Mathematics

Extended Essay

"How well can a simplified, two-person zero-sum poker game analysis help to interpret basic decisions and moves made by companies and small businesses?"

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Abstract

In this essay a simplified game of poker, a two-person one-card poker with unitary bet size was analyzed in means of every possible scenario possible to occur between to rational players. The outcomes of these scenarios were properly placed together to find an equation for one of the player's expected value. Then this expected value was briefly analyzed for each player. It was found out that one of the players, the one in the dealer position, had an unbeatable strategy. Due to this fact, the dealer simply had the advantage. It was stated that this problem, for this game, can be solved by replacing the roles of the players each round.

A real situation concerning the expansion of airline transportation in Turkey was inspected and features and the moves of the poker game were easily transposed into that situation. After that, a scenario was created including two rival markets in the same street. By accepting a multiple of assumptions, many moves from the poker game were interpreted into this scenario and the opener, who had the disadvantage in the poker game, at some point, succeeded to gain a positive EV. During the period of changing strategies, it never looked the most beneficial for the dealer to activate their unbeatable strategy due to the content of the scenario. It was seen that the basic poker model taken was good enough to analyze and explain a multiple of decisions and moves made by real-life companies or small businesses. It was also seen that it is the untouchable strategies do not always seem to be the most beneficial for the player.

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1. Introduction

1.1. Game Theory

Game Theory studies the interactions between rational players which produce outcomes according to combination of moves made by each individual player. Purpose of using game theory is to generate the optimum strategy for a player which will produce the most beneficial payoff for the player. The game theory can be used for any type of strategic interactions between individuals, groups or companies which allow this study apply to many social sciences such as economics or business yet I will be using game theory to analyze and find optimal strategies for a simplified, two-person zero-sum poker game. Then I will explain a multiple of campaigns or moves made by companies and prove the profitability of each move using the calculations from the poker analysis. Therefore, the research question is: How well can a simplified, two-person zero-sum poker game analysis help to interpret basic decisions and moves made by companies and small businesses?

2. Poker

The card game of poker has always been an interest for me since I learned to play the game. When I was introduced to the Game Theory and learned about the poker application, I thought this would be a good opportunity for me to analyze the Poker game more deeply and an interesting topic for my extended essay. Then it came to my mind that explaining some moves made by companies by referencing a poker game might be an interesting way to show the use of game theory in economics.

Poker is considered as a multi-player sequential zero-sum game with imperfect information. In this type of game, a sequential game, the action of the player can be strategized not only in the beginning but also at any point of the game. The player does not know each action every other player took, ∴ there is no best move for the player. Poker is a zero-sum game where winning of a player can only be attained through losing of other players.

Optimal strategies can theoretically be computed using game theory. Although this is the case, in the real-world application of poker, outside the experimental environment, there are too many variables to consider and it would be unwise and uneasy to base the entire playing strategy only on these optimal strategies gained through this type of analysis.

2.1. Strategic Form:

It is the form of representing a game using a matrix where each box represents the payoff for a set of strategies. This method can be used for a simple analysis yet it is a compact view of the game and should be used for simultaneous games where no player is aware of other's strategies, ∴ there won't be any dominated strategies which should have been removed. This way, a mixed strategy can be found, which is basically what move to do and how frequently to do this move to optimize the average payoff. The representation is also known as the Normal form, as seen in the example:

		Player A	
		option 1	option 2
Player B	option 1	$+a$	$-a$
	option 2	$-b$	$+b$

Yet, this form has no use for our purposes since it can only show the payoff of the decisions, it is the steps leading to it we are interested in.

2.2. Extensive form:

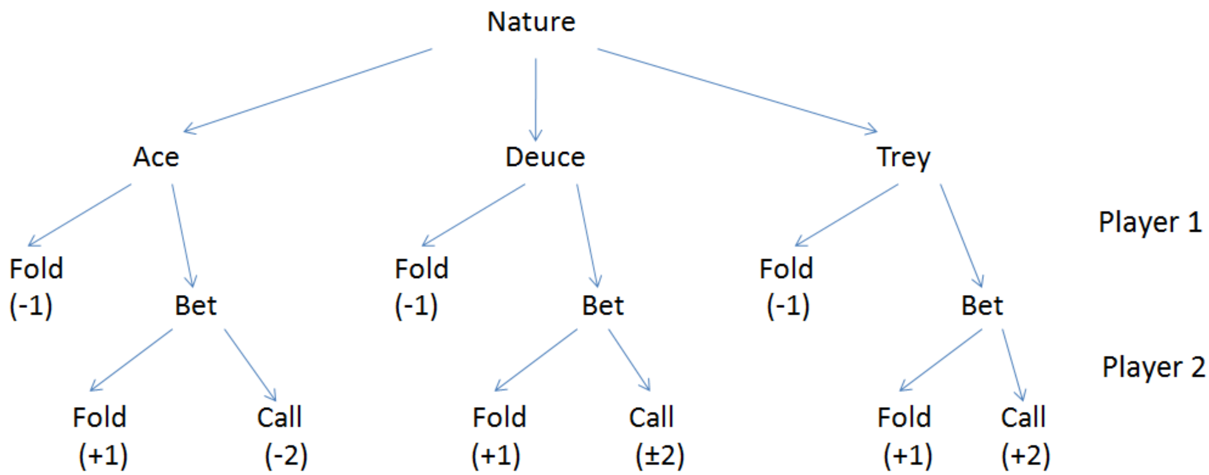
Also referred as the Game Tree, it is the graphical representation of sequential, a multiple game. It includes information about the players, strategies and the payoffs of those strategies. Each decision point is shown with a node and there are edges showing the results of these decisions. Every set of edges in the tree ends with one last node, terminal node, which indicates ending of the game. An example will be given in the next section.

2.3. Nature:

Not necessarily in every game, there are moves for each player that is not determined by any variables, are out of players' control and is completely random. In the game of poker cards dealt to each player can be seen as the nature which does not depend on anything.

3. One-Card Poker With Unitary Bet Size:

This is a two-player poker game played with a deck of three cards; an ace, a deuce and a trey. The players will be referred as the dealer and the opener. Each player places 1 unit of money into the pot, which is also known as ante, before the dealer deals one card to each of the player. Then according to their card, the opener checks or bets 1 more unit of money. If the opener bets, the dealer may either call or fold, if he folds, the opener wins the single unit of money which the dealer put into the pot as ante. If the dealer calls, there would be showdown and the player with the higher card would get all of the pot. But if the opener does not bet, checks, the dealer gets the opportunity to bet for the opener to either call the bet or fold. Each player can only gain a maximum of two units of money since the other two were placed by themselves in order to raise the pot. Since a player can only win only when the other player loses, the game is considered as a zero-sum game. This one-card poker game can be presented in extensive form as in the diagram below.



In the diagram above first three nodes from above are the options for the player and they all have payoffs or next nodes. The second set of nodes is the options for the opponent to move and the edges show the actions of the opponent and their possible outcomes. But it should be remembered this diagram is also a shortened version of the full 1-card poker game. For example you might check with your card in your hand and if the opponent bets you must either call or fold which would be same as betting or folding with your card in the first place.

For example you are player 1 and you have the deuce in your hand. You choose to bet which is the resolution of the first node below deuce. From this point, fate of the game will be decided by the opponent, player 2. Opponent folds, you would win (+1) or opponent calls: there is a chance for either winning or losing 2 units.

3.1. Dominated Strategies and Irrational Moves:

Each unique line in the tree represents a different strategy that can be used for the game. But for our purposes, I won't be analyzing the game through the extensive form with traditional ways. In order to transpose the game into economic moves; I will be inspecting every possible scenario in the game with their outcomes. In the diagram, there are some obvious strategies which should be eliminated from the available moves a player can make to decrease the number of scenarios that would not be occurring between two rational players. This next analysis was inspired by an article by Jason Swanson, "Game Theory and Poker". There are a multiple of irrational moves that should be discarded from choice of options:

→ Neither player should fold when the trey is in the hand. Trey would win against any other card in this game. The player should always bet with the trey if possible.

→ Neither player should call or bet with the ace, all the time. In order to optimize a strategy the player should not: never call or bet with the ace or always call or bet with the ace. But now, in order to have the next rule to make sense, I will just say that the player should never call or bet with the ace. (The optimal frequency of this move will be dealt with in the next section.)

→ Neither player should bet with the deuce. When the previous rule is considered, if the player bets with the deuce, the opponent will fold with the ace and call with the trey. If the opponent has an ace, betting with the deuce would have no meaning since the opponent would fold. If the opponent has a trey, the opponent would not hesitate to call this bet and instead of losing one unit of money if the player just checked, now the player would lose two units of money through the showdown.

By following these rules, dominated strategies such as folding with the trey would be removed from the possible strategies.

3.2. Expected Values:

Considering that both of the players play according to these rules, the possible course of events is reduced. Before explaining these scenarios, the frequencies of actions each player take should be denoted. From this point players will be referred as the dealer and the opener to establish the scenarios. For the dealer:

→ Frequency of bluffing with the ace: d_1

→ Frequency of calling with the deuce: d_2

And for the opener:

→ Frequency of bluffing with the ace: o_1

→ Frequency of calling with the deuce: o_2

→ Frequency of betting with the trey: o_3

It must be remembered that these are frequency values, ∴ they can only be between 0 and 1. It should be known that giving a frequency to betting with the trey is not an irrational move for the opener since betting might scare an opponent with holding an ace from bluffing which in this case would have been beneficial for the opener.

3.2.1. Possible Scenarios and Their Outcomes:

Considering that the antes placed before the game starts also counts for gained or lost money, making this a pre-ante expected value calculation, there are four possible outcomes to all of these scenarios for the opener: losing 2 units of money, losing 1 unit of money, winning 1 unit of money and winning 2 units of money. These scenarios are as followings:

→ The opener has the trey, the dealer has the deuce.

- The opener bets and the dealer calls, so the opener wins 2 units of money. This is d_2 x o_3.
- If the dealer does not call the bet, the opener wins a single unit of money which is (1-d_2) x o_3.
- If the opener does not bet and the dealer checks, the game will go to a showdown with the antes and the opener wins a single unit of money. This is (1-o_3).

→ The opener has the trey and the dealer has the ace.

- If the opener checks and the dealer bluffs and the bluff is called by the opener, the opener wins 2 units of money which is (1 - o_3) x d_1.
- If the opener checks and the dealer also checks, the opener wins one unit of money via the showdown. This is (1-o_3) x (1-d_1).
- If the opener bets, the dealer would be forced to fold since he would be holding the ace. The opener wins one unit of money with a probability of o_3.

→ The opener has the deuce and the dealer has the trey.

- If the opener calls the dealer's bet, he loses 2 units of money which has a possibility of o_2.
- If the game goes to a showdown when the opener checks and is followed by the dealer's check which is (1 - d_3), the opener loses one unit of money.

→ The opener has the deuce and the dealer has the ace.

- If the opener checks and the dealer bluffs and with a probability of d_1 x o_2 the opener calls and wins 2 units of money.
- The opener might not call the dealers bluff by a probability of d_1 x (1 - o_2) and loses 1 unit of money.
- The other starting situation, the opener checks and the dealer also checks leading the game to a showdown with a probability of (1 - d_1) and the opener wins 1 unit of money.

→ The opener has the ace and the dealer has the trey.

- If the opener bluffs and is called with a probability of o_1, he loses 2 units of money.

- If the opener checks and the dealer bets with the trey in his hand forcing the opener to call with a probability of $(1-o_1)$, the opener is forced to fold, and loses 1 unit of money.

→ The opener has the ace and the dealer has the deuce.

- If the opener bluffs and the dealer folds with the deuce with a probability of $o_1 \times (1 - d_2)$ the opener wins 1 unit of money.
- If the opener's bluff is called with the probability $o_1 \times d_2$, he loses 2 units of money.
- If the opener does not bluff and checks; and the dealer also checks leading the game to a showdown with a probability $(1 - o_1)$, the opener loses 1 unit of money.

These cards are received by the players by nature which gives each of these scenarios a probability of 1/3 x ½ = 1/6 to occur. The entire opener's winning or losing scenarios can be put together.

→ Winning 2 units of money:

$$P_1 = 1/6 \times [(d_2 \times o_3) + ((1 - o_3) \times d_1) + (d_1 \times o_2)]$$

→ Winning 1 unit of money:

$$P_2 = 1/6 \times [(1 - d_2) \times o_3 + (1-o_3) + (1-o_3) \times (1-d_1) + o_3 + (1 - d_1) + o_1 \times (1 - d_2)]$$

→ Losing 1 unit of money:

$$P_3 = 1/6 \times [(1 - o_2) + (1-o_2) \times d_1 + (1 - o_1) + (1 - o_1)]$$

→ Losing 2 units of money:

$$P_4 = 1/6 \times [o_2 + o_1 + o_1 \times d_2]$$

So this gives the opener an expected value of:

$$EV_O = 2 \times P_1 + P_2 - P_3 - 2 \times P_4$$

$$= 1/6 \times (2d_2o_3 + 2d_1 - 2d_1o_3 + 2d_1o_2 + o_3 - d_2o_3 + 1 - o_3 + 1 - d_1 - o_3 + d_1o_3 + o_3 + 1 - d_1 + o_1 - d_2o_1$$
$$- 1 + o_2 - d_1 + d_1o_2 - 1 + o_1 - 1 + o_1 - 2o_2 - 2o_1 - 2d_2o_1)$$

$$= 1/6 \times (d_2o_3 - d_1o_3 + d_1o_2 - d_2o_1 - o_2 + d_3o_1)$$

This is, at the same time, the converse of the dealer's expected value, EV_D.

What game theory does is that, studying this equation to find a possible gap which will allow either player to keep their frequencies at a constant rate that cannot be affected by the opponent. Mathematically speaking, find a set of values for openers or dealers frequencies to eliminate the other set of values from the equation which creates an indifferent, consistent strategy. Such values cannot be always found. Initial values can be given for a game and the moves of the players can be inspected to see the EV changes.

3.2.2. The opener's expected value:

$EV_O = 1/6 \times [d_1 \times (3o_2 - o_3 - 1) + d_2 \times (o_3 - 3o_1) + o_1 – o_2]$

For the opener to have an indifferent strategy what he should do is to find a set of values for o_1, o_2 and o_3 that would make d_1's and d_2's multiplier in

"$1/6 \times [d_1 \times (3o_2 - o_3 - 1) + d_2 \times (o_3 - 3o_1) + o_1 – o_2]$" equal to zero:

$3o_2 - o_3 – 1 = 0$ and $o_3 - 3o_1 = 0$

Where we should start is the $o_3 - 3o_1 = 0$ equation. It can be seen that $o_3 = 3o_1$ and from that you can give the values $o_3 = 0$, $o_1 = 0$ which makes $o_2 = 1/3$ for the opener to have an indifferent strategy. However these set of values give the opener an expected value of -1/18 which is for the dealer's benefit. Other set of values can be found from the previous equation system providing that:

$o_3 = 3o_1$ and

$3o_2 - o_3 - 1 = 0$.

For example the opener can bluff with the ace 1/3 of the time where he should always bet with the trey, ∴ call with the deuce ⅔ of the time. But again it is seen that this also gives the opener an EV of -1/18, again which is beneficial for the dealer.

3.2.3. The dealer's expected value:

$EV_D = 1/6 \times [o_3 \times (d_1 - d_2) + o_1 \times (3d_2 - 1) + o_2 \times (1-3d_1) + d1]$

What should be done is:

$d_1 - d_2 = 0$,

$3d_2 - 1 = 0$ and

$1-3d_1 = 0$.

It can be easily seen that the only solution set for the system is $d_1 = d_2 = 1/3$. By doing this the dealer would have an EV of 1/18 which is beneficial for the dealer. Since it s indifferent, it does not depend on the opener's moves so it is the optimal strategy for the dealer. If the players do not change positions, the dealer would always be winning in the long term. That simply would not be fair since the opponent can also see this. To avoid this situation, the players can change positions and each time the opponent who has become the dealer would also use the same optimal strategy and in the end, each player would have an EV of 0, ∴ no one wins and the game is stuck. So this is possibly not a path that will be chosen by the players since no one is winning.

There are many scenarios where initial ratios for players' moves can be set and then investigated to make it beneficial for either player but those situations will not be dealt here. They will be transposed into real life situations to interpret some decisions and actions of companies or small business'.

4. Real World Situations:

4.1. The Expansion of Airline Transportation in Turkey:

Until a few years ago, the airline transportation was not popular and was considered luxurious in Turkey. Pegasus Airlines, a company which had its management renewed decided to start a campaign to initiate the "low cost model". For every single domestic flight in the country, they reserved 5 seats and gave those seats to who tried to purchase a seat first, for free. This, of course lured the customers, the passengers, who were used to pay less for transportation and take a bus instead of a plane. After some time, the number of customers who knew the comfort and ease of flying on a plane instead of taking a bus increased and ignoring the price difference (by the way which was getting smaller and smaller), people started to choose airline transportation instead of land transportation. In fact, today, some domestic flights -thanks to the low cost model they started in Turkey- are cheaper than bus tickets to the same destination from the same place.

In this case, the airline company can be considered as the opener and the dealers moves can be considered as the customers' choices. And the EV of the dealer would be what happens to the land transportation.

$EV_O = 1/6 \times [d_1 \times (3o_2 - o_3 - 1) + d_2 \times (o_3 - 3o_1) + o_1 - o_2]$

At first we would take the customers as a consistent player one that does not bluff with the ace that often and that is afraid of calling with the deuce. Let's give the values $d_1 = 1/6$ and

d_2 = 1/3. Remember now this is the society's behavior and the company is aware of this behavior and approaching to this. At first, the opener should not call with the deuce and bluff a lot with the ace to encourage the dealer (the customers) to call with the deuce and bluff with the ace too. The company started the 5 seat for free campaign to encourage people to try using the plane for transportation. Of course the airline company has a group of royal customers who are aware of the comfort of travelling via plane. Let's give the probabilities o_3=1 o_2=0 and o_1=1/2.

This would give the opener an EV of +1/12.

This is not much, but the airline is still earning money from this campaign and what they are doing is changing the view of the customer to use the airline transportation more which is, in a way bluffing with the ace and calling with the deuce, ∴ this EV will probably be consistent. Although it is more expensive for the customer, the customer is now aware of the comfort of travelling with the plane and will keep doing it.

4.2. A Battle of Two Markets:

Consider this: There are two markets, considered rivals in the street. Let's say every single product they have import rates for different categories of products: The expensive products that both markets consider to be risky, in means of consumption, rate as they might not get all sold; the middle priced products and the low priced products which are guaranteed to be sold.

This first market, called the "opener" market, receives this new expensive product that is not being imported to the opposite "dealer" market. The opener market is worried that this new product would not be well received and gives this item an import rate of o_1=¼ similar to bluffing with an ace knowing that it might be called. To secure their business, the opener market gives a high ratio of o_3=¾ for its import of cheap products which are guaranteed to be sold, similar to betting with the trey which is guaranteed to beat any hand in the previously discussed poker game. And then sets o_2=¼, rate of importing mid-priced products in order to, let's say, divert the customers' attention to the other two groups of products.

As a response to this act, the dealer market keeps their own pricey product import rate at low as d_1=¼ and, relatively, increase their mid-priced product import rate to d_2=½. Again, let's say their cheap products are trusted and efficiently sold, ∴ the rate of importing should be 1 for them. Yet as it will be seen from the equation, it has no effect on the opener market's EV. Again let's consider, this is due to the fact that the dealer market's cheap products are more trustworthy. And this can actually be interpreted from the poker game: the opener knows that if the dealer has the trey, he will definitely call or bet since at that point there is nothing for the opener to do. The trey in dealer's hand is more useful than the trey in opener's hand.

$$EV_O = 1/6 \times [d_1 \times (3o_2 - o_3 - 1) + d_2 \times (o_3 - 3o_1) + o_1 - o_2]$$

When the values are placed in the equation, it is found that the opener market's EV becomes -1/24. This indicates that opener market is, in the long term, losing money. At this point opener market may try to prompt their customers to the expensive products by increasing o_1 to ½ and by decreasing o_3 to ½ while keeping the mid-priced product import rate still at low by o_2=¼. To this move the dealer market would react by keeping its mid-priced product import rate d_2=½ and letting the opener market take the risk by decreasing its own expensive product import rate to d_1=1/8.

It is seen that the new EV didn't worked for opener market. The new EV is -11/192, opener market is still losing money; going on with the new expensive product is not working for the opener market. What they should have done was to decreasing o_1 to 1/8 and maximizing the other import rates as $o_2 = o_3 = 1$. To this, dealer market would have tried to put their expensive products forward by making $d_1 = 1$ and decreasing d_2 to ¼. This works for the opener market and gives the opener market the EV of +3/64.

But of course this will be seen by the dealer market and a counter strategy will be improvised and the battle of the two markets will keep going on.

5. Conclusion:

The initial analysis was used to examine and transpose the moves talked about in the real world situations. The moves in the poker game were successfully adapted to the real world situations and every move made by the players in the real world situations could have been explained by referring to the poker game. The EV equation found was usable to calculate the payoffs of the players and produce conclusions of from their decisions.

From the expected value calculations it was seen that the dealer had an indifferent strategy that gave him an EV of +1/18. The dealer had the advantage in the two-player one-card poker game. Therefore in the real world situations, beneficial situations for the opener were tried to be achieved. Both players altered their move rates but it was never seemed as the most beneficial for the dealer to use their indifferent strategy. The EV was constantly being altered due to changes made by the players as a reaction to the opposing player's movement changes. All these changes were easily explained as this was the poker game previously analyzed.

This brings to my mind a saying that I ran into multiple times during my research by a famous poker player David Sklansky: "*Poker is the only game where it is correct to play incorrect.*" Considering what is tried to be shown in this essay, it can be said that same thing applies to many situations that can be referred to as a "game". And from what is seen by having the opener a positive EV in a game that is disadvantageous for them, it can be said that it is not important what you have or where you are in a game, what's important is how you can make use of what you have and where you are. The same thing can be said about life itself, and in the end, life is a game, isn't it?

6. Appendix

Bet: It is to increase the amount of money each player should place by placing a fixed amount of money in the pot.

Ante: It is the initially placed money by each player in order to receive a hand.

Bluff: To falsely persuade the opponent to imply that the player has a good hand, by betting while holding a bad hand.

Call: Accepting the opponent's bet and placing same amount of money as the bet.

Check: Not betting or calling when it is the player's turn to move.

Fold: Refusing to call the opponents bet and forfeiting the hand at the cost of the player's own money in the pot.

Showdown: Each player shows their cards to decide who wins the pot after the betting is finished.

Dealer: The player in the game who deals the card and moves last in every round.

Opener: The player who moves first each round.

End Game: The term refers to the final round of a poker game where no more cards are given to any players and the players bet for last time.

Rational Player: Name given to a player in any game who would never use dominated strategies and use indifferent strategies if possible.

Payoff: It is the result of actions or a strategy, winnings or loss' in a game.

Expected Value: It is the weighted average of every possible result in a game.

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