Extended Essay

Physics

Title: Investigation of how the moment of inertia in a rolling body gets affected by the rolling bodies inner diameter

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Introduction:

Many systems rely on rotational motion to help drive their processes; however, it does not always occur as expected with mass distribution. For instance, when it comes to linear movement, mass is a greater factor, while in rotational motion, moment of inertia is more relevant - how much of a given cylinder mass is present or how it's distributed from the axis of motion. This is especially true for hollow cylinders, as the inner dimension makes a difference in rotational functionality in terms of how easy or difficult something is to get moving.

Many applications and theories rely on the application of a hollow cylinder. From mechanical parts in mechanical engineering to everyday wheels, pipes and rollers, the presence of an inner radius vs. non-inner radius makes a difference for mechanical advantage and application. Therefore, understanding how and why dimensions that can be changed influence how something rolls in applied settings allows for people to understand variable behaviors and adapt.

The purpose of this experiment is to determine how the inner diameter of a hollow cylinder affects final velocity down an inclined plane. By assessing the inner construction of the cylinder and relating it to applied moment of inertia and distributed energy, a means to best ascertain how expectations relate to reality through applied rotational motion will be explored. This expands on theoretical aspects of physics along with implications for the practical considerations that use such variables for best means of application.

This investigation will explore the theoretical background needed, implementation processes to find results and discussion of findings based on relativistic comparisons of trials measuring substantially the same cylinder with different interior radius measurements.

Research Question: How Does the Diameter of a Hole in a Hollow Cylinder Affect the Hollow Cylinder's Moment of Inertia During Its Motion Down an Inclined Path?

Background Information:

First, we must understand how a cylinder behaves on an inclined plane and the forces on the cylinder during the cylinder's motion

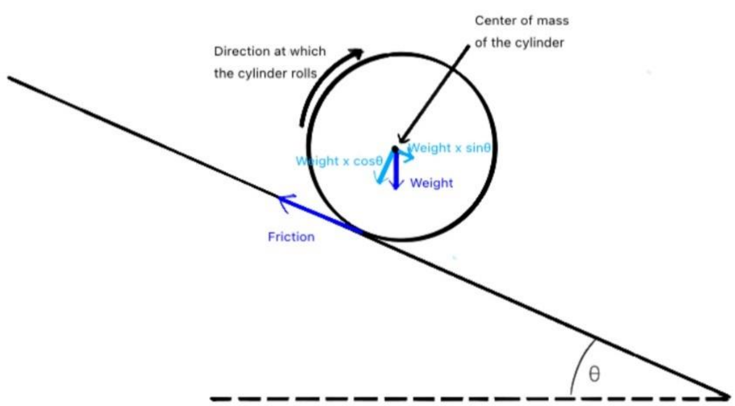


Figure 1: Free-Body diagram of the forces acting on a cylinder during its motion on an inclined plane

The two forces on the cylinder during the cylinder's motion on the plane are the weight of the cylinder and the drag force between the cylinder and the inclined plane. Weight will act on the cylinder's center of mass with its two components that make it up which results in a downward acceleration on the cylinder. However, friction acts on the contact point of the cylinder and doesn't originate from the center of mass, The two forces are not on the same plane which results in the addition of these two vectors creating a non-orthogonal force or in other words Torque. Torque is defined as the force acting on an object multiplied by the perpendicular distance from where the force acts to the rotation axis. The Torque in this case creates the rotating motion of the cylinder.

With the understanding of how the cylinder rolls down the slope, we are unable to utilize linear motion equations for the same reason we were able to determine the rotating movement of the cylinder, therefore we are required to utilize rotational motion equations. The rotational motion counterpart for linear velocity is angular velocity, which can be seen as in equation 1.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta l}{r\Delta t} = \frac{v}{r} = 2\pi f$$

Equation 1: The relationship between angular velocity and frequency

Where ω represents the angular velocity of a rotating point, Δθ denotes the change in the angle and Δl constitutes the change in arc length over Δt, r depicts the radius and v represents the tangential velocity of the rotating point, and f denotes the frequency of the point.

Rather than using mass as it would be used in linear dynamics, moment of inertia is used, which is defined as a body's tendency to resist angular acceleration. The moment of inertia of a singular point mass is expressed as shown in Equation 2.

$$I = mr^2$$

Equation 2: A singular point masses moment of inertia

Where I represents the moment of inertia, m represents the mass, and r represents the distance of the mass from the rotation axis.

For an object taken from reality where said object would have way more points of contact than a point mass, let's say this imaginary object has number n of point masses and points of contact. We can find the moment of inertia of the object by adding up all the point masses moments of inertia. The sum of the moments of inertia of any given number of points of contact and point masses is presented in Equation 3.

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2 = \sum mr^2 = \int r^2 \, dm$$

Equation 3: The sum of the moments of inertia of any given number of points of contact and point masses

From equation 3, the calculation of the moment of inertia for a hollow cylinder, where the total mass is M, the volume density is ρ, the height is h, the outer radius is R, and the inner radius is R', can be determined using the sum of the moments of inertia for an infinite number of infinitesimally thin rings, which is equivalent to the integration of r^2dm, where dm represents the mass of each ring ranging from R to R' as shown in Figure 2 and illustrated by the equation 4.

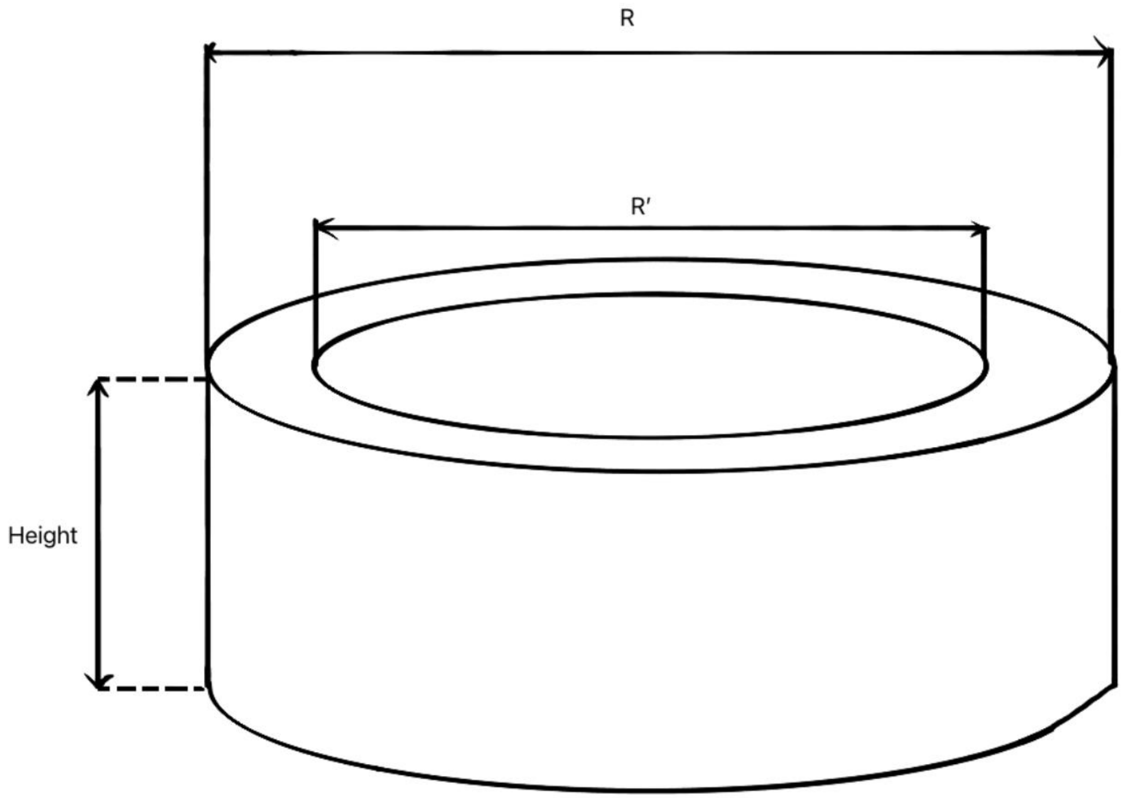


Figure 2: An illustration showing a hollow cylinder with outer radius of R inner radius of R' and a given height

$$I = \int_{R\prime}^{R} r^2 \ dm = \int_{R\prime}^{R} r^2 \ p \ dV = \int_{R\prime}^{R} r^2 \ p \ 2\pi r \ dr \ h = \rho 2\pi \mathrm{h} \int_{R\prime}^{R} r^3 \ dr = p2\pi h \left(\frac{R^4}{4} - \frac{R'^4}{4}\right)$$
$$= \frac{\pi}{2} ph(R^4 - R'^4) = \frac{\pi}{2}\left[\frac{M}{\pi(R^2 - R'^2)h}\right] h(R^2 - R'^2)\,(R^2 + R'^2) = \frac{1}{2} M(R^2 + R'^2)$$

Equation 4: The calculations required to find the moment of inertia of a hollow cylinder with outer radius of R, inner radius of R' and a given Height

Using angular velocity and moment of inertia we can replace their counterparts in the kinetic energy formula to obtain rotational kinetic energy. Which is given in equation 5

$$E_{Kr} = \frac{1}{2} I w^2$$

Equation 5: the rotational kinetic energy of a rigid rotating object.

Since the cylinders movement is originated from its center of mass due to its weight the cylinder is Rolling without slipping. Therefore, it can be noted that the body's kinetic energy includes both its translational and its rotational counterparts which gives us a formula of total kinetic energy which can be given by equation 6

$$E_{KT} = \frac{1}{2} MV^2 + \frac{1}{2} IW^2$$

Equation 6: The total kinetic energy of a rotating and translating object.

Since there also is a change in height throughout the motion our total energy equation has to include potential energy which can be given by equation 7

$$E_T = \frac{1}{2} MV^2 + \frac{1}{2} IW^2 + Mgh$$

Equation 7: The total rotational energy of a rotating and translating object which changes in height

Therefore, the final velocity of a hollow cylinder identical to that in Figure 2 released from rest from the top of an inclined plane with a vertical height h, can be calculated as in Equation 8.

$$Mgh = \frac{1}{2} MV^2 + \frac{1}{2} IW^2$$

$$Mgh = \frac{1}{2} MV^2 + \frac{1}{2} \left[\frac{1}{2} M\left(R^2 + R'^2\right)\right] \left(\frac{V}{R}\right)^2$$

$$4gh = 2V^2 + \left(R^2 + R'^2\right) \left(\frac{V^2}{R^2}\right)$$

$$V = \sqrt{\frac{4gh}{3 + \left(\frac{R'}{R}\right)^2}}$$

Equation 8: Derivation of the final velocity of a hollow cylinder

Hypothesis:

The moment of inertia of the cylinder will be lesser as the inner radius is decreased; Since the rotational kinetic energy will be higher, and the translational kinetic energy will be lower. As such, we can hypothesize that as the inner radius for a hollow cylinder gets smaller when it is rolled down an inclined path the moment of inertia will decrease and as a result its angular velocity will increase.

Apparatus:

Equipment	Number of equipment
Hollow resin cylinder with an outer diameter of 0.1 m and inner diameter 0.01,0.02,0.03,…,0.09 m $\pm 1e^{-5}$ m	1 of each
Data logger	1
Velocity sensor ± 1% m/s	1
Stand	1
Rectangular plate with a rolling path of 0.35 m	1
Ruler ± 0.0005 m	1
Protractor ± 0.5°	1

Table 1: List of equipment and the quantities required for the experiment with their respective uncertainties

Variables:

Kind of variable	**Variables**
Dependent variable	Averaged velocity of the cylinder rolling down the inclined plane
Independent variable	Inner radius of the cylinder

Table 2: Dependent and independent variables of the experimental process

Controlled Variables	**Purpose of Control**	**How it is Controlled**
The cylinders outer radius	A difference in the R value of the rolling body will affect its moment of inertia due to the R' value not being the only independent variable in the experimental process. Therefore, the outer radius must stay the same	Cylinders all have the same R value of 0.1 m
Materials of the cylinder and the plane	Differences in materials in both the plane and the cylinder will result in different volume densities or different coefficients of frictions, which will result in different moments of inertia. Because of this both materials of the cylinder and the plane are required to be kept constant	The same incline was used for all trials of the experimental process. The material used to make the cylinders were kept constant throughout each iteration.
The inclines height	The cylinder's potential energy will be higher if it is released from rest at a higher point, therefore the elevation in which the cylinder is released from must be kept constant for a concise answer to my research question	The inclined plane used is always at the same angle, which keeps the height of the slope constant.

Table 3: Variables that are controlled during the experimental process.

Procedure:

1. Set up the equipment listed as presented in photo 1 and figure 3
2. Turn on the data logger and the velocity sensor
3. Mark the top of the inclined plane where the cylinder will be placed and put the cylinder in the marked spot
4. Release the cylinder from the marked spot and record the motion until the cylinder has rolled off the inclined plane
5. Repeat step 4, 5 times for each trial of the experiment
6. Repeat steps 4-5 for each respective R' value of a given cylinder
7. Collect and note the data for further processing

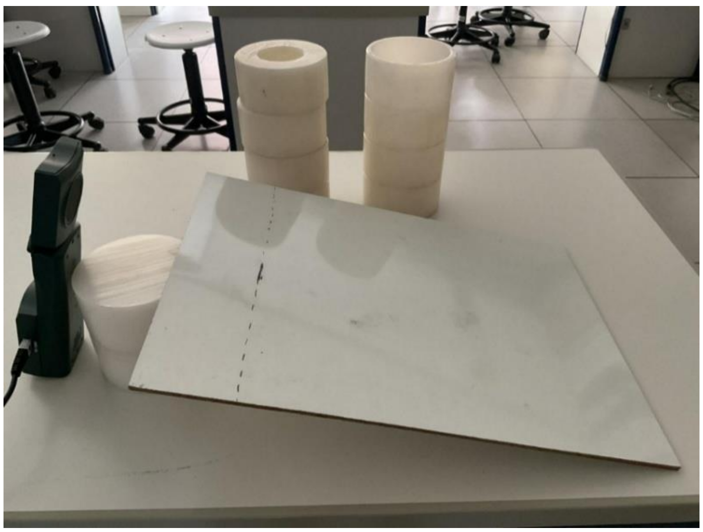


Photo 1: Photo of the experimental setup.

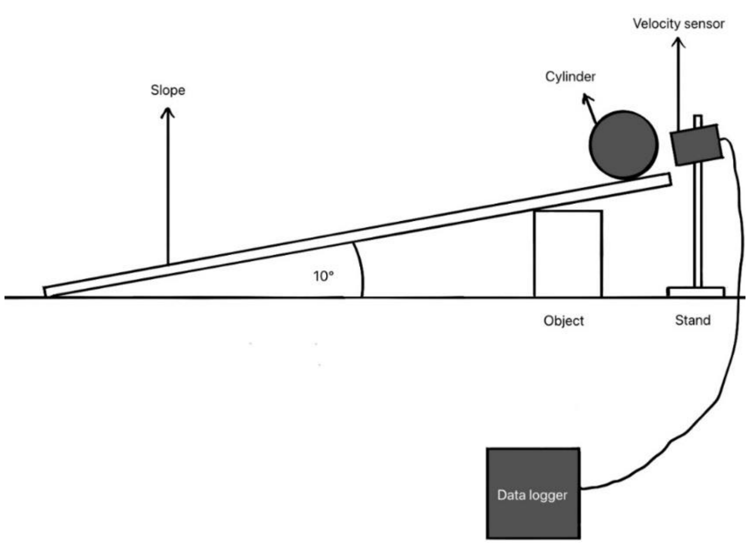


Figure 3 An illustration of the setup of the experiment.

Environmental, Safety, and Ethical Concerns:

Precautions must be taken during the production of the hollow resin cylinders since they are made by an industrial machine. Wearing precautionary equipment such as nitrile gloves, safety goggles, and a respirator to prevent the adverse effects of the toxicity of resin. Furthermore, the materials used for the hollow cylinder were later sent to a recycling center to prevent material waste and rerelease of toxic gas to the environment and upon humans. There are no ethical concerns during the preparation and process of this experiment.

Data Analysis

Raw Qualitative data

By observations, as the inner diameter of the cylinder increases, the velocity of the objects decent on the inclined plane decreases.

Diameter of the hole (D') $\pm 1e^{-5}$ m	1st Output ±1% m/s	2nd Output ±1% m/s	3rd Output ±1% m/s	4th Output ±1% m/s	5th Output ±1% m/s	6th Output ±1% m/s	7th Output ±1% m/s	8th Output ±1% m/s	9th Output ±1% m/s	10th Output ±1% m/s	Final Velocity ±1% m/s
1.00	0.506	0.513	0.514	0.515	0.517	0.518	0.519	0.522	0.522	0.524	0.517
2.00	0.461	0.468	0.470	0.475	0.473	0.475	0.482	0.485	0.487	0.489	0.478
3.00	0.478	0.479	0.486	0.488	0.489	0.496	0.497	0.498	0.499	0.500	0.491
4.00	0.400	0.401	0.421	0.433	0.434	0.438	0.443	0.446	0.447	0.457	0.432
5.00	0.400	0.402	0.403	0.414	0.424	0.431	0.440	0.452	0.456	0.458	0.428
6.00	0.391	0.392	0.394	0.95	0.413	0.423	0.431	0.445	0.447	0.449	0.418
7.00	0.336	0.338	0.344	0.357	0.365	0.372	0.373	0.384	0.385	0.386	0.364
8.00	0.312	0.313	0.331	0.347	0.350	0.352	0.353	0.362	0.364	0.366	0.345
9.00	0.265	0.268	0.280	0.283	0.289	0.290	0.295	0.301	0.303	0.306	0.288

Table 4 Trial 1 of raw quantitative data

The rest of the trials of the raw qualitative data can be to give final velocities can be found in appendix 2

Sample calculations

The final velocities of the cylinders can be calculated by averaging every data point measured by the velocity sensor. The sensors sensitivity is 50 hertz or in other words the sensor takes a measurement every 0.02 seconds. The data collected for when the R' value is 1 cm will be used for sample calculations for the rest of the measurements.

$$v_{final} = \frac{1}{10} * (0.506 + 0.513 \ldots + 0.522 + 0.524) = 0.517$$

The mean final velocities for each of the 5 trials can be calculated by getting the average of each respective trial for each different value of the diameter of the hole. The error calculations will be done when the diameter of the hole is 2 cm as a representative for the rest of the data points.

$$v_{average} = \frac{1}{5} * (0.478 + 0.488 + 0.489 + 0.485 + 0.490) = 0.486$$

Uncertainty calculations for the calculations done can be given by the below calculations

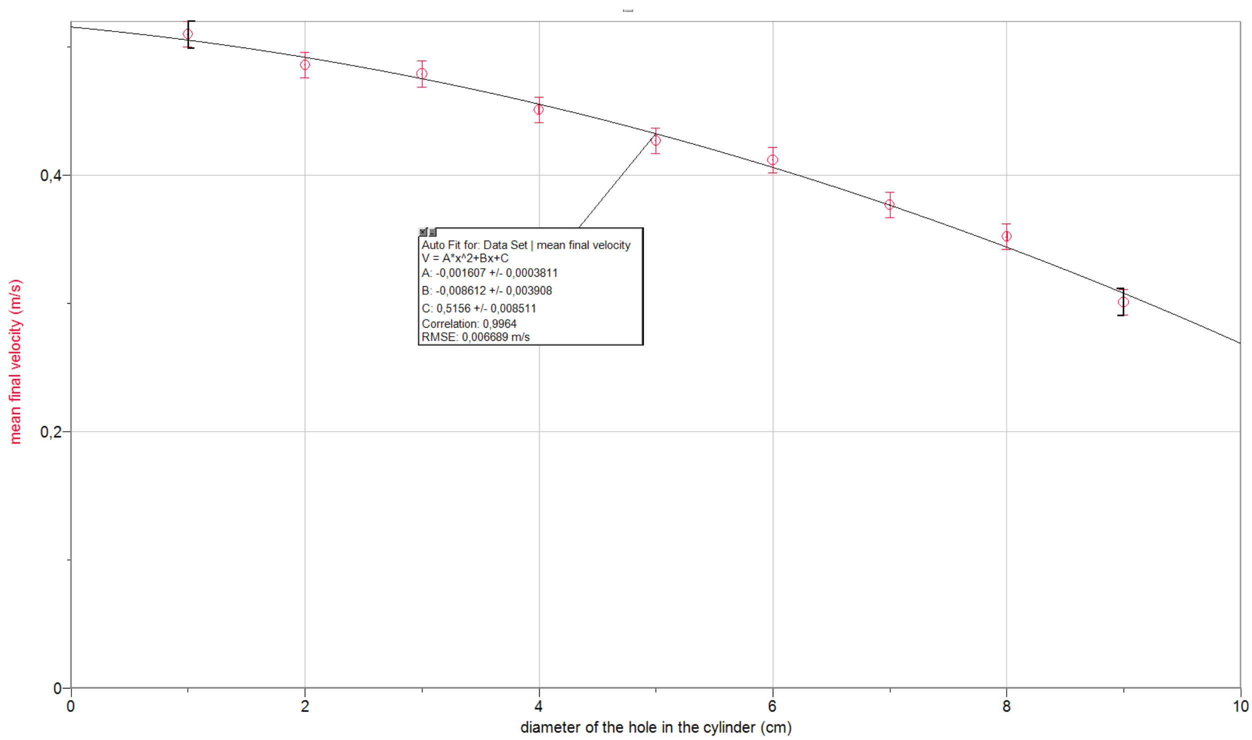
$$v_{final} = \frac{1}{10} * (1\% . 10) = 1\%$$

$$v_{average} = \frac{1}{5} * (1\% . 5) = 1\%$$

When applied similarly to the rest of the D' values we obtain table 5 which represents the velocity data points for each trial and the mean final velocity for its respective R' value.

Diameter of the hole (D') $\pm 1e^{-5}$ m	1st Trials Final Velocity ±1% m/s	2nd Trials Final Velocity ±1% m/s	3rd Trials Final Velocity ±1% m/s	4th Trials Final Velocity ±1% m/s	5th Trials Final Velocity ±1% m/s	Mean final velocity ±1% m/s
1.00	0.517	0.538	0.478	0.494	0.523	0.510
2.00	0.478	0.488	0.489	0.485	0.490	0.486
3.00	0.491	0.470	0.483	0.463	0.488	0.479
4.00	0.432	0.447	0.448	0.438	0.430	0.439
5.00	0.428	0.426	0.416	0.427	0.438	0.427
6.00	0.418	0.408	0.406	0.450	0.428	0.422
7.00	0.364	0.379	0.397	0.393	0.352	0.377
8.00	0.345	0.355	0.369	0.362	0.364	0.359
9.00	0.288	0.281	0.328	0.289	0.319	0.301

Table 5: Processed raw quantitative data



Graph 1: The relationship between the diameter of the hole in the cylinder and the averaged final velocity

Experimental graph analysis

From the graph, it can be observed that as the R' value of the cylinder increases the mean final velocity of the cylinder decreases at an increasing rate. With this information we can infer that with the use of a parabolic curve an equation can be obtained that represents the relationship between these two factors. The equation that fits the trends of the graph best is the parabola

$$y = -0.001607x^2 - 0.008612 + 0.5156$$

Linearization of the relationship

According to equation 4 that was used to derive an expression for the final velocity of a hollow cylinder rolling down a slope, the expression indicates that the final velocity is commensurate to the inverse of the root of the square of the ratio of the internal diameter to the external diameter of the hollow cylinder plus 3 (this will be referred to as the correspondence factor in this investigation) when the gravitational acceleration and vertical height are kept constant. The relationship is indicated in Equation 9.

$$V = \sqrt{\frac{4gh}{3 + \left(\frac{R'}{R}\right)^2}} \rightarrow V \propto \frac{1}{\sqrt{3 + \left(\frac{R'}{R}\right)^2}}$$

Equation 9: The proportionality of the final velocity to the correspondence Factor.

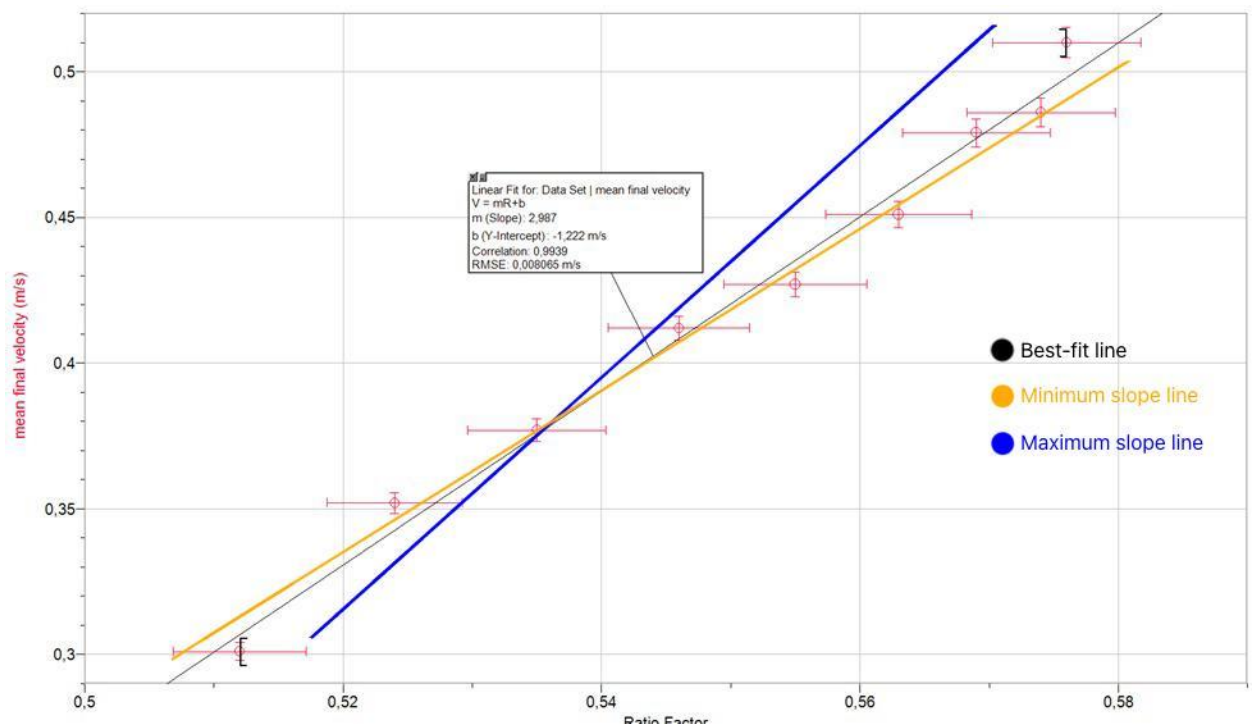
To see if the data collected from the experimental procedure aligns with the theoretical toy-data set the data will be plot against the correspondence factor of the cylinder. If the collected data produces a linear relationship, it signifies that the data obtained aligns with ideal conditions.

The correspondence factors of the hollow cylinder must be calculated. The R' value of 3 will be used as a sample calculation.

$$\frac{1}{\sqrt{3 + \left(\frac{3/2}{10/2}\right)^2}} = 0.569\ (3SF)$$

By using the same method, the rest of the correspondence factors for all other R' values can be calculated. These values can be seen in the appendix.

Then the mean final velocities of each R' value will be graphed against the correspondence factor of the respective R' value. The relationship is given in graph 2



Graph 2: The relationship between Correspondence Factor and the averaged final velocity

Linearized Graph Analysis

From graph 2 we can see a very high correlation coefficient of R^2=0.988 (3 sf) when mean final velocity is plotted against the correspondence factor. On top of this the line of best fit is between the line with the maximum and the minimum slope line. This shows that the gathered data from the experimental process fits the hypothesized results.

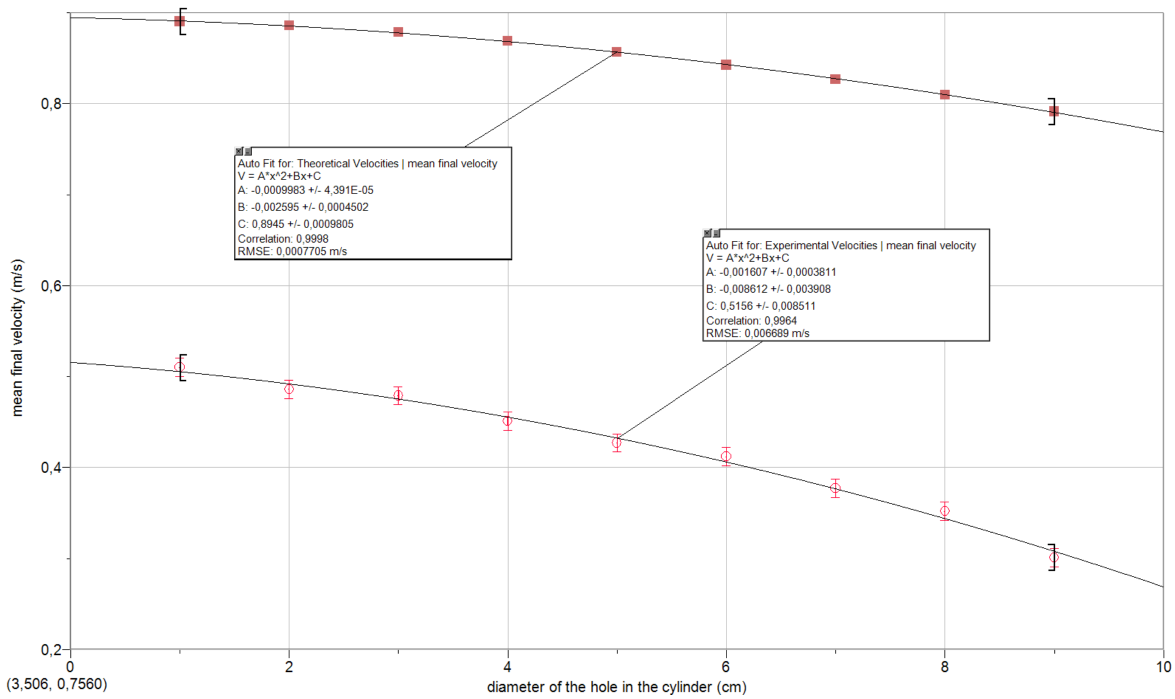
Comparison With Theoretical Results

By using equation 8, the velocity at which the hollow cylinders roll down in the height h medium in ideal, and perfect conditions can be calculated. A stand-in D' value of 4 cm will be used.

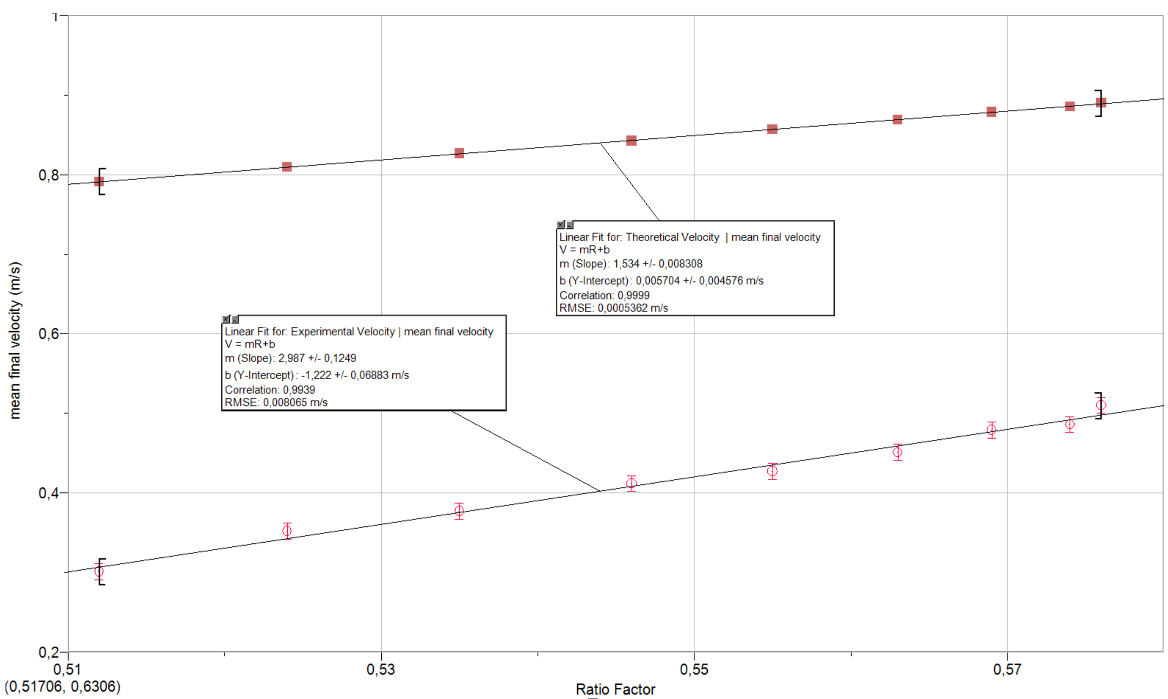
$$V = \sqrt{\frac{4gh}{3+\left(\frac{R'}{R}\right)^2}} = \sqrt{\frac{4*(9.81)*(0.35*\sin(10°))}{3+\left(\frac{4/2}{10/2}\right)^2}} = 0.869(3sf)$$

Using this formula the rest of the theoretical velocities of each R' value can be calculated and is shown in the appendix.

To collate between the experimental data and the calculated theoretical values, the theoretical and experimental relationships between the final velocities, the R' values and the correspondence factors will be graphed in the same plane to compare the differences. These relationships can be seen as in graphs 3 and 4



Graph 3: Experimental and Theoretical relationship between the R' value and the final velocity



Graph 4 Experimental and Theoretical relationship between the Correspondence Factor and final velocity

Comparison of The Theoretical and Experimental Graphs

From graphs 3 and 4, we can see that the theoretical values calculated are usually placed higher vertically on the graph than those of the experimental ones. However, there is little to no dissimilitude between the two. Both follow the same trends in their respective graphs, meaning that the values gathered by experimentation are compatible with their theoretical counterparts.

Conclusion and Evaluation

Conclusion

This investigation's purpose was to observe and analyze how the size of a hole inside a rolling body affects its moment of inertia and resultantly its velocity after rolling down an inclined plane. Based upon the research it can be inferred that if the hole in a cylinder gets bigger, then the hollow cylinder will decent the incline with a lower speed than a cylinder with a lesser diameter. The basis for this relationship is since as the inner radius of the hollow cylinder gets larger, its moment of inertia increases proportionally. Ergo, for the rotational kinetic energy of the rolling body to increase it must take away from its counterpart translational kinetic energy lowering it. Which results in the hollow cylinder traveling slower than that of a cylinder with a smaller hole when both are released from the top of the same incline.

The results obtained from analyzing the data collected during the experiments indicated that as the diameter of the hole in the hollow cylinder increased the final velocity also decreased at an increasing rate. The parabolic function denoted by the equation of the parabola $y = -0.001607x^2 - 0.008612 + 0.5156$ shown in Graph 1 demonstrates that the proposed hypothesis had been tested and proved to be correct.

To demonstrate that the experimental data corresponds to the mathematical model used to represent the motion of the cylinder in the introduction, the experimental data was linearized by graphically representing the velocity values that were measured during the experiments versus the correspondence factors of the cylinders. The linearization indicates that as the correspondence factor increases the final mean velocity of each respective hollow cylinder will increase linearly and this relationship is expressed by the equation $y = 2.987x - 1.222$, with a very high coefficient of determination value $R^2=0.988$ indicated by the linearized graph of experimental data to correspondence factors, further indicating that the experimental results are suitable to be modeled using the theoretical model of the experiment.

The obtained graphs relating the experimental and the theoretical counterparts have been compared to the theoretical relationships on graphs 3 and 4. from the results it is apparent that the theoretical values were generally above those of the experimental values in the resulting graphs; however, both sets of values showed the same trend, thus the experimental data has shown itself to be reasonably accurate. Therefore, it can be concluded that this investigation into how the size of a hole inside a cylinder relates to its moment of inertia when said cylinder descends down an incline has produced satisfactory results

Evaluation

Systematic Error	Repercussions on The Experimental Process	Technique for Refinement
Manufacturing issues with the hollow cylinders.	Manufacturing errors during the process of creating the resin cylinders may have arisen non accurate data points during the experimental process. Issues such as the existence of air bubbles in the resin during manufacturing or mechanical errors such as an uneven cylinder. This may have created an uneven center of mass leading to wobbling or different values of velocities than what should have happened.	A thorough analysis of each cylinder should be carried out to make sure there are no inconsistencies with the desired product.
Slipping of the cylinders during their motion	Slipping will occur if there is a momentary velocity of the contact point between the medium and the rolling body which supports it, and that instantaneous velocity is not zero. When slipping happens it causes both sliding and rotational motion for the cylinder down the slope, therefore the measured experimental velocities were less than the expected results had no slipping occurred. In the case of this experiment, we assume that no case of slipping occurred during the motion	The slope or the cylinder material can be chosen such that the coefficient of friction between the two materials are high enough such that the slipping is prevented
Energy loss due to drag forces	The only drag force in this case being air resistance acts against the movement of the cylinders going down an inclined path leading to the velocity value measured through experimentation being lower than that of the theoretical value, as shown in graph 3,4. But since the motion of the cylinders are at relatively low speeds and at a relatively short distance, the air resistance was assumed to be negligible	Accounting for the energy loss caused by drag forces during the estimation of the theoretical values of the final velocities to better compare between the theoretical and experimental relationships

Table 6: Sources, effects and refinement techniques for the systematic errors in the experiment

Random Error	Repercussions on The Experimental Process	Technique for Refinement
Small variation in rolling path	In a few of the trials during the experiment the cylinders rolling direction didn't follow a linear path leading to a lower reading in the velocity sensor than the true velocity of the motion	Alteration of the slope to add 2 barriers with distance of 0.11 m to fit the cylinder but also not slow down its motion
Inconsistent release point	Even though a spot was marked on the path the cylinders were rolled on to prevent this issue there wasn't a fool proof method to prevent this issue in the experiments set up. Resulting in a slight variation between every mistakenly placed cylinder	A small chamber to fit the hollow cylinders at the peak of the slope could have been added

Table 7: Sources, effects and refinement techniques for the random errors in the experiment

Further Investigations

Firstly, an investigation on comparison between hollow cylinders of equal mass but different inner radii can be conducted. In which mass of each hollow cylinder would be kept constant using materials with different densities while changing the inner radius. This investigation would see if mass still plays a role in the motion of the cylinders through the slope, which may allow a more direct test of the relationship between moment of inertia and translational velocity.

Secondly, instead of investigating the final velocity of the motion, an investigation on how different radii of the hollow cylinder effects the distribution of translational and rotational kinetic energy. In which a deeper understanding of energy partitioning in rotational dynamics could be created.

Thirdly, an investigation on the opposite of my experiment could be created by keeping the inner radius same or 0 while increasing or decreasing the outer radius of the cylinder. This investigation would uncover how overall size of the rolling cylinder affects its moment of inertia.

Lastly, by applying every method of improvement that has been listed in the evaluation section to prevent systematic and random errors as much as possible. The relationship between the calculated theoretical values and the data's collected through experimentation found may be made clearer by removing the discrepancy of the different velocity values.

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Appendix

Appendix 1

Photos related to the experimental process

Photo 2: Image of the hollow resin cylinders.

Appendix 2

Trials 2-5 of the raw qualitative data

Diameter of the hole (D') $\pm 1e^{-5}$ m	1^{st}Output ±1% m/s	2^{nd}Output ±1% m/s	3^{rd}Output ±1% m/s	4^{th}Output ±1% m/s	5^{th}Output ±1% m/s	6^{th}Output ±1% m/s	7^{th}Output ±1% m/s	8^{th}Output ±1% m/s	9^{th}Output ±1% m/s	10^{th}Output ±1% m/s	Final Velocity ±1% m/s
1.00	0.510	0.511	0.522	0.532	0.542	0.544	0.546	0.547	0.558	0.568	0.538
2.00	0.462	0.463	0.482	0.485	0.487	0.489	0.490	0.503	0.506	0.513	0.488
3.00	0.441	0.457	0.466	0.468	0.470	0.473	0.474	0.477	0.482	0.492	0.470
4.00	0.420	0.421	0.427	0.428	0.447	0.452	0.463	0.464	0.472	0.476	0.447
5.00	0.400	0.401	0.402	0.417	0.423	0.426	0.434	0.443	0.455	0.459	0.426
6.00	0.382	0.383	0.385	0.392	0.415	0.418	0.420	0.425	0.426	0.434	0.408
7.00	0.356	0.361	0.364	0.372	0.378	0.386	0.387	0.389	0.392	0.405	0.379
8.00	0.325	0.326	0.327	0.332	0.357	0.369	0.371	0.377	0.382	0.384	0.355
9.00	0.252	0.254	0.255	0.278	0.280	0.282	0.295	0.300	0.305	0.309	0.281

Table 8: Trial 2 of the raw qualitative data

Diameter of the hole (D') $\pm 1e^{-5}$ m	1^{st}Output ±1% m/s	2^{nd}Output ±1% m/s	3^{rd}Output ±1% m/s	4^{th}Output ±1% m/s	5^{th}Output ±1% m/s	6^{th}Output ±1% m/s	7^{th}Output ±1% m/s	8^{th}Output ±1% m/s	9^{th}Output ±1% m/s	10^{th}Output ±1% m/s	Final Velocity ±1% m/s
1.00	0.450	0.453	0.463	0.477	0.479	0.480	0.490	0.492	0.497	0.499	0.478
2.00	0.463	0.465	0.466	0.469	0.473	0.503	0.507	0.510	0.515	0.519	0.489
3.00	0.454	0.455	0.458	0.478	0.490	0.493	0.494	0.499	0.503	0.506	0.483
4.00	0.423	0.432	0.436	0.443	0.444	0.450	0.459	0.460	0.461	0.472	0.448
5.00	0.391	0.393	0.394	0.399	0.414	0.427	0.430	0.436	0.437	0.439	0.416
6.00	0.382	0.384	0.385	0.393	0.399	0.411	0.415	0.426	0.427	0.438	0.406
7.00	0.370	0.371	0.378	0.382	0.394	0.406	0.409	0.413	0.423	0.424	0.397
8.00	0.341	0.353	0.360	0.365	0.368	0.369	0.374	0.378	0.388	0.394	0.369
9.00	0.302	0.303	0.305	0.311	0.327	0.334	0.339	0.351	0.353	0.355	0.328

Table 9: Trial 3 of the raw qualitative data

Diameter of the hole (D') $\pm 1e^{-5}$ m	*1st*Output ±1% m/s	*2nd*Output ±1% m/s	*3rd*Output ±1% m/s	*4th*Output ±1% m/s	*5th*Output ±1% m/s	*6th*Output ±1% m/s	*7th*Output ±1% m/s	8*th*Output ±1% m/s	*9th*Output ±1% m/s	*10th*Output ±1% m/s	Final Velocity ±1% m/s
1.00	0.466	0.467	0.484	0.490	0.492	0.493	0.497	0.514	0.517	0.520	0.494
2.00	0.463	0.465	0.467	0.469	0.477	0.492	0.496	0.500	0.509	0.512	0.485
3.00	0.431	0.433	0.434	0.467	0.468	0.472	0.474	0.479	0.484	0.488	0.463
4.00	0.410	0.411	0.412	0.414	0.443	0.447	0.448	0.460	0.467	0.468	0.438
5.00	0.400	0.405	0.406	0.418	0.425	0.431	0.436	0.444	0.447	0.458	0.427
6.00	0.421	0.422	0.430	0.439	0.454	0.456	0.460	0.467	0.474	0.477	0.450
7.00	0.374	0.379	0.386	0.388	0.390	0.391	0.394	0.404	0.410	0.414	0.393
8.00	0.336	0.342	0.355	0.358	0.362	0.365	0.366	0.369	0.377	0.390	0.362
9.00	0.261	0.263	0.276	0.288	0.290	0.298	0.299	0.301	0.302	0.312	0.289

Table 10: Trial 4 of the raw qualitative data

Diameter of the hole (D') $\pm 1e^{-5}$ m	*1st*Output ±1% m/s	*2nd*Output ±1% m/s	*3rd*Output ±1% m/s	*4th*Output ±1% m/s	*5th*Output ±1% m/s	*6th*Output ±1% m/s	*7th*Output ±1% m/s	8*th*Output ±1% m/s	*9th*Output ±1% m/s	*10th*Output ±1% m/s	Final Velocity ±1% m/s
1.00	0.494	0.495	0.511	0.524	0.525	0.527	0.530	0.534	0.542	0.548	0.523
2.00	0.463	0.464	0.467	0.483	0.486	0.490	0.505	0.509	0.516	0.517	0.490
3.00	0.460	0.461	0.462	0.473	0.488	0.495	0.499	0.513	0.514	0.515	0.488
4.00	0.400	0.407	0.412	0.425	0.426	0.437	0.440	0.444	0.451	0.458	0.430
5.00	0.404	0.431	0.433	0.435	0.436	0.437	0.440	0.450	0.454	0.460	0.438
6.00	0.403	0.404	0.418	0.419	0.427	0.429	0.439	0.441	0.443	0.457	0.428
7.00	0.321	0.341	0.346	0.347	0.348	0.349	0.361	0.367	0.369	0.371	0.352
8.00	0.336	0.340	0.345	0.347	0.367	0.370	0.380	0.381	0.385	0.389	0.364
9.00	0.290	0.291	0.303	0.316	0.324	0.329	0.330	0.332	0.333	0.342	0.319

Table 11: Trial 5 of the raw qualitative data

Appendix 3

The Correspondence Factors for the respective D' values.

Diameter of the hole (D') $\pm 1e^{-5}$ m	Correspondence Factor (±1%)
1	0.576
2	0.574
3	0.569
4	0.563
5	0.555
6	0.546
7	0.535
8	0.524
9	0.512

Table 12: The Correspondence Factors for the respective D' values.

Appendix 4

Theoretical final velocities.

Diameter of the hole (D')	Theoretical Final Velocities
1	0.8901237497
2	0.8857208022
3	0.8785255506
4	0.8687405536
5	0.8566273891
6	0.8424885318
7	0.8266485781
8	0.8094367992
9	0.7911724368

Table 13: Calculated theoretical final velocities.