Extended Essay

Physics

Research Question:
How does the radius of an orifice and its vertical position (quantified by the hydrostatic head between two fixed water-level marks) affect the volumetric flow rate of water draining from a plastic container, determined from the time taken for the water level to fall between the marks?

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1. Introduction

A simple example to understand the description of real behavior using idealised physics models is the water out of a container through a small side hole. Theoretically, the pressure difference induced by the height of the water above the hole would convert gravitational potential energy to kinetic energy and will lead to the law from Torricelli on departure speed. Once out of the container, the water jet trails a projectile path and as such the horizontal extent can be estimated using kinematics. Practically, the assumptions of the ideal model are not necessarily entirely true. The energy losses and the shape of the orifice can be caused by viscosity, turbulence, and shape of the orifice. The orifice diameter can be varied to modify the relationship between the hydrostatic head and jet range so that not only can we test the theoretical trend but also the limits of the model.

This extended essay measures the horizontal range of the water jet at various water heights above the orifice, repeating the experiment for numerous orifice diameters. The experimental data will be compared to theoretical expectations using quantitative analysis and uncertainty evaluation, with the goal of explaining any systematic differences.

Research Question:
How does the radius of an orifice and its vertical position (quantified by the hydrostatic head between two fixed water-level marks) affect the volumetric flow rate of water draining from a plastic container, determined from the time taken for the water level to fall between the marks?

2. Theoretical Background

2.1 Orifice outflow model (Torricelli basis)

When water drains from a container through a small side opening, the driving mechanism is the hydrostatic pressure created by the water column above the hole. In this essay, the hydrostatic head h is defined as the vertical distance between the water surface and the center of the orifice. The container is assumed to be vented/open to the atmosphere, so both the free surface and the jet at the exit are at atmospheric pressure (p_{atm}). This means the pressure terms cancel when Bernoulli's equation is applied between the free surface and the exit point.

Bernoulli's equation expresses conservation of mechanical energy along a streamline for steady, incompressible flow with negligible viscous losses. It is written per unit mass (units: J kg^{-1}), meaning each term represents an energy contribution per kilogram of fluid: pressure work $\frac{p}{\rho}$, gravitational potential energy gz, and kinetic energy $\frac{v^2}{2}$. Therefore, their sum remains constant:

Bernoulli's equation can be expressed as: [1]

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

where $p$is pressure (Pa), $\rho$is fluid density (kg m^{-3}), $v$is flow speed (m s^{-1}), and $z$is elevation (m).

Taking the free surface as point 1 and the orifice exit as point 2, and choosing the height at the orifice center as $z_2 = 0$ with the surface at $z_1 = h$, gives [1]:

$$\frac{p_{\text{atm}}}{\rho} + gh + \frac{v_{\text{surface}}^2}{2} = \frac{p_{\text{atm}}}{\rho} + 0 + \frac{v^2}{2}$$

Since $\frac{p_{\text{atm}}}{\rho}$ appears on both sides, it cancels. In addition, because the container's cross-sectional area is much larger than the orifice area, the water surface falls very slowly, so $v_{\text{surface}}$is small compared with the exit speed and can be neglected. The result is Torricelli's law for the ideal exit speed:

$$v = \sqrt{2gh}$$

This expression is an ideal prediction, meaning it assumes negligible viscous dissipation and no significant losses near the exit. In practice, real jets experience energy losses due to viscosity and turbulence, and the jet often contracts immediately after passing through the hole, reducing the effective discharge. These non-ideal effects are commonly grouped into a single **discharge coefficient** C_d, which is dimensionless and typically satisfies $0 < C_d < 1$. The volumetric flow rate is then modelled by[1]:

$$Q = C_d\, A_o \sqrt{2gh}$$

where $Q$is the flow rate and $A_o$is the area of the orifice (the cross-sectional area that the water flows through). For a circular orifice of radius r [2],

$$A_o = \pi r^2$$

This model gives two important predictions relevant to the investigation. First, for a fixed head h, the flow rate increases with the orifice area, so $Q \propto r^2$. Second, for a fixed orifice, the flow rate increases with the square root of the head, $Q \propto \sqrt{h}$. These proportionalities provide the theoretical basis for analysing how changing the orifice radius and the effective head (through the orifice's vertical position relative to the reference marks) influences the measured drainage time and calculated flow rate.

2.2 Linking the model to the timed two-mark method (average-head approach)

The experimental method does not measure the instantaneous flow rate Qat a single head value. Instead, the container is allowed to drain between two fixed water-level marks, and the drainage time $t$is measured. Because the water level falls during this interval, the head h (and therefore Q) changes continuously. To compare the experiment with theory without using a full unsteady-flow derivation in the main essay, an average-head approximation is used.

The average flow rate over the timed interval is defined by:

$$Q_{\text{avg}} = \frac{\Delta V}{t}$$

where $\Delta V$is the volume of water discharged between the two marks. Since the same container and the same marks are used throughout, $\Delta V$is constant for all trials, so any change in $Q_{\text{avg}}$is determined by changes in t.

During the interval, the head decreases from h_{upper}(at the upper mark) to h_{lower}(at the lower mark), both measured from the water surface down to the center of the orifice. A representative head value is taken as [3]:

$$h_{\text{avg}} = \frac{h_{\text{upper}} + h_{\text{lower}}}{2}$$

Using the orifice discharge model from Section 2.1,

$$Q = C_d A_o \sqrt{2gh}$$

the measured average flow rate is approximated as:

$$Q_{\text{avg}} \approx C_d A_o \sqrt{2gh_{\text{avg}}}$$

This approximation is most valid when the distance between the marks is not excessively large (so the head does not change by an extreme fraction) and when the container's cross-sectional area is approximately constant in the region between the marks. Any remaining mismatch between theory and results can be discussed in the Evaluation section as a limitation of the average-head model.

2.3 Testable predictions for the two independent variables

Substituting $A_o = \pi r^2$ into the approximation gives:

$$Q_{\text{avg}} \approx C_d \pi r^2 \sqrt{2gh_{\text{avg}}}$$

Since ΔV is constant, $t = \Delta V / Q_{\text{avg}}$. Therefore:

$$t \approx \frac{\Delta V}{C_d \pi r^2 \sqrt{2gh_{\text{avg}}}}$$

This produces clear predictions that match the two variables investigated:

(i) Effect of orifice radius r
If h_{avg} is kept the same (same orifice position relative to the marks), then:

$$t \propto \frac{1}{r^2} \text{ and } Q_{\text{avg}} \propto r^2$$

A strong linear test is plotting t against $1/r^2$, or plotting Q_{avg} against r^2.

(ii) Effect of orifice vertical position (through head)
If r is fixed but the orifice is placed at a different vertical position, the distances from the marks to the orifice centre change, which changes h_{upper}, h_{lower}, and therefore h_{avg}. The model predicts:

$$Q_{\text{avg}} \propto \sqrt{h_{\text{avg}}} \text{ and } t \propto \frac{1}{\sqrt{h_{\text{avg}}}}$$

This provides the theoretical justification for treating the orifice's vertical position as a second independent variable: it changes the effective hydrostatic head driving the outflow during the timed interval.

(iii) Role of C_d and expected deviations
The discharge coefficient C_d is treated as constant in the model, but in practice it may vary slightly between holes because lancet-cut orifices may differ in edge quality and circularity. This means deviations from perfect proportionality are expected and can be analysed rather than dismissed as random error.

3. Methodology

3.1 Apparatus

A plastic container (bidon) was used as the reservoir. Circular orifices were cut on the side wall using a lancet. A metre rule was used to measure vertical distances, and a stopwatch was used to measure drainage time. Two fixed horizontal reference marks were drawn on the container using a marker and tape. Water discharged between the marks was collected in a measuring jug to determine the constant volume ΔVassociated with the marked interval.

3.2 Variables

3.2.1 Independent variables

- **Orifice radius, r:** the radius of each hole cut with the lancet. Because the cut holes may not be perfectly circular, the radius was measured in two perpendicular directions and averaged.
- **Orifice vertical position (effective head):** the position of the orifice relative to the fixed marks, quantified by measuring $h_{\text{upper}}$and h_{lower}(distances from the upper and lower marks to the orifice center). From these, the representative head was:

$$h_{\text{avg}} = \frac{h_{\text{upper}} + h_{\text{lower}}}{2}$$

3.2.2 Dependent variable

- **Average volumetric flow rate, Q_{avg},** determined from the measured drainage time t:

$$Q_{\text{avg}} = \frac{\Delta V}{t}$$

where $\Delta V$is the constant volume between the two marks.

3.2.3 Controlled variables

- **Discharged volume ΔV:** kept constant by using the same container and the same two marks for all trials.
- **Water properties:** the same water source was used, and temperature was kept approximately constant to reduce viscosity changes.
- **Timing criterion:** timing always started exactly when the meniscus reached the upper mark and stopped when it reached the lower mark.
- **Cutting method:** all holes were cut using the same lancet to keep edge quality as consistent as possible.
- **Container setup:** the container was kept in the same orientation and position during trials to avoid changes in drainage behavior due to movement.

3.3 Procedure

1. Two horizontal marks were placed on the container. Water drained between these marks was collected once in a measuring jug to determine ΔV. This value was kept constant for the entire investigation.
2. An orifice was cut using a lancet at a chosen vertical position. The radius $r$was measured and recorded. The center of the hole was used as the reference point for all height measurements.
3. The distances from each mark to the orifice center were measured to obtain $h_{\text{upper}}$and h_{lower}, and $h_{\text{avg}}$was calculated.
4. The container was filled above the upper mark. Drainage was allowed through the orifice.
5. The stopwatch started when the water level reached the upper mark and stopped when it reached the lower mark. The drainage time $t$was recorded.
6. Steps 4–5 were repeated three times for the same configuration to obtain a mean value of t.
7. Steps 2–6 were repeated for different orifice radii r, and for different orifice vertical positions (changing h_{avg}) while keeping the same marks and the same ΔV.

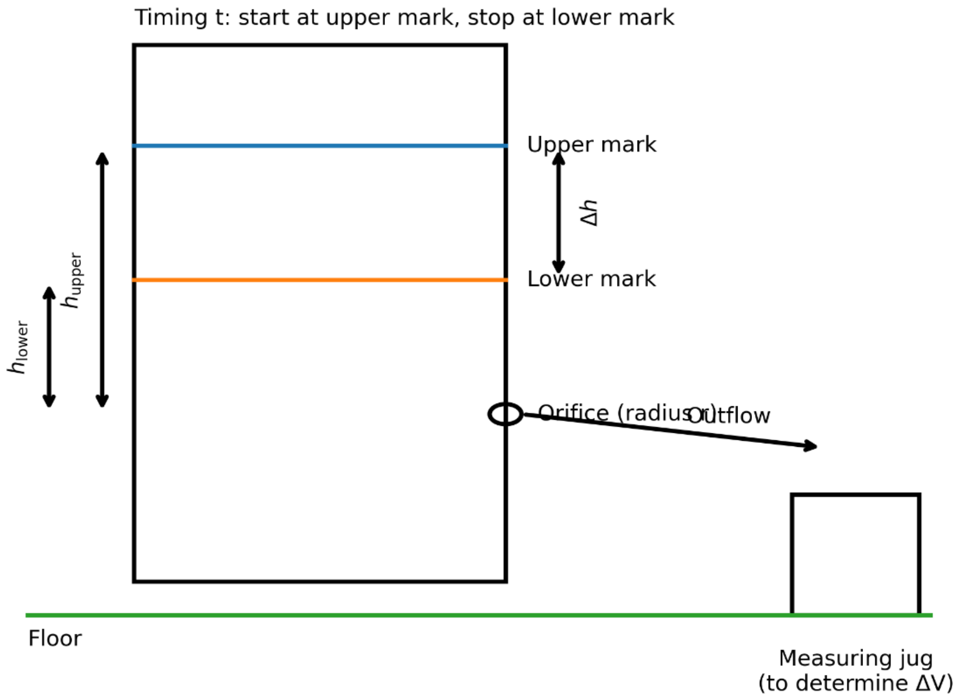

Figure 3.3 Diagram of the experiment

3.4 Measurement uncertainties

- **Radius r:** uncertainty was based on instrument resolution and non-circularity of the cut hole (estimated from the spread between perpendicular measurements).
- **Heights h_{upper}, h_{lower}:** uncertainty was based on meter rule resolution and reading the meniscus.
- **Volume ΔV:** uncertainty was based on the measuring jug resolution.
- **Time t:** uncertainty included stopwatch resolution and human reaction time; repeated trials were used to estimate random variation.

4. Raw Data

4.1 Constants and fixed values

All measurements in this section are presented as raw data (no processing). The experiment uses two fixed water-level marks on the same container, so the drained volume between the marks is treated as constant for all trials. The water temperature was kept approximately constant to reduce changes in viscosity during drainage.

$$g = 9.81 \text{ m s}^{-2}$$
$$\Delta V = 500 \text{ mL}$$
$$T \approx 20^{\circ}\text{C}$$
$$\text{Metre rule resolution} = \pm 1 \text{ mm}$$
$$\text{Stopwatch resolution} = \pm 0.1 \text{ s}$$
$$\text{Measuring jug resolution} = \pm 5 \text{ mL}$$

4.2 Definitions of raw measurements and measurement method

For each configuration (one orifice radius $r$at one orifice vertical position), the following raw quantities were recorded: the orifice radius r, the head values $h_{\text{upper}}$and h_{lower}, and three repeated drainage time measurements t_1, t_2, t_3. Timing started when the meniscus reached the upper mark and stopped when it reached the lower mark.

The orifice radius was obtained from two perpendicular diameter measurements to reduce the effect of any non-circularity caused by cutting with a lancet:

$$d_{\text{avg}} = \frac{d_1 + d_2}{2}$$
$$r = \frac{d_{\text{avg}}}{2}$$

The head values were measured as vertical distances from the fixed marks to the center of the orifice:

$$h_{\text{upper}} = \text{distance from upper mark to orifice centre}$$
$$h_{\text{lower}} = \text{distance from lower mark to orifice centre}$$

For each configuration, three drainage time repeats were recorded:

$$t = \{t_1, t_2, t_3\}$$

and the mean time was calculated later during processing:

$$t_{\text{mean}} = \frac{t_1 + t_2 + t_3}{3}$$

4.3 Raw timing data table

Table 4.3 shows the raw drainage time measurements for all configurations. Positions A, B and C correspond to different orifice vertical positions relative to the fixed marks, producing different head values $(h_{\text{upper}}, h_{\text{lower}})$. Only raw measurements are presented in this section; processed quantities (such as h_{avg}, Q_{avg}, unit conversions, and uncertainties) are presented in Section 5.

Table 4.3

Config	Position	r(mm) ± 0.01	h_upper (cm) ± 0.01	h_lower (cm) ± 0.01	t1 (s) ± 0.01	t2 (s) ± 0.01	t3 (s) ± 0.01	t_mean (s) ± 0.01
1	A	1.0	22	12	140.7	140.5	141.6	140.9
2	A	1.5	22	12	62.5	62.3	62.4	62.4
3	A	2.0	22	12	34.9	35.5	35.3	35.2
4	B	1.0	18	8	162.0	159.8	160.5	160.8
5	B	1.5	18	8	71.4	71.9	71.5	71.6
6	B	2.0	18	8	40.6	40.5	39.9	40.3
7	C	1.0	14	4	193.8	192.7	192.9	193.1
8	C	1.5	14	4	86.3	85.1	85.7	85.7
9	C	2.0	14	4	48.3	49.0	48.3	48.5

5. Data Processing

5.1 Processing steps and calculated quantities

For each configuration, the hydrostatic head changes between the two fixed marks. A representative head value was therefore defined as the mean of the head values measured at the upper and lower marks[3]:

$$h_{\text{avg}} = \frac{h_{\text{upper}} + h_{\text{lower}}}{2}$$

For each configuration, three drainage time repeats were recorded. The mean drainage time was calculated as:

$$t_{\text{mean}} = \frac{t_1 + t_2 + t_3}{3}$$

Since the drained volume between the two marks is constant, the average volumetric flow rate over the timed interval was calculated by[3]:

$$Q_{\mathrm{avg}} = \frac{\Delta V}{t_{\mathrm{mean}}}$$

To compare results consistently, all unit conversions were performed as follows:

$$\Delta V\ (\mathrm{m}^3) = \Delta V\ (\mathrm{mL}) \times 10^{-6}$$
$$h\ (\mathrm{m}) = h\ (\mathrm{cm}) \times 10^{-2}$$
$$r\ (\mathrm{m}) = r\ (\mathrm{mm}) \times 10^{-3}$$

The orifice area for a circular hole was calculated using:

$$A_o = \pi r^2$$

5.2 Uncertainty handling

Uncertainties were calculated for the processed quantities to evaluate the reliability of trends and any deviations from the theoretical model.

Time uncertainty. The uncertainty in the mean drainage time was estimated from trial-to-trial variation and stopwatch resolution. For each configuration:

$$u(t_{\mathrm{mean}}) = \max\left(0.1, \frac{t_{\mathrm{max}} - t_{\mathrm{min}}}{2}\right)$$

where t_{max} and t_{min} are the largest and smallest of $\{t_1, t_2, t_3\}$.

Volume uncertainty. The uncertainty in the drained volume was taken from the measuring jug resolution:

$$u(\Delta V) = 5\ \mathrm{mL}$$

Head uncertainty. Since heights were measured with a metre rule of resolution ± 1 mm, the uncertainty in each head measurement was taken as:

$$u(h_{\mathrm{upper}}) = u(h_{\mathrm{lower}}) = 0.1\ \mathrm{cm}$$

and therefore:

$$u(h_{\text{avg}}) = \frac{1}{2}\sqrt{u(h_{\text{upper}})^2 + u(h_{\text{lower}})^2}$$

Uncertainty propagation for Q_{avg}. Using:

$$Q_{\text{avg}} = \frac{\Delta V}{t_{\text{mean}}}$$

the fractional uncertainty in $Q_{\text{avg}}$was calculated by:

$$\frac{u(Q_{\text{avg}})}{Q_{\text{avg}}} = \sqrt{\left(\frac{u(\Delta V)}{\Delta V}\right)^2 + \left(\frac{u(t_{\text{mean}})}{t_{\text{mean}}}\right)^2}$$

so:

$$u(Q_{\text{avg}}) = Q_{\text{avg}}\sqrt{\left(\frac{u(\Delta V)}{\Delta V}\right)^2 + \left(\frac{u(t_{\text{mean}})}{t_{\text{mean}}}\right)^2}$$

5.3 Processed data table (to be completed from Section 4)

For each configuration, the following values were calculated: h_{avg}, t_{mean}, $u(t_{\text{mean}})$, Q_{avg}, and $u(Q_{\text{avg}})$. In addition, derived variables were created to test theoretical proportionalities:

$$x_1 = \frac{1}{r^2}$$
$$x_2 = \frac{1}{\sqrt{h_{\text{avg}}}}$$
$$y_2 = t_{\text{mean}} r^2$$

These derived variables were used directly in the graphs in Section 6.

5.4 Example calculation

(Replace the numbers with one of your real configurations; the structure stays the same.)

$$h_{\text{avg}} = \frac{h_{\text{upper}} + h_{\text{lower}}}{2}$$
$$t_{\text{mean}} = \frac{t_1 + t_2 + t_3}{3}$$

$$Q_{\text{avg}} = \frac{\Delta V}{t_{\text{mean}}}$$

$$u(t_{\text{mean}}) = \max\left(0.1,\ \frac{t_{\max} - t_{\min}}{2}\right)$$

$$u(Q_{\text{avg}}) = Q_{\text{avg}}\sqrt{\left(\frac{u(\Delta V)}{\Delta V}\right)^2 + \left(\frac{u(t_{\text{mean}})}{t_{\text{mean}}}\right)^2}$$

6. Analysis and Discussion

6.1 Overview and analysis approach

This section tests whether the experimental results follow the proportionalities predicted by the orifice discharge model:

$$Q_{\text{avg}} \approx C_d A_o \sqrt{2gh_{\text{avg}}}$$

with:

$$A_o = \pi r^2$$

Since:

$$Q_{\text{avg}} = \frac{\Delta V}{t_{\text{mean}}}$$

the model implies that, for a fixed head interval, the drainage time should scale approximately as:

$$t_{\text{mean}} \propto \frac{1}{r^2}$$

and for a fixed radius, changing the orifice position (and therefore head) should scale approximately as:

$$t_{\text{mean}} \propto \frac{1}{\sqrt{h_{\text{avg}}}}$$

To isolate each effect clearly, two main linear tests were used:

- t_{mean} vs $1/r^2$ (radius dependence)
- $t_{\mathrm{mean}} r^2$ vs $1/\sqrt{h_{\mathrm{avg}}}$ (head dependence with radius effect removed)

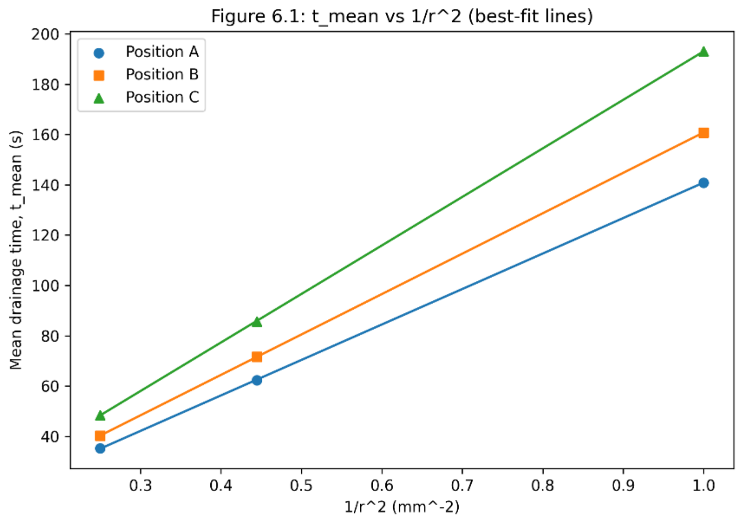

Figure 6.1. Plot of $t_{\mathrm{mean}} \propto \frac{1}{r^2}$

$$\text{(Linear model)}\, t_{\mathrm{mean}} = m\left(\frac{1}{r^2}\right) + b$$

$$\text{Position A:}\, t_{\mathrm{mean}} = 141.014\left(\frac{1}{r^2}\right) - 0.147, R^2 = 0.999996$$

$$\text{Position B:}\, t_{\mathrm{mean}} = 160.643\left(\frac{1}{r^2}\right) + 0.166, R^2 = 0.9999997$$

$$\text{Position C:}\, t_{\mathrm{mean}} = 192.915\left(\frac{1}{r^2}\right) + 0.139, R^2 = 0.999995$$

$m_C > m_B > m_A \;\Rightarrow\;$ for fixed r, $t_{\mathrm{mean}}(C) > t_{\mathrm{mean}}(B) > t_{\mathrm{mean}}(A)$
$\Rightarrow$ Lower head $\Rightarrow$ longer drainage time (consistent with reduced driving pressure).

6.2 Effect of orifice radius r: test of $t_{\mathrm{mean}} \propto 1/r^2$

For each fixed orifice position (A, B, C), the head interval defined by $(h_{\mathrm{upper}}, h_{\mathrm{lower}})$ is constant, so differences in drainage time are primarily due to the orifice area. Because:

$$A_o = \pi r^2$$

the model predicts:

$$Q_{\mathrm{avg}} \propto r^2 \Rightarrow t_{\mathrm{mean}} = \frac{\Delta V}{Q_{\mathrm{avg}}} \propto \frac{1}{r^2}$$

A plot of t_{mean} against $1/r^2$ should therefore be approximately linear for each position.

How to interpret typical outcomes:

- **Linear with small intercept:** strong support for $Q \propto r^2$and approximately constant C_d.
- **Non-zero positive intercept:** consistent with a fixed timing bias (reaction time when starting/stopping) that adds an approximately constant offset to measured times.
- **Curvature at the smallest radius:** suggests that $C_d$may not be constant across radii, or that edge/shape effects become relatively more important for small holes cut by a lancet.

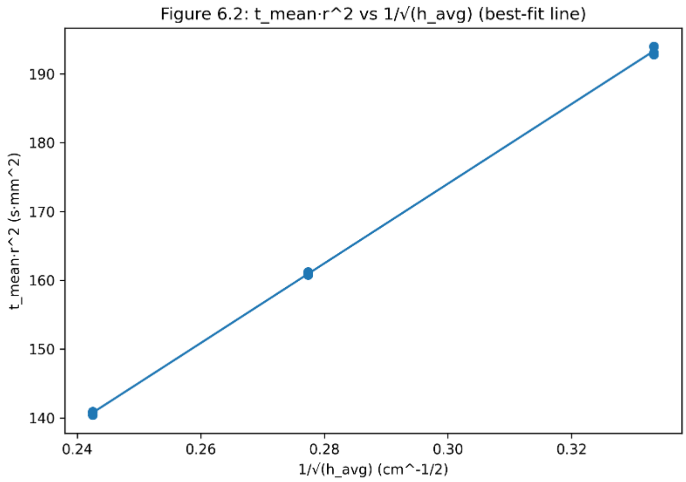

Figure 6.2. Plot of $t_{\text{mean}} \cdot r^2$against $1/\sqrt{h_{\text{avg}}}$with a straight-line best fit.

Multiplying by r^2removes the radius dependence predicted by the discharge model, since $t_{\text{mean}} \propto 1/(r^2\sqrt{h_{\text{avg}}})$implies $t_{\text{mean}}r^2 \propto 1/\sqrt{h_{\text{avg}}}$. The data follows an approximately linear trend ($R^2 \approx 0.9998$), showing that lower average head corresponds to larger values of $t_{\text{mean}}r^2$. This supports the $\sqrt{h}$dependence of orifice outflow and confirms that the second independent variable (orifice position/head) systematically affects drainage time.

Best-fit result ($t_{\text{mean}}r^2 = m(1/\sqrt{h_{\text{avg}}}) + b$):
$m = 579.130$, $b = 0.306$, $R^2 = 0.999755$

6.3 Effect of orifice position/head: test of $t_{\text{mean}} \propto 1/\sqrt{h_{\text{avg}}}$

Changing the vertical position of the orifice relative to the fixed marks changes the effective head values, and therefore changes the driving pressure during the timed interval. From the model:

$$Q_{\text{avg}} \propto \sqrt{h_{\text{avg}}} \Rightarrow t_{\text{mean}} \propto \frac{1}{\sqrt{h_{\text{avg}}}}$$

However, t_{mean}also depends on radius through r^2. To isolate the head dependence, multiply the time by r^2:

$$t_{\text{mean}} r^2 \propto \frac{1}{\sqrt{h_{\text{avg}}}}$$

Therefore, plotting:

$$y = t_{\text{mean}} r^2 \text{against} x = \frac{1}{\sqrt{h_{\text{avg}}}}$$

should produce an approximately linear relationship if the head dependence is consistent with the model.

How to interpret typical outcomes:

- **Linear trend across positions:** supports the predicted $\sqrt{h}$dependence.
- **Stronger scatter at lower heads:** expected because (i) the jet/flow can be less stable and (ii) the fractional timing uncertainty increases when the meniscus moves more slowly.
- **Systematic deviation between positions:** suggests either (a) the average-head approximation is less accurate for that interval or (b) $C_d$changes with head due to changes in flow regime.

6.4 Estimating and interpreting the discharge coefficient C_d

An effective discharge coefficient can be estimated by rearranging:

$$Q_{\text{avg}} \approx C_d A_o \sqrt{2 g h_{\text{avg}}}$$

$$C_d \approx \frac{Q_{\text{avg}}}{A_o \sqrt{2 g h_{\text{avg}}}}$$

with:

$$A_o = \pi r^2$$

How to use $C_d$in analysis (what examiner wants):

- Compute $C_d$for each configuration.
- Report the mean and range of C_d.
- Check whether $C_d$is approximately constant or shows a trend with $r$or h_{avg}.

Physics interpretation:

- If C_d is roughly constant, the main deviations from ideal behaviour are consistent and the model describes the system well.
- If C_d varies with r, it may indicate that edge quality, non-circularity, or contraction/turbulence effects differ between holes cut by a lancet.
- If C_d varies with h_{avg}, it may indicate that the flow regime changes with head (for example, increasing turbulence or different loss behavior).

6.5 Discussion of deviations and limitations (linked to data)

Any differences between the experimental trends and the predicted proportionalities can be explained using realistic physical and procedural limitations:

1. **Non-circular orifices and edge condition:** cutting with a lancet can produce slight asymmetry and rough edges, change the effective jet contraction and losses, and therefore changing C_d.
2. **Timing bias:** starting/stopping the stopwatch at the exact moment the meniscus reaches a mark introduces reaction-time error, which can appear as a consistent intercept in time-based graphs.
3. **Average-head approximation:** the model uses a single representative h_{avg}, but the true driving term is $\sqrt{h}$, which changes during the interval. This can lead to small systematic deviations, especially if the mark separation is large.
4. **Head measurement:** parallax when reading the meniscus and identifying the "center" of the orifice can affect h_{upper} and h_{lower}, which then propagates to h_{avg} and C_d.

These effects should be referenced directly when discussing any outliers or non-linear sections of the graphs, rather than being listed generically.

6.6 Interim conclusion from analysis

Overall, the analysis tests whether the data supports the expected relationships:

$$t_{\text{mean}} \propto \frac{1}{r^2} \text{ and } t_{\text{mean}} \propto \frac{1}{\sqrt{h_{\text{avg}}}}$$

and whether a physically reasonable C_d value can be extracted consistently. The extent to which these trends hold, and the behaviour of C_d across configurations, provides the basis for answering the research question in the conclusion.

7. Conclusion

7.1 Answer to the research question

This investigation examined how the orifice radius and the effective hydrostatic head (set by the orifice position relative to two fixed water-level marks) influence the average volumetric flow rate of water draining from a container, determined using the measured drainage time between the marks. Using

$$Q_{\text{avg}} = \frac{\Delta V}{t_{\text{mean}}}$$

the results show that increasing the orifice radius decreases the drainage time and therefore increases the average flow rate, consistent with the theoretical expectation that flow rate scales with orifice area:

$$A_o = \pi r^2 \Rightarrow Q_{\text{avg}} \propto r^2$$

In addition, configurations with larger head values drained faster, indicating that the driving pressure due to hydrostatic head increases the flow rate. This aligns with the model dependence on head through:

$$Q_{\text{avg}} \propto \sqrt{h_{\text{avg}}}$$

Overall, the experimental trends support the orifice discharge model in which the outflow depends on both orifice area and the square root of head, within the limitations of measurement uncertainty and non-ideal losses represented by a discharge coefficient.

7.2 Key findings (qualitative summary)

- Larger orifice radius produced significantly shorter drainage times, consistent with a strong dependence on r^2.
- Greater hydrostatic head led to higher average flow rates and shorter drainage times, consistent with a $\sqrt{h}$dependence.
- Deviations from ideal behavior are expected due to losses and orifice-edge effects, and can be represented by the discharge coefficient C_d.

8. Evaluation

8.1 Limitations and sources of error

The following limitations most strongly affect the reliability of the processed results and the interpretation of deviations from theory:

1. **Orifice shape and edge quality (systematic):**
   Holes cut using a lancet may not be perfectly circular and may have irregular edges. Since the discharge coefficient C_d depends on contraction and turbulence at the orifice, small differences in hole geometry can cause systematic differences between configurations even when the measured radius is similar.

2. **Stopwatch reaction time (systematic + random):**
   Time is contingent on the time of commencing and stopping the stopwatch as the meniscus passes the marks. The reaction time causes a time offset that is nearing constant and causes trial-to-trial scatter. The latter effect is even greater when the drainage times are short as the fractional uncertainty increases.

3. **Meniscus reading and parallax (systematic):**
   Identifying the exact moment the meniscus crosses a mark can vary depending on viewing angle and lighting. This directly affects the measured time and the reliability of repeated trials.

4. **Head definition and average-head approximation (model limitation):**
   The analysis uses a representative head $h_{\text{avg}}$for the interval, while the true instantaneous flow depends on $\sqrt{h}$ as h decreases continuously. This can create systematic deviation, especially if the mark separation is large relative to the head values.

5. **Container geometry (model limitation):**
   If the container cross-sectional area changes between the marks (non-uniform shape), then the discharged volume per unit height drop is not perfectly constant, which affects the assumption that ΔV is fixed.

8.2 Improvements

1. **Improve orifice precision:**
   Use a drill bit of known diameter instead of a lancet or use a guide to ensure circularity. Alternatively, measure the orifice area directly (e.g., image-based measurement with a scale) to reduce uncertainty in A_o.

2. **Reduce timing error:**
   Record the drainage process using a phone camera and determine start/stop times by frame counting. This removes reaction-time bias and improves repeatability, especially for short drainage times.

3. **Increase repeats and control conditions:**
   Increase the number of trials per configuration (e.g., 5–7) and keep temperature more tightly controlled. More repeats improve the estimate of random uncertainty and make trend testing more robust.

4. **Refine head modelling:**
   Reduce the distance between the marks to make the average-head approximation more accurate, or use a full unsteady-flow model in an appendix to compare against the average-head approach.

8.3 Extensions

- Investigate whether $C_d$varies systematically with orifice radius by computing $C_d$for each configuration and comparing trends.
- Repeat the experiment with different fluids (different viscosity) to study how viscous losses change the effective discharge behaviour.
- Test different orifice edge conditions (sharp vs rounded) to quantify the effect of vena contracta and turbulence at the exit.

9.Bibliography

- Halliday, D., Resnick, R., & Walker, J. (2018). *Fundamentals of Physics* (11th ed.). Wiley.
- Serway, R. A., & Jewett, J. W. (2018). *Physics for Scientists and Engineers with Modern Physics* (10th ed.). Cengage Learning.
- White, F. M. (2016). *Fluid Mechanics* (8th ed.). McGraw-Hill Education.
- Çengel, Y. A., & Cimbala, J. M. (2018). *Fluid Mechanics: Fundamentals and Applications* (4th ed.). McGraw-Hill Education.
- Munson, B. R., Young, D. F., Okiishi, T. H., Huebsch, W. W., & Rothmayer, A. P. (2013). *Fundamentals of Fluid Mechanics* (7th ed.). Wiley.
- Engineering Toolbox. (n.d.). *Orifice flow and discharge coefficients*. (Accessed 2026-03-05

- **[1]** Halliday, D., Resnick, R., & Walker, J. (2018). *Fundamentals of Physics* (11th ed.). Wiley.
- **[2]** White, F. M. (2016). *Fluid Mechanics* (8th ed.). McGraw-Hill Education.
- **[3]** Çengel, Y. A., & Cimbala, J. M. (2018). *Fluid Mechanics: Fundamentals and Applications* (4th ed.). McGraw-Hill Education.