

## **May 2026 Physics HL Extended Essay**

### **Research Question:**

**How does the internal gauge pressure of a basketball (0.55–1.00 bar) affect its coefficient of restitution when dropped from different heights (80–200 cm) onto a polished concrete surface?**

**Word Count: 3216**

## Introduction

A basketball is composed of a rubber outer layer, internal carcass layers, and an inflated bladder. The basketball performs a partially elastic collision behaviour, recovering some of the energy it lost upon impact with a surface. The basketball forms a flat contact region when compressed which leads to a rapid rise in internal air pressure during the collision. This deformation of the basketball creates a practical system to investigate the affects of internal pressure on rebound in partially elastic collisions such as a basketball's collision with a surface.

Rebound height depends on the amount of energy conserved during the deformation of the basketball. Gravitational potential energy is converted into translational kinetic energy during the fall. The amount of energy returned after the collision determines the rebound height. When the basketball contacts a surface some of its energy is stored as elastic strain energy and some of it is lost during the collision. The stored elastic strain energy then helps the ball bounce back. The energy lost is lost mainly through internal friction within the basketball, acoustic emission, and small heat losses at the contact region. However, these energy conversions are not constant and they are determined by the rate of deformation on the ball. As a result, different behaviours can be observed depending on conditions such as inflation pressure and impact speed.

To observe such different behaviours, different drop heights and different pressures are used. To compare these behaviours, we use the coefficient of restitution  $e$ . For a vertical collision,  $e$  is calculated as the ratio of separation speed to approach speed. Using kinematic equations and drop-height and rebound-height measurements, both measured from the centre of mass of the basketball, we can calculate the coefficient of restitution without a speed camera. As a result, changes in rebound height can be calculated with the coefficient of restitution rather than a test from every height.

Ball's behaviour upon impact such as its rebound height is known to be affected by changes in its internal pressure. Internal pressure is expected to change the ball's stiffness and its contact time with the rough surface therefore changing its rate of deformation. High inflation pressure is expected to reduce ball's deformation upon impact, thereby reducing energy loss. The increase in conserved energy will then result in a higher rebound height.

Impact speed is another factor that affects behaviour. A higher drop height results in a higher impact speed, which may have effects on rebound height even under constant internal pressure. So changing the drop height allows to choose the main determinant of the rebound height and bounciness.

## **Background and Theory**

### **Basketball as a pressurised and deformable system**

A basketball is an outer rubber layer strengthened by some carcass layers and an internal bladder filled with air. When in contact with a rigid surface, the basketball forms a flat contact region that compresses the air inside, deforming its outer layer both elastically. Amount of energy stored during compression and deformation and returned during re-expansion determines the ball's rebound height. This is called a partially elastic collision where some of the mechanical energy is recovered and some of it is lost.

### **Drop Height, Impact Speed and Rebound Height**

In a vertical drop of the basketball from rest, neglecting air resistance as it is a short distance, the impact speed can easily be calculated with kinematics:

$$v_{\text{impact}} = \sqrt{2gh_d}$$

Where  $g$  is the acceleration due to gravity and  $h_d$  is the release height that is consistently measured from the centre of the ball's mass. After contact with the surface, the ball will rebound into the air to a maximum height  $h_r$ , where its speed is zero, which is again measured from the centre of the basketball's mass. The upward speed immediately after the contact is:

$$v_{\text{rebound}} = \sqrt{2gh_r}$$

### **Coefficient of Restitution**

A widely used measure of the bounciness of an object is the coefficient of restitution  $e$ , defined as the ratio of the speed of separation to the speed of approach, in this case, impact and rebound speeds:

$$e = \frac{v_{\text{rebound}}}{v_{\text{impact}}}$$

For a vertical bounce on a fixed rigid surface, this can be found by the rebound speed divided by the impact speed:

$$e = \sqrt{\frac{h_r}{h_d}}$$

This equation relates rebound performance to drop height, allowing results from different heights to be compared without the ambiguity that higher release height implies better bounce performance.

### Energy Loss

The coefficient of restitution is closely related to energy restitution, as it is defined kinematically. Since kinetic energy is proportional to  $v^2$  and  $e$  is proportional to  $v$ , the ratio of kinetic energies before and after the collision/impact is:

$$\frac{K_{\text{after}}}{K_{\text{before}}} = \frac{v_{\text{rebound}}^2}{v_{\text{impact}}^2} = e^2$$

Therefore, the part of kinetic energy lost during the collision is:

$$\Delta = 1 - e^2$$

The lost energy is not lost during the collision, it is converted into other forms of energy because energy cannot be created or destroyed, it can only be changed from one form to another. The energy is mainly converted to internal heating, acoustic emission, and some small losses from friction from collisions and the motion in the air. A larger  $e$  value will show a smaller energy loss during impact.

### Effect of Internal Pressure on Restitution

The basketball's stiffness and rate of deformation can change with its internal pressure. A higher internal pressure requires more work to deform because the ball resists volume changes and its shell has more restoring forces. Reduced deformation will result in reduced viscoelastic dissipation in the ball, thereby increasing the energy returned for the rebound motion and, as a result, the coefficient of restitution  $e$ . Investigations on the inflated ball's report that  $e$  increases with pressure, with a limiting value at higher pressure levels, consistent with an exponential relationship rather than a linear increase. The most practical way to analyse the data is to test whether it is consistent with:

- A linear trend  $e = mP + b$ , or
- An exponential trend  $e(P) = e_0 + \frac{mP}{P + P_c}$

where  $P$  is the inflation pressure,  $e_0$  is the baseline coefficient of restitution,  $m$  is the maximum possible increase in restitution and  $P_c$  is the characteristic pressure where  $e$  reaches half of its total increase.

## Impact Speed as a Second Variable

In this experiment impact speed is determined by the drop height because of the equation:

$v = \sqrt{2gh_d}$  For viscoelastic materials such as a basketball, deformation is expected to be dependent on the impact speed even at constant pressure. Multiple tests from different heights determine whether restitution is constant with speed or changes with speed. Experimental studies show that the impact of a ball on a rigid surface is not instantaneous. This makes the impact speed of the ball a very important variable in collision experiments such as this one.

## Assumptions

The experiment is build on some assumptions such as a perfectly vertical motion, negligible air resistance because of the short heights, and a stationary, rigid surface. Any changes, such as spins, measurement errors, surface problems or temperature changes can affect the heights and the calculations.

## Methodology

### Variables

- **Independent:** Gauge Pressure  $P$ (bar)
- **Dependent:** Coefficient of Restitution( $e$ )
- **Controlled:** Type of Surface, Temperature, Orientation of the Ball, Release Method, Measurement Method, Same Basketball, Same Location

### Apparatus

- Pump and Pressure Gauge( $\pm 0.01$ bar)
- Measuring Tape( $\pm 0.05$ cm)
- Phone Camera(120fps)
- Tape
- Tripod
- Pen

### Setup

- Surface: Polished Concrete Indoors
- Measuring Tape taped to the wall from each drop height
- Phone Camera fixed perpendicular to the motion plane

## Procedure

- 1. Prepare the area:** Select a rigid floor such as polished concrete near a wall and mark a spot where impact will happen with tape.
- 2. Install the measure:** Tape a measuring tape on the wall behind the marked impact spot. Align the 0cm mark on the measuring tape with the floor for best measurements.
- 3. Mark the Drop Heights:** Use a pen to mark each drop height on the same wall.
- 4. Install the Camera:** Place the phone camera on a tripod or a stable support perpendicular to the motion plane. Level the lens at the mid-height of the measuring tape. Lock the focus.
- 5. Set the Pressure Level:** Inflate/deflate the basketball to the first target gauge pressure(1.00 bar). Wait a minute to stabilise, and verify that it is correct.
- 6. Perform The Drops:**
  - Align the ball's centre of mass(middle) with the 200cm mark on the wall.
  - Release without pushing or spinning by opening your hands and letting the basketball fall.
  - Record the bounce in slow motion, allowing the ball to finish its motion.
  - Repeat the drop 4 more times for a total of 5 trials
- 7. Recheck Pressure:** After the drops, recheck the pressure to make sure it didn't drift.
- 8. Repeat For Other Drop Heights:** Repeat steps 5-7 for the remaining drop heights (170cm, 140cm, 110cm, 80cm)
- 9. Repeat For Other Pressure Levels:** Repeat Steps 5-8 for the remaining pressure levels (0.85, 0.70, 0.55 bar). Keep the camera position, impact point and the measuring tape the same.
- 10. Extract Rebound Heights From Videos:** For each trial, use frame-by-frame playback to identify the rebound height.

## Data Collection

$P = 1.00$  bar (Table 1)

| $h_d$<br>(cm)( $\pm 0.05$ ) | T1<br>(cm)( $\pm 0.05$ ) | T2<br>(cm)( $\pm 0.05$ ) | T3<br>(cm)( $\pm 0.05$ ) | T4<br>(cm)( $\pm 0.05$ ) | T5<br>(cm)( $\pm 0.05$ ) |
|-----------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 200                         | 140                      | 142                      | 139                      | 140                      | 139                      |
| 170                         | 118                      | 121                      | 116                      | 117                      | 118                      |
| 140                         | 93                       | 96                       | 92                       | 92                       | 91                       |
| 110                         | 70                       | 74                       | 69                       | 70                       | 70                       |
| 80                          | 48                       | 50                       | 48                       | 46                       | 47                       |

$P = 0.85$  (Table 2)

| $h_d$<br>(cm)( $\pm 0.05$ ) | T1<br>(cm)( $\pm 0.05$ ) | T2<br>(cm)( $\pm 0.05$ ) | T3<br>(cm)( $\pm 0.05$ ) | T4<br>(cm)( $\pm 0.05$ ) | T5<br>(cm)( $\pm 0.05$ ) |
|-----------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 200                         | 133                      | 134                      | 133                      | 134                      | 133                      |
| 170                         | 113                      | 114                      | 112                      | 113                      | 111                      |
| 140                         | 89                       | 82                       | 87                       | 89                       | 87                       |
| 110                         | 64                       | 67                       | 63                       | 63                       | 64                       |
| 80                          | 45                       | 47                       | 44                       | 43                       | 43                       |

$P = 0.70$  (Table 3)

| $h_d$<br>(cm)( $\pm 0.05$ ) | T1<br>(cm)( $\pm 0.05$ ) | T2<br>(cm)( $\pm 0.05$ ) | T3<br>(cm)( $\pm 0.05$ ) | T4<br>(cm)( $\pm 0.05$ ) | T5<br>(cm)( $\pm 0.05$ ) |
|-----------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 200                         | 127                      | 129                      | 125                      | 126                      | 125                      |
| 170                         | 107                      | 109                      | 105                      | 106                      | 106                      |
| 140                         | 83                       | 84                       | 82                       | 85                       | 82                       |
| 110                         | 60                       | 64                       | 58                       | 58                       | 61                       |
| 80                          | 41                       | 42                       | 40                       | 39                       | 40                       |

$P = 0.55$  (Table 4)

| $h_d$<br>(cm)( $\pm 0.05$ ) | T1<br>(cm)( $\pm 0.05$ ) | T2<br>(cm)( $\pm 0.05$ ) | T3<br>(cm)( $\pm 0.05$ ) | T4<br>(cm)( $\pm 0.05$ ) | T5<br>(cm)( $\pm 0.05$ ) |
|-----------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 200                         | 121                      | 122                      | 120                      | 121                      | 118                      |
| 170                         | 102                      | 105                      | 101                      | 103                      | 102                      |
| 140                         | 78                       | 80                       | 76                       | 79                       | 76                       |
| 110                         | 56                       | 59                       | 54                       | 55                       | 58                       |
| 80                          | 37                       | 38                       | 37                       | 36                       | 36                       |

## Data Processing

### Processing 1 Statistics for Rebound Height

Five trials were made for each combination of drop height and internal pressure, and five rebound heights  $h_r$  were recorded ( $n = 5$ ). The mean rebound height was then calculated using:

$$\bar{h}_r = \frac{1}{n} \sum_{i=1}^n h_{r,i}$$

The standard deviation was calculated using:

$$s_{h_r} = \sqrt{\frac{\sum_{i=1}^n (h_{r,i} - \bar{h}_r)^2}{n - 1}}$$

For the error bars on the mean graph, the standard error of the mean was calculated:

$$\text{SEM}_{h_r} = \frac{s_{h_r}}{\sqrt{n}}$$

### Worked Example

Condition:  $h_d = 200\text{cm}$ ,  $P = 0.70$  bar

Five Trials(Rebound Heights): 127cm, 129cm, 125cm, 126cm, 125cm

**Mean:**

$$\bar{h}_r = \frac{127 + 129 + 125 + 126 + 125}{5} = \frac{632}{5} = 126.40\text{cm}$$

**Standard Deviation:**

Deviations from the mean: +0.60, +2.60, -1.40, -0.40, -1.40

Squares: 0.36, 6.76, 1.96, 0.16, 1.96

Sum of Squares: 11.20

$$s_{h_r} = \sqrt{\frac{11.20}{4}} = \sqrt{2.80} = 1.67$$

**Standard Error of the Mean(SEM):**

$$\text{SEM}_{h_r} = \frac{1.67}{\sqrt{5}} = \frac{1.67}{2.24} = 0.75$$

### Mean Rebound Heights (Table 5)

| $h_d$<br>(cm)( $\pm 0.05$ ) | 1.00 bar          | 0.85 bar          | 0.70 bar          | 0.55 bar          |
|-----------------------------|-------------------|-------------------|-------------------|-------------------|
| 200                         | 140.00 $\pm$ 1.22 | 133.40 $\pm$ 0.55 | 126.40 $\pm$ 1.67 | 120.40 $\pm$ 1.52 |
| 170                         | 118.00 $\pm$ 1.87 | 112.60 $\pm$ 1.14 | 106.60 $\pm$ 1.52 | 102.60 $\pm$ 1.52 |
| 140                         | 92.80 $\pm$ 1.92  | 86.80 $\pm$ 2.86  | 83.20 $\pm$ 1.30  | 77.80 $\pm$ 1.79  |
| 110                         | 70.60 $\pm$ 1.95  | 64.20 $\pm$ 1.64  | 60.20 $\pm$ 2.49  | 56.40 $\pm$ 2.07  |
| 80                          | 47.80 $\pm$ 1.48  | 44.40 $\pm$ 1.67  | 40.40 $\pm$ 1.14  | 36.80 $\pm$ 0.84  |

**Standard Error of the Mean Heights(SEM)(Table 6)**

| $h_d$<br>(cm)( $\pm 0.05$ ) | 1.00 bar          | 0.85 bar          | 0.70 bar          | 0.55 bar          |
|-----------------------------|-------------------|-------------------|-------------------|-------------------|
| 200                         | 140.00 $\pm$ 0.55 | 133.40 $\pm$ 0.24 | 126.40 $\pm$ 0.75 | 120.40 $\pm$ 0.68 |
| 170                         | 118.00 $\pm$ 0.84 | 112.60 $\pm$ 0.51 | 106.60 $\pm$ 0.68 | 102.60 $\pm$ 0.68 |
| 140                         | 92.80 $\pm$ 0.86  | 86.80 $\pm$ 1.28  | 83.20 $\pm$ 0.58  | 77.80 $\pm$ 0.80  |
| 110                         | 70.60 $\pm$ 0.87  | 64.20 $\pm$ 0.73  | 60.20 $\pm$ 1.11  | 56.40 $\pm$ 0.93  |
| 80                          | 47.80 $\pm$ 0.66  | 44.40 $\pm$ 0.75  | 40.40 $\pm$ 0.51  | 36.80 $\pm$ 0.37  |

**Processing 2 Coefficient of Restitution**

Coefficient of restitution is calculated from the measured heights for each trial using:

$$e = \sqrt{\frac{h_r}{h_d}}$$

Then the Mean:

$$\bar{e} = \frac{1}{n} \sum_{i=1}^n e_i$$

Sample Standard Deviation :

$$s_e = \sqrt{\frac{\sum_{i=1}^n (e_i - \bar{e})^2}{n - 1}}$$

And Standard Error of the Mean:

$$SEM_e = \frac{s_e}{\sqrt{n}}$$

**Worked Example**

Condition:  $h_d = 200\text{cm}$ ,  $P = 0.70$  bar

Trial 1 Rebound Height: 127cm

$$e = \sqrt{\frac{127}{200}} = \sqrt{0.64} = 0.80$$

Using all five trials, the mean restitution for this condition was:

$$\bar{e} = \sqrt{\frac{\bar{h}_r}{h_d}} = \sqrt{\frac{126.40}{200}} = \sqrt{0.63} = 0.79$$

For this condition, the spread across trials is:

$$s_e \approx 0.01 \quad \text{SEM}_e \approx 2.35 \times 10^{-3}$$

### Mean Coefficient of Restitution and Its Standard Deviation (Table 7)

| $h_d$<br>(cm)( $\pm 0.05$ ) | 1.00 bar          | 0.85 bar          | 0.70 bar          | 0.55 bar          |
|-----------------------------|-------------------|-------------------|-------------------|-------------------|
| 200                         | $0.837 \pm 0.004$ | $0.817 \pm 0.002$ | $0.795 \pm 0.005$ | $0.776 \pm 0.005$ |
| 170                         | $0.833 \pm 0.007$ | $0.814 \pm 0.004$ | $0.792 \pm 0.006$ | $0.777 \pm 0.006$ |
| 140                         | $0.814 \pm 0.008$ | $0.787 \pm 0.013$ | $0.771 \pm 0.006$ | $0.745 \pm 0.009$ |
| 110                         | $0.801 \pm 0.011$ | $0.764 \pm 0.010$ | $0.740 \pm 0.015$ | $0.716 \pm 0.013$ |
| 80                          | $0.773 \pm 0.012$ | $0.745 \pm 0.014$ | $0.711 \pm 0.010$ | $0.678 \pm 0.008$ |

### Standard Error of the Mean Coefficient of Restitution (Table 8)

| $h_d$<br>(cm)( $\pm 0.05$ ) | 1.00 bar                        | 0.85 bar                        | 0.70 bar                        | 0.55 bar                        |
|-----------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 200                         | $0.837 \pm 1.63 \times 10^{-3}$ | $0.817 \pm 0.75 \times 10^{-3}$ | $0.795 \pm 2.35 \times 10^{-3}$ | $0.776 \pm 2.19 \times 10^{-3}$ |
| 170                         | $0.833 \pm 2.95 \times 10^{-3}$ | $0.814 \pm 1.84 \times 10^{-3}$ | $0.792 \pm 2.51 \times 10^{-3}$ | $0.777 \pm 2.56 \times 10^{-3}$ |
| 140                         | $0.814 \pm 3.76 \times 10^{-3}$ | $0.787 \pm 5.86 \times 10^{-3}$ | $0.771 \pm 2.70 \times 10^{-3}$ | $0.745 \pm 3.83 \times 10^{-3}$ |
| 110                         | $0.801 \pm 4.91 \times 10^{-3}$ | $0.764 \pm 4.34 \times 10^{-3}$ | $0.740 \pm 6.81 \times 10^{-3}$ | $0.716 \pm 5.88 \times 10^{-3}$ |
| 80                          | $0.773 \pm 5.35 \times 10^{-3}$ | $0.745 \pm 6.24 \times 10^{-3}$ | $0.711 \pm 4.48 \times 10^{-3}$ | $0.678 \pm 3.44 \times 10^{-3}$ |

### Processing 3 Instrument Uncertainty

The uncertainty of the measuring tape was ( $\pm 0.05\text{cm}$ ) for both  $h_d$  and  $h_r$ . So for:

$$e = \sqrt{\frac{h_r}{h_d}}$$

propagation of uncertainty is:

$$\frac{u(e)}{e} = \frac{1}{2} \sqrt{\left(\frac{u(h_r)}{h_r}\right)^2 + \left(\frac{u(h_d)}{h_d}\right)^2}$$

$$u(e) = e \cdot \frac{1}{2} \sqrt{\left(\frac{u(h_r)}{h_r}\right)^2 + \left(\frac{u(h_d)}{h_d}\right)^2}$$

### Worked Example:

Using the same condition:  $h_d = 200\text{ cm}$ ,  $\bar{h}_r = 126.40$ ,  $u(h_r) = u(h_d) = 0.05$

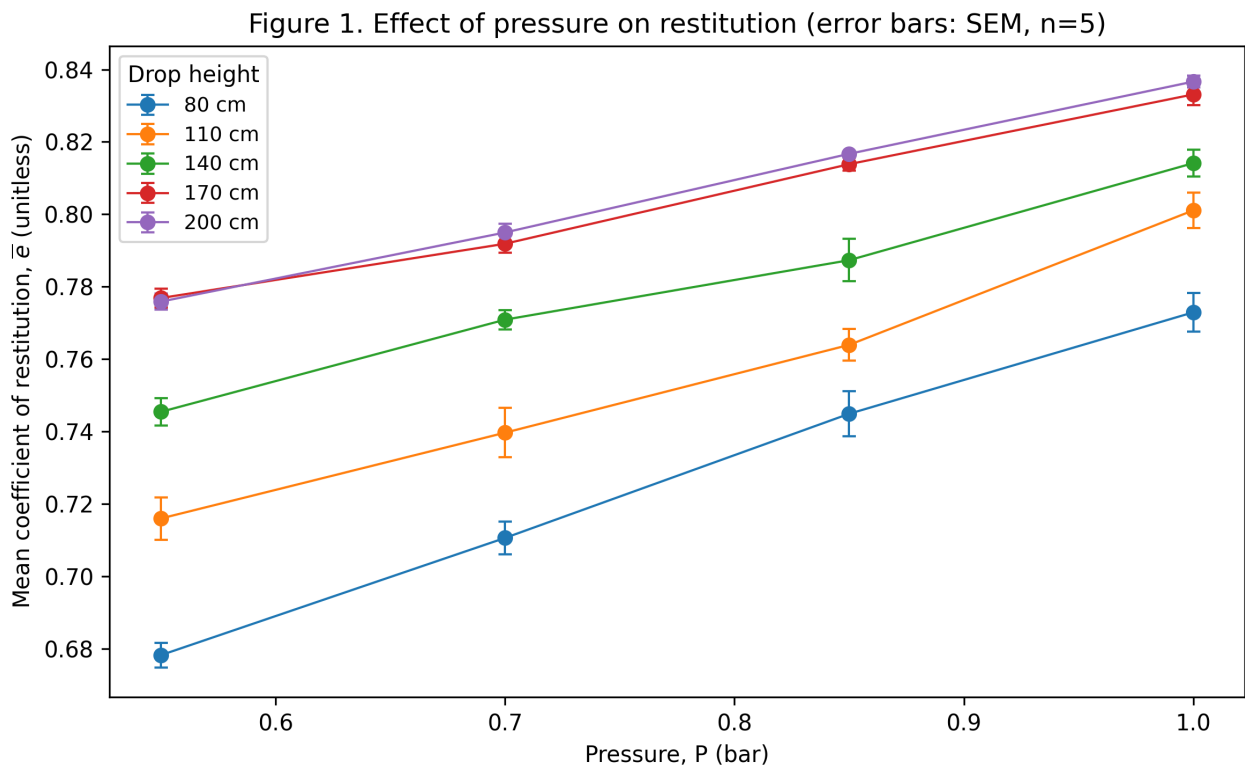
$$e = \sqrt{\frac{126.40}{200.00}} = 0.79$$

$$u(e) = 0.79 \cdot \frac{1}{2} \sqrt{\left(\frac{0.05}{126.40}\right)^2 + \left(\frac{0.05}{200.00}\right)^2} \approx 1.86 \times 10^{-4}$$

Instrument uncertainty is minimal compared to trial-to-trial scatter and doesn't really contribute to the overall uncertainty.

### Results and Analysis

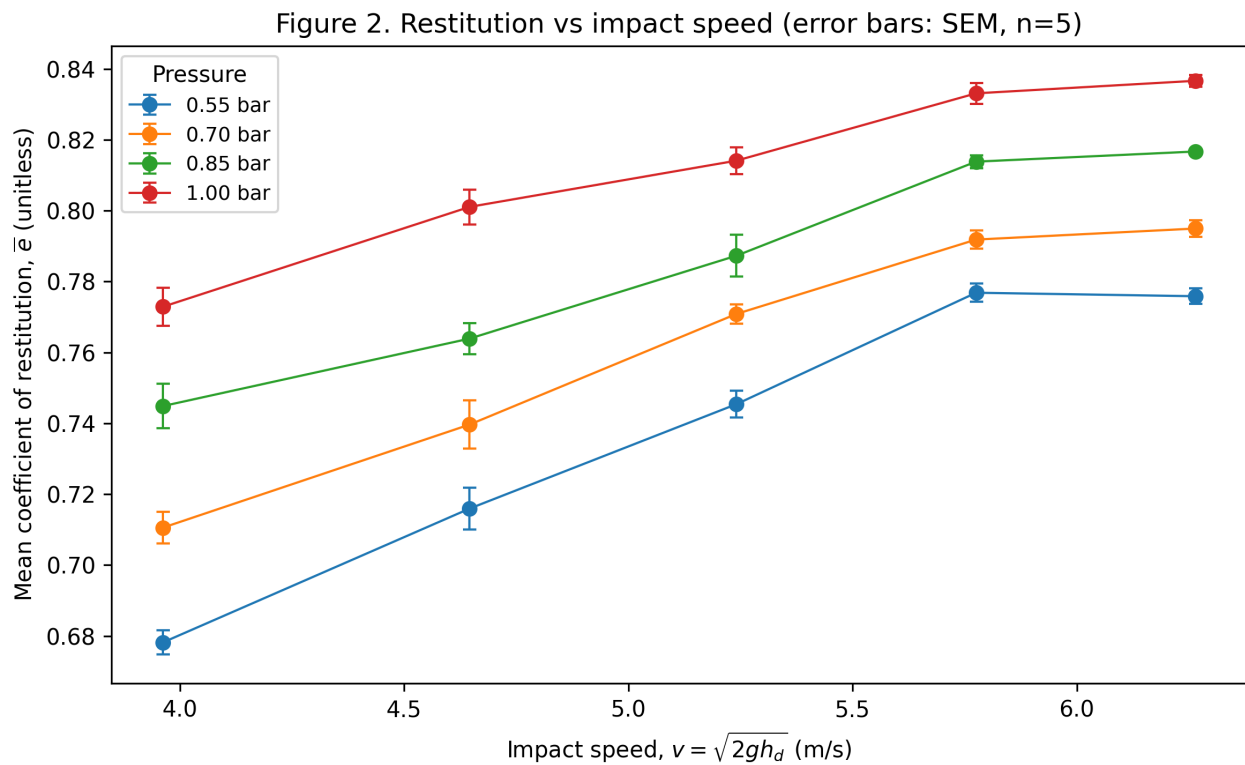
Results show that the coefficient of restitution depends highly on internal pressure. To examine this relationship for all drop heights, the average coefficient of restitution,  $\bar{e}$ , is plotted against internal pressure,  $P$ . Standard error of the mean (SEM) error bars show the uncertainty in the means.



Mean coefficient of restitution  $\bar{e}$  as a function of pressure  $P$  for five drop heights. Error bars show  $\pm$ SEM. ( $n = 5$ )

Figure 1 shows that the mean coefficient of restitution,  $\bar{e}$ , increases with internal pressure,  $P$ , in all drop heights. Higher internal pressure decreases the amount of energy lost during impact and results in greater recovery of energy and a higher rebound height. The graph seems to fit a more saturating model but the appearance is not reliable with only 4 levels of internal pressure.

Coefficient of restitution can also change with impact speed, which changes with drop height. Plotting  $\bar{e}$  against  $v$  allows for the comparison of pressure and impact speed as determinants of the rebound height and coefficient of restitution. Figure 2 shows the influence of the impact speed on



Mean coefficient of restitution  $\bar{e}$  as a function of impact speed  $v = \sqrt{2gh_d}$  for four pressures.

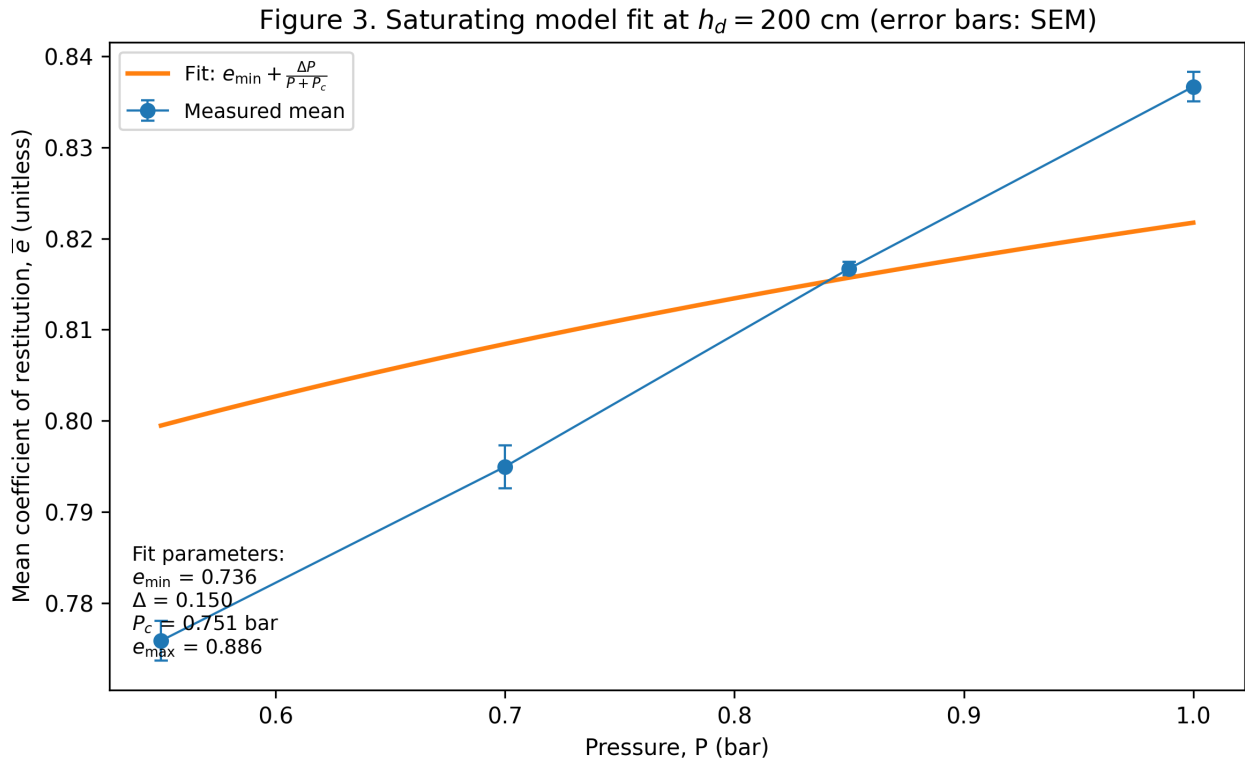
Error bars show  $\pm$ SEM. ( $n = 5$ )

the coefficient of restitution. If  $\bar{e}$  decreases as impact speed  $v$  increases this suggests that rate-dependent energy loss is more significant at higher deformation rates. However if  $\bar{e}$  remains relatively constant as  $v$  increases, internal pressure is the dominant determinant in this dataset. The influence of impact speed in Figure 2 can be compared with the effects of pressure in Figure 1 to understand which variable is the main determinant. In this dataset  $\bar{e}$ , changes slightly with impact speed. But the effect of pressure is higher, showing that pressure is the main determinant in this dataset.

The pressure dependence in Figure 1 shows a saturating affect in higher pressure levels so a saturating model fit was tested using its formula:

$$e(P) = e_{\min} + \frac{\Delta P}{P + P_c}$$

where  $e_{\min}$  is the low-pressure limit,  $\Delta$  is the total possible increase in restitution, and  $P_c$  is the level of pressure where half the total increase is achieved. The fit was tried on the  $h_d = 200\text{cm}$  dataset to keep the height-reading uncertainty at its lowest.



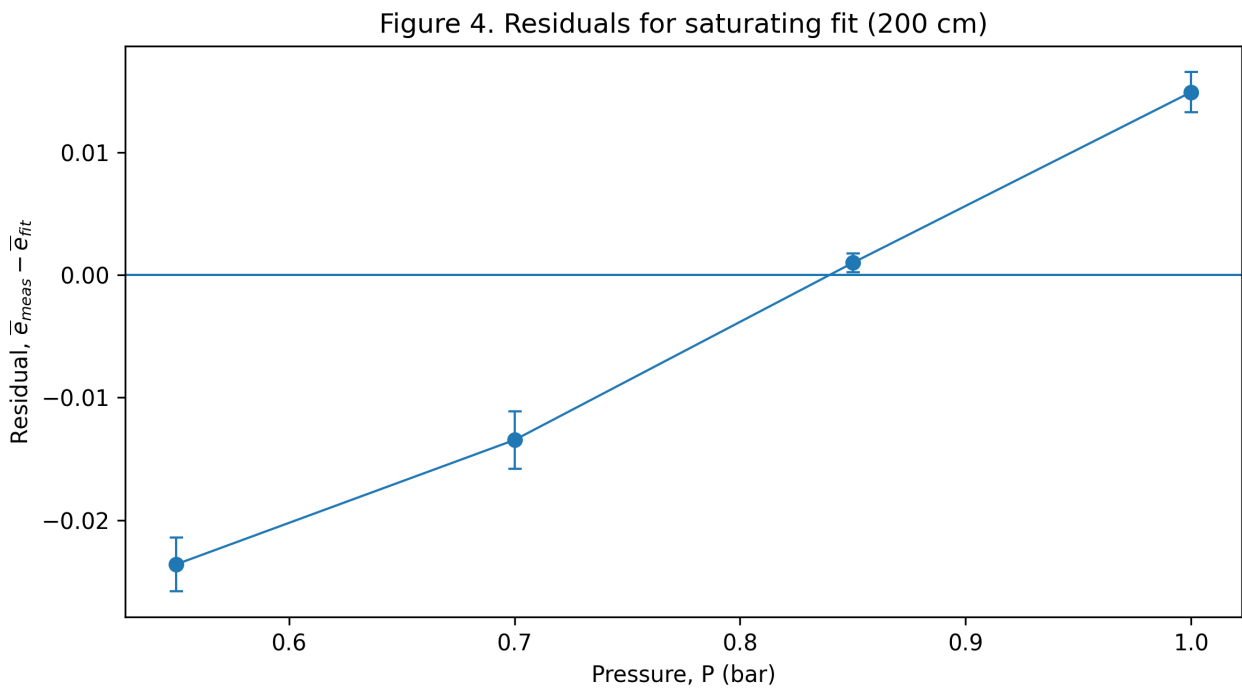
Saturating model fit at  $h_d = 200\text{cm}$ :  $e(P) = e_{\min} + \frac{\Delta P}{P + P_c}$ . Error bars show  $\pm\text{SEM}$ . ( $n = 5$ )

Figure 3 confirms that  $\bar{e}$  increases with pressure. However, in this internal pressure range (0.55 bar to 1.00 bar), points show a linear line rather than a saturating line. A saturating line may better suit a bigger pressure range but not in this one. A saturating model and a linear model will be compared to determine whether the pressure range was enough to see a saturating trend.

To evaluate the line in Figure 3, a residual analysis was performed and graphed. The residuals for each pressure point was calculated with the formula:

$$r = \bar{e}_{\text{measured}} - \bar{e}_{\text{fit}}$$

where  $\bar{e}_{\text{measured}}$  is the experimentally determined coefficient of restitution, and  $\bar{e}_{\text{fit}}$  is the value predicted by the fitted model. Graphing the residuals against pressure shows if the deviations are consistent with trial-to-trial variation or the model has systematic bias in this pressure range (0.55-1.00 bar).



Residuals for the saturating fit at  $h_d = 200\text{ cm}$ . Error bars show  $\pm\text{SEM}$ . ( $n = 5$ )

Figure 4 shows that the relationship is not consistent with the saturating model, because the residuals show a shift with pressure. The residuals graph shows model bias rather than only trial-to-trial variation. This leads us to test an alternative model such as the linear line and compare the fits.

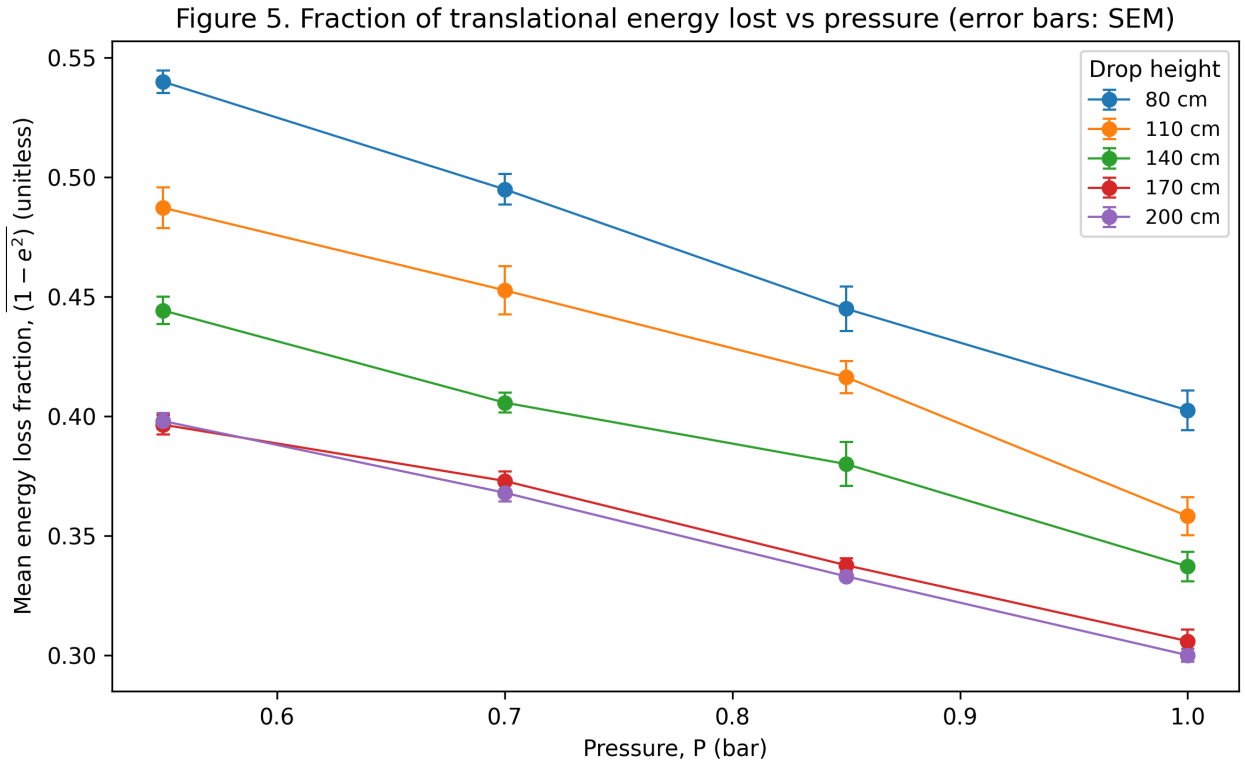
While  $\bar{e}$  is a kinematic measure of bounciness, it can be better understood in terms of energy transfer during impact. Since:

$$\frac{K_{\text{after}}}{K_{\text{before}}} = e^2$$

The amount of translational kinetic energy lost in the collision is:

$$\Delta = 1 - e^2$$

Therefore, plotting  $\Delta$  against pressure shows how inflation pressure affects the amount of energy lost in the ball, without any particular model for  $e(P)$ .



Mean translational energy loss amount  $(1 - e^2)$  as a function of pressure for five drop heights.

Error bars show  $\pm$ SEM. ( $n = 5$ )

Figure 5 shows that the mean translational energy loss  $(1 - e^2)$  decreases as the internal pressure increases in all drop heights. Higher inflation pressure results in less kinetic energy being lost into internal heating, acoustic emission, and other types of energy losses during deformation. The separation between curves for different drop heights shows that impact speed and deformation rate may also influence energy loss. However, the pressure trend is consistent in all heights, making it the

main determinant of the basketball's bounciness. This will be evaluated in the later sections, when considering (SEM) and some potential systematic errors, such as spin and peak-frame selection.

## Evaluation and Limitations

### Effect of Pressure Across Heights

Across all drop heights (80cm, 110cm, 140cm, 170cm, 200cm), the coefficient of restitution  $e$  increases with the pressure (0.55 bar to 1.00 bar). This result is consistent with the interpretation that the coefficient of restitution is a measure of how much kinetic energy is conserved after impact. For a vertical drop and rebound motion on a rough surface:

$$e = \sqrt{\frac{h_r}{h_d}}$$

One way to understand the amount of pressure effect is to compare  $\bar{e}$  at 1.00 bar and 0.55 bar for each drop height. Which gives the observed increases in the mean coefficient of restitution as:

- 200cm: +0.06
- 170cm: +0.06
- 140cm: +0.07
- 110cm: +0.09
- 80cm: +0.09

The mean loss fraction ( $1 - e^2$ ) decreases approximately by 0.09-0.14 across the same pressure change. This supports the claim that greater pressure leads to greater bounce, resulting in less dissipation into internal heating and acoustic emission.

### Model Testing Using All Height

Dataset of drop height 200cm was used in figures 3 and 4 to show model fitting and residual analysis. The model comparison can be repeated for each drop height dataset by fitting a dataset to

$\bar{e}(P)$  graph at the four different pressures (0.55, 0.70, 0.85, 1.00) and examine fit quality using the SEM and the residuals.

The basketball could have blown away at a higher pressure level or could not have bounced at a lower pressure level. This is why there was only 4 different internal pressures which is a small number for this experiment. This tells that a saturating model may appear visually good in this small dataset while being statistically unstable.

A better fit with this dataset is a linear model with a general formula:

$$e(P) = aP + b$$

This model was evaluated by comparing its deviations from the mean to the mean's uncertainty.

$$r_i = \bar{e}_i - (aP_i + b)$$

where  $\bar{e}_i$  is the experimentally measured mean coefficient of restitution at a certain pressure. The residuals were then divided by the standard error of the mean to achieve a number that shows how close those two numbers are.

$$z_i = \frac{r_i}{\text{SEM}_{e,i}}$$

Because  $\text{SEM}_{e,i}$  shows the uncertainty in the mean  $\bar{e}_i$  from repeated trials ( $n = 5$ ), the unitless number  $z_i$  shows whether the error of the model is small or big compared to experimental uncertainty. If the residuals are generally in the same order with SEM (most  $|z_i| \lesssim 1$ ) and do not show a systematic pattern with pressure the linear model can be considered reliable. However, if multiple points give  $|z_i| \gg 1$  or the residuals show a clear pressure-dependent pattern (such as all residuals positive at low pressure and negative at high pressure), this would show that a linear model is not reliable and there is curvature.

## Conclusion

This investigation examined how internal pressure affects a basketballs rebound height. The independent variables were the balls internal pressure (0.55 bar, 0.70 bar, 0.85 bar, 1.00 bar) and drop heights (80cm, 110cm, 140cm, 170cm, 200cm). The drop height was tested to consider the affects of impact speed on rebound heights of the basketball. For each combination of internal pressure and drop height 5 trials were made. The trials were recorded with slow motion from a fixed vertical plain. Rebound heights were then extracted from the slow motion video recordings. And the dataset of 100 measurements were achieved.

These rebound height datas were then used to calculate the coefficient of restitution to have a real measure of the bounciness of the basketball. Mean values, standard uncertainties and standard errors of the mean's were also calculated for every combination of internal pressure and drop height. The results showed clear pressure dependence for the rebound heights. An increase in internal pressure resulted in increased rebound height and therefore coefficient of restitution in every drop height tested. These results also showed that higher internal pressure decreased the amount of translational kinetic energy lost during the collision with the formula  $(1 - e^2)$ .

Coefficient of restitution was also plotted against impact speed to examine and understand whether it was a determinant of rebound height or not. The graph proved that the dependence on impact speed was very small compared to dependence on changes in internal pressure. Impact speed calculations are also more open to limitations such as possible small spins and errors in peak frame selection. As a result internal pressure is found to be the main determinant of the rebound height in the given dataset. As a final step the graph was tried to be fitted to a saturating model. However the residuals showed that a linear model was a better fit with the given dataset.

Overall, the investigation was successful on showing the relationship between the bounciness of a basketball and its internal pressure. The investigation can be expended to a wider range of pressures and drop heights with a different ball and a different system as this ball did not allow for any higher or lower pressures and we could not get the ball higher in indoor areas. A bigger dataset will match these results and strengthen them.

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