

Resistance and Damping in a Series RLC Circuit

Extended Essay in Physics

International Baccalaureate Diploma Programme

Research Question:

“How does the total resistance in an underdamped series RLC circuit affect the damping coefficient of its voltage oscillations?”

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Contents

1	Introduction	3
2	Background Theory	3
2.1	The Series RLC Circuit	3
2.2	Governing Differential Equation	4
2.3	The Underdamped Solution	5
2.4	Physical Interpretation and Component Values	6
2.5	Extracting the Damping Coefficient from Experimental Data	6
3	Methodology	8
3.1	Equipment	8
3.2	Resistance Values	8
3.3	Independent, Dependent, and Controlled Variables	9
3.4	Circuit Configuration and Excitation Method	10
3.5	Data Collection Procedure	12
3.6	Data Processing	12
3.7	Other Sources of Uncertainty and Systematic Error	12
4	Results	13
4.1	Damping Coefficient Measurements	14
4.2	Signal Degradation and Fanning Uncertainty	15
4.3	Linear Regression: γ vs. R	16
4.4	Cross Check Using the Logarithmic Decrement Method	19
5	Evaluation	22
5.1	Success of the Investigation	22
5.2	Comparison to Theory	22
5.3	Parasitic Resistance	22
5.4	Error Analysis	23
5.4.1	Propagation of Resistance Uncertainty	23

5.5	Method Comparison	24
5.6	Limitations and Improvements	24
6	Conclusion	25
A	Per-Resistance Diagnostic Plots	27

1 Introduction

Oscillatory behavior is a widespread phenomenon in physics, from mechanics to harmonic motion and from acoustics to electromagnetic systems. In electrical circuits, oscillations arise naturally when energy is exchanged between an inductor and a capacitor in an LC loop. In reality, everything from the wires to the components has resistance and dissipates energy during each cycle. As a result, the oscillations gradually decay. This process, known as damping, is central to the behavior of the series RLC circuit. An RLC circuit consists of a resistor, inductor, and capacitor connected in series.

I became interested in this topic as I was working on a relevant personal electronics project, for which I was building a custom LCR meter from scratch in order to get a better understanding of impedance. As I was developing it, I frequently observed damped oscillations on my oscilloscope like transient voltage waveforms decaying over time after I switched a resistor on with a transistor. I became curious about what controlled the rate at which those oscillations died out, and whether I could isolate and measure the effect of a single variable, resistance, on the damping behavior. This led me to formalize this research question for this extended essay:

How does the total resistance in an underdamped series RLC circuit affect the damping coefficient of its voltage oscillations?

Classical theory predicts that the damping coefficient γ is directly proportional to the total circuit resistance, $\gamma = R/(2L)$.² To test this, I built a series RLC circuit driven by a microcontroller. I then systematically varied the resistance while keeping other things constant, and extracted the damping coefficient from each captured waveform using two distinct methods: logarithmic decrement and nonlinear curve fitting.

2 Background Theory

2.1 The Series RLC Circuit

A series RLC circuit consists of a resistor (of resistance R), an inductor (of inductance L), and a capacitor (of capacitance C) connected in a closed loop, as shown in Figure 1. When the

capacitor is initially charged and the circuit is closed, energy bounces between the electric field of the capacitor and the magnetic field of the inductor. The resistor dissipates energy as heat.

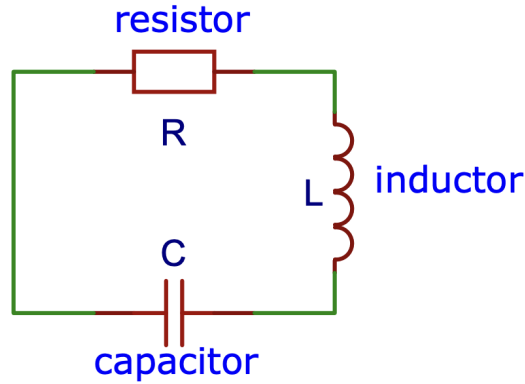


Figure 1: Series RLC circuit that consists of a resistor, inductor and a capacitor in series.

2.2 Governing Differential Equation

Applying Kirchhoff's voltage law around the loop, the sum of voltage drops across each component equals zero:

$$V_R + V_L + V_C = 0 \quad \implies \quad L \frac{di}{dt} + Ri + \frac{q}{C} = 0 \quad (1)$$

Since current $i = dq/dt$ and $V_C = q/C$, this can be rewritten as a second-order ODE for the capacitor voltage:³

$$\frac{d^2 V_C}{dt^2} + \frac{R}{L} \frac{dV_C}{dt} + \frac{1}{LC} V_C = 0 \quad (2)$$

This is mathematically identical to the equation governing a damped harmonic oscillator in mechanics like a pendulum, where L plays the role of mass, R acts as friction, and $1/C$ corresponds to the spring constant.² I define two key parameters:

$$\gamma = \frac{R}{2L} \quad (3)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (4)$$

where γ is the **damping coefficient** (s^{-1}) and ω_0 is the **natural angular frequency** (rad s^{-1}).

2.3 The Underdamped Solution

The nature of the solution depends on whether γ is greater than, equal to, or less than ω_0 , corresponding to overdamped, critically damped, and underdamped regimes respectively.⁴ Only the underdamped case ($\gamma < \omega_0$) produces oscillations, so it is the focus of this investigation. Substituting a trial solution $V_C(t) = Ae^{st}$ into Equation 2 and solving the resulting characteristic equation gives complex roots $s = -\gamma \pm j\omega_d$, where

$$\omega_d = \sqrt{\omega_0^2 - \gamma^2} \quad (5)$$

is the **damped angular frequency**. Applying Euler's formula to this and combining the complex exponentials results in the general solution:

$$V_C(t) = V_0 e^{-\gamma t} \cos(\omega_d t + \phi) \quad (6)$$

where V_0 is the initial amplitude and ϕ is the phase from the initial conditions. This describes a sinusoidal oscillation at frequency ω_d with an amplitude that decays exponentially with rate γ . The underdamped regime requires $R < R_{crit}$, where $R_{crit} = 2\sqrt{L/C}$ is the critical resistance.⁵

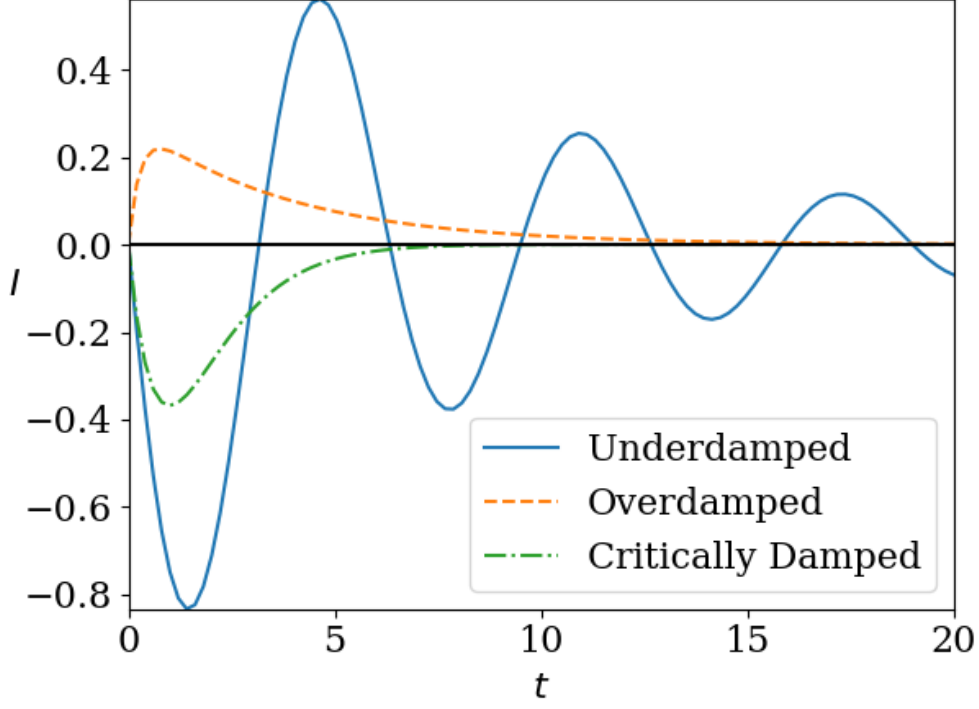


Figure 2: The voltage response of an underdamped (solid), overdamped (dashed) and critically damped (dashed-dotted) oscillator.⁵

2.4 Physical Interpretation and Component Values

My key prediction coming from Equation 3 is that γ is directly proportional to R for constant L . This is the basic relationship that I target to confirm. Note that even though γ is independent of C , ω_0 depends on C so it indirectly affects ω_d .

For my chosen components with $L = 10 \text{ mH}$ and $C = 100 \text{ nF}$, the resonant frequency is $\omega_0 = \frac{1}{\sqrt{LC}} \approx 31\,623 \text{ rad s}^{-1}$ ($f \approx 5.03 \text{ kHz}$) and the critical resistance is $R_{\text{crit}} = 2\sqrt{\frac{L}{C}} \approx 632 \, \Omega$. So if the resistance is less than around $632 \, \Omega$ it will produce an underdamped response. This will give me a wide range to test different R values and observe how γ changes.

2.5 Extracting the Damping Coefficient from Experimental Data

In practice, the damping coefficient can be determined by taking the amplitude of the first few peaks in a measured voltage waveform and seeing how it decays. If V_n and V_{n+k} are the amplitudes of two peaks separated by k complete cycles, then from the exponential relationship:

$$\frac{V_n}{V_{n+k}} = e^{\gamma k T_d} \quad (7)$$

where $T_d = 2\pi/\omega_d$ is the period of the damped oscillation. Taking the natural logarithm:

$$\ln\left(\frac{V_n}{V_{n+k}}\right) = \gamma k T_d \quad (8)$$

This quantity is related to the **logarithmic decrement** δ , defined for successive peaks ($k = 1$) as:⁶

$$\delta = \ln\left(\frac{V_n}{V_{n+1}}\right) = \gamma T_d = \frac{2\pi\gamma}{\omega_d} \quad (9)$$

By measuring several peak amplitudes from a single waveform, I can linearize the y-axis by taking the natural logarithm of the amplitudes. I can then perform a linear regression of $\ln(V_n)$ against peak number to extract γ with greater precision than a single pair of peaks would allow. With this method I can also perform a visual check on the exponential decay assumption as if the data points fall on a straight line, the model is consistent with the empirical data.⁷

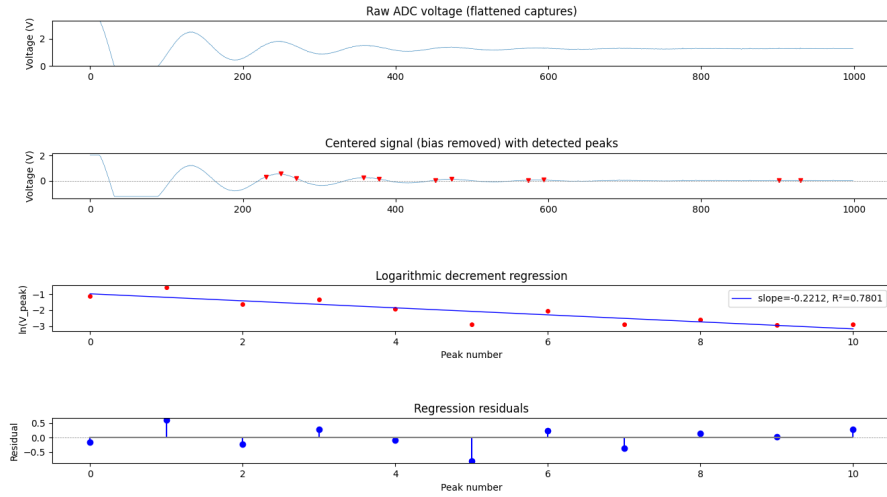


Figure 3: An empirical application of the logarithmic decrement method I sampled. Successive peak amplitudes V_n are identified from the captured waveform, and a plot of $\ln(V_n)$ against peak number n yields a straight line whose slope is proportional to γ .

However, the logarithmic decrement method only uses peak amplitudes, discarding the vast majority of the sampled data points. A more robust alternative is to fit the full analytical solution

(Equation 6) directly to the entire waveform using nonlinear least-squares regression. This method calculates γ , ω_d , V_0 , and ϕ simultaneously from every sample point. So I use the curve-fit method as the primary analysis tool for this investigation and apply the logarithmic decrement as a cross-check. As a result I can compare and contrast their robustness.

3 Methodology

3.1 Equipment

The circuit was assembled on a breadboard using a 10 mH fixed inductor, a 100 nF ceramic capacitor, and carbon film resistors in various series combinations (see Table 1). I chose a ceramic capacitor for its negligible equivalent series resistance (ESR), which prevents uncontrolled resistance from contaminating the damping measurement.⁴ An STM32F103C8T6 (“Blue Pill”) microcontroller served as both signal generator and data recorder, using a GPIO pin to produce the voltage step and its 12-bit ADC (up to 1 Msps) to capture the resulting waveform.⁸ I verified the waveforms visually using my hand-soldered DSO138 oscilloscope. Actual resistance values were measured with a digital multimeter (Uni-T UT33D+) before each data set, and data was streamed over serial to a computer running Python for analysis.

3.2 Resistance Values

Eight resistance values spanning $5\ \Omega$ to $470\ \Omega$ were selected, all well below $R_{\text{crit}} \approx 632\ \Omega$ (Table 1).

Table 1: Resistance values and their series combinations.

Trial set	R (Ω)	R_{Actual} (Ω)	Uncertainty of R_{Actual} (Ω)
1	5	4.7	± 0.1
2	10	10.3	± 0.1
3	22	22.8	± 0.1
4	47	46.9	± 0.1
5	100	99.6	± 0.1
6	220	216	± 1
7	330	321	± 1
8	470	462	± 1

Each combination was measured with a digital multimeter before data collection, bypassing the wide 5% nominal tolerance. The digital multimeter is not auto-ranging, so there are different resistance modes corresponding to different ranges of resistance. The 200 Ω mode displays up to a single decimal place whereas the 2000 Ω mode cannot show decimal precision. Therefore, the absolute uncertainties of the measurements performed with them are ± 0.1 Ω and ± 1 Ω , respectively.

3.3 Independent, Dependent, and Controlled Variables

- **Independent variable:** Total resistance R in the series RLC loop, varied across eight values from 5 Ω to 470 Ω .
- **Dependent variable:** The damping coefficient γ , extracted from the exponential decay envelope of the measured voltage oscillation.
- **Controlled variables:**
 - Inductance $L = 10$ mH
 - Capacitance $C = 100$ nF
 - STM32 output voltage step (same GPIO pin, same firmware, same supply voltage)

- ADC sampling configuration (same sampling rate, same resolution, same number of samples per capture)
- Ambient temperature (measurements conducted in a single session and with short energized periods to minimise thermal drift in component values)

3.4 Circuit Configuration and Excitation Method

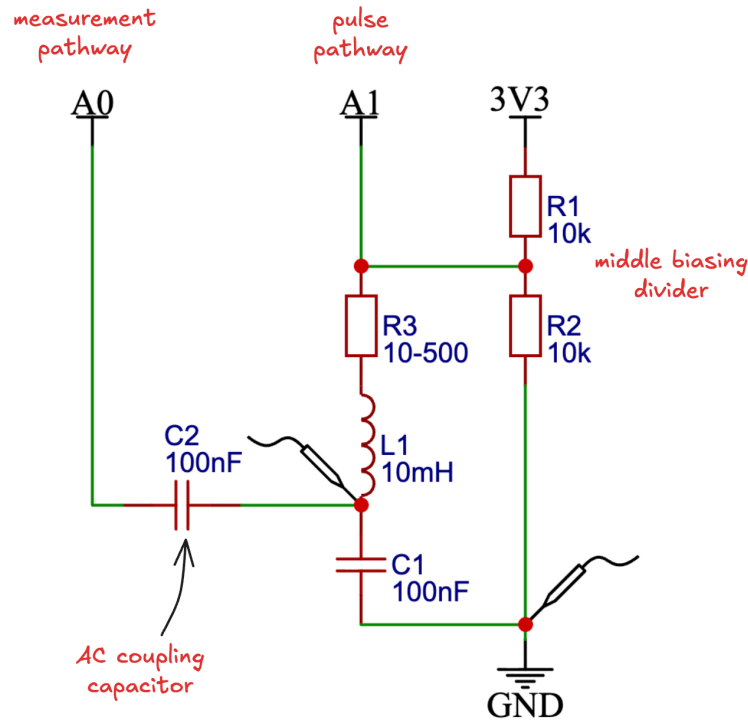


Figure 4: Schematic of the experimental setup. The STM32 Blue Pill generates the step voltage input and captures the transient waveform via its ADC. Data is transmitted over serial to a computer for analysis. The DSO138 oscilloscope probing across the capacitor provides visual verification.

The series RLC circuit forms the main pathway that generates the damped oscillation. A resistor divider biases the signal to $\frac{V_{CC}}{2} \approx 1.62V$ so that the troughs don't sink to negative voltages and the ADC of the STM32 can measure the negative portion of the signal. The STM32 Blue Pill both generates the excitation signal and records the resulting waveform. The middle of the inductor and capacitor is connected to the measurement pin through a 100 nF AC coupling capacitor (C2 in Figure 4) instead of a direct short. This capacitor blocks the DC bias set by the

resistor divider and passes only the AC oscillation to the ADC pin, otherwise the STM32 could not read the negative voltages.⁴ However, first C2 must charge before it can reliably pass the oscillation so it introduces a brief transient, to be discussed in Section 4.1.

The firmware drives output pin A1 HIGH (3.3 V) for a very brief period and then pulls it to LOW. This produces a voltage step that initiates the transient oscillation. The STM32's GPIO rise and fall times are on the order of 5–25 ns,⁸ which is negligible compared to the oscillation period of approximately 200 μ s, so it produces a clean step.

At the same time, the STM32's 12-bit ADC begins sampling the voltage across the capacitor at up to 1 Msps through pin A0. At the calculated oscillation frequency of around 5 kHz, this yields roughly 120 samples per cycle, which is detailed enough for each waveform. The ADC captures a buffer of 1000 samples (making around 8 to 9 complete cycles), which is then transmitted to my computer for storage and analysis. This approach yields a precise numerical time series $V_C(t_i)$ for each trial which is far better than trying to read peak values from an oscilloscope display.

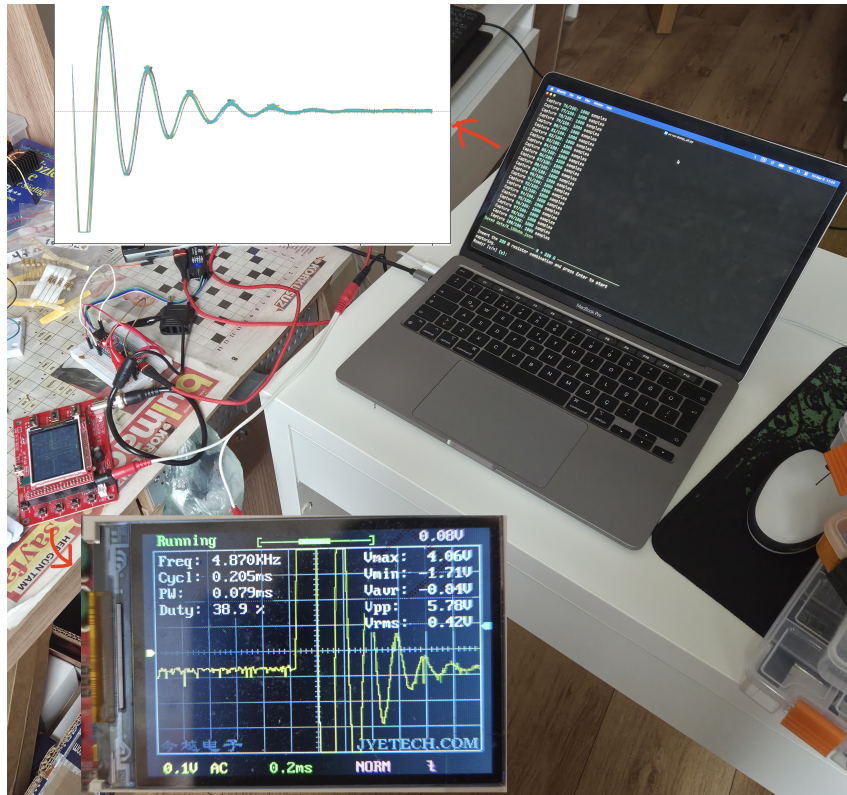


Figure 5: The setup in action.

3.5 Data Collection Procedure

I assembled the circuit shown in Figure 1 on a breadboard, and tested it with each resistor accordingly. The STM32 firmware generated a voltage step and captured the result from the ADC buffer. The data was then transmitted over serial to my computer. This process was repeated 100 times per resistance value (of which 98 were used, discarding the first two to remove the initial transients).

3.6 Data Processing

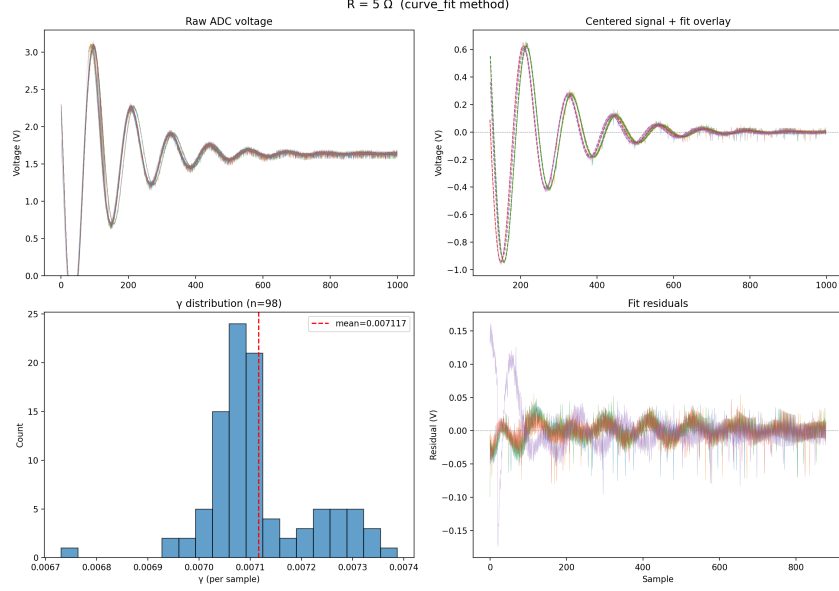
Each captured waveform has 1000 data points. I processed the data in Python using NumPy and SciPy.⁹ I converted the raw 12-bit ADC values to voltages using the 3.3 V reference. For the analysis, I fit the damped sinusoid model stated in Equation 6 using SciPy's `curve_fit` function, which performs nonlinear least-squares optimization. This method directly calculates γ , ω_d , V_0 , and ϕ from every sample point. I also applied the logarithmic decrement method described in Section 2.5 to the detected peak amplitudes, and calculated the mean $\bar{\gamma}$ and standard deviation σ_γ for each resistance value.

3.7 Other Sources of Uncertainty and Systematic Error

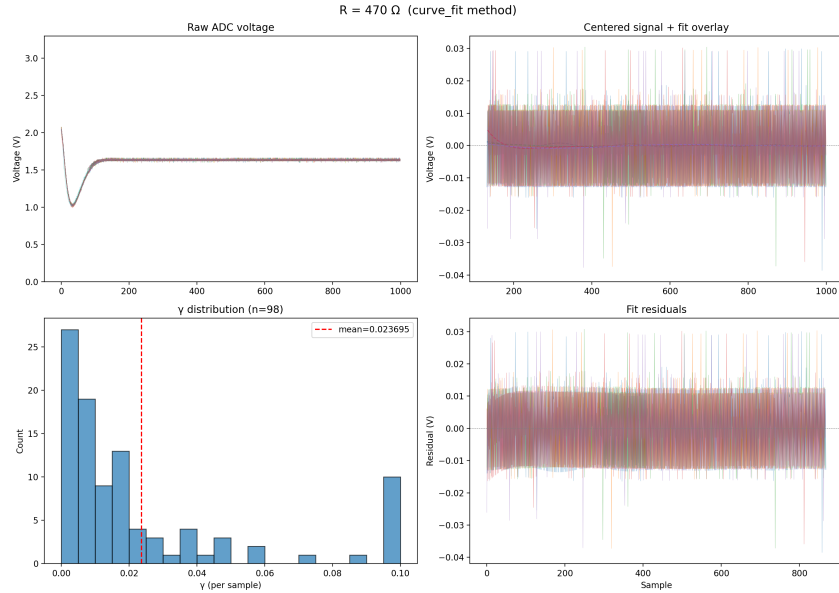
I expect the following sources of error and uncertainty: (1) The parasitic resistance from the inductor's DC winding resistance, the resistance of the breadboard contacts and the output impedance of the STM32 GPIO. These implicitly add to the R value and systematically increase the γ measured. (2) ADC limitations. The 12-bit STM32 ADC has a resolution of 0.8 mV per bit.⁸ As there is a finite sampling rate the true peak may fall between samples, but with roughly 120 points per cycle this effect is below 0.1%. (3) External electromagnetic interference. There are many electronic gadgets in the room where the experiment was conducted, so the long jumper wires may act as antennae, introducing unwanted high-frequency noise from the environment.

4 Results

Figure 6 illustrates the progression from clean signal to noise-dominated captures at the two extremes of the resistance range. The full set of per-resistance diagnostic plots for both methods is collected in Appendix A.



(a) $R = 5 \Omega$: many oscillation cycles survive, producing a tight overlay and a narrow γ distribution.



(b) $R = 470 \Omega$: oscillations vanish within one cycle, the overlay is dominated by noise, and the γ distribution is spread across a wide range.

Figure 6: Curve-fit diagnostic panels at the lowest and highest resistance values, showing the progressive collapse of signal quality. Each panel shows (clockwise from top-left): raw ADC voltage overlay, centered signal with fit overlay, fit residuals, and γ histogram.

4.1 Damping Coefficient Measurements

The damping coefficient γ was extracted from each of the 98 valid captures per resistance value using nonlinear least-squares fitting of the damped sinusoid model (Equation 6) via SciPy's `curve_fit`.

I first applied this procedure on the entire waveform, including before the first detected peak. This produced bizarre and bimodal γ distributions at intermediate resistance values (22 Ω to 100 Ω).

After looking at the first samples closely, I found that the coupling capacitor between the LC junction and the ADC pin caused a charging transient during the first ~ 50 to 100 samples after the microcontroller excitation that settled after roughly 44 to 78 samples. Including this region threw off both algorithms into producing inflated γ values. Therefore I started the fitting after the first 100 samples, which eliminated the bimodality and produced clean approximately Gaussian distributions at every resistance except 470 Ω . At that value the signal was so weak that neither algorithm could extract meaningful results.

The fitted values of γ are in units of per-sample. To convert to SI units (s^{-1}), I calibrated the effective sampling rate using the $R = 5 \Omega$ data, where damping is minimal and $\omega_d \approx \omega_0$. The measured ω_d at $R = 5 \Omega$ was 0.0538 rad/sample, giving $f_s = \omega_0/\omega_{d,\text{meas}} \approx 588\,000$ samples/s. All per-sample γ values were then multiplied by this f_s to obtain γ in s^{-1} .

Table 2 presents the measured and theoretical damping coefficients for all eight resistance values.

Table 2: Measured and theoretical damping coefficients for each resistance value. σ_γ is one standard deviation across 98 captures, $\Delta\gamma_R$ is the uncertainty propagated from resistance measurement, and $\Delta\gamma_{\text{tot}}$ is their quadrature sum.

$R_{\text{actual}} (\Omega)$	$\gamma_{\text{meas}} (\text{s}^{-1})$	$\sigma_\gamma (\text{s}^{-1})$	$\Delta\gamma_R (\text{s}^{-1})$	$\Delta\gamma_{\text{tot}} (\text{s}^{-1})$	$\gamma_{\text{theory}} (\text{s}^{-1})$
4.7	4182.7	60.8	5.0	61.0	235.0
10.3	4450.6	84.1	5.0	84.2	515.0
22.8	5110.1	81.1	5.0	81.2	1140.0
46.9	6407.7	146.5	5.0	146.6	2345.0
99.6	9250.6	183.1	5.0	183.2	4980.0
216 [†]	14 970.4	1828.9	50.0	1829.6	10 800.0
321 [†]	23 059.4	9007.0	50.0	9007.1	16 050.0
462 [†]	13 926.0	17 856.9	50.0	17 857.0	23 100.0

[†]Unreliable: fewer than two clean oscillation cycles available after the transient skip. See Section 4.2.

The γ values measured consistently show larger values compared to the theoretically predicted value $\gamma = \frac{R}{2L}$ for all values of R . This is because there is some significant value of parasitic resistance involved in the circuit, which is in addition to the external resistance R . See Section 5.3.

4.2 Signal Degradation and Fanning Uncertainty

A pattern is evident in Table 2. As R goes up, the standard deviation of the 98 fitted γ values first increases slowly, then very quickly. The relative standard deviation (RSD) remains below 2.5% for $R \leq 100 \Omega$ and γ increases monotonically with R . However, when $R = 220 \Omega$, the RSD is 12%. At $R = 330 \Omega$, it reaches 39%. At $R = 470 \Omega$, the standard deviation fully passes over the mean (RSD = 128%) and the measured γ is actually lower compared to $R = 220 \Omega$, violating the trend and the theory entirely.

Looking at the overlay plots for each resistance makes it easy to see how it degrades. The centered fitted curves come together to form a very thin overlay at $R = 5 \Omega$ because there is an almost perfect overlap. At $R = 100 \Omega$, the shadow thickens noticeably toward the tail. At $R = 220 \Omega$, the fitted curves scatter apart and the overlay becomes a broad smear. At

$R = 470\ \Omega$, the overlay is solid noise with no identifiable features. In fact the residual plots also demonstrate the same pattern of being tight and structured at low R and widening into a featureless band at high R .

The underlying mechanism is straightforward. Higher resistance means stronger damping, which means fewer oscillation cycles survive above the noise floor, which leaves less information for the fit to work with. The 12-bit ADC resolves 0.8 mV per bit. At $R = 5\ \Omega$, peak amplitudes in the fit window are hundreds of millivolts (hundreds of least-significant-bit). By $R = 220\ \Omega$, they have fallen to tens of millivolts (a few tens of LSB), and by $R = 470\ \Omega$, the oscillation amplitude is comparable to the quantisation step itself. The optimizer is no longer converging to a physically meaningful minimum; it is fitting noise, and the resulting γ values scatter in all directions.

The gamma distributions demonstrate this well. For $R \leq 100\ \Omega$, the distributions are tightly clustered and Gaussian. For $R = 220\ \Omega$ and up, the distributions develop a positive skew as the mode ends up being less than the average because the occasional capture where the noise acts like a rapid decay causes the average to be higher than the mode. And in the $R = 470\ \Omega$ case, the distribution is no longer unimodal as the large cluster of γ values near zero is present in addition to the long right tail. This is consistent with the idea that the optimizer is having trouble finding the global minimum and is instead finding local minima depending on the noise in the individual experiments.

4.3 Linear Regression: γ vs. R

Figure 7 shows γ plotted against R for all eight data points, together with the linear regression and the theoretical prediction $\gamma = R/(2L)$.

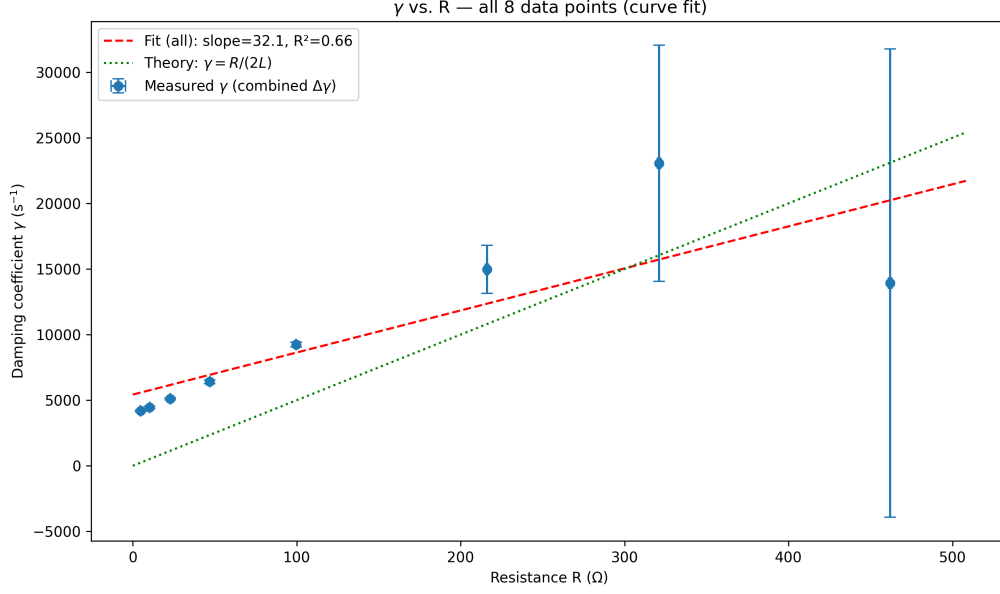


Figure 7: Plot shows damping coefficient in the y -axis and resistance R in the x -axis for all eight R values. Error bars represent one standard deviation. The points demonstrate fanning with increasing resistance. The green dotted line is the theory prediction $\gamma = R/(2L)$ whereas the red dashed line is the fit to the entire dataset ($R^2 = 0.67$).

The full eight-point regression yields:

$$\gamma = (31.5 \pm 9.8) \cdot R + (5430 \pm 1960) \quad (\text{s}^{-1}) \quad (10)$$

with $R^2 = 0.67$. I found that the bad correlation is mostly caused by the three highest resistance samples. Specifically, the $470 \, \Omega$ point is both a strong outlier and has strong leverage. As a result it is a strong contributor to the fact that the regression slope is off by 37%.

To determine which points were reasonable and others outliers numerically, I performed a test that assessed the internal consistency of the model. The underdamped solution has the equation $\omega_0^2 = \omega_d^2 + \gamma^2 = 1/(LC)$. So the underdamped frequency ω_0 depends only on the inductance and capacitance, which were held constant in the experiments. Therefore, as long as the damped sinusoid model was a good fit to the signal, the value of ω_0 ought to be the same regardless of the resistance. The value of ω_0 for each fit is shown in Table 3.

Table 3: Natural frequency consistency check. ω_0 is computed from the fitted ω_d and γ at each resistance value and compared to the reference value at $R = 5 \Omega$.

$R (\Omega)$	ω_0 (rad/sample)	Deviation (%)
5	0.0543	0.0
10	0.0545	0.5
22	0.0546	0.6
47	0.0548	1.0
100	0.0554	2.1
220	0.0558	2.9
330	0.0586	7.9
470	0.0376	-30.7

For $R = 5 \Omega$ to 100Ω , the computed ω_0 is consistent within 2%, confirming that the damped sinusoid model accurately describes the waveform in this range. At $R = 220 \Omega$, the deviation reaches 3% (borderline). At $R = 330 \Omega$ and 470Ω , it exceeds 8% and 31% respectively, confirming that the fit can no longer reliably separate γ from ω_d when too few oscillation cycles constrain the parameters.

After this check, I decided to restrict the primary linear regression to the five data points at $R \leq 100 \Omega$, where data points have low variance. Using the actual measured resistance (see Table 1) instead of the advertized values and the combined uncertainty $\Delta\gamma_{\text{tot}}$ (see Section 5.4.1) produces:

$$\gamma = (53.6 \pm 0.2) \cdot R + (3905 \pm 9) \quad (\text{s}^{-1}) \quad (11)$$

with $R^2 = 0.9999$. The measured slope of $53.6 \Omega^{-1} \text{s}^{-1}$ is 7.2% above the theoretical $50.0 \Omega^{-1} \text{s}^{-1}$, and the positive y -intercept of 3905s^{-1} indicates that there is damping even when there is zero external resistance. This corresponds to a parasitic resistance of $R_{\text{parasitic}} \approx 73 \Omega$.

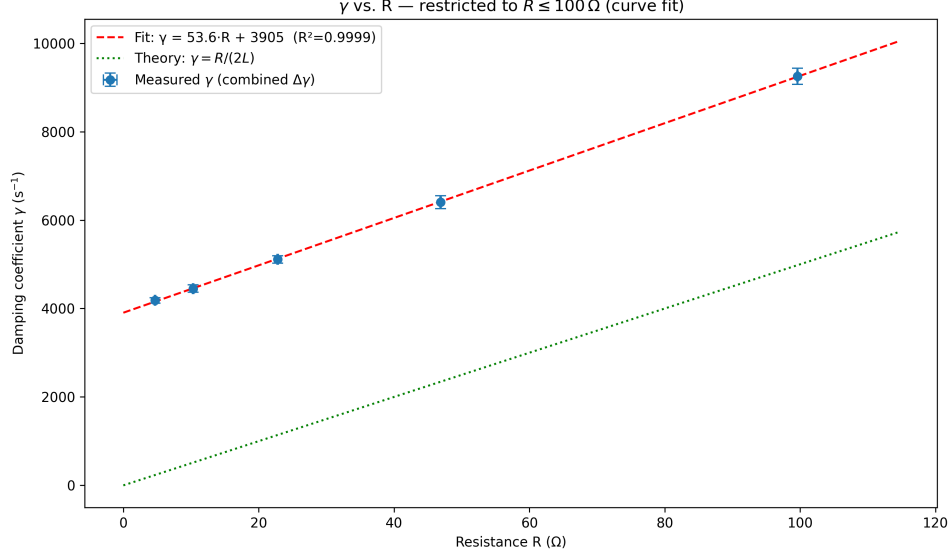


Figure 8: Measured damping coefficient γ as a function of resistance R for $R \leq 100 \Omega$. A much better correlation is observed after removing the outliers.

4.4 Cross Check Using the Logarithmic Decrement Method

As a secondary analysis, I applied the logarithmic decrement method (Section 2.5) to all eight resistance values. This method detects peaks in the waveform and extracts γ from the slope of $\ln(V_n)$ against peak number.

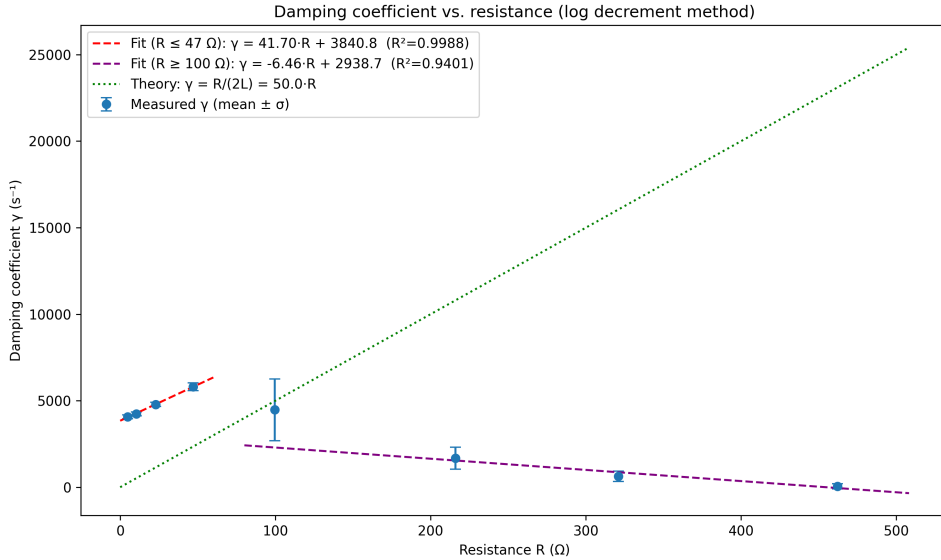
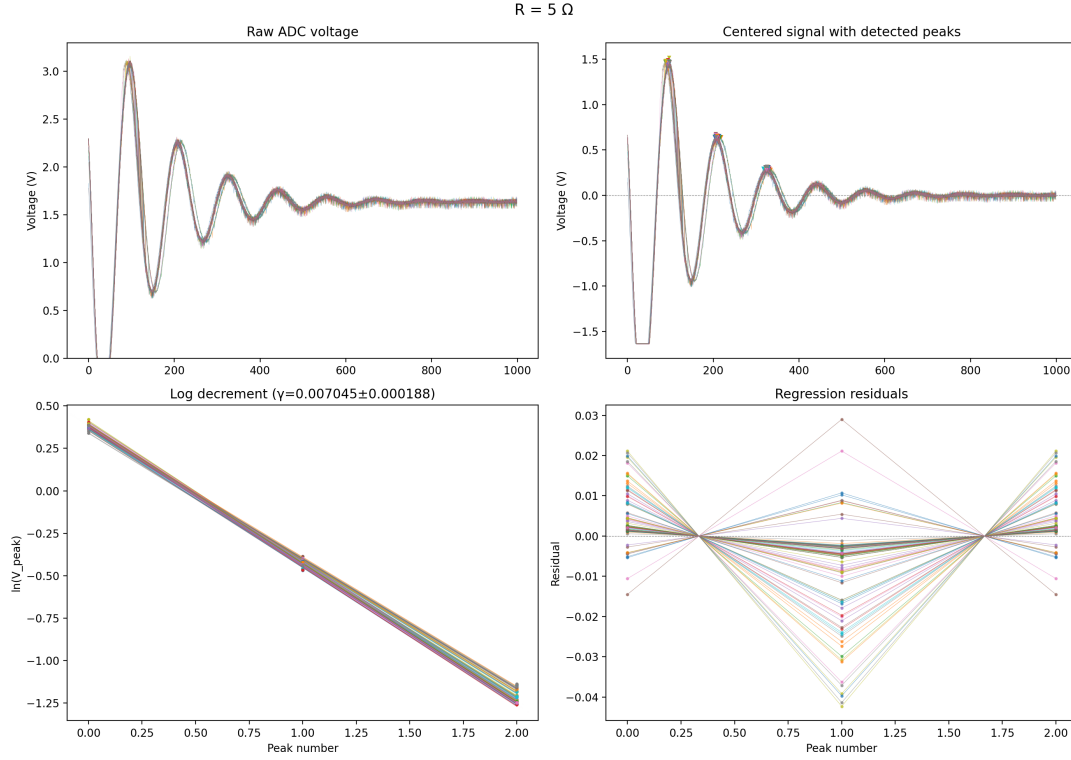


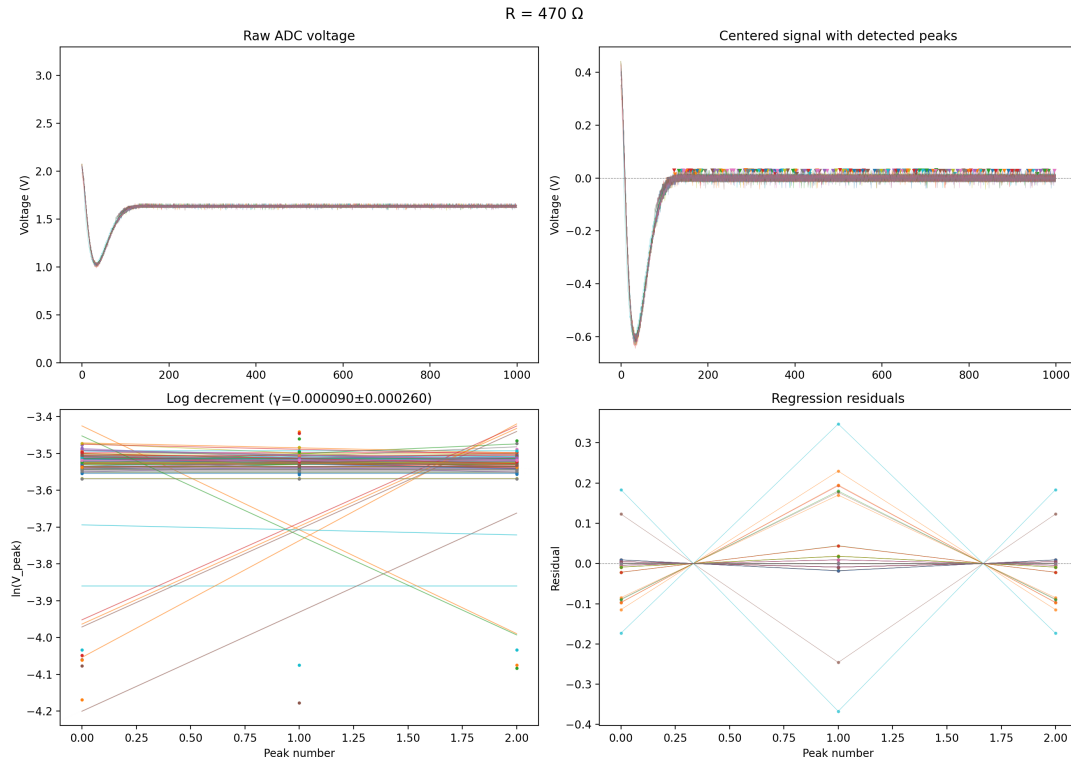
Figure 9: The damping coefficient γ using the logarithmic decrement method and partial linear regression for $R \leq 47 \Omega$ (red dashed, $R^2 = 0.999$). This shows that the slope indeed follows the theory. The trend remarkably changes sign for $R \geq 100 \Omega$ (purple dashed) as the damping coefficient γ decreases with R . This is physically not possible and is indicative of systemic failure.

Three or more peaks are present for low resistance cases, where $R \leq 47 \Omega$. This signifies that such results are comparable to the curve-fitting method. The partial regression has a strong positive relationship with an R^2 value of 0.999. However, as the resistance increases, the method fails at a much faster rate than the curve-fitting method. This is shown nicely in the per-resistance overlay plots: for low R , the markers for the detected peaks cluster tightly along the decay envelope, but as R increases, the markers for the later cycle positions scatter over an increasingly wider range. For $R = 100 \Omega$, the third "peak" is actually a noise bump instead of a legitimate oscillation peak. The regression slopes also begin to scatter wildly: some captures produce positive slopes, which is physically impossible. For $R = 330 \Omega$ and $R = 470 \Omega$, all three of the "peaks" actually have amplitudes that are essentially at the noise floor, which results in near zero or even negative slopes and nonsensical γ values in the main regression.

The result is a really striking reversal visible in Figure 9. The regression line is positively-sloped just as expected for the first four points. Nevertheless, it plateaus at 47Ω and then collapses towards zero. Figure 10 demonstrates this clearly: at $R = 5 \Omega$ the three detected peaks line up beautifully on the $\ln(V)$ regression, while at $R = 470 \Omega$ the "peaks" are actually just noise bumps scattered all over with no coherent slope.



(a) $R = 5 \Omega$: detected peaks align with the decay envelope and the $\ln(V)$ regression lines are parallel and steep.



(b) $R = 470 \Omega$: all detected “peaks” sit at the noise floor, the regression lines scatter in all directions, and the extracted γ values are meaningless.

Figure 10: Logarithmic decrement at the smallest and largest resistance values. Each plot, clockwise from the top-left: raw ADC voltage overlay, centered signal with detected peaks, regression residuals, and $\ln(V_{\text{peak}})$ versus peak number regression.

5 Evaluation

5.1 Success of the Investigation

The main prediction of this investigation was that the damping coefficient γ is linearly related to the resistance R in an underdamped series RLC circuit. I verified this with the restricted regression ($R \leq 100 \Omega$, $R^2 = 0.9999$).

5.2 Comparison to Theory

The measured slope of $53.6 \Omega^{-1} \text{s}^{-1}$ is 7.2% above the theoretical $50.0 \Omega^{-1} \text{s}^{-1}$, implying an effective inductance of $L_{\text{eff}} = 1/(2 \times 53.6) \approx 9.3 \text{ mH}$, approximately 7% below the nominal 10 mH. The inductor I used has a tolerance of $\pm 20\%$, so this mismatch is well within specification. I could not independently verify L using a precision LCR meter, which would have resolved this uncertainty.

Yet another cause of the slope discrepancy is the sampling rate calibration. The logic in converting the sampling rate to an SI unit when calculating ω_0 for deriving f_s is based on an exact $L = 10 \text{ mH}$, but if L is 9.3 mH as predicted in this model, it means that ω_0 is about 4% higher, hence shifting the sampling rate as well as all SI converted values. This circularity cannot be resolved without independently measuring L or f_s . Granted, since $R_p = \text{intercept/slope}$ is a ratio of quantities from each sample it does not affect the parasitic resistance estimate so it cancels out the sampling rate.

5.3 Parasitic Resistance

The y -intercept of the restricted fit corresponds to a parasitic resistance of $R_p \approx 73 \Omega$. I was able to identify the main contributors:

- **Inductor DC winding resistance:** measured directly with the multimeter at 27.8Ω .
- **STM32 GPIO output impedance:** estimated at 25Ω to 50Ω in push-pull mode, derived from the output voltage specifications ($V_{OL} \leq 0.4 \text{ V}$ at 8 mA) in the STM32F103 datasheet.⁸

- **Breadboard contact resistance and wiring:** estimated at $5\ \Omega$ to $10\ \Omega$ across multiple contact points.

The interval of these contributions ($57.8\ \Omega$ to $87.8\ \Omega$) contains the measured $73\ \Omega$. Leaving no significant unexplained resistance, this is a marked improvement over the full eight-point regression. The previous analysis yielded an inflated $R_p \approx 172\ \Omega$ because the unreliable high- R points demonstrated a bizarrely negative trend and shifted the y -intercept upward by a large margin.

5.4 Error Analysis

The biggest systematic problem was the coupling capacitor transient, which I identified and corrected as described in Section 4.1. My calibrated sampling rate of $588\,000$ samples/s is lower than the nominal 1 Msps because of ADC conversion time in the polling loop, accounted for by the calibration procedure which also creates the circularity mentioned earlier.

5.4.1 Propagation of Resistance Uncertainty

The multimeter measurements of resistance carry their own uncertainty ($\pm 0.1\ \Omega$ for $R \leq 100\ \Omega$ and $\pm 1\ \Omega$ for $R > 100\ \Omega$, as discussed in Table 1). Since the theoretical relationship is $\gamma = R/(2L)$, the contribution of resistance uncertainty to the uncertainty in γ is:

$$\Delta\gamma_R = \frac{\Delta R}{2L} \quad (12)$$

This was combined in quadrature with the experimental standard deviation σ_γ to yield a total uncertainty:

$$\Delta\gamma_{\text{tot}} = \sqrt{\sigma_\gamma^2 + \Delta\gamma_R^2} \quad (13)$$

As shown in Table 2, the resistance-propagated contribution $\Delta\gamma_R$ is $5.0\ \text{s}^{-1}$ for the low- R points and $50.0\ \text{s}^{-1}$ for the high- R points. In all cases, $\Delta\gamma_R$ is negligible compared to σ_γ : for the most favorable case ($R = 4.7\ \Omega$, where σ_γ is smallest), the resistance contribution accounts

for only 8% of the total uncertainty. For all other points it is below 6%. The dominant source of uncertainty in γ is therefore the capture-to-capture variation, not the resistance measurement.

5.5 Method Comparison

I found that the nonlinear curve fitting method was much more robust compared to the logarithmic decrement method. I attribute this to the fact that it uses all sample points available (several hundred) rather than 3 peaks, making it less susceptible to noise. The optimization directly calculates all physical parameters in one run and provides helpful goodness-of-fit metrics.

Interestingly, even though both start to fail as R increases, they do so differently. The curve fit produces high variance estimates which at least scatter around a reasonable central value (up to around $220\ \Omega$) which makes the breakdown obvious. In contrast, the logarithmic decrement method produces invalid "peak" detections everywhere, especially to the end of the signal, and gives answers that are confidently wrong. By doing so it flattens the regression slopes and underestimates γ for increasing R .

It is better to have the method that fails loudly (wide error bars) than the one that fails quietly (narrow error bars on the wrong answer). This was a key factor that made me decide to go with the curve fit as my primary analysis.

5.6 Limitations and Improvements

The main limitation is that the usable range of resistance values is very narrow. Out of those eight resistance values, only five ($R \leq 100\ \Omega$) were confirmed with the ω_0 check and actually produced a reliable fit. The reasons for this are straightforward. First, the initial 100 data points get skipped, reducing the available number of points. Second, the microcontroller's default ADC is not well suited for dealing with the wide range of values that is needed for the high R setups, especially when close to being over-damped.

Possible improvements for the future: (1) Using a custom-made PCB as opposed to a breadboard to eliminate parasitic effects. (2) Using an external ADC to eliminate calibration problems. (3) Using a commercial LCR meter to verify L . (4) Redesigning the circuit to eliminate the coupling capacitor, perhaps with an OpAmp. The experiment could also be extended by

making L and C variable and observing how γ changes.

6 Conclusion

My original research question was “How does the total resistance in an underdamped series RLC circuit affect the damping coefficient of its voltage oscillations?” This experiment has clearly shown that the damping coefficient is positively correlated with the resistance. As this is exactly the result that $\gamma = R/(2L)$ predicts, the theory matches the results.

For the five resistance values for which the damped sinusoid model fits very closely ($R = 4.7\ \Omega$ to $99.6\ \Omega$), the result is quite confident with $R^2 = 0.9999$. The fact that the y-intercept is not exactly zero and the slope is 7.2% off is due to the presence of the parasitic resistance and the inductor tolerance, as discussed earlier. However, as the values of the resistance increase, the signal-to-noise ratio collapses, as the system gets too close to the critically damped case and the results cease to be sensible.

It was really the long process of the analysis that taught me as much as the final result. I had to carefully diagnose the coupling capacitor problem that originally caused the bimodal γ distributions and the increasing fanning out of uncertainty with increasing resistance. The failure modes of the two extraction methods also required careful analysis. Ultimately, the curve fitting technique was more robust. But this wasn’t enough to get past the significant damping based noise problem. It was during the investigation that I realized the physics behind damped oscillation and how, in the end, a measurement can be no better than the signal you have to measure from.

References

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A Per-Resistance Diagnostic Plots

This appendix collects the full set of diagnostic plots for all eight resistance values, produced by both the nonlinear curve-fit method and the logarithmic decrement method. Each four-panel figure shows (clockwise from top-left): the raw ADC voltage overlay of all 98 captures, the centered signal with either the fit overlay (curve fit) or detected peaks (log decrement), fit residuals or regression residuals, and the γ histogram or $\ln(V_{\text{peak}})$ regression.

$$R = 5\ \Omega$$

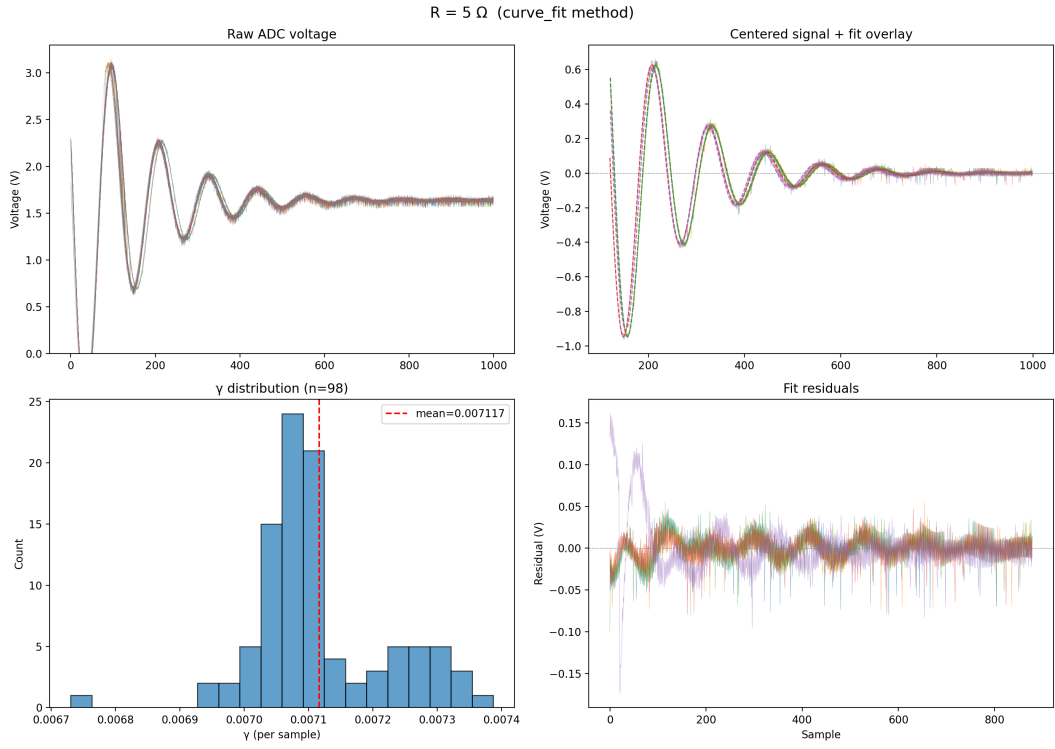


Figure 11: Curve-fit diagnostics for $R = 5\ \Omega$.

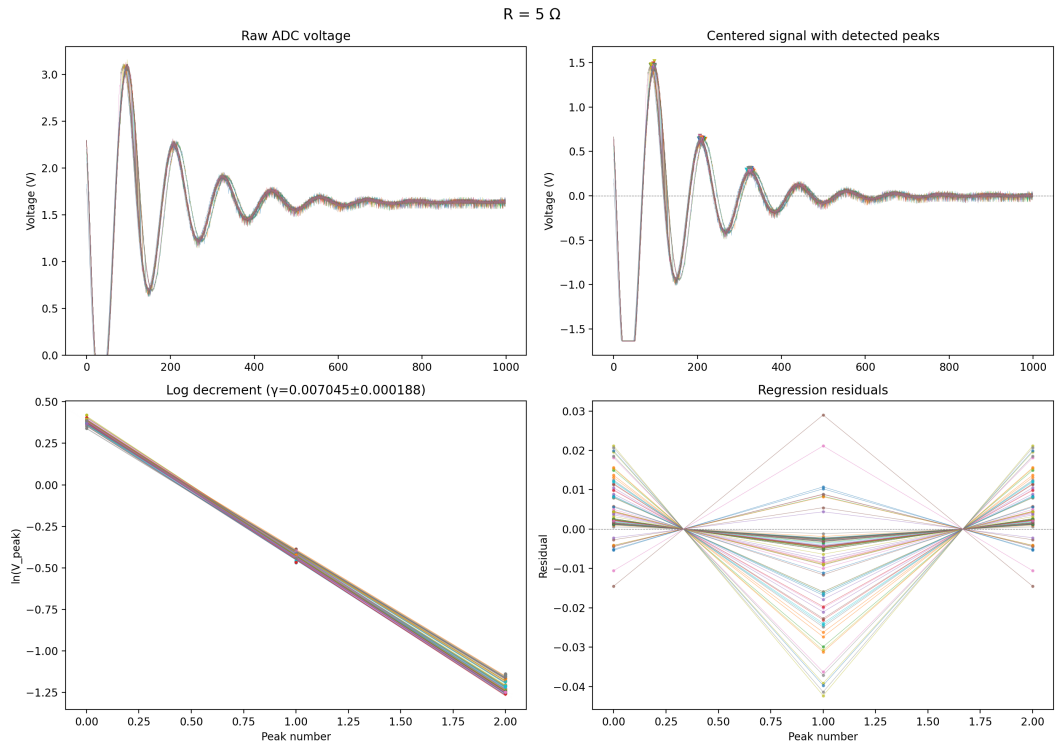


Figure 12: Log decrement diagnostics for $R = 5\ \Omega$.

$$R = 10\ \Omega$$

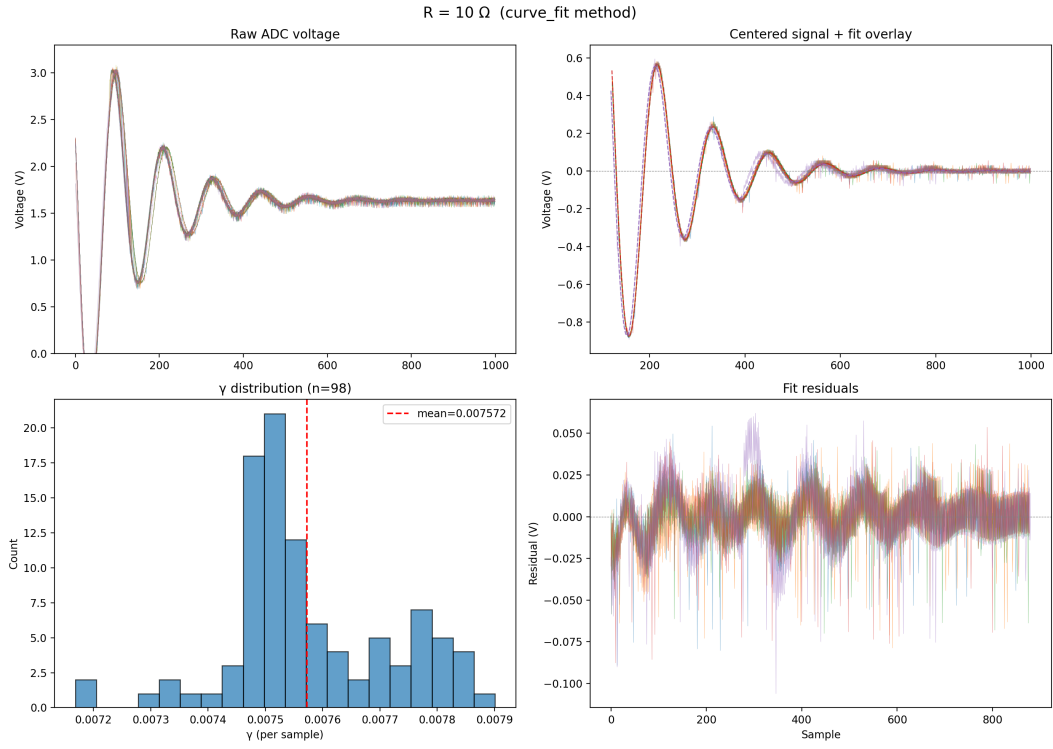


Figure 13: Curve-fit diagnostics for $R = 10\ \Omega$.

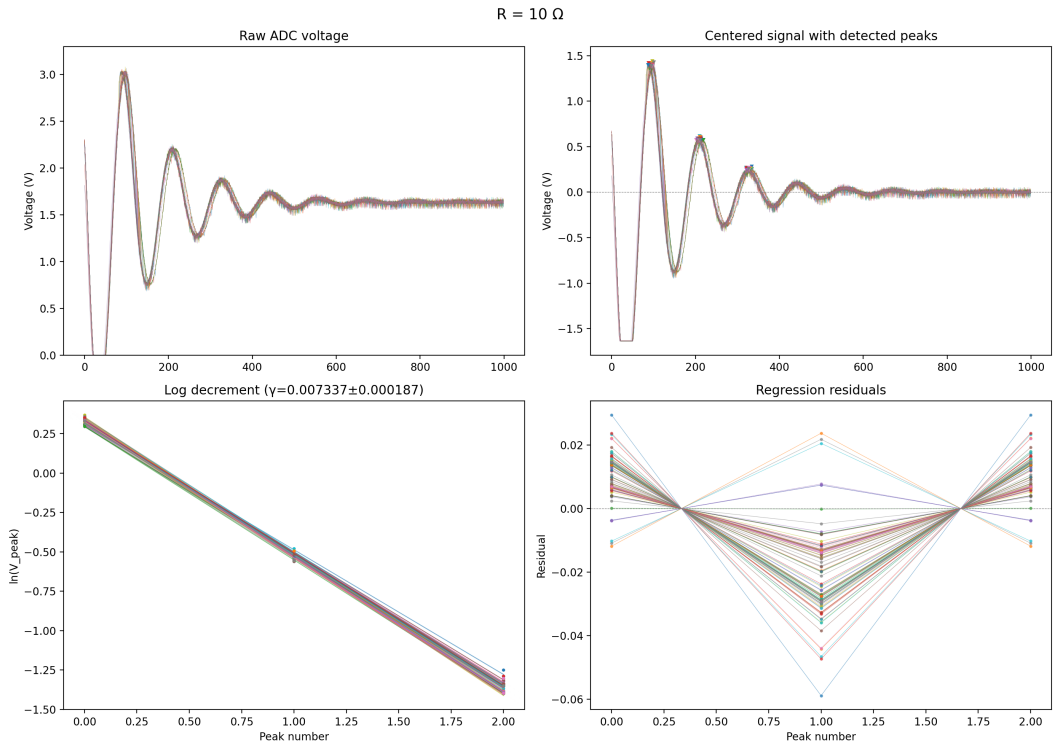


Figure 14: Log decrement diagnostics for $R = 10\ \Omega$.

$$R = 22\,\Omega$$

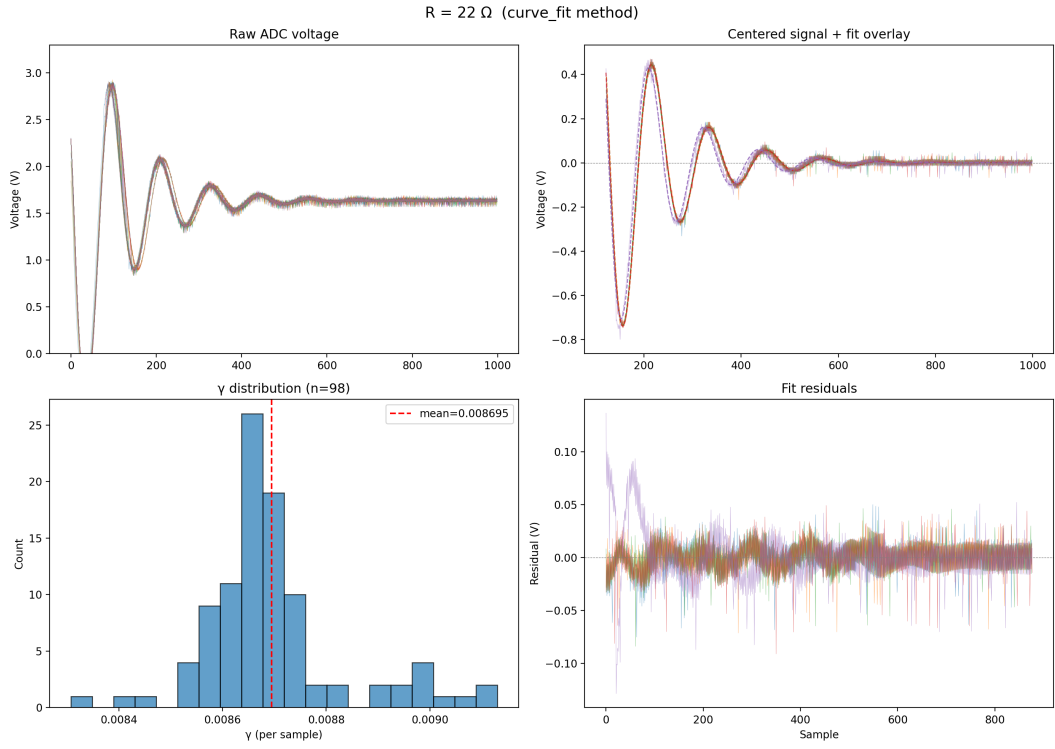


Figure 15: Curve-fit diagnostics for $R = 22\,\Omega$.

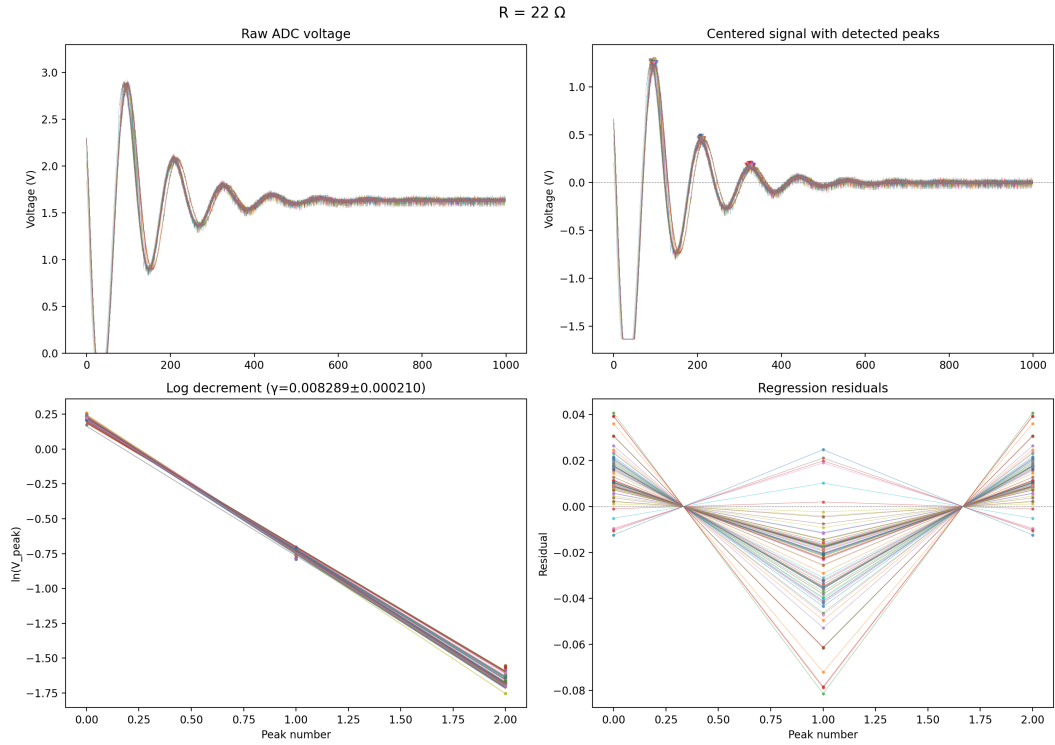


Figure 16: Log decrement diagnostics for $R = 22\,\Omega$.

$$R = 47\,\Omega$$

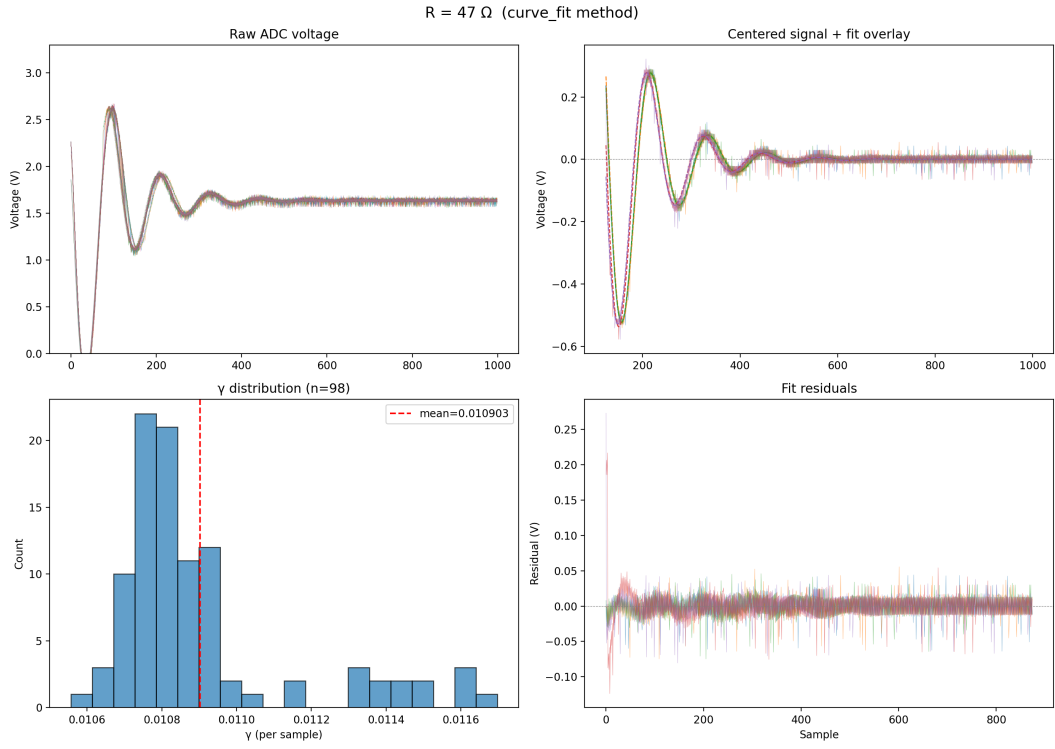


Figure 17: Curve-fit diagnostics for $R = 47\,\Omega$.

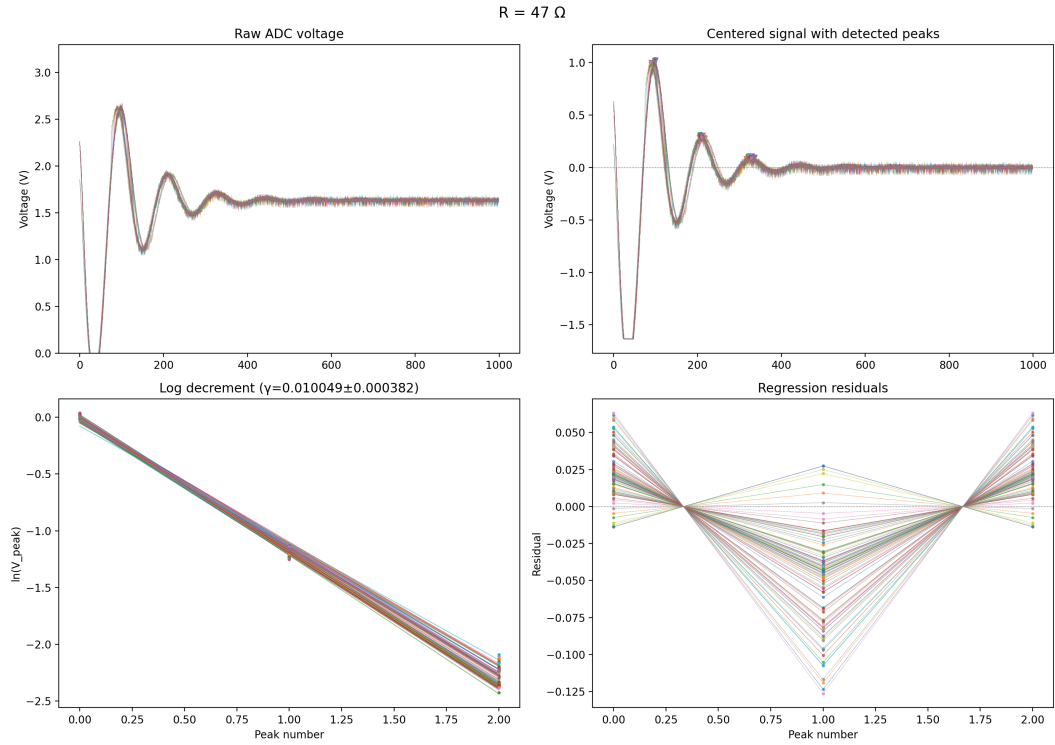


Figure 18: Log decrement diagnostics for $R = 47\,\Omega$.

$$R = 100 \Omega$$

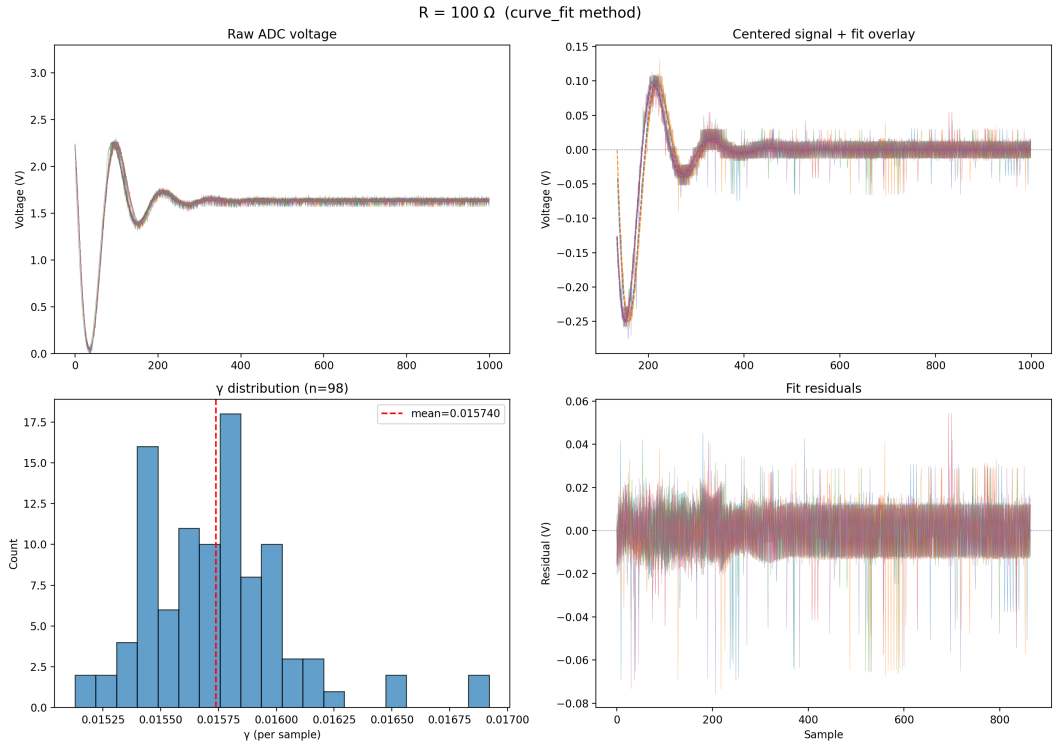


Figure 19: Curve-fit diagnostics for $R = 100 \Omega$.

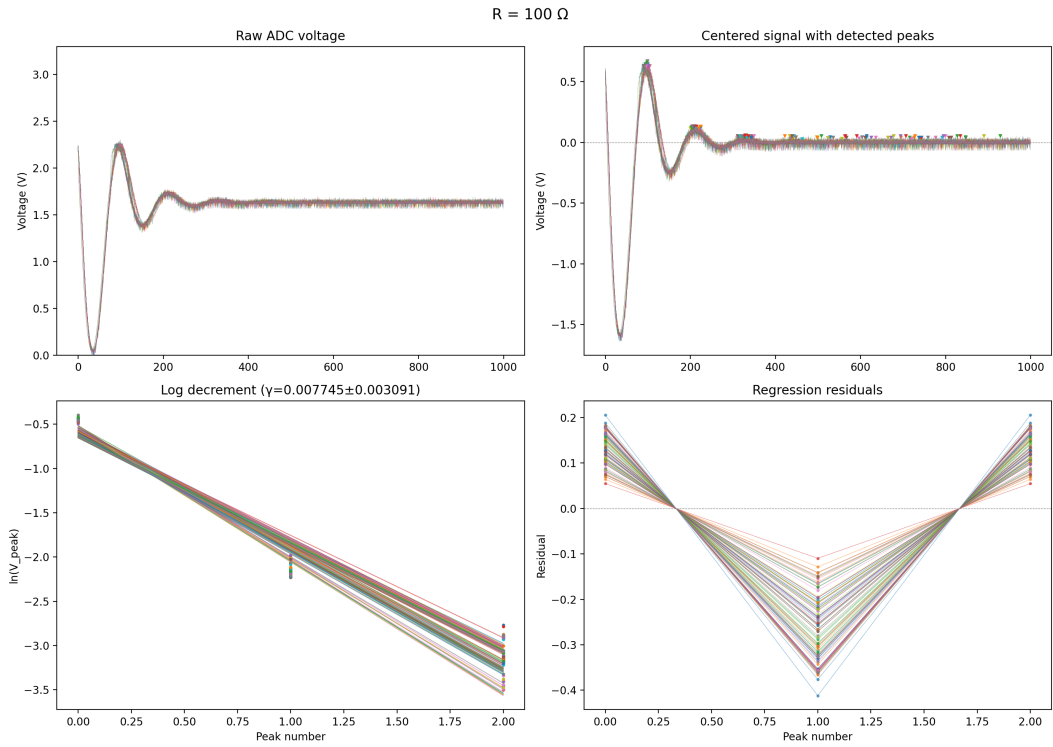


Figure 20: Log decrement diagnostics for $R = 100 \Omega$.

$$R = 220\ \Omega$$

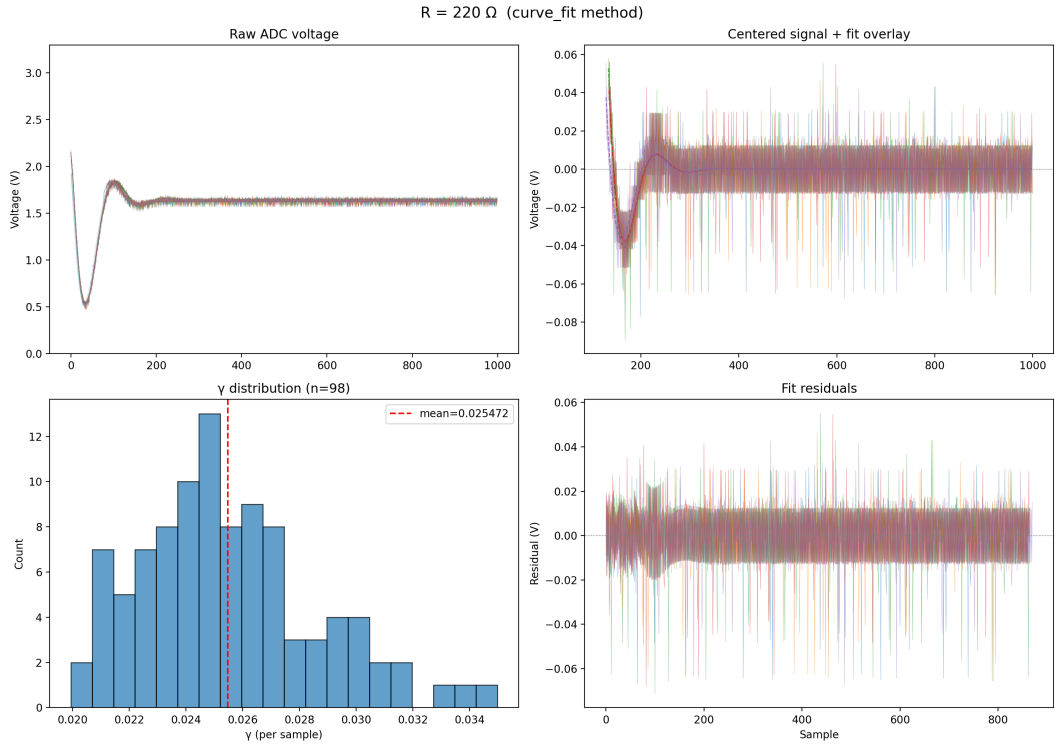


Figure 21: Curve-fit diagnostics for $R = 220\ \Omega$.

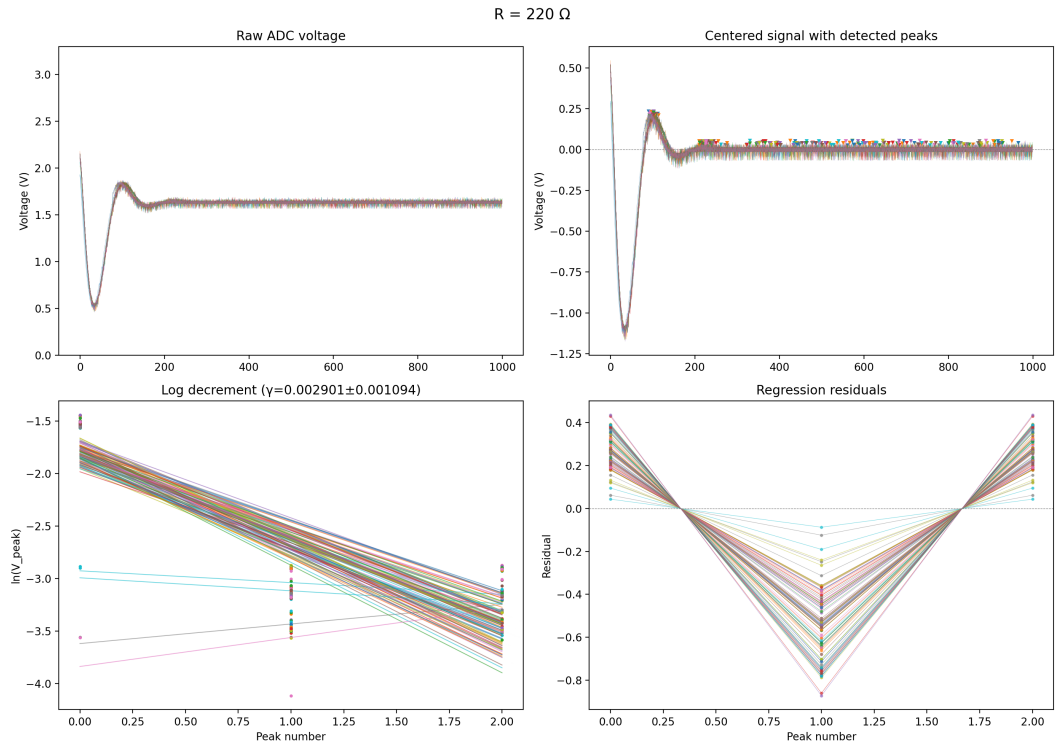


Figure 22: Log decrement diagnostics for $R = 220\ \Omega$.

$$R = 330 \Omega$$

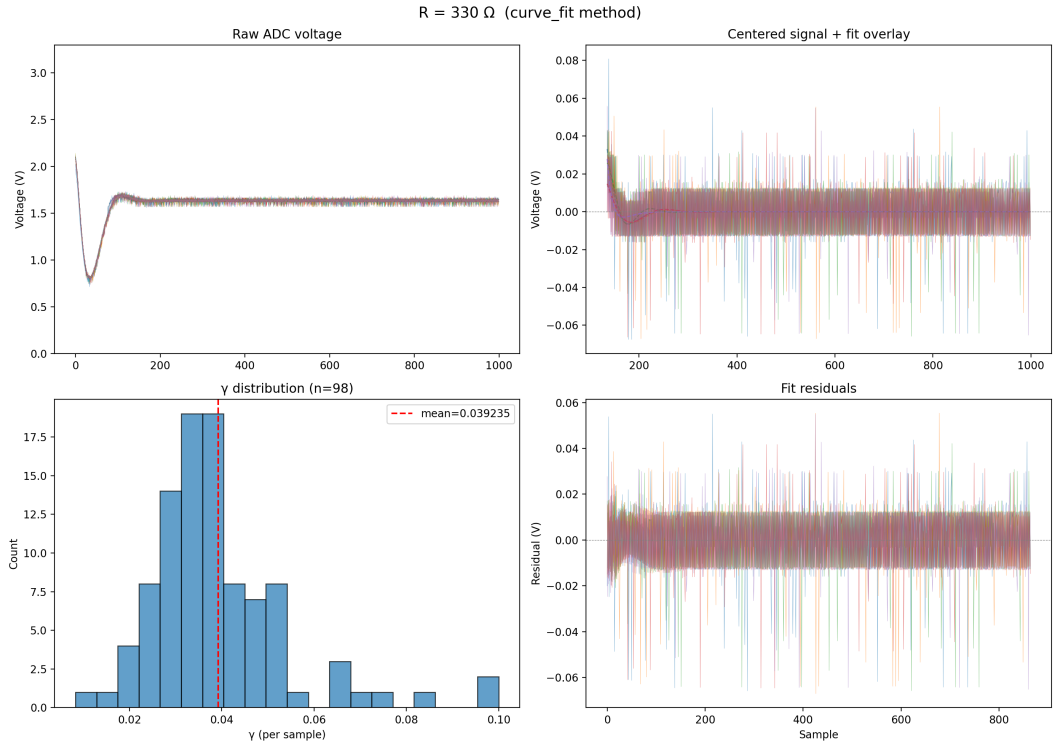


Figure 23: Curve-fit diagnostics for $R = 330 \Omega$.

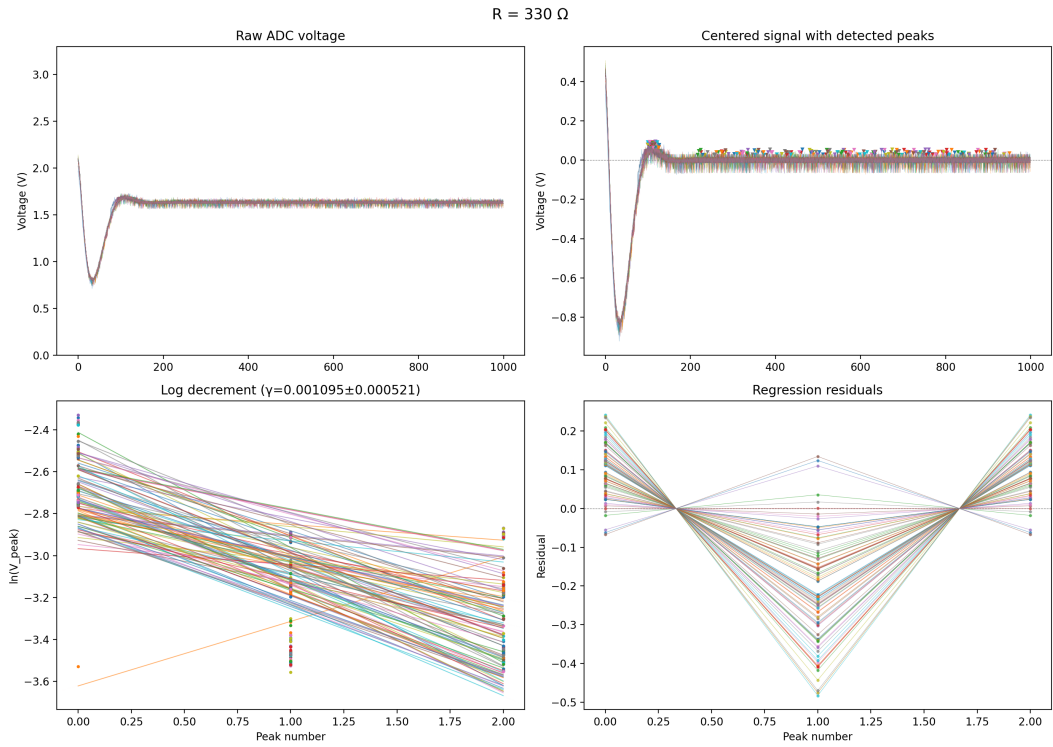


Figure 24: Log decrement diagnostics for $R = 330 \Omega$.

$$R = 470\ \Omega$$

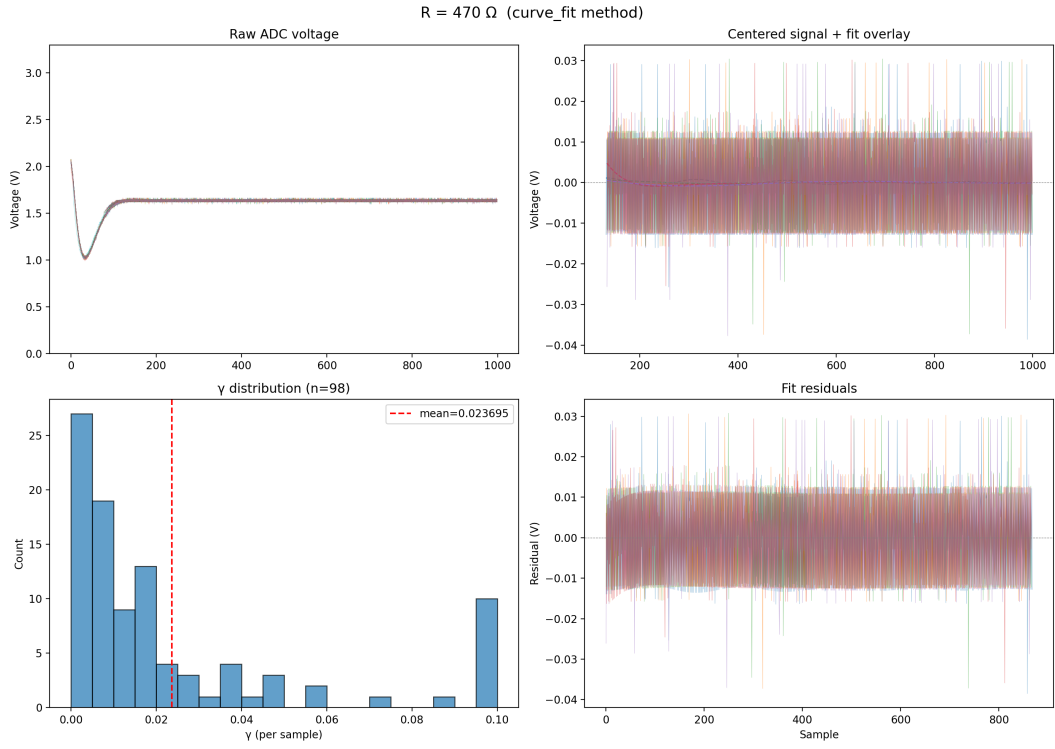


Figure 25: Curve-fit diagnostics for $R = 470\ \Omega$.

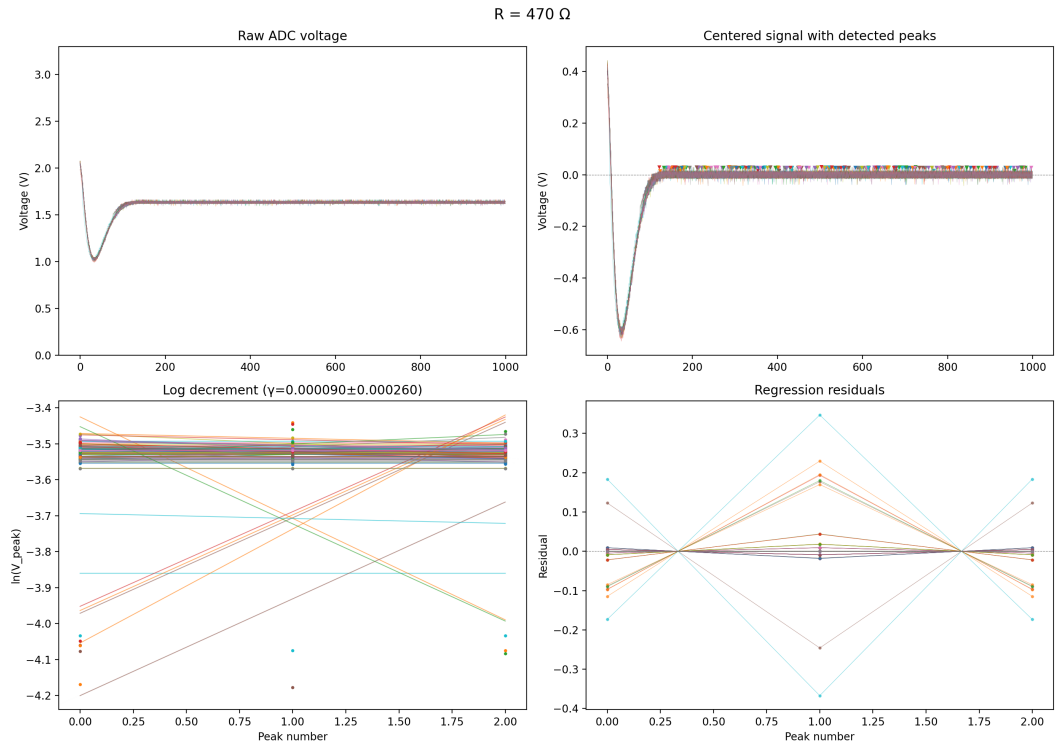


Figure 26: Log decrement diagnostics for $R = 470\ \Omega$.