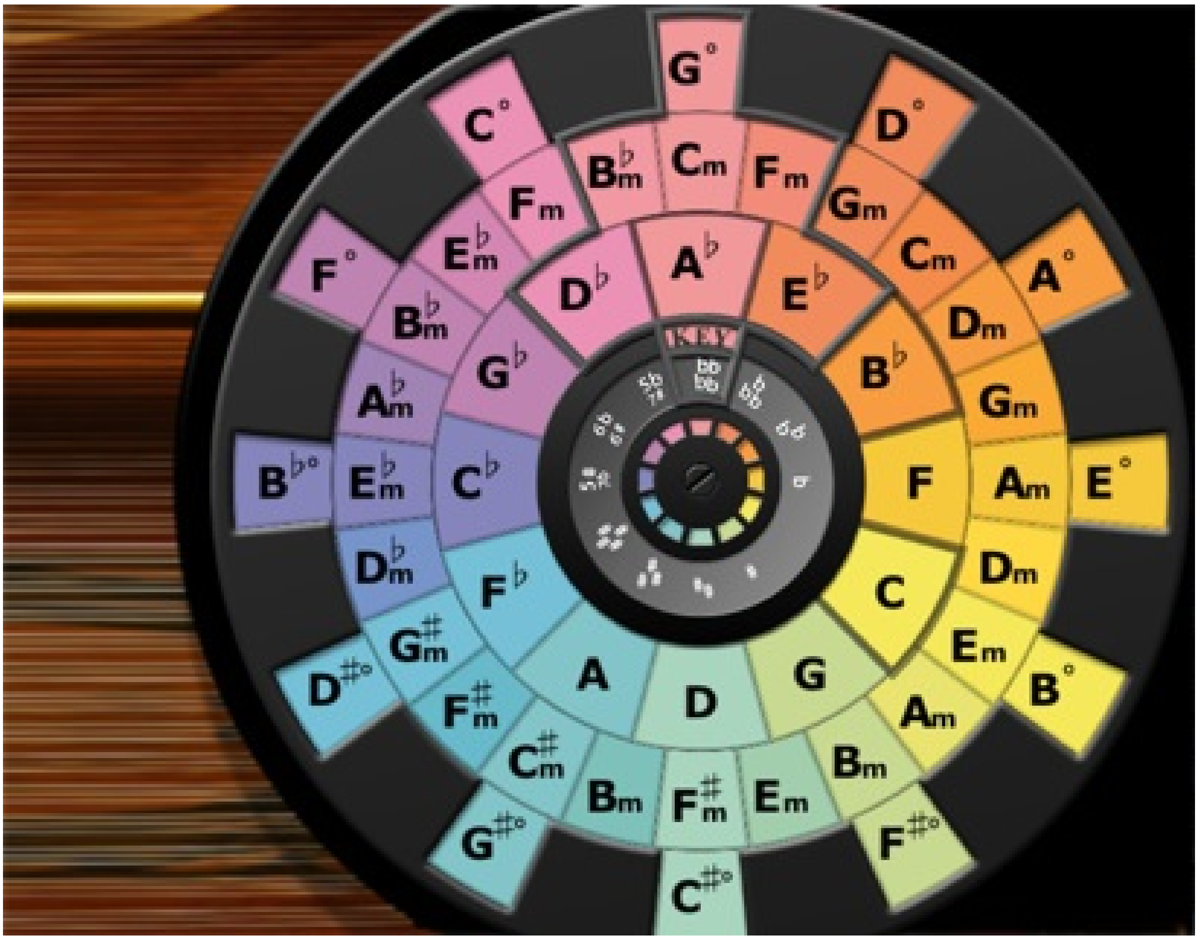

Figure 16 : Chord Wheel

In this part, mathematical relationships between the chords and their properties are introduced.

The **chord wheel** (figure 16) perfectly represents the relationship between the chords. (Reference 2) The inner circle represents the tonic chord. The middle circle represents the minor chords, i.e. ii, iii and vi chords, of the tonic chords. The outer circle represents the diminished chords of the tonic chords.

Here is the crucial pattern of the chord wheel for tonic key **F**. The neighbourhood chords are **IV** and **V** chords which are major chords whereas **ii**,**iii** and **vi** chords are minor chords and **vii°** chords in the family.

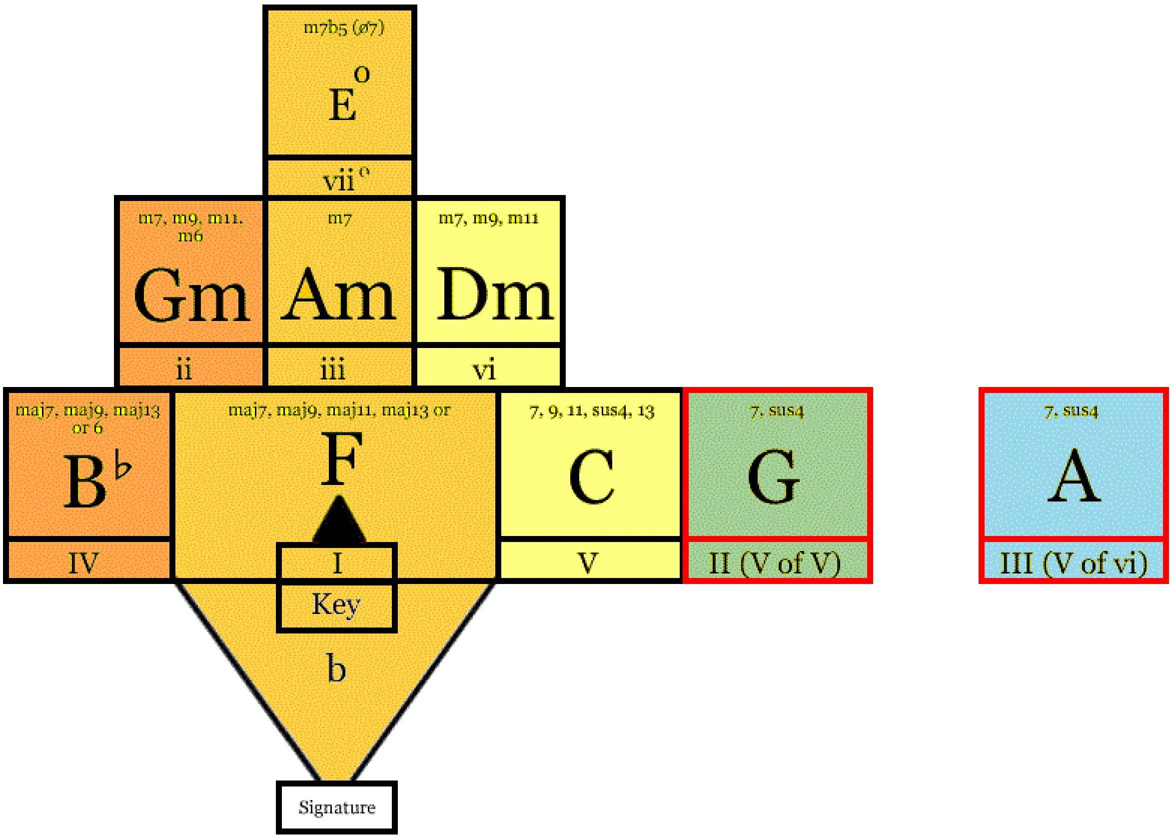

Figure 17 : Interpretation of the chord Wheel (Reference 2)

Some other underlying patterns in the chord wheel are:

- The inner chord circle goes with **perfect fifths** clockwise, **perfect fourths** counter-clockwise i.e. Clockwise Db -> Ab -> Eb (pattern of fourths), counter-clockwise Eb -> Ab -> Db (pattern of fifths).
- The middle chord circle goes with **major twos** succeeding with **perfect fourths** clockwise, **minor sevenths** succeeding **perfect fifths** counter-clockwise i.e. Clockwise Fm -> Gm -> Cm -> Dm -> Gm -> Am -> Dm(pattern of major twos and perfect fourths), counter-clockwise Dm -> Am ->Gm -> Dm -> Cm -> Gm -> Fm (pattern of perfect fifths and major twos).
- The outer chord circle goes perfect fourths clockwise and perfect fifths counter-clockwise.

- Key signature patterns given in the inner gray circle are as follows:
  - Starting with C, one # is added clockwise, one ♭is added counter-clockwise. Hence, the next key that is fourth of a current key has one more ♭, whereas fifth of the current key has one more #. (Observe that # + ♭= natural sound)
  - One more important pattern:
    - E has 4#s, E ♭ has 3 ♭s.
    - C has nothing, C# has 7#, C ♭has 7 ♭.
    - B has 5#, B ♭ has 2 ♭.

  Where they are all add up to 7.

2.2.SCALES

Scale is a simply sequence of notes in some predefined order. The order of the notes determines the type of the scale. There exists many scales in music, however the most important and fundamental ones will be explained here. These are; (Reference 3)

- Chromatic
- Modal System; Ionian, Dorian, Phyrgian, Lydian, Mixolydian, Aeolian, Locrian
- Major
- Minor
- Pentatonic

Note: Below, W stands for Whole step, H stands for half step

(i)Chromatic Scale:

This scale consists of 12 notes from the key to the key one octave higher/lower. The steps are all *H.*

(ii)Modal System:

- **Ionian Scale** ***:*** *W-W-H-W-W-W-H*
- **Dorian Scale :** *W-H-W-W-W-H-W*
- **Phyrgian Scale :** *H-W-W-W-H-W-W*
- **Lydian Scale :** *W-W-W-H-W-W-H*
- **Mixolydian Scale :** *W-W-H-W-W-H-W*

- **Aeolian Scale :** *W-H-W-W-H-W-W*
- **Locrian Scale :** *H-W-W-H-W-W-W*

(iii)Major Scale:

It is the Ionian scale made up of 7 notes. The steps are: *W-W-H-W-W-W-H*

(iv)Minor Scale:

It is the Aeolian scale made up of 7 notes. The steps are: *W-H-W-W-H-W-W*

CONNECTIONS WITH VARIOUS FIELDS OF MATHEMATICS

3.1. Connections With Logarithms

In the science of audio, logarithms are used in sound intensity and frequency.

(i) Sound Intensity (Reference 9)

The intensity of sounds depends on the vibration frequency of the object. Intensity is an important feature with the pitch and frequency. Because of the even considerably high changes in pressure, power and voltage values cause relatively low changes in the sound intensity, logarithmic scale is used instead of linear scale for the measurements of the sound and signals.

(ii)Desibel (dB): Desibel is the unit measure used in the sound and signal processing. It is easier to denote considerably high and low values. It is always used for to express the **ratio** of two values. In a technical way of expressing; dB is used for expressing the logarithmic ratios between the electricity, acoustic and power measurements.

Here are the formulas:

$$\mathbf{dB = 10 \log (P / P_{ref})}$$

where **P** is power in *watts,* $\mathbf{P_{ref}}$ is the reference power in *watts.*

Let's calculate the ratio of 2 Watt to 1 Watt:

dB = 10 log(2 / 1)

= 10 log2

dB = 3

Now let's calculate the ratio of 10 Watt to 1 Watt:

dB = 10 log(10 / 1)

= 10 log10

dB = 10

So these numbers tell us some important facts:

- If the power is doubled, we got an increase of +3 dB, if the power is being halved, we got a decrease of -3 dB.
- If the power is multiplied by 10, we got an increase of +10 dB, if the power is divided by 10, we got a decrease of -10 dB.

(iii) Frequency (Revisited) (Reference 9)

Remember an octave higher has 2 times as much frequency as the reference pitch. Thus, if the two tones have frequency ratio of four, that would give us a 2 octave difference.

$$\textbf{Difference in Octaves} = \log_2(f1 / f2)$$

where **f1** and **f2** are the frequencies of the pitches being compared.

Let's illustrate ths wih an example by calculating the octave difference between f1 = 440Hz and f2 = 1760Hz i.e. pitch 'A' in different octaves;

DiO = $\log_2 1760 / 440$

DiO = 2, hence A;440 Hz is 2 octaves lower than the A;1760Hz.

3.2. Connections With Trigonometry[2]

When sound is generated, the vibrations in the air causes for sound waves to be produced. Depending on the frequency and how loud the music is playing, the amplitude will change over time(*See Part I: Sound Basics*). As the French heat engineer Fourier proved that any wave can be interpreted with some combinations of sines and cosines, which can be also applied to sound waves since they are ordinary waves indeed. Here is the basic formula of a sound generated from a musical instrument: (Reference 10)

$$f(t) = A \sin(2 \pi w t) + B \cos(2 \pi w t)$$

[2] The graphics in this chapter are generated using MATLAB software. (Reference 5)

where **t** represents time, **A,B** represents the amplitude and **w** represents the frequency i.e. pitch. Some examples:

- $f(t) = \sin(2 \pi t)$

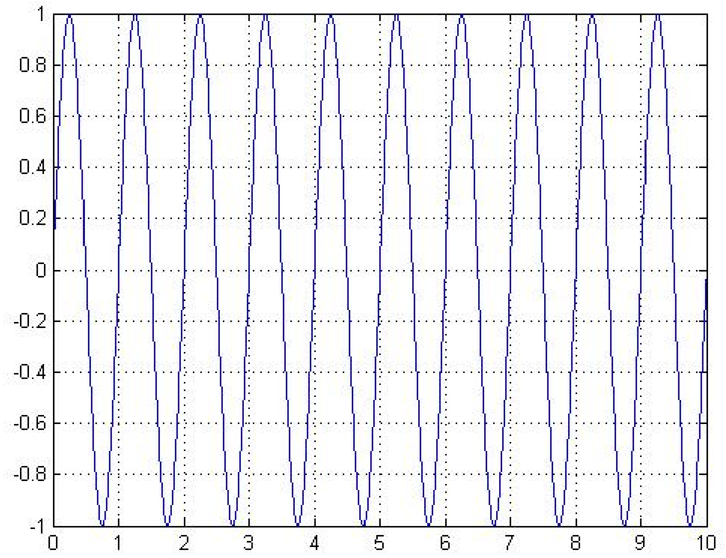

Figure 18

- $f(t) = 2 * \sin(2 \pi t)$ *(the amplitude is two times greater than above)*

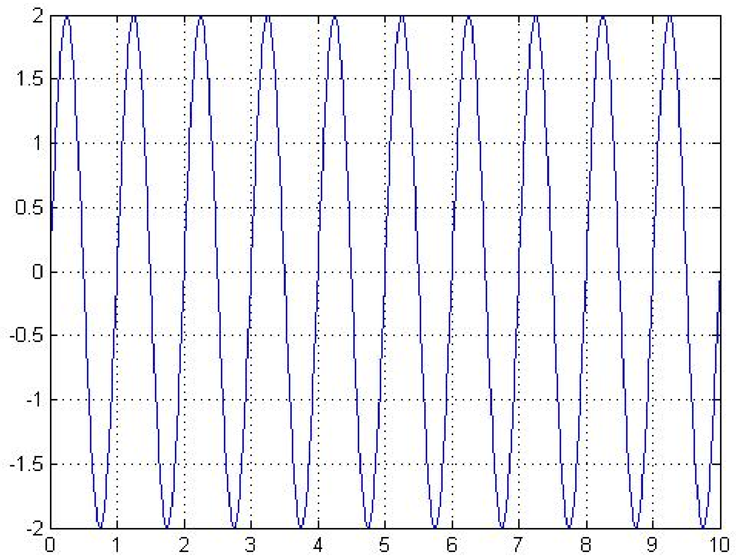

Figure 19

- f(t) = sin (2 π t * 0.5) *(note that the cycle time i.e. period doubles, since the frequency halves, with respect to the first example)*

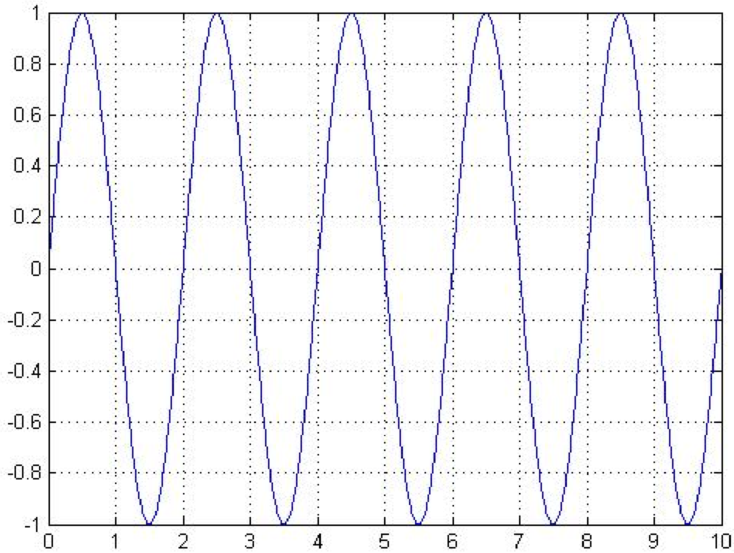

Figure 20

- f(t) = cos (2 π t)

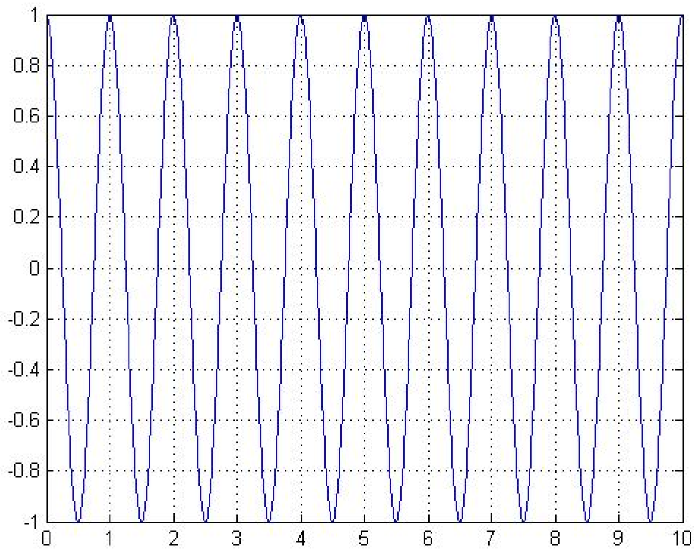

Figure 21

- f(t) = sin (2 π t) + cos (2 π t) *(addition of the first example and the preceding one)*

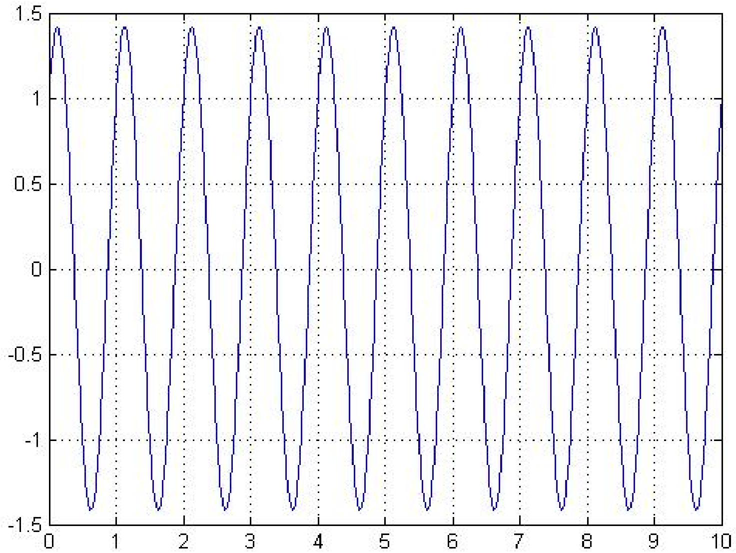

Figure 22

Synthesizer players use sinusoidal waves (and some other shapes like triangle, square) to produce new sounds like the sounds in electronic/techno music from this very basic level!

3.3. Connections With Set Theory

Here is the Venn Diagram Of The Scales C- Chromatic and C-Dorian;

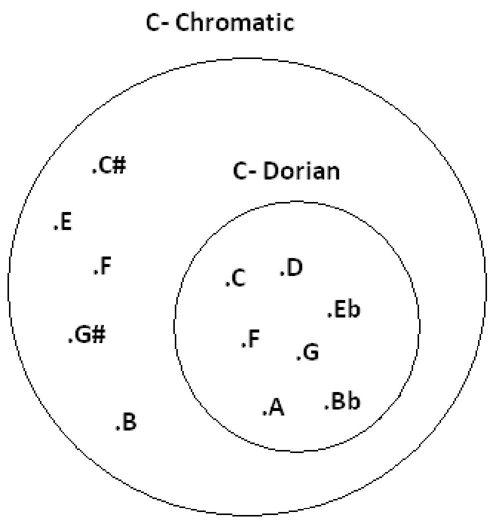

Figure 23: Venn schema of chromatic scale

From the Venn schema above;

C – Chromatic Scale ⊃ C – Dorian

However, if we view the notes left behind [C-Chromatic Scale] – [C – Dorian] again; it will be seen that the set made by those elements form the **E major pentatonic** scale. So in mathematical terms;

E – Major Pent. = (**C – Dorian**)'

This patterns are very useful for musicians for tone modulation jumping from one chord to another throughout the verses during a particular piece.

3.4.Connections With Golden Ratio and Fibonacci Numbers

(İ)What Is Golden Ratio?

Golden ratio was discovered by ancient greeks and by ancient egyptians a long time ago. Golden ratio is a special number which exist in the shape and structure of most of the living and non living organisms in the nature. It is shown by the symbol phi (φ) and its value is equal to 1.61803398874989484820...

Let AB be a line divided by the point C.

AC = x

CB = y

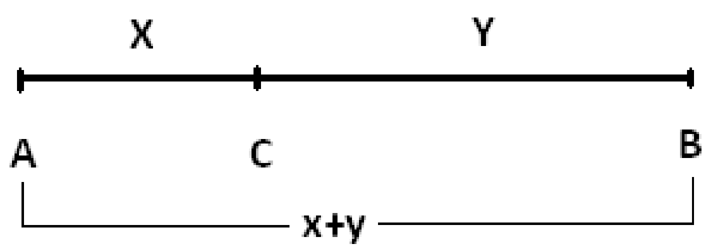

Figure 24: Line AB divided by point C with Golden Ratio. (Reference 13)

If the ratio of the length of the longest part (Y) to the smallest part (X) is equal to the ratio of the length of the line (x+y) to the length of the longest part (Y), then the value of the ratio will be equal to THE GOLDEN RATIO which is equal to 1.61803398874989484820... Reference (17)

$$Y / X = (X + Y) / Y = 1.61803398874989484820...$$

(ii)Fibonacci Numbers:

Fibonacci discovered the famous Fibonacci Numbers when he was trying to solve the rabbit problem. Fibonacci numbers are;

1,1,2,3,5,8,13,21,34,55,89,144…

If these numbers are analyzed, it can be seen that starting from the third term of the serial, all numbers are tequal to the sum previous two numbers before themselves.

2= 1+1

3= 1+2

5= 2+3

…

The relationsip between the golden ratio and the fibonacci numbers is that when we divide the numbers of the serial to each other, the result gets c loser to the golden ratio which is 1.61803398874989484820…

1/1 = 1

2/1 = 2

3/1 = 1.5

5/3 = 1.667

…

89/55 = **1.618**

The Golden ratio can be seen in the shape of the instruments.

In Violin;

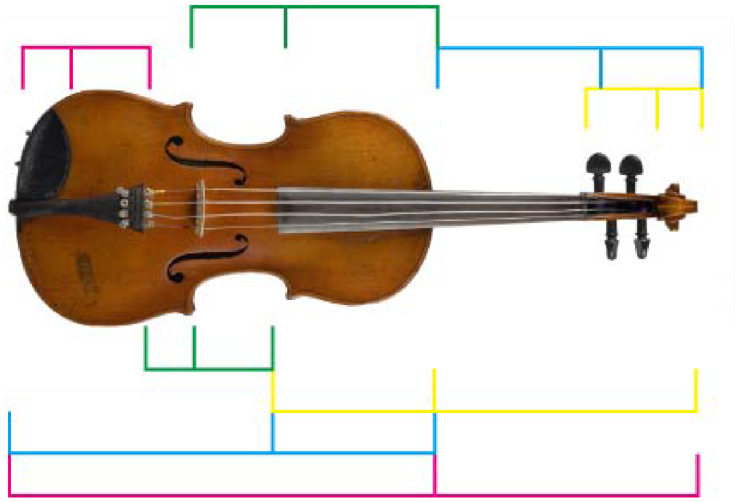

Figure 25: Golden ratio on the shape of violin. (Reference 14)

In Trumpet;

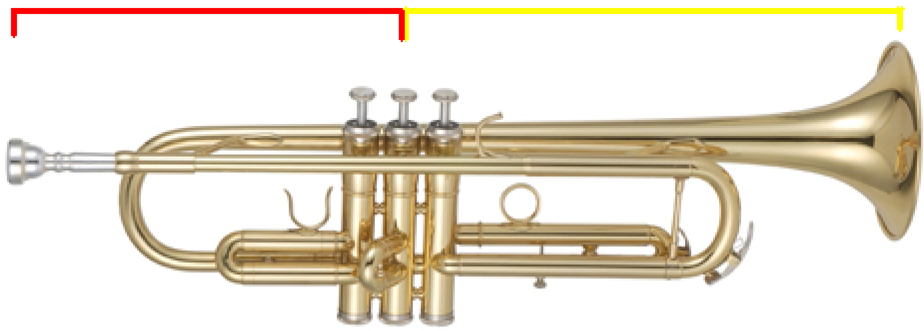

Figure 26: Golden ratio on the shape of trumpet. (Reference 15)

* The instruments such as piano, french horn has golden ratio in their shapes too.

If the piano keys are analyzed, it will be seen that the intervals between the keys are equal to the Fibonacci numbers.

In one octave, 13 keys exist.(Black and white keys are counted together.) 8 of them are white keys and 5 of them are black keys. There are 2 black keys exist between three white keys and 3 black keys exist between four white keys. (Reference 16)

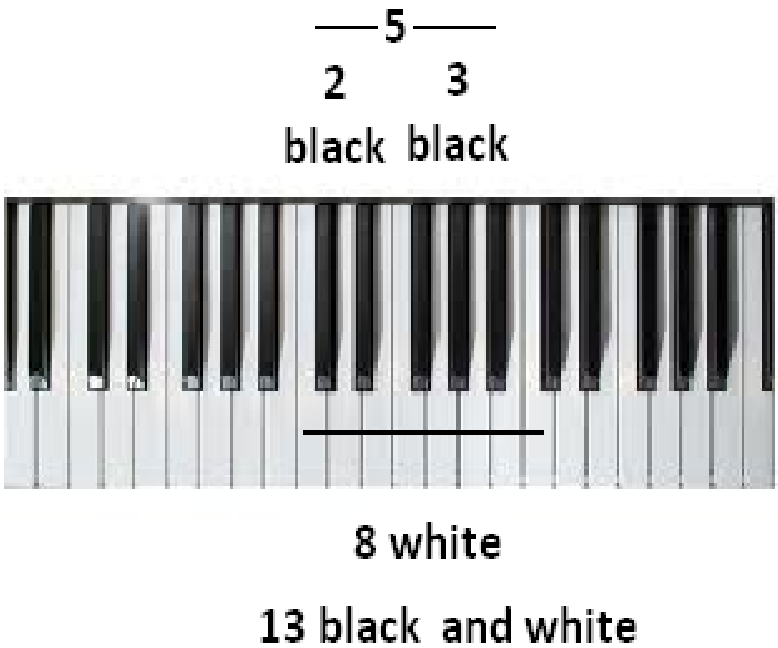

Figure 27: Fibonacci numbers in the intervals of the piano key. ()

As it can be seen from figure 27, the number are 2,3,5,8,13 which belong to Fibonacci numbers.

Most of the famous composers such as; Johann Sebastian Bach, Wolfgang Amadeus Mozart, Ludwig Van Beethoven, Franz Liszt, Frederic Chopin, Bela Bartok…etc. used golden ratio in their compositions.

Conclusion

As being surveyed, this paper clearly investigated the connections between the music and mathematics. It started with the basics of sound investigated important concepts like pitch, frequencies etc… for stating the music in the physical world have also important underlying mathematics and preparing an important background for the rest of the paper.

In the second part the chord theory is investigated in patternist perspective. This new perspective clearly broadens the way of looking the mathematics underlying the music, offering very useful theoretical and practical knowledge. Moreover, the findings in this part of the paper can be definitely applied while performing music. If these patterns can be learnt clearly, one can easily learn chord theory without memorizing too many rules which leads to produce better and more creative improvisation lines.

In the final part of the paper relationships between music and some various fields of mathematics are presented. However these relationships hardly have applications during a musical performance unlike the findings in the preceding part. Logarithms and trigonometry are used in sound engineering whereas fibonacci numbers are used in musical compositions. Nevertheless, applications of set theory in musical theory presented here could be very useful during a performance.

To sum up, music can be taught as an abstraction constituting from various fields of mathematics. Many connections and patterns are presented in this paper, although some connections not presented in here like geometry, algebra, number theory and even logic, should be studied in a much more detailed manner. Finally, more research should be made for new connections and patterns since it is clear that many more patterns and connections are waiting to be discovered.

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Note: In figures 15 and 16, the additions were made by Microsoft Paint.

APPENDIX

Chord Namings:

Table x	
Name Of The Chords	**Chords Abbreviations**
Major	
Minor	m
7th	7
minor 7th	m7
Major 7th	maj7
suspended 4th	sus4
seventh suspended 4th	7sus4
Diminished	°
Augmented	+ or #
augmented 7th	7+5
dominant 7th with flat(ted) 5th	7-5
6th	6
minor 6th	m6
5th	5
9th	9
minor 9th	m9
Major 9th	maj9
augmented 9th	9+5
augmented 11th	9-5
11th	11
13th	13
added 9th	add9
added 4th	add4
minor added 9th	madd9
minor added 4th	madd4
minor 7th added 4th	m7add4
suspended 2nd	sus2
minor, Major 7th	m(maj7)
minor 11th	m11
6th 9th	69
dominant 7th with flat(ted) 9th	7-9
7th with sharpened 9th	7+9
suspended 2nd, suspended 4th	sus2,sus4
minor 7th with flat(ted) 5th	m7-5
minor 7th with sharpened 5th	m7+5
minor , Major 9th	m(maj9)
minor sixth ninth	m69
minor 13th	m13
Major 9th with sharpened 11th	maj9+11
Major 13th	maj13
Major 13th with sharpened 11th	maj13+11
Major 7th with flat(ted) 5th	maj7-5
Major 7th with sharpened 5th	maj7+5
seventh suspended 2nd	7sus2
7th added 4th	7add4
minor suspended 4th	msus4
minor 7th suspended 4th	m7sus4