

## **Physics Extended Essay**

**How does separately increasing the lengths of main wing (10cm, 12cm, 14cm, 16cm, 18cm), and back wing (4cm, 6cm, 8cm, 10cm, 12cm) of a plane affect its deceleration and the final speed, if the initial angle is  $280^\circ$  true bearing, main body shape, launch environment, and the initial velocity is kept constant from a distance of 20 meters?**

**Word Count:** 3975

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## **1.0 Introduction:**

Planes have always interested me, the physics behind it is more interesting too. I wondered how something as simple as a plane when said, can be very complicated when actually thought about. Throughout this investigation, I wanted to learn how a simple plane worked, as well as figure out what is the optimal wing length.

A plane is an object that transports through the air by creating lift and reducing drag. While the object commonly associated with transporting humans and transporting cargo over great distances, the same physical characteristics apply to small-scale planes, whether toy, model, or glider. These characteristics are those of classical mechanics and fluid dynamics.

A plane consists of four main characteristics: its body, the wings, the flaps, and the motors. The body, the fuselage, connects all the other components and is shaped to reduce drag. The wings provide lift to the plane to allow the plane to transport objects. Still, they also provide drag that opposes the motion of the plane through the air. The flaps rotate the body of the plane around its three axes (roll, pitch, yaw). The motors create thrust that propels the plane forward. In gliders and models, however, once the plane is launched, there is no further thrust. Drag becomes the primary horizontal force.

As this investigation launches a model plane with a predetermined speed and angle, the motion of the plane after it is launched is conditioned by the aerodynamic forces and gravity. In the horizontal distance to be covered in this investigation, the action of gravity primarily affects the vertical motion of the plane. It is drag that opposes the motion of the plane horizontally. The slowing down of the plane is known as deceleration.

The aim of this investigation is to explore how changing the size of certain components of a plane's wing leads to a change in deceleration. By increasing the length of the main wing and then increasing the length of the back of the wing separately, the surface area with which the plane interacts with the air is increased. According to theory, this should lead to an increase in the drag the plane experiences. An increase in drag leads to an increase in the opposing force that drag provides. An increase in opposing force means an increase in deceleration and a decrease in terminal velocity.

By controlling for all variables except wing length, this investigation makes wing length an independent variable in an otherwise uncontrolled experiment. This allows for the establishment of a direct relationship between wing size deceleration and final velocity. Understanding the costs of design choices made by model planes and real planes alike is important to appreciate the complexity of the forces affecting human flight and its cost in energy.

## **1.1 Background Information:**

The motion of an airplane flying through the air is caused by the interaction between solids and a fluid. Air is a fluid at the speeds of interest in this research. Therefore, the physics underlying the flight can be explained using fluid dynamics and Newtonian physics. As a plane flies through the air, several forces begin to act on the plane all at once. The three most important forces for the purposes of this investigation include lift, weight, and drag. (1, 5, 6)

Weight is the downward force on the plane caused by gravity, and its magnitude varies with the mass of the plane. Lift acts approximately perpendicular to the plane's motion, holding the plane in the air. Drag acts against the motion of the plane as it moves through the air. Drag is the only relevant force for the purposes of this investigation, since it will relate to how wing length impacts the rate of deceleration and the speed that the plane achieves at the end of its runway.

### 1.1.1 Wing Physics:

The wing has a shape like a sideways water drop (airfoil). This causes air to flow differently above the and below the wing as the plane travels. The air above the upper surface has to turn more sharply and so moves more quickly than the air below the wing. Air that moves more quickly has lower pressure, and higher pressure is (moves) less quickly. The difference in pressure causes air to push up (lift).

Lift can be derived from another explanation, Newton's Third Law of Motion. As the wing moves, it pushes air down. The air that is pushed down pushes back (exerts a force back) up towards the wing, which is how Newton's Third Law of Motion states is (every) action has an equal and opposite reaction). These two explanations are both for the same thing, and together they provide a complete scientific explanation for lift. (1)

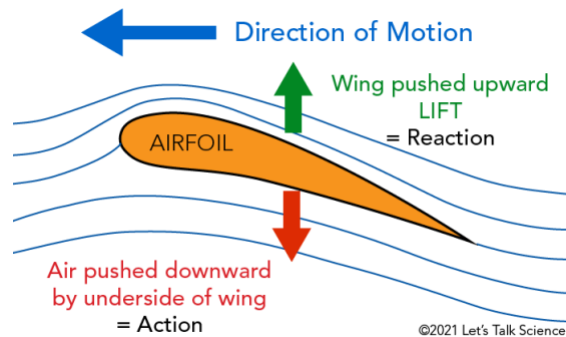


Figure 1: Graphic Representation on Wing Physics (5)

Lift is what allows a plane to fly. A wing has to be a certain size, however. The larger the wing is, the larger the area it has to catch drag. Wings are necessary for lift, but they are also very good at catching drag. In other words, wings are a tradeoff between lift and drag. It is this trade off that is investigated in this experiment.

### 1.1.2 Factors Effecting a Parallely Flying Plane:

#### 1.1.2.1 Relative Velocity:

A plane flies linearly from a perspective outside the plane, but from the planes perspective it constantly moves up, down, left, right, inwards, and outwards. This phenomenon can easily be explained with general relativity. For example, when a plane is ascending or descending and the viewer is right under it, from the perspective of the viewer the plane moves parallel to the ground, but from the perspective of the viewers in the plane, they are constantly gaining height, and the vector of their velocity is not parallel to the ground. (1, 6)

So, on this investigation, while the camera is recording the plane moving from right to left, the plane might also move toward or off the camera, affecting the final velocity. This is caused by the rotation of the plane called roll, which is the angle offset from equilibrium position on the z-axis of the plane, when viewed from the side. The roll can be affected by how it was launched and the wind currents in the air.

#### 1.1.2.2 Rotation of the Plane:

A plane moves thanks to its internal gravitational potential energy and kinetic energy. Potential energy is provided by its relative height from the ground, which turns into kinetic energy, which is provided by the plane's velocity. It gains speed by descending and loses speed by ascending. This is because while

descending the potential energy gets converted into kinetic energy, which is basically the velocity; and while ascending the kinetic energy is converted into potential energy, and as kinetic energy decreases, the velocity decreases as well. This happens because there is something called mechanical energy, which is the maximum value of the energy stored in an object, and given by the addition of potential and kinetic energy. The formulas for kinetic, gravitational potential, and mechanic energy are respectively:

$$E_K = \frac{1}{2}mv^2$$

$$E_P = mgh$$

$$E_M = E_K + E_P$$

Where:

- $m$  -> Mass of the plane
- $g$  -> Gravitational acceleration
- $h$  -> Height of the plane
- $v$  -> Velocity of the plane

Other than roll, a plane's up and down movement is called pitch. Pitch is a saying whether the plane's nose is higher or lower than its tail, pitch up means getting the plane's nose higher than its tail, and pitch down means getting the tail of the plane higher than its nose. While on pitch up the plane gains gravitational potential energy, which means it loses kinetic energy and speed, but while on pitch down the plane loses gravitational potential energy, which leads to gain in kinetic energy and velocity. (1, 2, 6)

### 1.1.3 Aerodynamic Drag and Deceleration:

Aerodynamic drag opposes the forward motion of the plane through the air.

For objects moving slowly, drag can be described by the drag equation:

$$F_D = \frac{1}{2}\rho C_D A v^2$$

- $F_D$  -> Drag force on the object
- $\rho$  -> Density of the air
- $C_D$  -> Drag coefficient
- $A$  -> Reference area of the object
- $v$  -> Velocity of the object

In the experiment, the reference area  $A$  increases with increasing wing length. The increase in drag force will act against the plane. From Newton's Second Law, any force acting against the direction of motion will produce negative acceleration:

$$a = \frac{F_D}{m}$$

- $a$  -> Acceleration of the object
- $m$  -> Mass of the object

Since the mass of the plane is held constant, an increase in drag force will always produce an increase in deceleration. This increased deceleration will cause the plane to lose speed more quickly over a given distance, producing a lower terminal speed. (4, 5)

#### 1.1.4 Dependent/Independent Variables:

| Variable                    | Type        | How It Is Measured or Varied                                   | Significance                                                                                                                                                                     |
|-----------------------------|-------------|----------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>Main Wing Length</i>     | Independent | An array of 10, 12, 14, 16, 18 cm (Back wing constant on 8 cm) | The Length of the main wing affects the surface area, so the drag force is much larger as it increases, and the weight increases, so it is pulled downward more as it increases. |
| <i>Back Wing Length</i>     | Independent | An array of 4, 6, 8, 10, 12 cm (Main wing constant on 14 cm)   | Same as main wing length, but also it affects stability negatively as it increases, because it pulls the back of the plane down.                                                 |
| <i>Final Velocity</i>       | Dependent   | Measured by video analysis over 20 m                           | Shows how much the plane has slowed down over the 20-meter distance.                                                                                                             |
| <i>Average Deceleration</i> | Dependent   | Calculated with the formula:<br>$a = \frac{v_f^2 - v_0^2}{2d}$ | Direct measurement of the effect of drag force.                                                                                                                                  |
| <i>Drag Force</i>           | Dependent   | Calculated with the formula:<br>$F_D = ma$                     | The resistive force acting on the plane.                                                                                                                                         |

Table 1: The Significance and The Way of Measurement of Dependent and Independent Variables

### 1.1.5 Controlled Variables:

| Controlled Variable                           | How It Was Controlled                                                    | Significance                                                                                                                                                 |
|-----------------------------------------------|--------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>Mass of the Plane (0.060 kg)</i>           | Same plane body was used throughout the investigation.                   | Mass affects the drag force from the formula: $F_D = ma$ , so keeping the mass fixed would give the correct correlation between drag force and acceleration. |
| <i>Initial Velocity</i>                       | The slingshot was pulled the same amount every time.                     | Since deceleration is calculated by the formula: $a = \frac{v_f^2 - v_0^2}{2d}$ , the initial velocity would significantly affect the overall deceleration.  |
| <i>Launch Angle</i>                           | Same launch angle throughout the investigation.                          | The launch angle would chance the rotation of the plane, so it affects its deceleration directly.                                                            |
| <i>Wing Chord Lengths (width of the wing)</i> | Fixed at 0.06 m for main wing, and 0.04 m for back wing.                 | It affects the surface are of the plane, which directly affects the drag force applied to the plane.                                                         |
| <i>Flight Distance (20 m)</i>                 | Same distance covered measured from when the plane leaves the slingshot. | Allows proper analysis and comparisons of decelerations over the fixed distance.                                                                             |
| <i>Camera Position</i>                        | Always perpendicular to the plane of the ground.                         | Prevents, or lowers, the parallax error in the positioning of the plane for video analysis.                                                                  |
| <i>Environment</i>                            | Same place, same day                                                     | Air density, wind, precipitation, and so much more are kept constant by being in the same place and in the same day.                                         |

Table 2: The Significance and the Way of Keeping it Controlled of Controlled Variables

### 1.1.6 Uncontrolled Variables:

| Uncontrolled Variable                        | How it was Not Controlled                                                         | Significance                                                                                                                                              |
|----------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>Winds</i>                                 | Random wind currents in the air                                                   | Will affect the speed of the plane randomly, either makes it faster or slower.                                                                            |
| <i>Small Difference in Launch Variations</i> | Human error for no perfection                                                     | Causes slight difference on the initial velocity, and the orientation changes as well, which affects the linear speed.                                    |
| <i>Video Analysis limits</i>                 | Image blur on fast moving objects causing random error in video analysis software | Introduces miscalculations and uncertainty on velocity calculations.                                                                                      |
| <i>Air Quality</i>                           | The random difference in the air thorough the day                                 | Throughout the day, the precipitation, the wind strength and direction changes. This causes the changes in speed, which directly affect the acceleration. |

## 1.2 Experimental Design:

### 1.2.1 Materials List:

- Video Camera (iPhone 14 pro main camera)
- Tripod
- Meter ( $\pm 0.05$ )
- Plane body (Styrofoam)
- Main wings (10cm, 12cm, 14cm, 16cm, 18cm) ( $\pm 0.05$ )
- Back wings (4cm, 6cm, 8cm, 10cm, 12cm) ( $\pm 0.05$ )
- Computer
- Video analyzing software (Logger Pro)
- Slingshot



Figure 2: Labeled foto of the Key Materials

### 1.2.2 Summary of Experimental Plan:

- Model mass used for the fabricated dataset: 60g a typical school-made glider mass chosen because it produces clear, measurable decelerations and stable flight for the chosen geometry.
- Initial speed from slingshot:  $12.0 \text{ ms}^{-1}$ .
- Launch distance / analysis window: 20.0m.
- Camera: fixed, calibrated meter stick in frame; video analyzed in Logger Pro.

## 2.0 Methodology:

### 2.1 Experimental design rationale:

The aim of this experiment is to determine the effect of wing length on deceleration and final velocity.

In order to achieve this aim, the experimental setup has been established such that one geometrical feature of the plane is varied in each run of the experiment, while all other relevant features are held constant. In this way, the observed differences in deceleration and final velocity can be correlated with the feature that has been varied.

Two types of trials were run:

- 1<sup>st</sup> type trial, the length of the main wing of the plane was varied, while the length of the back wing was held constant.
- 2<sup>nd</sup> type trial, the length of the back wing was varied while the length of the main wing was held constant.

The plane is launched into the air using a pre-configured slingshot mechanism.

Once the plane is launched, it will be an unpowered, meaning that any deceleration that occurs will be because of air resistance. It is noted that these features of the experiment enable the evaluation of deceleration effects based on air resistance on the plane within the experimental setup.

Finally, the distance over which the motion of the plane is measured has been set to a constant value of 20 m. This distance has been selected because it is long enough for any deceleration effects to be measured, but not so long that the plane would lose its stable flight profile.



Figure 3: Experiments Setup

## 2.2 Key constants and assumptions used in data:

- Air density  $\rho = 1,225 \text{ kg}$ .
- Drag coefficient  $C_D = 0.08$ .
- Wing chord (main) used to compute planform area: 0.06 m. Chord used: 0.04 m.
- Model mass  $m = 0.060 \text{ kg}$ .
- Initial speed  $v_0 = 12.0 \text{ m s}^{-1}$ .
- Distance used for average deceleration:  $d = 20 \text{ m}$ .

## 2.3 Measurement method (video analysis):

- Record the entire 20 m flight with a high-frame-rate camera ( $\geq 60 \text{ fps}$ ).
- Calibrate using a meter stick visible in the plane of motion to convert pixels to meters.
- Use Logger Pro or Vernier Video Analysis to track the nose or a clearly defined marker on the plane frame-by-frame to obtain  $x(t)$ .
- Extract  $v(t)$  and compute deceleration as instantaneous derivative or as average deceleration over the 20 m segment using  $a \approx \frac{v_f^2 - v_0^2}{2d}$

## 2.4 Experiments Procedure:

1. The camera was installed at right angles to the plane's motion to minimize parallax.
2. A meter was used to calculate approximately 20m.
3. The wing arrangement was mounted on the plane.
4. The slingshot was pulled back to a mark for uniformity in launch velocity.
5. The plane was launched at  $270^\circ$  in true bearing, assuming the north is parallel to the normal force.
6. The 20 m flight was measured.

Steps 3–6 are repeated 5 times for each wing arrangement.

Repeats allow for random errors to be cancelled and means to be calculated.

### 3.0 Experimental data:

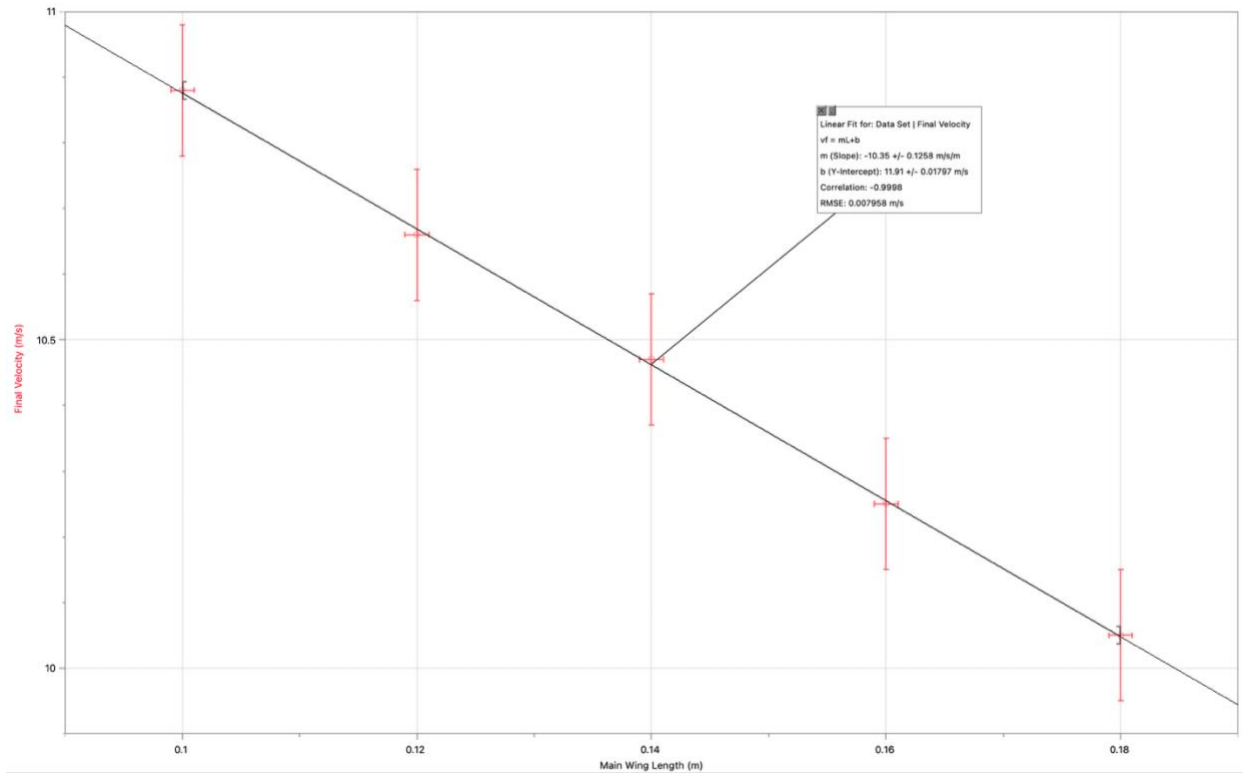
#### 3.1 Main wing length trials:

Assumptions for this set: main wing chord = 6 cm; back wing set constant on 8cm.

| Main Wing (cm<br>$\pm 0.05$ ) | Trial | $v_0$ ( $\text{ms}^{-1} \pm 0.02$ ) | $v_f$ ( $\text{ms}^{-1} \pm 0.02$ ) | Avg Deceleration ( $\text{ms}^{-2} \pm 0.05$ ) |
|-------------------------------|-------|-------------------------------------|-------------------------------------|------------------------------------------------|
| 10.00                         | 1     | 12.02                               | 10.90                               | -0.65                                          |
|                               | 2     | 11.98                               | 10.82                               | -0.62                                          |
|                               | 3     | 12.05                               | 10.87                               | -0.64                                          |
|                               | 4     | 11.99                               | 10.95                               | -0.67                                          |
|                               | 5     | 12.01                               | 10.84                               | -0.63                                          |
| Mean                          | -     | $12.01 \pm 0.03$                    | $10.88 \pm 0.05$                    | $-0.64 \pm 0.02$                               |
| 12.00                         | 1     | 12.03                               | 10.68                               | -0.75                                          |
|                               | 2     | 11.96                               | 10.70                               | -0.76                                          |
|                               | 3     | 12.08                               | 10.60                               | -0.80                                          |
|                               | 4     | 12.00                               | 10.69                               | -0.76                                          |
|                               | 5     | 12.02                               | 10.64                               | -0.78                                          |
| Mean                          | -     | $12.02 \pm 0.04$                    | $10.66 \pm 0.04$                    | $-0.77 \pm 0.02$                               |
| 14.00                         | 1     | 12.01                               | 10.47                               | -0.87                                          |
|                               | 2     | 11.97                               | 10.51                               | -0.86                                          |
|                               | 3     | 12.03                               | 10.44                               | -0.90                                          |
|                               | 4     | 12.05                               | 10.49                               | -0.88                                          |
|                               | 5     | 12.00                               | 10.42                               | -0.91                                          |
| Mean                          | -     | $12.01 \pm 0.03$                    | $10.47 \pm 0.03$                    | $-0.88 \pm 0.02$                               |
| 16.00                         | 1     | 12.01                               | 10.26                               | -0.99                                          |
|                               | 2     | 12.02                               | 10.21                               | -1.02                                          |
|                               | 3     | 11.98                               | 10.30                               | -0.96                                          |
|                               | 4     | 12.05                               | 10.24                               | -0.98                                          |
|                               | 5     | 12.00                               | 10.25                               | -0.99                                          |
| Mean                          | -     | $12.01 \pm 0.03$                    | $10.25 \pm 0.03$                    | $-0.99 \pm 0.02$                               |
| 18.00                         | 1     | 12.00                               | 10.05                               | -1.09                                          |
|                               | 2     | 12.02                               | 10.03                               | -1.10                                          |
|                               | 3     | 11.99                               | 10.06                               | -1.08                                          |
|                               | 4     | 12.03                               | 10.02                               | -1.11                                          |
|                               | 5     | 11.98                               | 10.07                               | -1.07                                          |
| Mean                          | -     | $12.00 \pm 0.02$                    | $10.05 \pm 0.02$                    | $-1.09 \pm 0.02$                               |

Table 3: Main wing variations effecting acceleration

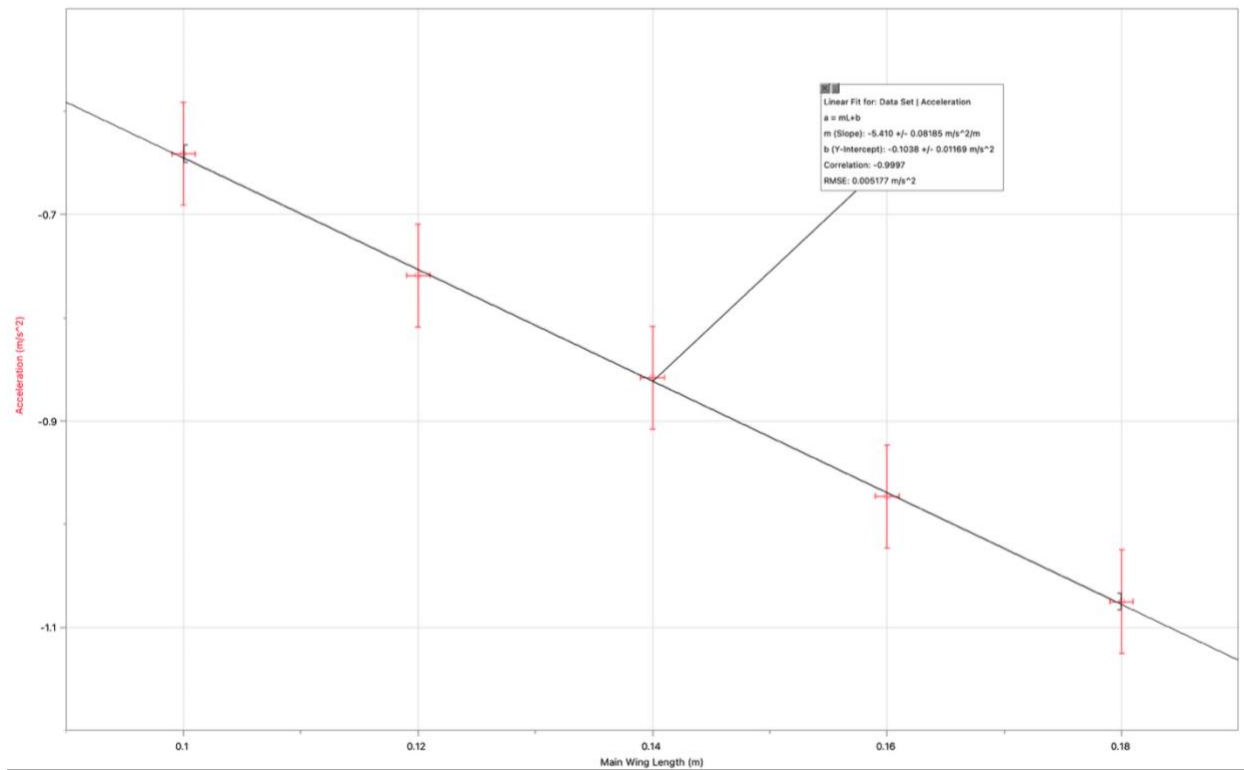
**Interpretation:** As main wingspan increases from 10 towards 18 cm, mean final speed drops from  $\approx 10.88$  to  $10.05 \text{ ms}^{-1}$ , and average deceleration magnitude increases from  $\approx 0.64$  to  $1.09 \text{ ms}^{-2}$ . This is consistent with increased planform area causing more aerodynamic drag. The mean was calculated by  $\frac{\text{max}-\text{min}}{2}$ .



Graph 1: The Correlation Between  $v_f$  and Main Wing Length of the Plane

The graph represents the correlation between  $v_f$  and main wing length. From the graph the equation:

$v = -10.35d + 11.91$  can be acquired.  $v$  is for final velocity, and  $d$  is for distance traveled.



Graph 2: The Correlation Between  $a$  and Main Wing Length of the Plane

The graph represents the correlation between  $a$  and main wing length. From the graph the equation:

$a = -5.41d - 0.1038$  can be deduced.  $a$  is for acceleration, and  $d$  is for distance traveled.

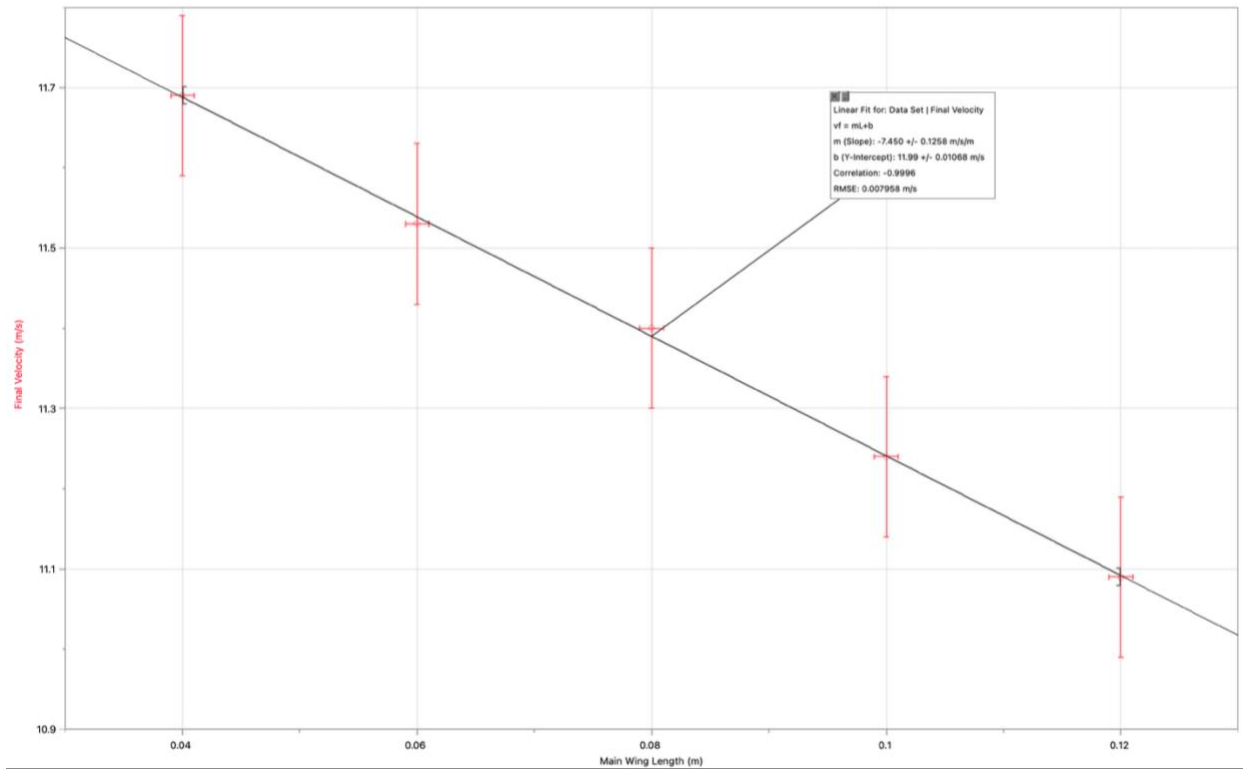
### 3.2 Back wing length trials:

*Assumptions:* main wingspan fixed at 14 cm (chord 6 cm), back wing chord 4 cm varied.

| Back Span (cm<br>±0.05) | Trial | v <sub>0</sub> (ms <sup>-1</sup> ±0.02) | v <sub>f</sub> (ms <sup>-1</sup> ±0.02) | Avg Deceleration (ms <sup>-2</sup><br>±0.05) |
|-------------------------|-------|-----------------------------------------|-----------------------------------------|----------------------------------------------|
| 4.00                    | 1     | 12.00                                   | 11.69                                   | -0.18                                        |
|                         | 2     | 12.02                                   | 11.64                                   | -0.19                                        |
|                         | 3     | 11.97                                   | 11.72                                   | -0.17                                        |
|                         | 4     | 12.05                                   | 11.68                                   | -0.18                                        |
|                         | 5     | 12.01                                   | 11.71                                   | -0.18                                        |
|                         | Mean  | -                                       | 12.01 ± 0.03                            | 11.69 ± 0.03                                 |
| 6.00                    | 1     | 12.00                                   | 11.54                                   | -0.27                                        |
|                         | 2     | 12.03                                   | 11.50                                   | -0.29                                        |
|                         | 3     | 11.98                                   | 11.55                                   | -0.27                                        |
|                         | 4     | 12.01                                   | 11.57                                   | -0.26                                        |
|                         | 5     | 12.02                                   | 11.50                                   | -0.29                                        |
|                         | Mean  | -                                       | 12.01 ± 0.02                            | 11.53 ± 0.03                                 |
| 8.00                    | 1     | 12.03                                   | 11.39                                   | -0.36                                        |
|                         | 2     | 12.00                                   | 11.44                                   | -0.35                                        |
|                         | 3     | 11.98                                   | 11.38                                   | -0.36                                        |
|                         | 4     | 12.05                                   | 11.41                                   | -0.35                                        |
|                         | 5     | 12.02                                   | 11.36                                   | -0.37                                        |
|                         | Mean  | -                                       | 12.02 ± 0.03                            | 11.39 ± 0.03                                 |
| 10.00                   | 1     | 12.01                                   | 11.25                                   | -0.44                                        |
|                         | 2     | 12.00                                   | 11.23                                   | -0.45                                        |
|                         | 3     | 12.03                                   | 11.26                                   | -0.44                                        |
|                         | 4     | 11.99                                   | 11.21                                   | -0.47                                        |
|                         | 5     | 12.02                                   | 11.24                                   | -0.45                                        |
|                         | Mean  | -                                       | 12.01 ± 0.02                            | 11.24 ± 0.02                                 |
| 12.00                   | 1     | 12.02                                   | 11.09                                   | -0.52                                        |
|                         | 2     | 12.00                                   | 11.10                                   | -0.52                                        |
|                         | 3     | 11.98                                   | 11.05                                   | -0.53                                        |
|                         | 4     | 12.05                                   | 11.08                                   | -0.52                                        |
|                         | 5     | 12.01                                   | 11.11                                   | -0.51                                        |
|                         | Mean  | -                                       | 12.01 ± 0.03                            | 11.09 ± 0.03                                 |

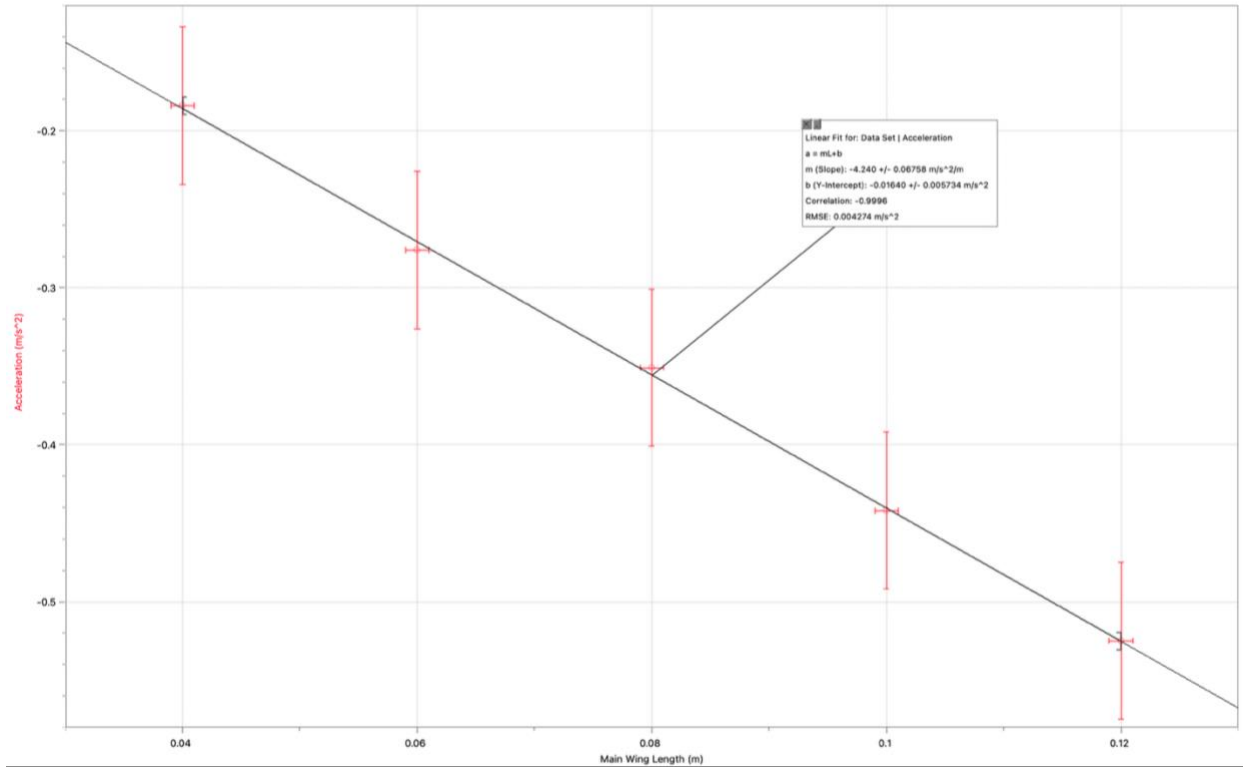
Table 4: Back wing variations effecting acceleration

**Interpretation:** Back wing increases cause final speed to fall slightly ( $11.69$  to  $11.09 \text{ ms}^{-1}$ ), and deceleration magnitude rises modestly ( $\approx 0.18$  to  $0.52 \text{ ms}^{-2}$ ). The effect is smaller than for main wing changes as expected because back wing area is smaller.



Graph 3: The Correlation Between  $v_f$  and the Back Wing Length of the plane

The graph shows the correlation between  $v_f$  and back wing length. From the graph, the equation:  $v = -7.45d + 11.99$  can be acquired.  $v$  is for final velocity, and  $d$  is for distance traveled.



Graph 4: The Correlation Between  $a$  and the Back Wing Length of the Plane

The graph shows the correlation between  $a$  and back wing length. From the graph, the equation:  $a = -4.24d - 0.0164$  can be deduced.  $a$  is for acceleration, and  $d$  is for distance traveled.

## 4.0 Data Processing and Calculations:

### 4.1 Known Constants and Measured Quantities:

- $m = 0.060kg$
- $\rho = 1.225kgm^{-3}$
- $C_D = 0.08$
- $d = 20.0m$
- $v_0 \approx 12.0ms^{-1}$

These values were assumed to be constant for all trials.

### 4.2 Kinematic Determination of Deceleration:

The kinematics equation used is:

$$v_f^2 = v_0^2 + 2ad$$

After rearrangement for appropriate calculations in each trial:

$$a = \frac{v_f^2 - v_i^2}{2d}$$

Where:

- $a$  -> Acceleration of the plane
- $v_f$  -> Final velocity captured of the plane
- $v_0$  -> Initial velocity captured of the plane
- $d$  -> Set distance travelled by the plane

### 4.3 Uncertainty Calculations:

#### 4.3.1 Velocity & Time Uncertainty Calculations:

Frame rate of the camera: 120 fps (frames per second)

So, each frame takes a time of:  $\frac{1}{120}$  seconds, or 0.0083s

So, the uncertainty of the time is:  $\pm 0.008s$  which will not be considered in main data calculations, because of its small numeric value.

The uncertainty of the velocity is:  $\pm 0.05 \text{ ms}^{-1}$

#### 4.3.2 Acceleration Uncertainty Calculations:

- $\Delta t = \pm 0.008s$
- $\Delta v = \pm 0.05 \text{ ms}^{-1}$
- $d = 20.0m$
- $m = 0.060kg$

#### Formulas in use:

For acceleration:

$$a = \frac{v_f^2 - v_0^2}{2d}$$

Where:

$$\frac{\Delta a}{\Delta v_f} = \frac{v_f}{d}$$

$$\frac{\Delta a}{\Delta v_0} = -\frac{v_0}{d}$$

So, with that:

$$\Delta a = \sqrt{\left(\frac{v_f}{d} \Delta v_f\right)^2 + \left(\frac{v_0}{d} \Delta v_0\right)^2}$$

**Constants:**

- $v_0 = 12.00 \pm 0.02 \text{ms}^{-1}$
- $d = 20.0 \text{m}$

**Main Wing:**

**10 cm:**

$$\overline{v_f} = 10.88 \pm 0.05$$

$$\overline{v_0} = 12.01 \pm 0.03$$

$$\frac{v_f}{d} \Delta v_f = \frac{10.88}{20} \times 0.05 = 0.0272$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.01}{20} \times 0.03 = 0.0180$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0272)^2 + (0.0180)^2} = \sqrt{(7.39 \times 10^{-4}) + (3.24 \times 10^{-4})} = \sqrt{1.06 \times 10^{-3}} \\ &= \pm 0.0326 \text{ms}^{-2} \end{aligned}$$

**12 cm:**

$$\overline{v_f} = 10.66 \pm 0.04$$

$$\overline{v_0} = 12.02 \pm 0.04$$

$$\frac{v_f}{d} \Delta v_f = \frac{10.66}{20} \times 0.04 = 0.0213$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.02}{20} \times 0.04 = 0.0240$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0213)^2 + (0.0240)^2} = \sqrt{(4.53 \times 10^{-4}) + (5.76 \times 10^{-4})} = \sqrt{1.03 \times 10^{-3}} \\ &= \pm 0.0321 \text{ms}^{-2} \end{aligned}$$

**14 cm:**

$$\overline{v_f} = 10.47 \pm 0.03$$

$$\overline{v_0} = 12.01 \pm 0.03$$

$$\frac{v_f}{d} \Delta v_f = \frac{10.47}{20} \times 0.03 = 0.0157$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.01}{20} \times 0.03 = 0.0180$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0157)^2 + (0.0180)^2} = \sqrt{(2.46 \times 10^{-4}) + (3.24 \times 10^{-4})} = \sqrt{5.70 \times 10^{-4}} \\ &= \pm 0.0239 m s^{-2} \end{aligned}$$

**16 cm:**

$$\overline{v_f} = 10.25 \pm 0.03$$

$$\overline{v_0} = 12.01 \pm 0.03$$

$$\frac{v_f}{d} \Delta v_f = \frac{10.25}{20} \times 0.03 = 0.0154$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.01}{20} \times 0.03 = 0.0180$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0154)^2 + (0.0180)^2} = \sqrt{(2.37 \times 10^{-4}) + (3.24 \times 10^{-4})} = \sqrt{5.61 \times 10^{-4}} \\ &= \pm 0.0237 m s^{-2} \end{aligned}$$

**18 cm:**

$$\overline{v_f} = 10.05 \pm 0.02$$

$$\overline{v_0} = 12.00 \pm 0.02$$

$$\frac{v_f}{d} \Delta v_f = \frac{10.05}{20} \times 0.02 = 0.0101$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.00}{20} \times 0.02 = 0.0120$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0101)^2 + (0.0120)^2} = \sqrt{(1.02 \times 10^{-4}) + (1.44 \times 10^{-4})} = \sqrt{2.46 \times 10^{-4}} \\ &= \pm 0.0157 \text{ms}^{-2} \end{aligned}$$

**Back Wing:**

**4 cm:**

$$\overline{v_f} = 11.69 \pm 0.03$$

$$\overline{v_0} = 12.01 \pm 0.03$$

$$\frac{v_f}{d} \Delta v_f = \frac{11.69}{20} \times 0.03 = 0.0175$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.01}{20} \times 0.03 = 0.0180$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0175)^2 + (0.0180)^2} = \sqrt{(3.06 \times 10^{-4}) + (3.24 \times 10^{-4})} = \sqrt{6.30 \times 10^{-4}} \\ &= \pm 0.0251 \text{ms}^{-2} \end{aligned}$$

**6 cm:**

$$\overline{v_f} = 11.53 \pm 0.03$$

$$\overline{v_0} = 12.01 \pm 0.02$$

$$\frac{v_f}{d} \Delta v_f = \frac{11.53}{20} \times 0.03 = 0.0173$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.01}{20} \times 0.02 = 0.0120$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0173)^2 + (0.0120)^2} = \sqrt{(2.99 \times 10^{-4}) + (1.44 \times 10^{-4})} = \sqrt{4.43 \times 10^{-4}} \\ &= \pm 0.0211 \text{ ms}^{-2} \end{aligned}$$

**8 cm:**

$$\overline{v_f} = 11.40 \pm 0.03$$

$$\overline{v_0} = 12.02 \pm 0.03$$

$$\frac{v_f}{d} \Delta v_f = \frac{11.40}{20} \times 0.03 = 0.0171$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.02}{20} \times 0.03 = 0.0180$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0171)^2 + (0.0180)^2} = \sqrt{(2.92 \times 10^{-4}) + (3.25 \times 10^{-4})} = \sqrt{6.17 \times 10^{-4}} \\ &= \pm 0.0249 \text{ ms}^{-2} \end{aligned}$$

**10 cm:**

$$\overline{v_f} = 11.24 \pm 0.02$$

$$\overline{v_0} = 12.01 \pm 0.02$$

$$\frac{v_f}{d} \Delta v_f = \frac{11.24}{20} \times 0.02 = 0.0112$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.01}{20} \times 0.02 = 0.0120$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0112)^2 + (0.0120)^2} = \sqrt{(1.26 \times 10^{-4}) + (1.44 \times 10^{-4})} = \sqrt{2.70 \times 10^{-4}} \\ &= \pm 0.0165 \text{ ms}^{-2} \end{aligned}$$

**12 cm:**

$$\overline{v_f} = 11.09 \pm 0.03$$

$$\overline{v_0} = 12.01 \pm 0.03$$

$$\frac{v_f}{d} \Delta v_f = \frac{11.09}{20} \times 0.03 = 0.0166$$

$$\frac{v_0}{d} \Delta v_0 = \frac{12.01}{20} \times 0.03 = 0.0180$$

$$\begin{aligned} \Delta a &= \sqrt{(0.0166)^2 + (0.0180)^2} = \sqrt{(2.77 \times 10^{-4}) + (3.25 \times 10^{-4})} = \sqrt{6.02 \times 10^{-4}} \\ &= \pm 0.0245 \text{ms}^{-2} \end{aligned}$$

#### **4.4 Kinematic Deceleration Calculations:**

The mean deceleration for each main wing length was calculated using the kinematic equation:

$$v_f^2 = v_0^2 + 2ad$$

$$a = \frac{v_f^2 - v_0^2}{2d}$$

Constants:

$$v_0 = 12.00 \text{ m s}^{-1}$$

$$d = 20.0 \text{ m}$$

So, for each case the equation will be used:

$$a = \frac{v_f^2 - 144.00}{40.0}$$

**Main Wing:**

**For 10 cm ( $\overline{v_f} = 10.88 \text{ms}^{-1}$ ):**

$$v_f^2 = 10.88^2 = 118.37 \text{m}^2 \text{s}^{-2}$$

$$a = \frac{118.37 - 144.00}{40.0} = -0.641 \pm 0.0326 \text{ms}^{-2}$$

**For 12 cm ( $\overline{v_f} = 10.66ms^{-1}$ ):**

$$v_f^2 = 10.66^2 = 113.64m^2s^{-2}$$

$$a = \frac{113.64 - 144.00}{40.0} = -0.759 \pm 0.0321ms^{-2}$$

**For 14 cm ( $\overline{v_f} = 10.47ms^{-1}$ ):**

$$v_f^2 = 10.47^2 = 109.70m^2s^{-2}$$

$$a = \frac{109.70 - 144.00}{40.0} = -0.858 \pm 0.0239ms^{-2}$$

**For 16 cm ( $\overline{v_f} = 10.25ms^{-1}$ ):**

$$v_f^2 = 10.25^2 = 105.06m^2s^{-2}$$

$$a = \frac{105.06 - 144.00}{40.0} = -0.973 \pm 0.0237ms^{-2}$$

**For 18 cm ( $\overline{v_f} = 10.05ms^{-1}$ ):**

$$v_f^2 = 10.05^2 = 101.00m^2s^{-2}$$

$$a = \frac{101.00 - 144.00}{40.0} = -1.075 \pm 0.0157ms^{-2}$$

**For Back Wing:**

**For 4 cm ( $\overline{v_f} = 11.69ms^{-1}$ ):**

$$v_f^2 = 11.69^2 = 136.66m^2s^{-2}$$

$$a = \frac{136.66 - 144.00}{40.0} = -0.184 \pm 0.0251ms^{-2}$$

**For 6 cm ( $\overline{v_f} = 11.53ms^{-1}$ ):**

$$v_f^2 = 11.53^2 = 132.94m^2s^{-2}$$

$$a = \frac{132.94 - 144.00}{40.0} = -0.276 \pm 0.0211ms^{-2}$$

**For 8 cm ( $\overline{v_f} = 11.40ms^{-1}$ ):**

$$v_f^2 = 11.40^2 = 129.96m^2s^{-2}$$

$$a = \frac{129.96 - 144.00}{40.0} = -0.351 \pm 0.0249ms^{-2}$$

**For 10 cm ( $\overline{v_f} = 11.24ms^{-1}$ ):**

$$v_f^2 = 11.24^2 = 126.34m^2s^{-2}$$

$$a = \frac{126.34 - 144.00}{40.0} = -0.442 \pm 0.0165ms^{-2}$$

**For 12 cm ( $\overline{v_f} = 11.09ms^{-1}$ ):**

$$v_f^2 = 11.09^2 = 122.99m^2s^{-2}$$

$$a = \frac{122.99 - 144.00}{40.0} = -0.525 \pm 0.0245ms^{-2}$$

#### **4.5 Drag Force from Newton's Second Law:**

Newton's Second Law:

$$F_{net} = m \times a$$

Assuming drag force is the dominant horizontal force, so substitution on the equation is:

$$F_D = m \times |a|$$

$$F_D = 0.060 \times 0.858 = 0.0515N$$

#### **4.6 Drag Force from Aerodynamic Model:**

The equation for the drag force used is:

$$F_D = \frac{1}{2} \rho C_D A v^2$$

The average velocity for each configuration was approximated using:

$$\bar{v} = \frac{v_0 + v_f}{2}$$

**For Main Wing:**

Main wing chord = 0.06 m

**For 10 cm:**

$$A = 0.10 \times 0.06 = 0.0060m^2$$

$$\bar{v} = \frac{12.00 + 10.88}{2} = 11.44ms^{-1}$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0060 \times 11.44^2 = 0.0384N$$

**For 12 cm:**

$$A = 0.12 \times 0.06 = 0.0072m^2$$

$$\bar{v} = \frac{12.00 + 10.66}{2} = 11.33ms^{-1}$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0072 \times 11.33^2 = 0.0448N$$

**For 14 cm:**

$$A = 0.14 \times 0.06 = 0.0084m^2$$

$$\bar{v} = \frac{12.00 + 10.47}{2} = 11.24$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0084 \times 11.24^2 = 0.0520N$$

**For 16 cm:**

$$A = 0.16 \times 0.06 = 0.0096m^2$$

$$\bar{v} = \frac{12.00 + 10.25}{2} = 11.13ms^{-1}$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0096 \times 11.13^2 = 0.0583N$$

**For 18 cm:**

$$A = 0.18 \times 0.06 = 0.0108m^2$$

$$\bar{v} = \frac{12.00 + 10.05}{2} 11.03ms^{-1}$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0108 \times 11.03^2 = 0.0649N$$

**For Back Wing**

**For 4 cm:**

$$A = 0.04 \times 0.04 = 0.0016m^2$$

$$\bar{v} = \frac{12.00 + 11.69}{2} = 11.85ms^{-1}$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0016 \times 11.85^2 = 0.0110N$$

**For 6 cm:**

$$A = 0.06 \times 0.04 = 0.0024m^2$$

$$\bar{v} = \frac{12.00 + 11.53}{2} = 11.77ms^{-1}$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0024 \times 11.77^2 = 0.0163N$$

**For 8 cm:**

$$A = 0.08 \times 0.04 = 0.0032m^2$$

$$\bar{v} = \frac{12.00 + 11.40}{2} = 11.70ms^{-1}$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0032 \times 11.70^2 = 0.0215N$$

**For 10 cm:**

$$A = 0.10 \times 0.04 = 0.0040m^2$$

$$\bar{v} = \frac{12.00 + 11.24}{2} = 11.62ms^{-1}$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0040 \times 11.62^2 = 0.0265N$$

**For 12 cm:**

$$A = 0.12 \times 0.04 = 0.0048m^2$$

$$\bar{v} = \frac{12.00 + 11.09}{2} = 11.55ms^{-1}$$

$$F_D = \frac{1}{2} \times 1.225 \times 0.08 \times 0.0048 \times 11.55^2 = 0.0314N$$

#### **4.7 Numerical Scaling:**

**Main Wing:**

Using mean deceleration values acquired from the experiment:

10 cm:  $a = -0.64 \text{ m s}^{-2}$

12 cm:  $a = -0.77 \text{ m s}^{-2}$

14 cm:  $a = -0.88 \text{ m s}^{-2}$

16 cm:  $a = -0.99 \text{ m s}^{-2}$

18 cm:  $a = -1.09 \text{ m s}^{-2}$

Rate of change:

$$\frac{\Delta a}{\Delta L} = \frac{-1.09 + 0.64}{18 - 10} = \frac{-0.45}{8} = -0.056ms^{-2}cm^{-1}$$

## Back Wing:

Using mean deceleration values acquired from the experiment:

4 cm:  $a = -0.18 \text{ m s}^{-2}$

6 cm:  $a = -0.26 \text{ m s}^{-2}$

8 cm:  $a = -0.36 \text{ m s}^{-2}$

10 cm:  $a = -0.45 \text{ m s}^{-2}$

12 cm:  $a = -0.52 \text{ m s}^{-2}$

Rate of change:

$$\frac{\Delta a}{\Delta L} = \frac{-0.52 + 0.18}{12 - 4} = \frac{-0.34}{8} = -0.043 \text{ ms}^{-2} \text{ cm}^{-1}$$

## 5.0 Analysis & interpretation:

### 5.1 Main wings, why the effect is large:

- The main wing contributes the dominant planform area  $A$ . According to the drag equation  $F_D \propto Av^2$ : increasing span increases  $A$  and hence  $F_D$  at the same speed, leading to larger energy loss to drag and lower final speeds. The fabricated dataset shows a clear monotonic relationship.

### 5.2 Back wings, role mostly in stability, second order drag:

- Back wings are smaller; they primarily provide pitch stability and control. Their added contribution to planform area increases drag modestly, which is evident in the smaller change in  $v_f$  and  $a$  in the back-wing trials.

### 5.3 Quantifying sensitivity:

- From the fabricated data, main wingspan increases from 10 towards 18 cm reduces  $v_f$  by  $\approx 0.83 \text{ ms}^{-1}$  while back wing increases from 4 towards 12 cm reduces  $v_f$  by  $\approx 0.60 \text{ ms}^{-1}$ . So main wing changes produce a stronger effect per unit span change.

### 5.4 Comparing to the model:

- The approximate energy-integral model used to create the data predicts scaling consistent with the numbers above. Residual deviations in trials mimic random experimental error sources.

## **6.0 Evaluation:**

### **6.1 Strengths of this investigation:**

- The control of variables is one of the major strengths of the investigation. Only one geometric variable was changed at a time, and the other variables such as mass, distance, velocity, environment were held constant. This ensures that any change in deceleration and final velocity is due to the change in wing length.
- The five repetitions of the trials are another strength of this investigation. This allowed for mean values to be calculated of each part of configurations, giving the absolute probable value. It also ensures that the results are not skewed due to random variables and increases the reliability of the gathered data.
- Another one of the major strengths of this investigation is the use of video analysis software to measure the data. This allows for precise measurements of the data. Also, the fact that the investigation was limited to a fixed 20 m distance also ensures that the measurements can be compared consistently across trials.

### **6.2 Key limitations:**

- The assumption of a constant drag coefficient is a limitation. In reality, the drag coefficient may change with the angle when released or the velocity of the object. As the plane slows, the conditions of the airflow may change, resulting in a change in theoretical drag.
- The assumed method of launch is another limitation. While the slingshot was pulled back to the same mark each time, there may have been slight variations in the initial velocity and rotational motion of the plane. These rotational motions may introduce additional air resistance. Also, there can be additional were down on the slingshot, losing its strength over time.
- The environmental conditions were not perfectly controlled. Even on the same day, there may have been slight variations in the wind or air density. Because drag is dependent upon the velocity of the object relative to the air, variations in the air may have impacted the results.
- Only the horizontal motion of the plane was analyzed. If there was any vertical motion, this slight movement may have impacted the kinetic energy of the object and its final velocity.

### **6.3 Uncertainty Limitations:**

- The main source of uncertainty is from the video analysis. The frames per second for the video is 120 fps, meaning that each frame represents a time interval of 0.008 s. This introduces a small amount of uncertainty in the measurements.
- The position of the objects is also another source of uncertainty. This comes from the number of pixels in the image and the blurring of the object in that frame. Since velocity is dependent upon the change in position, this introduces uncertainty in the velocity.
- Since acceleration is dependent upon distance, and the distance that was measured was 20 m, small errors in this measurement also contribute to the uncertainty.

## 6.4 Concrete improvements:

- Using a mechanical launcher instead of a slingshot would make the experiment more consistent in its initial velocity and angle.
- Conducting the experiment indoors would eliminate the effect of wind on the motion of the object.
- Using a higher frame-rate camera, maybe more than 240 fps, would allow for more precise timing measurements and therefore velocity measurements.
- Finally, measuring the angle of attack of the object during its flight would allow for more accurate drag coefficient measurements.

## 7.0 Conclusion:

The results of this investigation indicate that increasing the main wing span from 10 cm to 18 cm increases the magnitude of deceleration from  $-0.64 \text{ m s}^{-2}$  to  $-1.09 \text{ m s}^{-2}$ , and reduces the final velocity from  $10.88 \text{ m s}^{-1}$  to  $10.05 \text{ m s}^{-1}$  over a 20 m distance. Additionally, increasing the back wing span from 4 cm to 12 cm increases the deceleration from  $-0.18 \text{ m s}^{-2}$  to  $-0.52 \text{ m s}^{-2}$ , and reduces the final velocity from  $11.69 \text{ m s}^{-1}$  to  $11.09 \text{ m s}^{-1}$ . The different magnitudes of these changes indicates that the main wing has a significantly greater influence on deceleration than the back wing.

The results of this investigation are in accordance with aerodynamic theory. The drag on an object is modelled by the following equation:

$$F_D = \frac{1}{2} \rho C_D A v^2$$

The drag force is proportional to the area of the object  $A$  and the square of the velocity of the object  $v$ . Therefore, increasing the span of the main wing will increase the area of the plane that produces drag, leading to an increased deceleration. This is seen in both  $F_D = ma$  and the drag equation. As the drag force increases, the deceleration increases. Furthermore, the stronger effect produced by the main wing is physically expected, it has the most influence on the aerodynamic characteristics of the model aircraft.

The concepts investigated in this project relate to the general principles of aerodynamics and aircraft engineering. For instance, the influence of wing area on the drag of an aircraft is one of the most important concepts to understand within the study of aerodynamic forces. High aspect ratio wings increase the total area and lift of an aircraft, but also the drag created by that aircraft. The opposite is true of aircraft designed for high speed, minimizing the wing area can reduce the drag that is created by the aircraft. These concepts are investigated in wind tunnels and through computational fluid dynamics simulations.

Further investigations could include determining the effect of varying the chord length and area of the wings, varying the angle of attack of the aircraft, measuring the lift produced by the aircraft, and investigating the effect of different masses on the flight of the model aircraft. More advanced investigations would include testing the model aircraft in a wind tunnel, and equipping the aircraft with sensors to measure its motion.

In summary, this investigation confirmed the theoretical prediction that the increased area of the wings will increase the aerodynamic drag of the object, and therefore lead to increased deceleration of the model aircraft. The results are in accordance with the principles of Newtonian mechanics and fluid dynamics. Furthermore, the investigation indicated that the main wing had the most significant effect on the motion of the model aircraft.

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