IB Physics Extended Essay

The effect of the fluid viscosity on the actuation time of a two-piston hydraulic system.

Research Question: How does the dynamic viscosity of a Newtonian fluid affect the actuation time of a two-piston hydraulic system, measured as the time required to transfer a fixed volume through a 7.0 cm IV tube with 2 mm inner diameter under a constant 1.00 kg load at room temperature?

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1.0 | Introduction

Hydraulic activation is one of the clearest demonstrations of how a fundamental physics theory is used practically in real-world engineering: pressure being applied to a confined liquid which then generates controlled power and motion elsewhere in the system. While the theory of hydraulic systems is simple, when it comes to real-world applications the same can not be said. When viscous fluid is run through tubing, valves and fittings, the system experiences viscous losses which reduces the flow-rate and weakens the response of the actuator. This is most commonly visible in cold conditions, where fluids become more viscous and have reduced flow rates, causing the actuator output to be sluggish and the system efficiency to decrease. This example, along with many other visible effects of viscosity on a system, cause it to expand from a theoretical term to an actual factor to be considered in engineering.

My interest in this topic stems from my interest in automotive engineering. From brake systems to power steering, hydraulics play a crucial part in the development and innovation of the automotive industry, and one of the dependencies of the systems is the viscosity of the liquid inside them. For this reason, I constructed my own small hydraulic system to measure the effect of viscosity on said system and predict how it affects hydraulic systems in general.

The components of my system consisted of two 60mL medical-grade syringes connected together with a 7cm IV tube with an inner diameter of 2mm. I chose to build my model as such because the geometry is simple to account for errors and have clear measurements and it includes all the core aspects of a hydraulic system: confined fluid, clear path to the actuator, airtight pistons, and marked cylindrical chambers so that I could clearly see the displacement in order to measure the time. Under conditions where flow in the tube remains laminar and the fluids behave Newtonian, internal flow theory predicts that the volumetric flow rate is inversely proportional to dynamic viscosity for fixed tube dimensions and pressure difference (Hagen–Poiseuille flow). Consequently, at fixed transferred volume the actuation time should increase with viscosity. This extended essay therefore asks:

How does the dynamic viscosity of a Newtonian fluid affect the actuation time of a two-syringe hydraulic system, measured as the time required to transfer a 50mL of a Newtonian liquid through a 7 cm tube under a constant 1.00 kg load at 24℃?

The chosen test liquids were pharmaceutical grade glycerol and distilled water. These liquids were chosen because they are readily and easily accessible, they are both fully Newtonian across a wide range of concentrations and shear rates (Carvalho et al., 2022) [6], and their viscosities at any temperature are clearly seen in Cheng (2008) empirical correlation [4]. The advantage of using two fluids in the experiment is that their chemical composition only changes in proportion to the concentration, allowing for predictable results and analysis. Four concentrations of glycerol were tested in this experiment in order to have a large range of different viscosities to analyse more precisely, these concentrations were evenly spaced to ensure predictable increases between trials. The concentrations were as follows: 0%, 15%, 30%, 45%. The aim of this investigation is to evaluate how actuation time scales in relation to viscosity and identify which external factors affect the hydraulic system and cause it to deviate from predicted results.

2.0 | Background Information

2.1 | Hydraulic Activation Beyond the Ideal Model

Ideal hydraulic systems are often introduced with the simple assumption that pressure applied to a confined fluid from an input will transfer the force to the output, creating movement. However, this assumption for hydraulic systems does not apply in practical use cases, where the fluid has many additional flow conditions to be considered. During the movement of the fluid through the tubing circuits of a hydraulic system pressure drops along the flow path, decreasing the flow-rate, causing the output pressure of the system to become lower than the input pressure. This pressure drop is caused by the shearing of the liquid while it flows along the tubing. Shearing means layers of liquid sliding against each other during movement, shearing occurs because the moving fluid goes through the center of the tubing while the fluid which is at the walls has zero effective velocity due to no-slip conditions. In Newtonian fluids shear stress, which is a measure of internal friction, is proportional to viscosity for a given shear rate. Therefore when talking about the shearing in a hydraulic system such as this one it is referred to as viscous shearing.

2.2 | Laminar internal flow and the Hagen–Poiseuille equation

For the system to be properly assessed, the IV tube will be treated as a straight circular tube. This allows the assumption of the tube being a pipe of 7cm length and circular cross-section to be made in order to make the calculations easier while still being viable to use. Since Newtonian fluids are being used in this system, it is assumed that in the experiment shear stress is proportional to shear rate, and the no-slip boundary condition which postulates that the velocity of the fluid layer directly in contact with the boundary is equal to the velocity of the boundary itself (0). Consequently, by these assumptions we can verify a laminar flow, which leads to the Hagen-Poeiseuille model being able to be used. This model calculates the pressure drop of a Newtonian fluid flowing through a straight, circular channel under steady state, laminar flow conditions. It can also be derived to calculate the flow rate of a hydraulic system, Q [1]:

$$Q = \frac{\pi r^4}{8\mu L}\Delta P$$

Where:

Q = volume flow rate of the system (m^3/s)

r = tube radius (m)

L = tube length (m)

μ = dynamic viscosity ($Pa \cdot s$)

ΔP = change in pressure from input to output

This expression makes two quantitative predictions which will be central to this experiment:

- $Q \propto \frac{1}{\mu}$, meaning that the flow rate is inversely proportional to the viscosity.
- $Q \propto r^4$, meaning that a small uncertainty in the radius of the IV tube may cause a large uncertainty in the flow rate.

2.3 | Linking flow rate to actuation time for a fixed transferred volume

In this investigation, actuation time is defined as the time to transfer a fixed volume of $V = 50mL$ between syringes. This can be found by dividing the volume by the flow rate:

$$t = \frac{V}{Q}$$

Substituting the Hagen–Poiseuille expression for Q yields:

$$t = \frac{8\mu LV}{\pi r^4 \Delta P}$$

Therefore, since the tube geometry and driving pressure is controlled, the model predicts:

$$t \propto \mu$$

In other words, increasing viscosity should increase actuation time approximately linearly, provided the flow remains laminar and the fluids behave Newtonian under the experimental shear rates.

2.4 | Newtonian behaviour and the choice of test fluids

Newtonian fluids are fluids in which viscosity is completely independent of shear rate and the relationship of shear rate and shear stress is linear and is annotated by μ. This behaviour allows the Hagen-Poiseuille equation to be applicable as it uses Newton's law of viscosity in cylindrical coordinates:

$$\tau = \mu(\frac{du}{dr})$$

Hagen-Poiseuille assumes μ to be constant throughout the flow regardless of flow conditions. Glycerol and water were picked for this reason; they satisfy the Newtonian requirement at a large range of shear rates and concentrations. Additionally, as stated before their viscosities at many temperature ranges and concentrations can be calculated through Cheng (2008) empirical correlation: The dynamic viscosity of the mixture is related to those of its two pure components through the power-law mixing formula (Cheng, 2008, Eq. 5):

$$\mu = \mu_w^{\alpha} \cdot \mu_g^{(1-\alpha)}$$

Where:

- $\mu_w =$ the dynamic viscosity of pure water
- $\mu_g =$ the dynamic viscosity of pure glycerol
- $\alpha =$ the weighing factor which varies with mass concentration (C_m) and temperature (T)

The weighting factor α is determined by a power-sum interpolating function whose temperature-dependent coefficients are (Cheng, 2008, Eqs. 12–13):

$$a = 0.705 - 0.0017T$$

$$b = (4.9 + 0.036T) \cdot a^{2.5}$$

Where:

- $T =$ degrees in celsius (℃)

At 24°C, a = 0.664 and b = 2.150. The viscosities calculated from this formula for each test fluid are presented in Section 4 [4].

2.5 | Temperature dependence of viscosity

Viscosity is heavily dependent on temperature; in mixtures of glycerol and water this can be seen clearly. Water's viscosity, as defined by the capillary flow measurements of the National Bureau of Standards, is 1.002 $(mPa \cdot s)$ at 20℃. This value decreases to 0.911 $(mPa \cdot s)$ at 24℃ according to reported experimental values. The value for glycerol at 24℃ was gotten from experimental values by Segur & Oberstar [5] as approximately 950 $(mPa \cdot s)$. Any change in temperature by external sources would directly affect these values so meticulous care was taken when determining the temperatures for each trial. [3]

2.6 | Ensuring laminar flow conditions

As the Hagen-Pouseuille model requires laminar flow conditions to be used, it needs to be ensured that said conditions are present. There exist two types of flow: laminar and turbulent. To distinguish between these, a dimensionless number called the Reynolds number, discovered by Osbourne Reynolds in 1883, needs to be calculated:

$$Re = \frac{pvd}{\mu}$$

Where:

- Re = Reynolds number
- ρ = density of the fluid (kg/m^3)
- v = mean flow velocity across the tube (m/s)
- d = inner diameter of the tube (m)
- μ = dynamic viscosity ($Pa \cdot s$)

For smooth walled circular pipes, the threshold where laminar flow is disrupted, according to Apsley, is $Re = 2,300$. The reason that the model is under this determined limit of 2,300 is because higher viscosity fluids generate more viscous resistance than the inertial forces generated by the flow itself. [2] Glycerol is comfortably under this limit and since glycerol has a higher viscosity than water, at higher concentrations of glycerol, it is well under the Reynolds limit. This ensures that this experiment was conducted with laminar flow.

2.7 | Hypotheses

- **Null Hypothesis:** Viscous changes of the hydraulic fluid have no effect on the actuation time of the hydraulic system.

- **Research Hypothesis:** Viscous changes of the hydraulic fluid affect the actuation time of the hydraulic system

3.0 | Strategy

3.1 | Variables

Variables	How the Variable was Controlled
Independent Variable	
Dynamic Viscosity of the Test Fluids	*By varying the concentrations of glycerol mixed with water for each concentration.*
Dependent Variable	
Actuation Time	*Measuring the elapsed time from the start of piston movement to the termination of piston movement.*
Controlled Variables	
Force Applied on Plunger	*A constant 1kg weight was placed on one piston for all trials*
Tube Length	*A silicone IV tube was cut to 7cm and that tube was used for all trials.*
Transfer Volume	*The syringe was filled to 50mL and was fully transferred every trial.*
Room Temperature	*The room temperature was set through the house thermometer and measured constantly with a room thermometer*
Syringe Type	*The syringes were identical medical grade syringes and the same ones were used for all trials.*

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Glycerol Concentration	*The glycerol and water were measured on a digital balance prior to mixing to ensure proper mass and concentrations.*
Fluid Temperature	*The mixture's temperature was measured after every trial, heated/cooled based on the temperature.*

|Table 1: Variables and How They Were Controlled|

3.2 | Materials & Apparatus

The following materials were required to conduct the experiment, the specifications of them were also listed:

Materials	**Specifications**
BD syringe $\times\ 2$	*Graduated to 60mL, inner radius 13.2 mm* $\pm\ 0.001\ cm$
Silicone IV tube	*2mm inner diameter, 7cm length*
Pharmaceutical Grade Glycerol	*>99.0% pure, 1L*
Distilled Water	*1L*
Digital Balance	$\pm\ 0.01g$ *uncertainty*
Smartphone Stopwatch	$\pm\ 0.01s$ *uncertainty*
Thermometer	$\pm\ 0.5°C$ *uncertainty*
Metal Weight	$1.0kg$, *solid molded steel class M1*

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Wooden Box	*20cm by 20cm by 30cm*
Hot Glue Gun + Hot Glue	*Pattex brand, 300mm silicone sticks*
100mL Graduated Cylinder	$\pm\ 0.5mL$ *uncertainty*
Callipers	$\pm\ 0.001cm$ *uncertainty*
Beaker $\times\ 4$	$500mL$ *capacity, numbered 1 – 4*
Pen	*Red, felt tip Sharpie*

|Table 2: Materials & Specifications|

4.0 | Methodology

4.1 | Solution Preparation

In this step of the experiment, glycerol mixtures of every concentration will be prepared. All of the fluid measurements will be done with the graduated cylinder and the digital scale and the numbered beakers will be used for each solution (3.2):

- ***4.1.1:*** Measure 100g of distilled water, pour into beaker 1, set beaker aside.
- ***4.1.2:*** Measure 15g of glycerol, pour into beaker 2.
- ***4.1.3:*** Measure 85g of distilled water, pour into beaker 2, set beaker aside.
- ***4.1.4:*** Measure 30g glycerol, pour into beaker 3.
- ***4.1.5:*** Measure 70g distilled water, pour into beaker 3, set beaker aside.
- ***4.1.6:*** Measure 45g glycerol, pour into beaker 4.
- ***4.1.7:*** Measure 55g distilled water, pour into beaker 4, set beaker aside.
- ***4.1.8:*** Mix the glycerol and distilled water thoroughly for 2 minutes, then set aside the beaker for 30 minutes before use. *(REPEAT THIS AFTER EVERY STEP BETWEEN 4.1.2 – 4.1.7)*

This produces around 100mL of all of the solutions which will be used in the experiment. The precise volume is not required to be measured as there is nearly twice as much fluid as needed for each concentration.

4.2 | Experimental Setup

Prior to the trials being conducted, the experimental setup needs to be prepared. This setup will mount the syringes to the box using hot glue, then connect their nozzles with the IV tube (3.2).

- ***4.2.1:*** Using the callipers and pen, measure and mark two mounting points 5cm apart from each other and 10cm up from the base of the wooden box.
- ***4.2.2:*** Apply hot glue to the back face of each syringe so that the graduated face is turned out. Mount the syringes onto the box on the same horizontal level, ensuring the nozzles are not blocked by hot glue and aligned with each other.
- ***4.2.3:*** After the hot glue dries, fit the IV tube on to the nozzle of one syringe using the luer slip to ensure an airtight seal.
- ***4.2.4:*** Measure the room temperature with the digital room thermometer to ensure 24℃

This setup was used to conduct all trials and was washed between use of the different concentrations of glycerol.

4.3 | Conducting Trials

- ***4.3.1:*** Measure 50mL of select glycerol solution concentration.
- ***4.3.2:*** Turn the box upside down, retract the piston of the inlet syringe fully and fill with measured 50mL of select solution from the nozzle, push the piston to remove excess air and tap to remove any air bubbles.
- ***4.3.3:*** Connect the unconnected end of the IV tube to the nozzle of the inlet syringe using the luer slip for an airtight seal.
- ***4.3.4:*** Flip the box back around, hold the extended plunger firmly with one hand and place the 1kg weight onto it with the other, ensuring it is centered.

4.3.5: Simultaneously let go of the plunger and start the smartphone stopwatch.

4.3.6: When the liquid is fully transferred into the other syringe, stop the timer and note the time down.

4.3.7: For the trials of the same solution, only steps 4.3.4 – 4.3.6 have to be repeated four more times as there will be no change in condition which will affect the trials. This was done to save on time and avoid possible experimental waste.

4.3.8: Repeat steps 4.3.1 – 4.3.7 one time for each concentration of glycerol solution.

5.0 | Data Analysis

5.1 | Raw Data

The following dataset is the actuation time of the system measured by timing the initial movement of the syringe piston to termination of movement. Each trial for different concentrations was measuring the time taken for a constant fluid volume 50mL to get transferred from input to output with a constant mass of 1kg applying force on it. The following table shows the raw data gathered for the experiment:

	Trials (s)				
Glycerol solutions (% concentration)	**T1**	**T2**	**T3**	**T4**	**T5**
0 (Pure Distilled Water)	*21.00*	*18.98*	*19.32*	*20.19*	*19.47*
15	*69.12*	*70.21*	*68.77*	*68.43*	*71.65*
30	*186.43*	*190.34*	*187.66*	*189.16*	*191.32*
45	*577.67*	*576.72*	*578.19*	*580.87*	*575.37*

|Table 3: Raw Data Values of Hydraulic Activation Time|

5.2 | Processed Data

The raw data was then further processed, finding the mean times, dynamic viscosities in $mPa \cdot s$, their standard deviations and their standard deviations. The results from these calculations will all be utilized during the plotting and further analysis.

5.2.1 | Mean Time, Standard Deviation & Half-Range Calculations

The mean times were calculated using the following equation to find the most reliable time these trials would have taken overall:

$$\bar{t} = \frac{\sum_{i=1}^{n} t_i}{n}$$

The standard deviations were calculated in order to determine the repeatability of these measurements and this experiment as a whole:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (t_i - \bar{t})^2}{n-1}}$$

The statistical uncertainty of the mean values was then estimated using a standard error equation which uses the spread of repeated measurements to estimate the precision of the mean actuation times:

$$u(\bar{t}) = \frac{s}{\sqrt{n}}$$

These values were then plotted in the table below:

Glycerol solutions (% concentration)	**Mean Times ($\bar{t}$) (s)**	**Standard Deviations (s) (s)**	**Standard Error SE (s)**
0 (Pure Distilled Water)	*19.79*	*0.81*	*0.36*
15	*69.64*	*1.31*	*0.59*
30	*188.98*	*1.97*	*0.88*
45	*577.76*	*2.04*	*0.91*

|Table 4: Mean times, Standard Deviations, Standard Errors of Actuation Times of Glycerol Solution Concentrations|

5.2.2 | Dynamic Viscosity Calculations

The dynamic viscosities of the fluids were calculated through the use of Eq. 5 of Cheng's paper determined in subsection 2.4:

$$\mu = \mu_w^{\alpha} \cdot \mu_g^{(1-\alpha)}$$

(example equation for 15% glycerol):

$$\mu = 0.911^{0.94445} \cdot 950^{0.05555} = 1.340$$

The definitions of everything were given in subsection 2.4. The values of μ_w and μ_g at 24℃ were $0.911\ mPa \cdot s$ and $950\ mPa \cdot s$ respectively. To find the viscosities of the different concentrations of glycerol solutions, the value of α needs to be found:

$$\alpha = 1 - C_m + \frac{abC_m(1-C_m)}{aC_m + b(1-C_m)}$$

Where:

$C_m =$ Concentration by mass

With the values of $a\,(0.664)$ and $b\,(2.150)$ as found in subsection 2.4 and the concentrations by mass for all of the solutions inserted into the calculation, the α values were calculated [4];
(example equation for 15% glycerol):

$$\alpha = 1 - 0.15 + \frac{0.664 \cdot 2.150 \cdot 0.15\,(1-0.15)}{0.664 \cdot 0.15 + 2.150(1-0.15)} = 0.94445 \approx 0.94$$

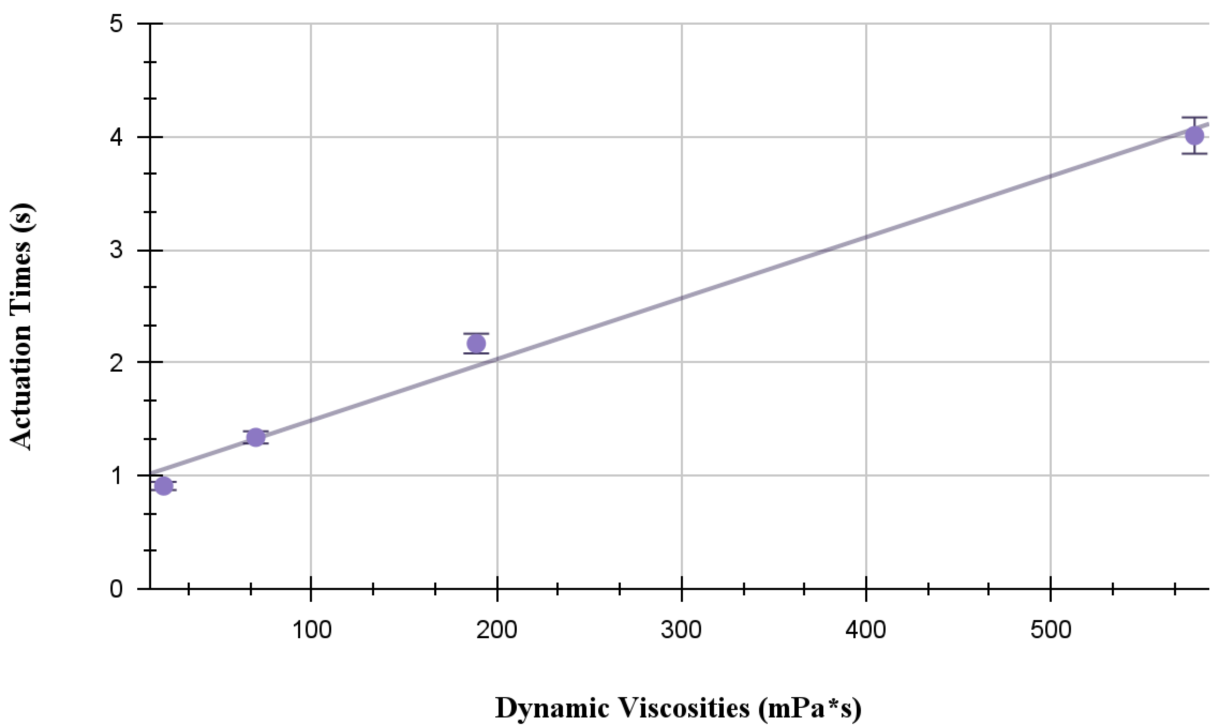
The results for all solutions are plotted below:

Glycerol solutions (% concentration)	**Mass fraction** (C_m)	**Weight factor** (α)	**Dynamic viscosity** (μ)($mPa \cdot s$)
0 (Pure Distilled Water)	*0.00*	*1*	*0.91*
15	*0.15*	*0.94*	*1.34*
30	*0.30*	*0.88*	*2.17*
45	*0.45*	*0.79*	*4.01*

|Table 5: Calculated Mass Fraction, Weight Factor, Dynamic Viscosity of all glycerol concentrations|

5.2.3 | Plotting Dynamic Viscosity Calculations

The results of the dynamic viscosity calculations calculated in subsection 5.2.2 along with the mean times measured in subsection 5.2.1 were plotted against each other in a graphical format in order to determine if they fit the prediction of them being proportional to each other proved to be true. The resulting graph can be seen below:



|Figure 1: Actuation Times Plotted Against Dynamic Viscosities at Different Glycerol Concentrations |

This graph clearly represents how the relationship between dynamic viscosity and hydraulic activation times is clearly proportional, an increasing trend being visible for the values. This trend will be further examined in the upcoming subsections 5.4-5.5.

5.3 | Uncertainty Calculations & Analysis

This section will analyze Hagen-Poiseuille and how accurate its predictions will be in comparison to the measured actuation times. It will also calculate the uncertainties which go along with the model.

5.3.1 | Defining and Calculating the Driving Pressure

Another factor which needs to be calculated is called the driving pressure, which is annotated by ΔP. Driving pressure describes the gravitational force of mass on the piston:

$$\Delta P = \frac{mg}{\pi R^2}$$

Where:

$\Delta P =$ driving pressure (Pa)

$mg =$ gravitational force of mass (N)

$R =$ inner radius of syringe barrel (m)

The values being inserted yields:

$$\Delta P = \frac{9.81}{\pi(0.0132)^2} = 17921.37\ Pa$$

The fractional uncertainty for this equation was calculated using the uncertainties of mass and syringe radius:

$$\delta\Delta P = \frac{\delta m}{m} + \frac{\delta R}{R}$$

(Uncertainty calculation equation for ΔP)

$$\frac{\delta\Delta P}{\Delta P} = 1\% + 0.08\% \approx 1.08\%$$

This value will be used to calculate the total uncertainty for the Hagen-Poiseuille model.

5.3.2 | Calculating Total Uncertainty

Since Hagen-Poiseuille is heavily dependent on measured factors, the instrumental error must be calculated in addition to the standard error. The calculations of all the instrumental margins and errors are listed below.

Unit Used in Equation	**Measuring Tool Used & Absolute Uncertainty**	**Fractional Error Contribution to Hagen-Poiseuille**
Tube Radius(1.00 mm)	*Callipers* $(\pm\ 0.001m)$	$4(\delta r/r)\ =\ 4.00\%$
Tube Length (7.00 cm)	*Callipers* $(\pm\ 0.001cm)$	$(\delta l/l)\ =\ 1.00\%$
Syringe Radius (13.2 mm)	*Callipers* $(\pm\ 0.001cm)$	$(\delta R/R)\ =\ 0.08\%$
Solution Volume (50mL)	*Graduated Cylinder* $(\pm\ 0.5mL)$	$(\delta V/V)\ =\ 2.00\%$
Piston Driving Mass (1kg)	*Metal Weight* $(\pm\ 0.001kg)$	$(\delta m/m)\ =\ 1.00\%$

|Table 6: Fractional Error Calculations For Hagen-Poiseuille|

The final uncertainty for Hagen-Poiseuille was calculated by adding all the instrumental fractional uncertainties which affect the equation. The uncertainty calculation is:

$$\frac{\delta t}{t} \approx \frac{\delta \mu}{\mu} + \frac{\delta l}{l} + \frac{\delta V}{V} + 4\frac{\delta r}{r} + \frac{\delta(\Delta P)}{\Delta P}$$

(Total uncertainty equation)

$$1.00\% + 1.00\% + 2.00\% + 4.00\% + 1.08\% \approx 9.08\%$$

(Solved total uncertainty equation)

From this calculation, instrumental uncertainty can be taken at a value of 9.08% for this model. To evaluate the magnitude of the effect of this uncertainty it was compared with the established standard errors in subsection 5.2.1. To do this, the percentage value was converted into time values for each concentration and plotted along with the standard errors in the table below:

Glycerol solutions (% concentration)	Mean Time ($\bar{t}$) (s)	Instrumental Uncertainty (δt) (s)	Standard Error (SE) (s)
0 (Pure Distilled Water)	*19.79*	*1.53*	*0.36*
15	*69.64*	*5.37*	*0.59*
30	*188.98*	*14.57*	*0.88*
45	*577.76*	*44.55*	*0.91*

|Table 7: Standard Deviations of Mean Actuation Time Compared with Standard Error|

These results show that instrumental uncertainty was the dominant error factor. The standard error being very low indicates this experiment is greatly repeatable with nearly all of its error being attributable to an issue in the setup, which will be addressed in subsection 5.4.

5.4 | Theoretical Predictions & Experimental Comparison

5.4.1 | Hagen-Poiseuille Predictions

Using the established factors in subsection 5.3, the Hagen-Poiseuille model can finally be calculated, yielding the following predicted actuation times (Equation presented in subsection 2.2):

Glycerol solutions (% concentration)	Dynamic viscosity $(\mu)(mPa \cdot s)$	Hagen Poiseuille Time (s)	Experimental Time (s)	Deviation	Factor
0 (Pure Distilled Water)	*0.91*	*0.45*	*19.79*	*+4268%*	*43.7*
15	*1.34*	*0.67*	*69.64*	*+10350%*	*104.5*
30	*2.17*	*1.07*	*188.98*	*+17509%*	*176.1*
45	*4.01*	*1.97*	*577.76*	*+29230%*	*293.3*

|Table 8: Hagen-Poiseuille "Ideal" Actuation Times Compared With Experimental Actuation Times|

Comparing these results to the experimental results, it can be seen that the two results deviate in large magnitudes. In addition to this, the error was not linear, increasing as the concentrations of glycerol did as well. Because of these two facts, further investigation was warranted to make this experiment a viable source of information.

5.4.2 | Seal Friction, Definition & Calculations

The Hagen-Pouiseuille model assumes that the driving pressure is transferred into the system fully without any losses in energy. This is untrue in the system used however as real hydraulic systems a significant proportion of the input force gets dissipated by the seal friction existent in the piston chambers, which is a force exerted in the opposite direction of piston movement, annotated here as F_f. In this case, the syringe plunger is fitted with an O-ring which creates the seal friction, which reduces the pistons ability to drive the fluid through the tubing [7]. The equation for the seal friction corrected driving pressure is given below:

$$\Delta P_c = \frac{(mg - F_f)}{A}$$

The corrected Hagen-Poiseuille model was found by substituting ΔP_c instead of ΔP:

$$t = \frac{8\mu LV}{\pi r^4 \Delta P_c}$$

The seal friction value was found by taking the experimental water trials' actuation times as a base and back-calculating from them using the corrected Hagen-Poiseuille equation. This allowed for precise predictions with minimal error when compared to the ideal Hagen-Pouiseuille's predictions:

$$\Delta P_c = \frac{8\mu LV}{\pi r^4 t_w} = \frac{(8 \cdot 9.11 \cdot 10^{-4} \cdot 0.07 \cdot 50 \cdot 10^{-6})}{((\pi \cdot 10^{-3})^4 \cdot 19.79)} = 410.28\ Pa$$

(Derived corrected driving pressure equation)

$$F_f = mg - \Delta P_c \cdot A = 9.81 - (410.28 \cdot 5.47 \cdot 10^{-4}) = 9.59N$$

(Derived seal friction force equation)

This result is within the break-loose forces of standard 60mL BD syringes, being between 8-15N. When compared against the total force applied onto the plunger, the frictional force returned 97.71% of the force, leaving only 0.223N pushing the syringe [9]. This explains most of the magnitude of overshot values, and the new values plotted based on the friction calculations done by taking water as a base which are plotted below:

Glycerol solutions (% concentration)	Dynamic viscosity $(\mu)(mPa \cdot s)$	Hagen Poiseuille Time (s)	Experimental Time (s)	Deviation	Factor
0 (Pure Distilled Water)	*0.91*	*19.79*	*19.79*	*+0%*	*1*
15	*1.34*	*29.11*	*69.64*	*+139.2%*	*2.39*
30	*2.17*	*46.88*	*188.98*	*+303.1%*	*4.03*
45	*4.01*	*86.05*	*577.76*	*+571.5%*	*6.71*

|Table 9: Corrected Hagen-Poiseuille Actuation Times Compared With Experimental Actuation Times|

Although this consideration allowed for the elimination of most of the error from the Hagen-Poiseuille times, there still remains an increasing factor as the concentration of glycerol increases. This suggests that there is another factor affecting and weakening the results of the experiment, motivating the further analysis. The reason for this could be that laminar flow was not reached, so using the equation from subsection 2.6 the Reynolds numbers of the solutions were calculated:

Glycerol solutions (% concentration)	Dynamic viscosity $(\mu)(mPa \cdot s)$	Mean Flow Velocity $(v)(m/s)$	Density (ρ) (kg/m^3)	Reynolds Number (Re)
0 (Pure Distilled Water)	*0.91*	*0.80*	*997*	*1760*

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15	*1.34*	*0.23*	*1035*	*340*
30	*2.17*	*0.08*	*1077*	*124*
45	*4.01*	*0.03*	*1108*	*43*

|Table 10: Reynolds Numbers, Dynamic Viscosities, Mean Flow Velocities, Density, for Concentrations of Glycerol|

These Reynolds numbers are all comfortably under the threshold of 2,300 for laminar flow in smooth circular pipes, indicating that a laminar flow issue was not the source of the increasing factors. [2] This will be further discussed in section 7, evaluation.

5.5 | Linearity Analysis

This final section in the data analysis will be analysing the linearity of the measured actuation times to the viscosity of solutions. The first step in doing that will be a linear regression.

5.5.1 | Correlation Calculations

The Pearson's correlation coefficient was calculated for the viscosities and the actuation times to assess the strength of the predicted linear relationship between the two in subsection 2.3:

$$R = \frac{\Sigma(x_i - \bar{x})(yi - \bar{y})}{\sqrt{\Sigma(x_i - \bar{y})^2 \Sigma(y_i - \bar{y})^2}}$$

(Pearson’s correlation coefficient equation)

Where:

$x = \mu$

$y = \bar{t}$

When the equation is performed it yields a value of 0.9943 for the Pearson's correlation coefficient R, which shows that the relationship is strongly linear. By taking the square of the R value, the amount of variability attributable to the linear relationship was found to be 0.9886, in other words 98.86% of the variability was due to the linear relationship. It is important to note however that since the trials all consisted of 4 data points it is more likely to present as linear when it is not in actuality, so it is unreliable to rely on solely these values. This will be further explored in section 7, Evaluation.

6.0 | Conclusion

This investigation aimed to evaluate how fluid viscosity affected the actuation time of a two piston hydraulic system, which was defined as the time taken to transfer 50mL of different concentrations of glycerol liquid between two 60mL medical grade BD syringes through a 7mm silicone IV tube with a constant weight of 1kg being applied at 24℃.

The viscosity of the fluid was determined by the glycerol concentration inside of the water based solution. As the concentration went up from 0% to 45%, the solution became more viscous and it was observed that the actuation time increased with the viscosity. This was later confirmed through the calculation of the Pearson's correlation coefficient. The calculation yielded results which numerically proved the existence of a linear relationship and attributed most of the error in the system to this relationship directly; this also validated the research hypothesis of varying viscosities affecting the flow rate of a hydraulic system.

However, there were some issues with the Hagen-Poiseuille model as it yielded predicted time and viscosity values which were significantly lower than the experimental results. This error was attributed to a main source of confusion in piston hydraulic systems, the seal friction. After the seal friction calculations and corrections, the results were much closer to the experimental results; the difference between the results were considerably less, with the remaining error being attributable to uncalculated factors.

In conclusion, this investigation proves that viscosity in hydraulic systems affects the actuation time in piston hydraulic systems and the small syringe mechanism was effective at depicting this, with the losses being attributable to frictional losses.

7.0 | Evaluation

The strengths of this investigation were evaluated and explained in detail:

Strengths	Explanation
Fluid Choice	The fluid choices being water and glycerol, two Newtonian fluids with differing viscosities, allowed for laminar flow and the use of Hagen-Poiseuille in the investigation which helped strengthen claims made.
Experimental Conditions	Consistent experimental conditions were preserved during each trial. This allowed for reliability in the experiment.
Easy Repeatability	The highest standard error was 0.91 which showed that this experiment can be reliably repeated with this apparatus.

|Table 11: Strengths and Explanation|

Following this, the main limitations were evaluated, explained and improvement suggestions were given:

Limitations	Explanation	Improvements
Experimental Errors Unaccounted For	While most of the overshot values were attributed to the piston friction existent in hydraulic systems, there was still a small portion of error unaccounted for. This could be due to flow restrictions in the experimental apparatus due to the tube length and laminar flow being disrupted leading to errors.	In a preliminary trial, test more tube lengths in order to find which one is best at minimizing the error caused by flow restrictions. This will allow for the determination of the actual actuation times and for more reliable calculations.
Human Timing Errors	Due to the timing being done with a stopwatch, errors could have occurred during the timing.	This could be mitigated by using a smartphone camera at a high framerate to film each trial then pull the times from

		an editing software.

|Table 12: Limitations, Explanation and Improvements|

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