The Effect of Distance from Axis of Rotation on Angular Acceleration in a Yoyo

Research Question: How does the radial distance of symmetrically placed standard masses from the axis of rotation of a yoyo affect its angular acceleration during descent?

Topic: Angular Motion

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Introduction

When designing rotating systems for things like mechanical flywheels or rotating sports gear, the way mass is distributed effects the performance of the system significantly. Distribution of mass can affect how easily the system can start rotating which is critical when energy efficiency matters. This phenomenon is because of the moment of inertia which is the quantified value of an object's resistance to rotational acceleration.

Yoyos are a great tool to observe rotational mechanics as they exhibit both linear and rotational motion as they unwind, also a yoyo is very easy to modify which makes the experiment that much easier to do. By attaching equal masses to a yoyo at different radial distances from its axis of rotation, it is possible to observe the impact of radial mass placement on angular acceleration. This is why mass distribution is very important in fields like mechanical engineering, robotics and even sports science as it is what determines the performance and energy conversion efficiency of rotational devices.

Background Information

In rotational motion, moment of inertia quantifies how much the object has to a change in its angular velocity. It depends on the object's mass and its mass distribution relative to its axis of rotation. As the distance of the standard masses increase from the axis of rotation, the moment of inertia will increase accordingly. For discrete masses treated as point masses, the moment of inertia is given by:

$$I = \sum mr^2$$

When a yoyo is thrown, it gains translational and rotational motion as it descends, and gravitational potential energy is turned into linear and rotational kinetic energy. The linear kinetic energy gain is due to the yoyo getting closer to the ground and the rotational kinetic energy gain is due to the tension in the string. The angular acceleration of the yoyo depends on the torque that is applied by the string and its moment of inertia that is due to the shape of the yoyo. Which can be shown as the rotational counterpart of Newton' Second Law.

$$\tau = I\alpha$$

Where: τ - torque of an object, I - moment of inertia of an object, α- angular acceleration of an object

The torque is created due to the tension force in the string acting on the yoyo's axle.

$$\tau = T r_{axle}$$

Where: τ - torque of an object, T - force of tension, r_{axle}– radius of axle

As torque is approximately constant during yoyo's movement, angular acceleration must change because of the moment of inertia. Moment of inertia is adversely proportional with angular acceleration, so a larger moment of inertia means a slower angular acceleration. In this experiment, this change in angular acceleration is due to symmetrically placed standard masses in different lengths from the centre of the yoyo. When additional standard masses are placed at different distances from the yoyo's axis, the moment of inertia changes accordingly. To calculate this, the parallel axis theorem is required.

$$I = I_{cm} + md^2$$

Where: I_{cm}- moment of inertia about the object's centre of mass, m - the mass of each coin, d - is the distance from the centre of mass to the axis of rotation of each coin

In this experiment, the angular acceleration is determined by measuring how long it takes the yoyo to unwind a fixed length of string. First, we need to find the angle the yoyo rotates to later calculate angular acceleration. This is for the experimentally calculated value of rotational acceleration.

$$\theta = \frac{L}{r_{axle}}$$

$$\alpha = \frac{2\,\theta}{t^2} = \frac{2\,L}{r_{axle}\,t^2}$$

Where: L - length of the string unwound, θ - angle the yoyo rotates, t - time it takes for the rope the unwind

To measure the theoretical angular acceleration:

$$\tau = I\alpha = T r_{axle}$$

$$I\alpha = m_{total}(g - \alpha r_{axle})r_{axle}$$

$$I\alpha = m_{total} g r_{axle} - \mathrm{m}_{\mathrm{axle}} r_{axle}^2 \alpha$$

$$\alpha(I + \mathrm{m}_{\mathrm{axle}} r_{axle}^2) = m_{total} g r_{axle}$$

$$\alpha = \frac{m_{total} g r_{axle}}{I_{total} + \mathrm{m}_{\mathrm{axle}} r_{axle}^2}$$

$$I_{total} = I_{yoyo} + 4\mathrm{m}_{\mathrm{standard}} d^2$$

$$\alpha = \frac{m_{total} g r_{axle}}{I_{yoyo} + 4\mathrm{m}_{\mathrm{standard}} d^2 + m r_{axle}^2}$$

Where: α – angular acceleration, t - time it takes for the rope the unwind, a- linear acceleration, m_{total}- total mass of yoyo and coins, $\mathrm{m}_{\mathrm{standard}}$- mass of a standard mass, $\mathrm{m}_{\mathrm{axle}}$- mass of axle, I- moment of inertia about the object's centre of mass, d - is the distance from the centre of mass to the axis of rotation of each coin, τ - torque of an object, T - force of tension, r_{axle}– radius of axle

Here we still cannot fully calculate the angular acceleration as we don't know the yoyo's moment of inertia. That is why the yoyo will be assumed to be a diabolo. A diabolo is a juggling prop that consists of an axle and two identical hollow frustums. Since a diabolo rotates about its symmetry axis, the moment of inertia must be calculated with respect to its axis of rotation.

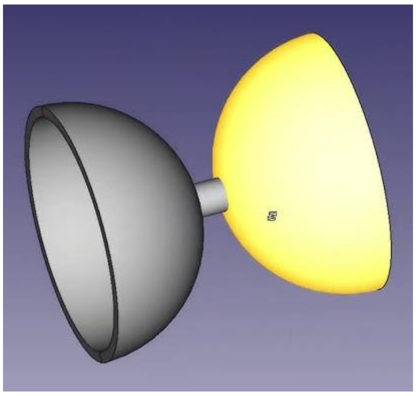

Figure 1 Shape of a Diabolo

Each cup has outer radius at the rim R, outer radius at the axle r, inner radius at the rim R_i, inner radius at the axle r_i, height of the cup h. We define a coordinate x along the axis of symmetry, measured from the small-radius end of the frustum. Because the cup wall is straight, the outer radius varies linearly with x. Therefore, by similar triangles,

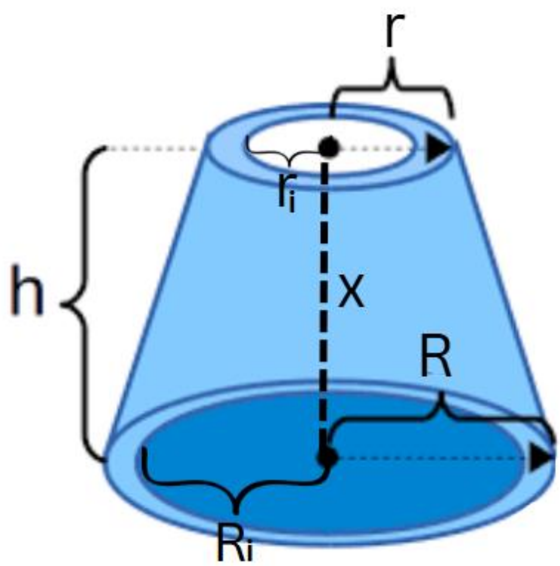

Figure 2 Geometry of a Hollow Frustum

$$y_{outer}(x) = r + \frac{R - r}{h}x$$

Similarly, the inner hollow radius also varies linearly:

$$y_{inner}(x) = r_i + \frac{R_i - r_i}{h}x$$

These expressions describe the geometry of the cup at any position along its height. Each disk has outer radius $y_{outer}(x)$, has inner radius $y_{inner}(x)$, and rotates about the central axis. This is valid because the frustum is symmetric. To calculate the moment of inertia, the frustum is divided into many infinitesimally thin disks of thickness dx. The volume of one thin hollow disk is:

$$V = \pi r^2$$

$$dV = \pi\left(y_{outer}^2(x) - y_{inner}^2(x)\right)dx$$

Assuming the frustum has uniform density ρ, the mass of the disk is:

$$m = \rho V$$

$$dm = \rho dV$$

$$dm = \rho\pi\left(y_{outer}^2 - y_{inner}^2\right)dx$$

The moment of inertia of a thin hollow disk about its central axis is:

$$dI = \frac{1}{2}dm\left(y_{outer}^2(x) + y_{inner}^2(x)\right)$$

Substituting dm:

$$dI = \frac{1}{2}\rho\pi\left(y_{outer}^4(x) - y_{inner}^4(x)\right)dx$$

The object is hollow, so inertia depends on the difference between the outer and inner volumes. As a result, the total moment of inertia of one hollow frustum is obtained by integrating over its full height.

$$I_{frustum} = \frac{1}{2}\rho\pi\int_0^h \left[\left(r+\frac{R-r}{h}x\right)^4 - \left(r_i+\frac{R_i-r_i}{h}x\right)^4\right]dx$$

As,

$$\int_0^h (a+bx)^4 dx = \frac{h}{5b}\left[(a+bh)^5 - a^5\right]$$

We find,

$$I_{frustum} = \frac{1}{10}\rho\pi h\left[\frac{R^5-r^5}{R-r} - \frac{R_i^5-r_i^5}{R_i-r_i}\right]$$

To find the mass of the frustum, we first look at the volume.

$$V_{solid\ frustum} = \frac{\pi h}{3}(R^2+Rr+r^2)$$

As the frustum is hollow:

$$V_{frustum} = V_{outer} - V_{inner}$$

$$V_{frustum} = \frac{\pi h}{3}\left[(R^2+Rr+r^2) - (R_i^2+R_ir_i+r_i^2)\right]$$

Substitute the volume of frustum for to find the mass formula:

$$m_{frustum} = \rho\pi h\left[\frac{R^2+Rr+r^2}{3} - \frac{R_i^2+R_ir_i+r_i^2}{3}\right]$$

Then find the density to put it into the inertia formula:

$$\rho = \frac{3m_{frustum}}{\pi h(R^2+Rr+r^2-R_i^2-R_ir_i-r_i^2)}$$

Where substituting it into the inertia expression gives:

$$I_{frustum} = \frac{3}{10}m_{frustum}\frac{\frac{R^5-r^5}{R-r} - \frac{R_i^5-r_i^5}{R_i-r_i}}{R^2+Rr+r^2-R_i^2-R_ir_i-r_i^2}$$

Since the diabolo consists of two identical cups and an axle:

$$I_{diabolo} = 2I_{frustum} + \frac{1}{2}m_{axle}r_{axle}^2$$

$$I_{diabolo} = \frac{3}{5}m_{cup}\frac{\frac{R^5 - r^5}{R - r} - \frac{R_i^5 - r_i^5}{R_i - r_i}}{R^2 + Rr + r^2 - R_i^2 - R_i r_i - r_i^2} + \frac{1}{2}m_{axle}r_{axle}^2$$

With this derived moment of inertia of the assumed diabolo shape, the theoretical value of the angular acceleration of each distance from the axis of rotation can be calculated. The comparison between the theoretical and experimental values of angular acceleration allows for real-world analysis like energy losses. Also, this experiment gives insight to how radial mass distribution affects rotational dynamics and provides knowledge for applications of torque, moment of inertia energy, conversion and angular acceleration in simple mechanic systems like automotive parts in which optimized mass distribution enhances or resists rotational acceleration.

Hypothesis

As the distance of the standard masses increases from the axis of rotation, the moment of inertia will increase, which will lower angular acceleration while unwinding. Therefore, the yoyo will take a longer time to unwind when the masses are placed further from the axis of rotation.

Materials

- 1 ruler (± 0.001m)
- 1 yoyo
- Four 10-gram standard masses
- 1 stopwatch (± 0.01s)
- Glue
- 1 Electrical Balance (± 0.001g)
- 2 wooden mixers
- 1 marker

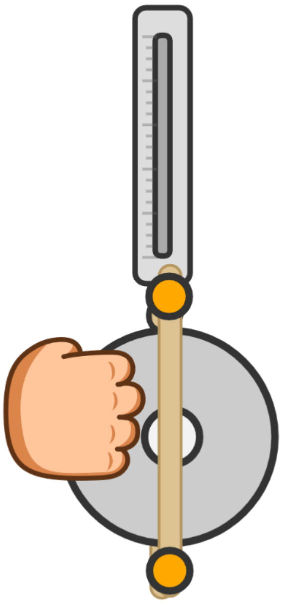

Figure 3 Before Release

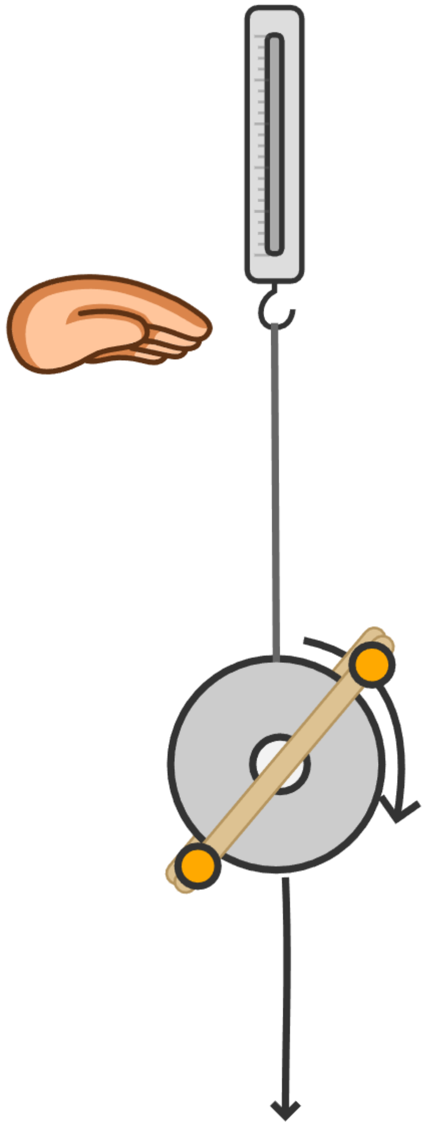

Figure 4 After Release

Variables

Independent Variable

The independent variable in this experiment is the distance of the attached standard masses from the axis of rotation of the yoyo which are 0.000 m, 0.015 m, 0.030 m, 0.045 m, 0.060 m, 0.075 m, 0.090 m, 0.105 m and they will be measured with a ruler. And 0 cm was chosen as a control group to compare the other results to clearly understand the effects of distance from the rotation axis and inertia.

Dependent Variable

The dependent variable in this experiment is the angular acceleration of the yoyo which was calculated by first calculating the angle the yoyo spun for the rope to completely unwind, then rotational a kinematic equation was used to calculate the experimental value of angular acceleration and the theoretical value of angular acceleration was calculated by calculating the moment of inertia and torque of the system to be compared with the experimental value.

Controlled Variables

Controlled Variable	**Importance of Control**	**Method of Control**
Mass of the Standard Masses	Changing the standard masses would affect the moment of inertia and make it unclear whether the change in angular acceleration is due to mass or distance.	Identical standard masses were used for all trials.
Radius of Axle (r_{axle})	Axle radius determines the torque so changing it would also alter the angular acceleration.	A yoyo with a fixed axle was used and its radius was carefully measured.
Release Method	Inconsistent release or additional push would affect the torque, kinetic energy and angular acceleration of the system.	The yoyo was dropped from the top without any external forces applied for every trial.
Mass of Yoyo	Varying mass would affect the moment of inertia and make it unclear whether the change in angular acceleration is due to mass or distance.	The same yoyo was used for every trial, and no parts were added except the coins.

Type of String	Different string thickness, elasticity and friction due to material would affect the energy loss, unwinding time and torque of the system.	Same type of string was used in every trial.
Number of Standard Masses	Changing the number of standard masses would change mass to the system and would affect the moment of inertia and make it unclear whether the change in angular acceleration is due to mass or distance.	Four identical standard masses were used in each trial.
String Length	Changing the string length would affect the kinetic energy gained and angular displacement, affecting angular acceleration.	Same length of string was used for every trial.
Material and Texture of Axle	Changing the material and surface texture of the axle would cause the system to lose different amounts of energy due to friction and different materials would cause the mass of the system to change.	Same type of axle was used for each trial.

Table 1 Controlled Variables

Procedure

1. Weigh the yoyo.
2. Weigh the wooden mixers.
3. Stick one wooden mixer to each side of the yoyo, positioning them symmetrically by aligning their centres with the yoyo's axis of rotation.
4. Measure the distances 0.000 m, 0.015 m, 0.030 m, 0.045 m, 0.060 m, 0.075 m, 0.090 m, 0.105 m from the axis of rotation with the ruler for each side of the wooden mixers.
5. Mark the points 0.000 m, 0.015 m, 0.030 m, 0.045 m, 0.060 m, 0.075 m, 0.090 m, 0.105 m on each wooden mixer with the marker.
6. Glue two standard masses per side symmetrically to one of the identical measured points.
7. Make sure the yoyo is not rotating or swinging before release.
8. Start the timer as soon as you drop the yoyo and stop the timer as soon as the string unwinds.
9. Record the time data.
10. Wind the string back.
11. Repeat steps 7-10 for the same marked point for 5 times.
12. Replace the coins symmetrically to another measured point.
13. Repeat steps 11-12 for each marked point.

Risk Assessment

Safety Concerns

Although the risk level is very low and won't cause any permanent harm, there are some small risks. First is the standard masses detaching while unwinding and they could fly off or bounce and hit someone causing minor injury. This could be solved by securing the coins firmly and ensuring their stability. Second is the yoyo hitting someone as it may swing uncontrollably or rebound during release causing minor injury. This could be solved by releasing yoyo carefully, directly vertical to the ground and with extended arms.

Environmental Concerns

There are no environmental concerns in this experiment, as no hazardous chemical or disposable single-use material was used and all materials used in this experiment were reusable and not polluting.

Ethical Concerns

There are no ethical concerns in this experiment, as no organism or moral problem was involved.

Data Processing

Time Taken for the String to Completely Unwind (±0.01 s)					
Distance From Axis of Rotation (± 0.001m)	**Trials**				
	1	**2**	**3**	**4**	**5**
0.000	1.78	1.81	1.89	1.84	1.72
0.015	1.96	2.01	2.04	1.93	2.07
0.030	2.24	2.29	2.16	2.30	2.15
0.045	2.61	2.68	2.56	2.54	2.70
0.060	2.87	2.93	2.84	2.80	2.96
0.075	3.05	3.13	3.02	3.17	3.10
0.090	3.54	3.44	3.58	3.39	3.49
0.105	3.84	3.78	3.93	3.88	3.76

Table 2 Raw Data of Time Taken fo the String to Completely Unwind

The Mean Time for Each Distance

$$\bar{t} = \frac{t_1 + t_2 + t_3 + t_4 + t_5}{5}$$

(0.030 m)

$$\bar{t} = \frac{2.24 + 2.29 + 2.16 + 2.30 + 2.15}{5} = 2.228s \approx 2.23s$$

Uncertainty

$$\Delta t_{\text{single}} = \sqrt{(0.01)^2 + (0.01)^2} = 0.0141 \text{ s}$$

$$\Delta t_{\text{mean}} = \frac{\Delta t_{\text{single}}}{\sqrt{n}}$$

$$\Delta t_{\text{mean}} = \frac{0.0141}{\sqrt{5}} = 0.0063 \text{ s}$$

Standard Deviation

$$\sigma = \sqrt{\frac{\sum(\mathrm{t_i} - \bar{\mathrm{t}})^2}{\mathrm{n} - 1}}$$

(0.030 m)

$$\sigma = \sqrt{\frac{(2.24-2.228)^2+(2.29-2.228)^2+(2.16-2.228)^2+(2.30-2.228)^2+(2.15-2.228)^2}{4}} = 0.070\text{s}$$

Distance From Axis of Rotation (± 0.001 m)	**Mean Time (±0.0063 s)**	**Standard Deviation**
0.000	1.808	±0.064
0.015	2.002	±0.057
0.030	2.228	±0.070
0.045	2.618	±0.069
0.060	2.880	±0.064
0.075	3.094	±0.058
0.090	3.488	±0.078
0.105	3.838	±0.070

Table 3 Processed Data of Mean Times

Experimental Angular Acceleration

$$\alpha = \frac{2L}{r_{\text{axle}} t^2}$$

as,

$$L = 1.00\ \text{m},\ r_{\text{axle}} = 0.005\ \text{m}$$

$$\alpha = \frac{400}{t^2}\ \text{rad s}^{-2}$$

(0.030 m)

$$t^2 = (2.228)^2 = 4.964$$

$$\alpha = \frac{400}{4.964} = 80.6\ \text{rad s}^{-2}$$

Uncertainty

$$\alpha \propto t^{-2}$$

So:

$$\frac{\Delta\alpha}{\alpha} = 2\frac{\Delta t}{t}$$

$$\Delta\alpha = \alpha\left(2\frac{\Delta t}{t}\right)$$

(0.030 m)

$$\Delta\alpha = 80.6\left(2 \times \frac{0.07}{2.228}\right) = 5.06\ \text{rad s}^{-2}$$

Percentage Uncertainty

$$\%\ \text{uncertainty} = \frac{\Delta\alpha_{\text{exp}}}{\alpha_{\text{exp}}} \times 100$$

(0.030 m)

$$\frac{\Delta\alpha_{\text{exp}}}{\alpha_{\text{exp}}} \times 100 = \frac{5.06}{80.6} \times 100 \approx 6.28\ \%$$

Distance From Axis of Rotation (± 0.001m)	Angular Acceleration (rad·s^{-2})	Uncertainty	Percentage Uncertainty
0.000	122	±8.66	±7.10%
0.015	99.8	±5.68	±5.69%
0.030	80.6	±5.06	±6.28%
0.045	58.4	±3.08	±5.27%
0.060	48.2	±2.14	±4.44%
0.075	41.8	±1.57	±3.76%
0.090	32.9	±1.47	±4.47%
0.105	27.2	±0.99	±3.64%

Table 4 Experimental Angular Accelerations

Theoretical Angular Acceleration Calculations

$$\alpha = \frac{m_{total} g r_{axle}}{I_{yoyo} + 4m_{standard} d^2 + m_{axle} r_{axle}^2}$$

Where:

$\mathrm{R} = 0.052\ \mathrm{m}$, $\mathrm{r} = 0.046\ \mathrm{m}$, $\mathrm{R_i} = 0.016\ \mathrm{m}$, $\mathrm{r_i} = 0.010\ \mathrm{m}$, $\mathrm{r_{axle}} = 0.005\ \mathrm{m}$, $\mathrm{m_{cup}} = 0.054\ \mathrm{kg}$,

$\mathrm{m_{axle}} = 0.012\ \mathrm{kg}$, $\mathrm{m_{standard}} = 0.010\ \mathrm{kg}$, $\mathrm{m_{total}} = 0.160\ \mathrm{kg}$

So,

$$I_{yoyo} = \frac{3}{5}(0.054)\frac{\frac{(0.052)^5 - (0.046)^5}{0.052 - 0.046} - \frac{(0.016)^5 - (0.010)^5}{0.016 - 0.010}}{(0.052)^2 + (0.052)(0.046) + (0.046)^2 - (0.016)^2 - (0.016)(0.010) - (0.010)^2} + \frac{1}{2}(0.012)(0.005)^2$$

$$I_{yoyo} \approx 1.40 \times 10^{-4} kg \cdot m^2$$

(0.030 m)

$$4md^2 = 4 * 0.01 * 0.03^2 = 0.04 * 0.0009 = 3.6 \times 10^{-5}$$

$$I_{denom} = 1.40 \times 10^{-4} + 3.6 \times 10^{-5} + 3.0 \times 10^{-7} = 1.763 \times 10^{-4}$$

$$\alpha = \frac{0.007848}{1.763 \times 10^{-4}} \approx 44.5\ rad/s^2$$

Distance From Axis of Rotation (± 0.001m)	Angular Acceleration (rad·s^{-2})
0.000	55.9
0.015	52.6
0.030	44.5
0.045	35.5
0.060	27.6
0.075	21.5
0.090	16.9
0.105	13.5

Table 5 Theoretical Angular Accelerations

Error Propagation

$$\% \text{ difference} = \frac{|\ \alpha_{\text{exp}} - \alpha_{\text{theory}}\ |}{\alpha_{\text{theory}}} \times 100$$

(0.030 m)

$$\frac{|\ 80.6 - 44.5\ |}{44.5} \times 100 = 81.1\%$$

Distance From Axis of Rotation (± 0.001m)	Percentage Difference
0.000	118%
0.015	89.7%
0.030	81.1%
0.045	64.5%
0.060	74.6%
0.075	94.4%
0.090	94.7%
0.105	101%

Table 6 Percentage Differences Between Experimental and Theoretical Angular Accelerations

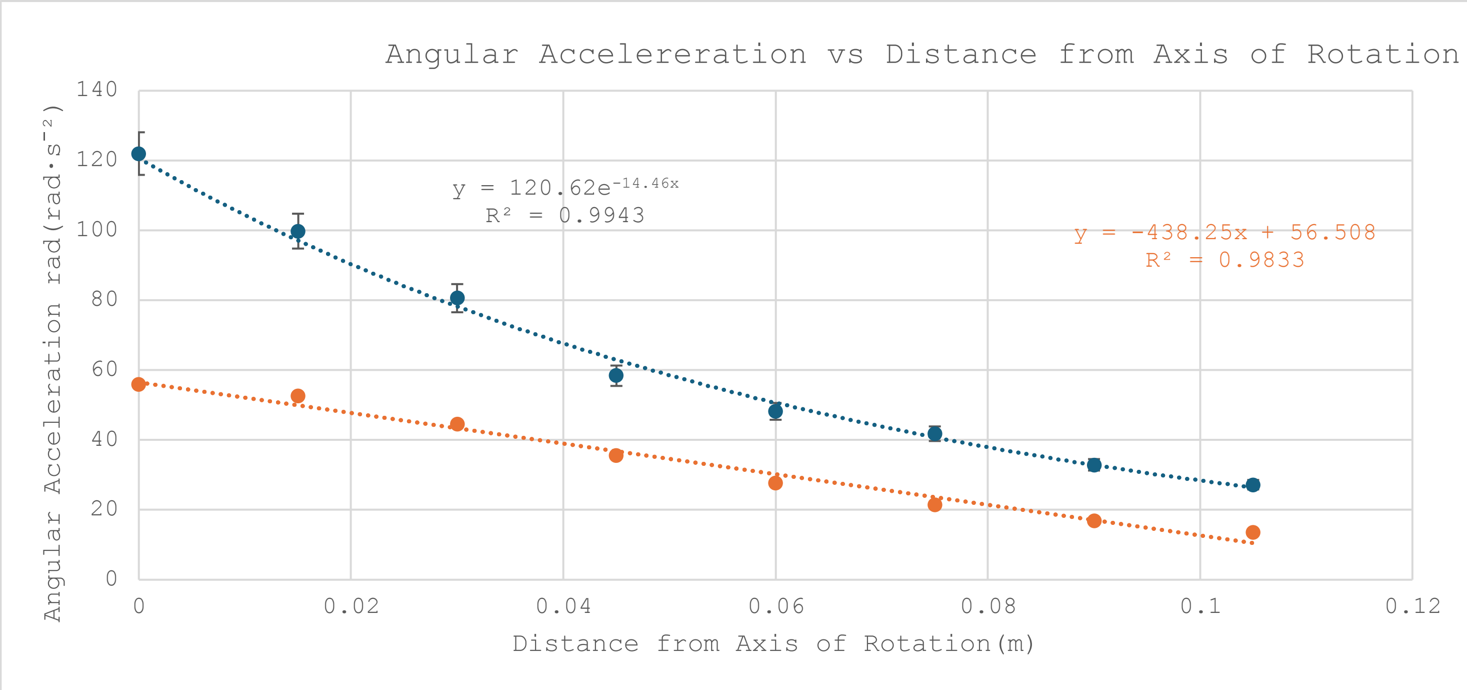

Figure 5 Angular Acceleration vs. Distance from Axis of Rotation

Data Analysis

From Figure 5, it can be seen that there is a relationship between the distance of the added standard masses from the axis of rotation and the angular acceleration of the system, where, as the distance increases, the angular acceleration decreases. This is supported by the negative exponential trend which is consistent with the inverse relationship of inertia and angular acceleration of the experimental data. This also produced a R^2 value of 0.9952, which means that there is a strong negative correlation. Since the R^2 value is close to 1, this implies that changes in distance from axis of rotation causes most of the difference in angular acceleration.

In the experiment, increasing the distance from axis of rotation of the standard masses increases the total moment of inertia of the system, which in turn reduces angular acceleration when torque is constant. This observed decrease in angular acceleration is consistent with the torque formula, which can be seen from this equation:

$$\tau = I\alpha$$

This confirms that redistributing mass further from the axis of rotation has an effect on the rotational behaviour of the yoyo.

In Table 4, the experimental data shows some variability across trials. Uncertainties decrease as the radial distances increase. The relatively larger percentage uncertainty at smaller distances can be attributed to shorter fall times, where small human reaction time errors have a larger effect on the angular acceleration of smaller distances due to the inverse square relationship between angular acceleration and time. As the fall time increases at larger distances, the effect of timing errors is reduced, which results in smaller uncertainties.

When the experimental data and the theoretical data are compared, it becomes clear that the theoretical angular accelerations are consistently lower than the experimental values in all different distances. Despite these differences in magnitude, the theoretical data also follows a similar decreasing trend but shows a linear relationship, with a R^2 value of 0.9833. This indicates that while the experimental model gives the expected relationship between distance from axis of rotation and angular acceleration, it exaggerates the angular acceleration due to systematic and random errors.

Table 6 shows the percentage difference between experimental and theoretical angular accelerations at each distance from axis of rotation. The large percentage differences between experimental and theoretical data and the percentage uncertainty being smaller than the percentage error suggests that these differences cannot be attributed to random errors alone and there are systematic errors alongside random errors. These are likely due to the assumptions made in the theoretical model, such as negligible friction, no string slipping, and perfectly constant torque. On the other hand, in the experiment there was friction between the string and axle, slight slipping during unwinding, and energy losses due to air resistance which reduced the accuracy of the theoretical model. However, since these effects influence all measurements similarly, the overall trend did not change and the results are valid.

Furthermore, the percentage uncertainty generally decreases as the distance from axis of rotation increases. This shows that as the unwinding times increase, the influence of human reaction time errors also decrease because the human reaction time gets proportionally smaller as time increases. However, even though the uncertainty got reduced, the percentage difference was still large, and it did not show a progression towards lower values. This suggests that improving the timing precision alone would not make the experimental results fit the theoretical model.

Conclusion

The aim of this investigation was to answer the research question "How does the radial distance of symmetrically placed standard masses from the axis of rotation of a yoyo affect its angular acceleration during descent?". This was achieved by changing the distances of the added standard masses, which showed a decreasing trend in angular acceleration as the distances increased. The strong negative correlation observed in the experimental data, supported by a R^2 value of 0.9952, confirms that mass distribution has a very important role in determining a system's rotational acceleration.

The error propagation shows that the percentage difference between the experimental and theoretical data is larger than the percentage uncertainty. This indicates there are systematic errors, which are axle friction, energy losses due to air resistance, non-uniform string tension during unwinding, string slipping, and assumptions made in the model in this case, alongside random errors such as human reaction time errors and equipment uncertainties.

Overall, the findings support the principles of rotational dynamics and highlight the importance mass distribution's effect on moment of inertia and angular acceleration. The error propagation confirms that the difference between the theoretical results and experimental results are not because of measurement imprecision but rather, it reflects fundamental assumptions made in the theoretical model. Since these errors influence all measurements similarly and apply to all trials, the validity of the conclusion is solidified.

Evaluation

Strengths

One strength of this investigation was the clear control of the controlled of variables. Especially, the total mass of the yoyo system was kept constant throughout all trials, so the changes in angular acceleration could only be only attributed to the variations in the distance from the axis of rotation of the added standard masses. This isolation of a single independent variable significantly increased the validity of the conclusions drawn.

Another strength was the conduction of multiple trials at each distance from the axis of rotation. With the data collected from multiple trials, the data processing was done to calculate mean values, uncertainties and standard deviations, which increased the reliability of the experimental data. Additionally, the relatively small uncertainties at larger radial distances show that the data became more precise and reliable as string unwinding times increased.

The experimental and theoretical angular acceleration trends were also in agreement with each other. Although the theoretical angular acceleration data was consistently lower than the experimental values and the trends were not identical, both data sets showed a strong and similar decrease in angular accelerations with increasing distances from the axis of rotation. This is shown by the high R^2 values of 0.9952 for the experimental data and 0.9833 for the theoretical data. This means that the experimental model is on par with the relationship between mass distribution and angular acceleration that the theoretical model predicted, even if it did not accurately predict the magnitude.

Limitations

One of the sources of uncertainty was the manually done measurement of time using a stopwatch. Since angular acceleration is inversely proportional to the square of time, small reaction time errors had a large impact on the calculated values, particularly at smaller radial distances where fall times were shortest.

However, error propagation showed that the percentage uncertainty in angular acceleration was significantly smaller than the percentage difference between experimental and theoretical values, which shows that the timing uncertainty was not the only error that was responsible for the observed differences.

Another significant limitation was that the yoyo was assumed to be an ideal diabolo when calculating its moment of inertia. Because the moment of inertia of the yoyo was not known, its geometry was assumed to be the same as a diabolo due to their similar shapes. Additionally, some other conditions were assumed to be ideal like the uniform mass distribution of each cup, perfect geometric symmetry, and that the axle behaves as a solid cylinder with constant radius in order to make the geometric optimisation more accurate. All these properties assumed may not be equal to the properties of the yoyos, so the inertia is not exact for the yoyo.

This systematic underestimation of the moment of inertia directly affects the theoretical angular acceleration, as angular acceleration is inversely proportional to the total moment of inertia. Since the theoretical model assumes a perfect geometry with ideal conditions, the predicted angular accelerations were different in magnitude from experimental values. Consequently, even small inaccuracies in the assumed geometry led to deviations in predicted angular acceleration.

Additionally, the theoretical model assumed negligible friction and no energy losses. In the experiment there was friction between the string and the axle, and air resistance which both reduced the net torque acting on the system. These effects caused a systematic discrepancy between experimental and theoretical values.

Another limitation was the assumption that the string unwound without slipping, and that the torque and axle radius remained constant throughout the fall. However, slipping caused the loss of some rotational kinetic energy and therefore reduced angular acceleration. Additionally, changes in string tension and the axle radius as the string unwound changed the net torque acting on the system. Furthermore, the string wound on itself rather than only around the axle which caused the effective axle radius to decrease while unwinding. Because of the changing radius and energy losses, the angular accelerations were non-uniform during the fall, which were left out by the simplified theoretical model.

Regardless, the high R^2 values of both the experimental and the theoretical data indicate that the investigation was successful in confirming the relationship between mass distribution and angular acceleration, even though the magnitudes were not completely accurate.

Improvements

The study could be furthered by measuring the time using a light gate timing system to prevent human reaction errors. If there are no light gate timing systems present, increasing the string length to increase the fall time would also reduce the proportional effect of reaction time errors. Additionally, the standard masses could be positioned more precisely to minimise asymmetries and reduce wobbling during motion. Finally, the theoretical model could be refined by experimentally determining the yoyo's moment of inertia, rather than assuming the yoyo to be something else and approximating the moment of inertia. Then, by considering friction, environmental factors and the known moment of inertia of the yoyo in the theoretical model, the theoretical predictions would be more realistic and improve the fit between theoretical and experimental angular acceleration data.

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