INVESTIGATION OF THE OSCILATORY MOTION OF THE SIMPLE PENDULUM

PHYSICS EXTENDED ESSAY

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ABSTRACT

While writing this essay, my prior aim was to investigate the factors affecting the oscilatory motion of the simple pendulum by specifying it's extent with the research question; "How do the length of the pendulum, initial angular amplitude of the pendulum and mass of the pendulum bob affect the period of the oscilatory motion of the pendulum?". Therefore, the investigation is surveyed through three seperate experiments; Experiments A, B and C; which examined the three effects respectively.

For each experiment, a pendulum is constructed by suspending a thread from the ceiling and attaching a mass hook with standard masses to it and the sinuodally varying acceleration of 20 oscilations are measured and recorded by Vernier Accelerometer. As for the results, predictedly, the period decreased dramatically when the length of the pendulum decreased. Yet, although the effects of the angle and mass aren't taken to consideration for the general formula of period, the period increased slightly as the initial angular amplitude increased and a beat is formed in the sinuodal function of the acceleration versus time when the mass of the bob is increased. Then, the theoretical periods of the oscilations are calculated and the experimental periods are compared with them. For each experiment, the experimental error is less than 5%, which indicated the accuracy of the data.

Overall, the investigation served for answering the further questions that are asked about the standard formulations about the simple pendulum. Yet, the investigation is limited with three effects which also affect the orbit of phase trajectory and distort the accuracy. Thus, a continuation can be made in order to relate the accuracy of the pendulum to the oscilatory motion by examining only the orbit of phase trajectory, as the pendulum undergoes an oscilatory motion that is closely related to the circular motion.

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INTRODUCTION

The characteristics of the simple harmonic motion are formulated by modelling the circular motion. Therefore, one of the fundemantals of the oscilatory motion is the simple pendulum, as it actually performs a small part of the circular motion, yet shows the characteristics of the simple harmonic motion. Therefore, I thought that investigating such a simple yet fundamental phenomenon could be interesting.

As described in the text books;

> "The simple pendulum is an idealization of a real pendulum. It consists of a point mass, m, attached to an infinitely light rigid rod of length l that is itself attached to a frictionless pivot point."(Baker & Blackburn, 2005, p. 9).
>
> "We assume that the cord doesn't stretch and that its mass can be ignored relative to that of the bob. The motion of a simple pendulum moving back and forth with negligible friction resembles simple harmonic motion: the pendulum oscilates along the arc of a circle with equal amplitude on either side of its equilibrium point (where it hangs vertically) and as it passes through the equilibrium point it has its maximum speed. (…) For small oscilations (less than 15°), the period of the pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$, where "T" is the period, "*l*" is the length of the string and "g" is the gravitational acceleration."(Giancoli, 2000, pp. 371-372)

On the other hand, despite the assumptions make the theoretical calculations easier, they cause the formulations to highly differ from the experimental values. Therefore, throughout this essay, my aim is to examine the factors affecting the period of oscilatory motion of the simple pendulum, which were assumed to be negligible while making the formulations; that are initial angular amplitude and mass of the bob, also the basic variable of the formula, that is length of the thread.

In order to examine these effects, a simple pendulum is constructed with a thread and cylindrical bob, which consists of a hook and standard cylindrical masses suspended from it. Theoretically, when the gravitational acceleration is kept constant, as the length of the thread from which the bob is suspended increases, the period of the oscilation increases, whereas the angular amplitude of the oscilation doesn't affect it for small angles, and the mass has no effect at all. Moreover, it is known that the acceleration of the simple pendulum versus time taken for the oscilations function is sinuodal and the time taken for the two consecutive acceleration peaks give the period of the oscilations (Haag, 1962, p. 1). Therefore, in order to examine the period accurately, each experiment is designed for examining the factors that effect the sinuodal function of acceleration versus time taken for the oscilations. For each and every experiment, a Vernier Lab Quest equipment, "Accelerometer" is used for measuring the sinuodal function of acceleration versus time taken of the simple harmonic oscilation of the pendulum for it's first 20 oscilations. Therefore, the effect on the period is examined along with the acceleration, too.

For Experiment A, the effect of length of the thread is investigated, whereas the angular amplitude and the mass of the bob are kept constant. Therefore, the hypothesis is "The period of the oscilations of the simple pendulum decreases as the length of the thread decreases.".

For Experiment B, the effect of angular amplitude is investigated, whereas the mass of the bob and the length of the thread are kept constant. Since as the angular amplitude increases, the angular displacement also increases, the hypothesis is "The period of the simple pendulum increases as the angular amplitude increases."

For Experiment C, the effect of mass of the bob is investigated, whereas the length of the thread and the angular amplitude are kept constant. Although the mass is irrelevant from the original period formula, as the mass increases, more vibrational energy is degraded as a result (Matthys 2004, p.4). Therefore, the beat of the plotted sinuodal function is expected to differ due to the energy loss and the bending torque that is stressed on the string (Matthys, 2004, p. 14). Hence, the hypothesis is "The mass of the bob affects the specific beats of the sinuodal function of acceleration versus time for each mass value." .

VARIABLES

As the investigation is divided into three experiments, it is crucial to keep in mind the dependent and independent variables of each experiment. Thus, the related variables are listed seperately for each of them below. Yet, although the investigation is surveyed by three seperate experiments, since the main aim is to investigate the oscilatory motion of the simple pendulum, the variables are set in a similar way in each experiment. For instance, in Experiment A and Experiment C, as controlled variables, the initial angular amplitudes of the pendulums are 10°, where as 10° is the angular amplitude of trial 1.1, 1.2, and 1.3 as the independent of Experiment B. Likewise, as a controlled variable, the total mass of the cylindrical masses that are attached to the mass hooks are 250g in Experiment A and Experiment B, while 250 g is used as an independent variable for trials 5.1, 5.2, 5.3 in Experiment C. By this way, the reliability of the overall conclusion of the experiments is increased as it would be related to each experiment's variables.

The following variables are kept constant for each trial of the experiments;

- Mass of the standard cylindrical masses (50g)
- Identical threads, that are made of cotton and have the same cross sectional area, are used.
- Number of oscilations of the pendulum (20 oscilations for each trial)
- As the vibration of the particles caused by the kinetic energy of the pendulum may affect the beat of the sinuodal function of acceleration versus time graph and the temperature may also affect the expansion of the bob and the elongation of the thread, the temperature of the room where the experiment takes place is also recorded and kept constant; which is 22.8°C ± 0.1 .

Independent Variables

- Experiment A : Length of the pendulum thread (m)
- Experiment B : Initial angular amplitude of the pendulum when it's released (°)
- Experiment C : Mass of the pendulum bob (kg), which is changed by increasing the number of cylindrical masses that are attached to the mass hook

Dependent Variables

- Experiment A & Experiment B : Period of the oscilations of the simple pendulum (s), derived from the plotted sinuodal funtion of acceleration of the simple pendulum versus time taken for 20 oscilations, that is measured by the *Accelerometer*
- Experiment C : The beat of the sinuodal function, derived from the plotted sinuodal funtion of acceleration of the simple pendulum versus time taken for 20 oscilations, that is measured by the *Accelerometer*

Controlled Variables

- Experiment A : - Initial angular amplitude of the pendulum when it's released (°)

  - Mass of the bob (kg), adjusted by using equal numbers of standard cylindical masses that are attached to the mass hook

- Experiment B : - Length of the thread of the pendulum (m)

  - Mass of the bob (kg), adjusted by using equal numbers of standard cylindical masses that are attached to the mass hook

- Experiment C : - Initial angular amplitude of the pendulum when it's released (°)

  - Length of the thread of the pendulum (m), measured by a ruler

OVERALL APPARATUS OF THE EXPERIMENTS

- Thread (9.175 m)
- Ruler (cm) (±0.1)
- Meter Stick (cm) (±0.1)
- Standard cylindrical masses of 50g (x 5)
- Mass hook
- Vernier Accelerometer (ms^{-2}) (±0.01)
- Vernier Motion Detector (m) (ms^{-1}) (±0.1)
- Vernier Lab Quest
- Protractor (°) (±0.5)
- Sellotape
- Thermometer (°) (±0.1)

EXPERIMENTAL METHOD

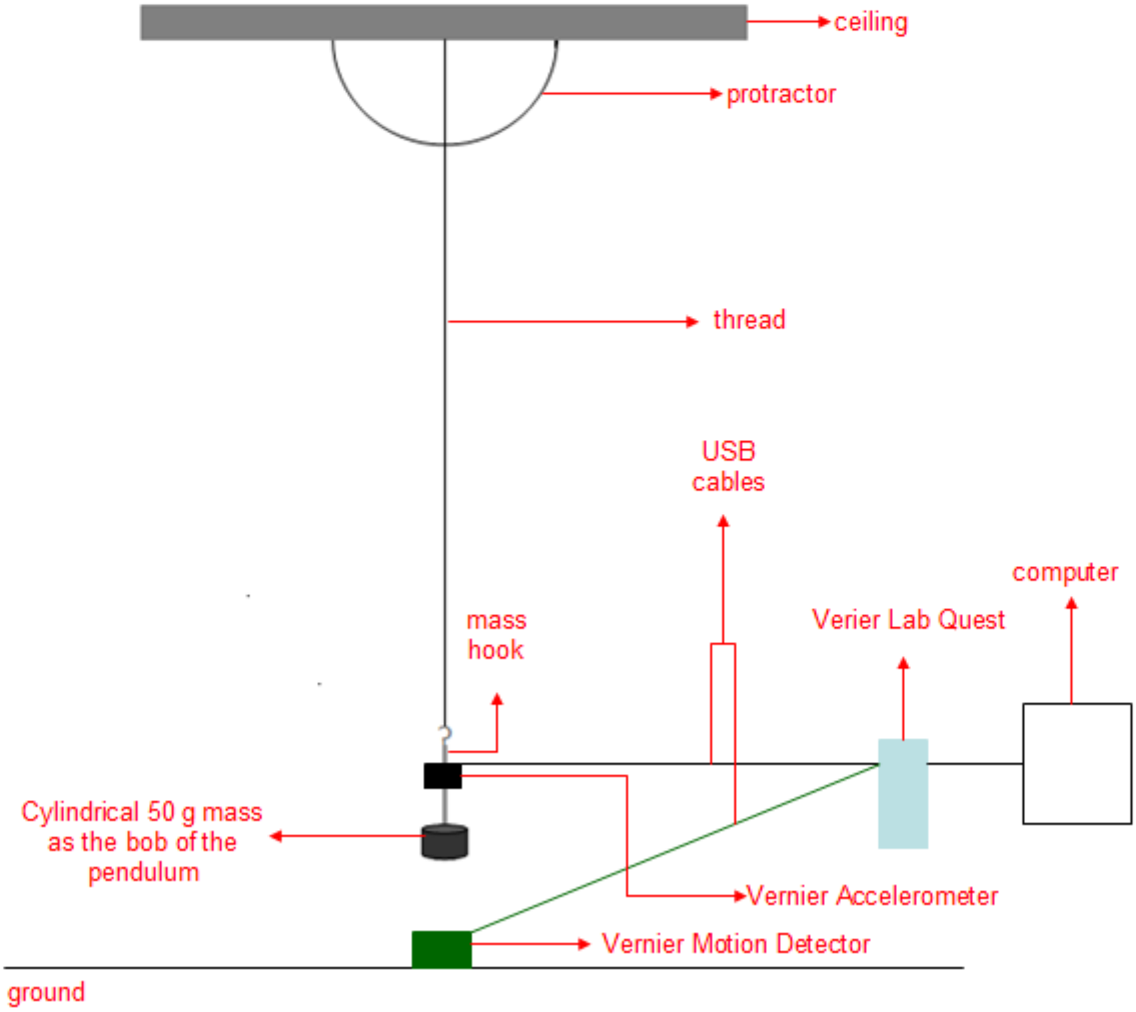


Figure 1 : Overall Experimental Setup

Although three seperate experiments are performed, the overall experimental setup is same for all of them, which is plotted in Figure 1. In order to construct the basic setup; these steps are followed;

1. A knot is tied in the tip of the thread.
2. By using sufficient amount of sellotape, the knot of the thread is stuck in the ceiling in a way that the thread is strong enough to carry the bob when it's oscilating. Since the height from the ceiling to the Vernier Motion Detector is used while making calculations, the possible source of systematic error is eliminated by this step.
3. By using sellotape, the protractor is stuck on the ceiling as shown in Figure 1, so that the 0 reading of it is in the same alignment with the thread. The protractor doesn't touch the thread, yet, the two are very close to eachother. In order to eliminate the possible error, attention is paid in order to set the protractor in a perfect horizontal state on the seiling instead setting it in a bent way.
4. Another knot is tied in the lower end of the thread and the mass hook is attached to it.

5. The rest of the thread is cut and the length of the pendulum thread is measured by a meter stick when the mass hook isn't attached.
6. When the mass hook is attached, 5 cylindrical standard masses of 50 g are held on it.
7. Vernier Motion Detector is placed on the ground right under the bob.
8. Vernier Accelerometer is attached to the upper tip of the mass hook.
9. By using the related USB cables, the electronic devices are attached to the Vernier Lab Quest and the computer, as shown in Figure 1.

After constructing the initial setup, the independent variables are adjusted by the following steps;

Experiment A

The certain amounts of thread are cut from the lower tip of the pendulum thread and the remaining length is measured by a meterstick when the mass hook is not attached.

Experiment B

The initial angular amplitude is increased from 10° to 50° by aligning the pendulum thread to the related readings of the protractor.

Experiment C

The mass of the bob is reduced by removing the standard masses from the mass hook.

The following steps are followed when the rest of the experiment is performed;

1. Length of the motion detector is measured by a ruler.
2. The room temperature is measured by a thermometer.
3. Vernier Motion Detector is used to measure the height from the ceiling to it.
4. The initial angular amplitudes are set by aliging the pendulum thread to the related reading of the protractor.
5. The "Collect" button of the Lab Quest is pressed at the same instant with the release of the pendulum. The same person does the both.
6. 20 oscilations are counted and at the end of the 20th, the "Stop" button of the Lab Quest is pressed.

In order to eliminate the random errors;

1. Identical equipment is used for all trials.
2. The USB cables are set loose enough so that they don't interfere the motion.

For each experiment, 5 different values of the dependent variables are applied and for each value, 3 trials are performed. The recorded data of the experiments are shown in Tables 1, 2 and 3 of Appendix II.

UNCERTAINTY CALCULATIONS AND DISCUSSIONS

- For "Temperature of the room where the experiment takes place" , "Length of the mass hook", "Height from ceiling to the motion detector", "Initial angular amplitude of the pendulum when it's released" and "Height from the bottom of the mass hook to the motion detector when the masses are attached", the minimum division of the measurement device is taken as the uncertainty of the measured values. Then, the significant figures are also converted when the measured values are converted to SI Units.
- In order to eliminate the error caused by the time taken for two people to coordinate for releasing the pendulum and pressing the related buttons of the measurement devices, the same person does the both. None the less, the error caused by the reflex reaction time taken to press the button at the end of the oscilations can not be vanished completely. For "Time taken for 20 oscilations (20T)", although the minimum division of the measurement device is 0.01s, the uncertainty of the measured value is taken as 0.25s (csm.jmu.edu , Date Accessed : 5 April 2010) due to the visual reflex reaction time since the experiment performer presses the device to end the measurement when she sees the oscilations end. Since this uncertainty would remain constant whether the measurement is for any number of oscilations, in order to reduce the uncertainty per period, 20 oscilations are measured. Then, the measured value is divided by 20 to obtain the duration of the period, as well as the uncertainty. So, uncertainty per period is reduced to; 0.25 / 20 = 0.0125 s per period and the precision and accuracy of the data is increased.
- The uncertainties of the periods are discussed in *Data Processing* section.
- Since standard cylindrical masses of 50g are used in the experiments, that are provided by the manifacturer, no uncertainy is taken in account for them and their significant values are irrelevant, only the unit conversions are made.
- The calculated values of the periods are rounded to have 4 decimals after the decimal points since the values are less than 5 seconds and a slight change in value may affect the precision and the percentage error dramatically.

DATA PROCESSING

- The processed data are presented in Table 4, Table 5 and Table 6 of Appendix III ; the units are converted to the units of SI Unit System. After SI Unit conversion, the related conversion of the signifigant figures are also made.
- Theoretically, a simple pendulum is depicted as a point mass suspended at the end of a vertical rod. Consequently, derived from the point mass theory (Woodward, 1995, p. 12), the weight of the bob should be considered to be centered at the center of gravity while making the calculations; however, since the bob consists of unit masses that are attached to a vertical mass hook, it is a non-uniform object. Therefore, the center of gravity is considered to be accumulated at the upper tip of the mass hook while making calculations.
- As the pendulum thread acts like a spring by streching back and forth vertically and elongates due to the mass hook that is attached with the masses, the length of the pendulum is not stabilized throughout the trials. That's why, after the SI Unit conversion is made, the maximum length of the elongated thread is calculated by the following formula in order to calculate the percentage of the error caused by the elongation (see Figure 3 of Appendix I for the explanations of the abbrevations);
  $h - l_{hook} - d = l_e$, where l_e is Length of the pendulum when the thread elongates due to the mass (when the pendulum doesn't oscilate). Since two subtractions are performed with three measured values, the uncertainties of the values are added to each other. Thus, the uncertainty of l_e is ±0.003.
  This formula is used to calculate the l_e because if a meterstick had been used, random parallax error may have occured since the meterstick should have been kept away from the thread which stretches continuosly. By this way, accuracy is preserved.
- For each trial, the experimental period is derived by dividing the measured value by 20. However, in order to discuss the accuracy and precision of the obtained data, a comparison should be made and for making that comparison, a reference should be defined. For these experiments, no literal values are listed for the period of each condition. Instead, a formula is applied to calculate the theoretical period; which is $T = 2\pi\sqrt{\frac{l_p}{g}}$,where l_p is the measured value of the length of non-elongated pendulum thread and g is the gravitational acceleration

of $9,81ms^{-2}$ (1) . By this way, in each experiment, for every different length of the pendulum, the theoretical period is calculated and used as the reference value.

Since l_p is the measured value of the length of non-elongated pendulum thread, which has the uncertainty of 0.001m ; when it's square root is calculated, the uncertainty is divided by 2, leaving 0.0005 as the uncertainty of the calculated theoretical period. For this experiment, as the π value and the gravitational acceleration are taken as literal values instead of measured ones, no uncertainties are shown for them. Due to these processes, the theoretical and the experimental values of the period have different uncertainties.

Then, the averages of the previously derived experimental values of the period are calculated and the uncertainty doesn't change for the average value. After that, the percentage error of the average experimental values are calculated by using the following formula;

$$Error_{percentage} = \frac{|Theoretical - Experimental|}{Theoretical} \times 100,$$

Moreover, the percentage error of the length of the elongated thread is also calculated in order to discuss it's contribution to the overall error of the period.

- For Experiment A, for each length, both of the theoretical and experimental values of period and length are calculated and the percentage error of each length is calculated by using it's own values.
- For Experiment B and Experiment C, since the length of the pendulum is a controlled variable and same for every trial, only one theoretical value of the periods are calculated and all experimental value of period averages are compared with that. Nevertheless, although only one elongate value is measured and processed for Experiment B; in Experiment C, the elongated values for each different mass are measured and processed since the weight applied on the material affects the amount of the elongation.

1

[1] 1 Constant taken from
http://www.unene.ca/un702-2007/qbank/Nomenclature%20for%20Reference%20Equations%202007.pdf
Date Accessed: 11 May 2010
No uncertainties are written since the value is a literature value

DATA INTERPRETATION AND EVALUTION OF THE HYPOTHESIS

The results, data processing and calculations are shown in Appendix III and as aforamentioned, there is no literal value for the applied circumstances of the simple pendulum. Therefore, the $T = 2\pi\sqrt{\frac{l_p}{g}}$ formula is used for obtaining reference values to make conjectures about the results of the experiments.

- For Experiment A; as stated in hypothesis, the period of the oscilations decreased as the length of the pendulum decreased. Yet, although the most obvious result was for this experiment, the percentage error of 4.8%, that is the greatest value on the whole, is reached in it's 5^{th} trials.This may have been caused by the contribution of the error of the elongated thread; as calculated, the percentage error caused it is less than 5% for almost all trials, yet it reaches the maximum values in Experiment A, particularly 5.7%, in the 4th trials. Most probably, it's because of the unstability of the thread as it's the longest in that trial since "The length of the rod is what determines the freqency of vibration, so the more stable its length the better." (Woodward, 1995, p. 13)
- For Experiment B, the period increased up to 0.03s as the initial angular amplitude increased. Although the average percentage error is 0.1%, when the data of the trials are examined seperately (see Appendix II and Appendix III), it can be seen that the error is almost 1% for each of them. As the data are also precise, it can be said that the effect of the initial angular amplitude shouldn't be underestimated.
- For Experiment C, when the graphs are analyzed (See Appendix IV), the unique beat of the each value can be seen. Yet, the average value of the periods deviated from the theoretical value as the mass is increased where the least error of 0.5% is formed when one standard mass was hung to the mass hook.

Since the average values are close to the medians of the data and very slight differences –almost less than 0.01- are occured within the trials of the same value, it can be said that the data are precise and the random errors are prevented. Moreover, the percentage error of the experiments are less than 5%, which indicates that despite the limitations, the experiments are performed accurately with least contribution of the systematic errors. Therefore, to conclude, it can be said that the results of the accurately performed experiments are highly reliable.

THE LIMITATIONS OF THE EXPERIMENTS

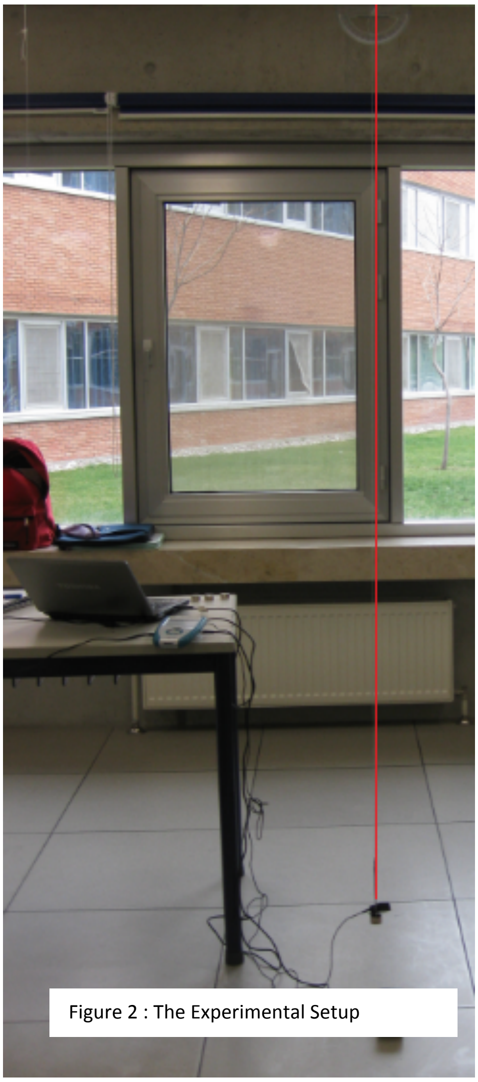


Figure 2 : The Experimental Setup

Although the experiments are performed according to the method meticulously, they still have the following limitations which may cause systemetic error and shift the accuracy of the results;

- The non-uniform shape and the multi-particled form of the bob also affects the vibration and the movement of the particles that is caused by the air drag, which lasts as an effect on the phase trajectory of the circular rotation and swing. "Over 90% of the drive energy put into a pendulum is dissipated in air drag losses as the pendulum swings through the air." (Matthys, 2004, p. 4).
- In spite of the fact that the oscilations of the ideal theoretical simple pendulum lasts infinitely, in practice, the pendulum undergoes damped harmonic motion because of the loss of energy caused by the friction; the amplitudes of the sinuodal function of acceleration versus time shrink as the oscilations last (See Appendix IV). In fact, in the end of the 20 oscilations, the swings become almost impossible to detect; even for an accurate device. Therefore, Vernier Accelerometer is needed to record the data. So, by recording 100 samples per every second, the error of the device is reduced to ±0.01 ms^{-2}. Moreover, thus, the "initial angular amplitude"s are recorded in the data tables instead of the label "angular amplitude". On the other hand, since the pendulum also undergoes circular rotation, the idealized sinuodal function can not be obtained, instead, a beat is formed. The meticulous 100 sample per second also made this beat form easier to detect on the graph.
- In order to avoid the unreliable disturbed data, the usb cable of the accelerometer is left loose enough to avoid the scatter of the pendulum. Moreover, the accelerometer is attached to the mass hook from the same point in each and every trial after the calibration of the device, in order to preserve the accuracy and precision. On the other hand, since the cable also swings, the acceleration affects on it may also disturb the accuracy of the data.
- "Extra air pressure also increases air resistance and reduces the amplitude of swing slightly."(Woodword, 1995, p. 10)

FURTHER OBSERVATIONS

In addition to the predicted data collections, further observertaions are made, which weren't predicted initially. These observations are listed below;

- In each experiment, in addition to it's oscilatory swings, the pendulum bob also rotates in a circular motion-like way, whose orbit of phase trajectory is the ellipse (Baker & Blackburn, 2005, p. 10) . On the other hand, in experiment B, the ellipse trajectory of this rotation gets distorted as the initial angular amplitude increases.

- In each trial of each experiment, when the masses are attached, before releasing the pendulum bobs, the pendulum acts like a spring and oscilates vertically slightly. However, in Experiment C, when the mass of the pendulum bob increases, this spring-like oscilatory motion decreases and becomes hard to be detected with naked eye.

- As the accelerometer that is attached to the mass hook has a cable that is plugged in to the Vernier Lab Quest, the possible distortion in the orbit of phase trajectory may occur although the cable is loose enough to prevent it.

- As the oscilations proceed, the amplitudes of their orbit of phase trajectory shrink.

These qualitative data indicates that each variable brings out more aspects to consider that aren't considered while making the theoretical conclusions and formulating the general formula of the period. For instance the spring-like behaviour of the pendulum shows a second type of oscilatory motion whereas a possible distortion of the phase trajectory may lead the pendulum to undergo a circular motion-like motion. Moreover, if the further investigations are proceeded, the significant effects may be found.

APPENDIX I

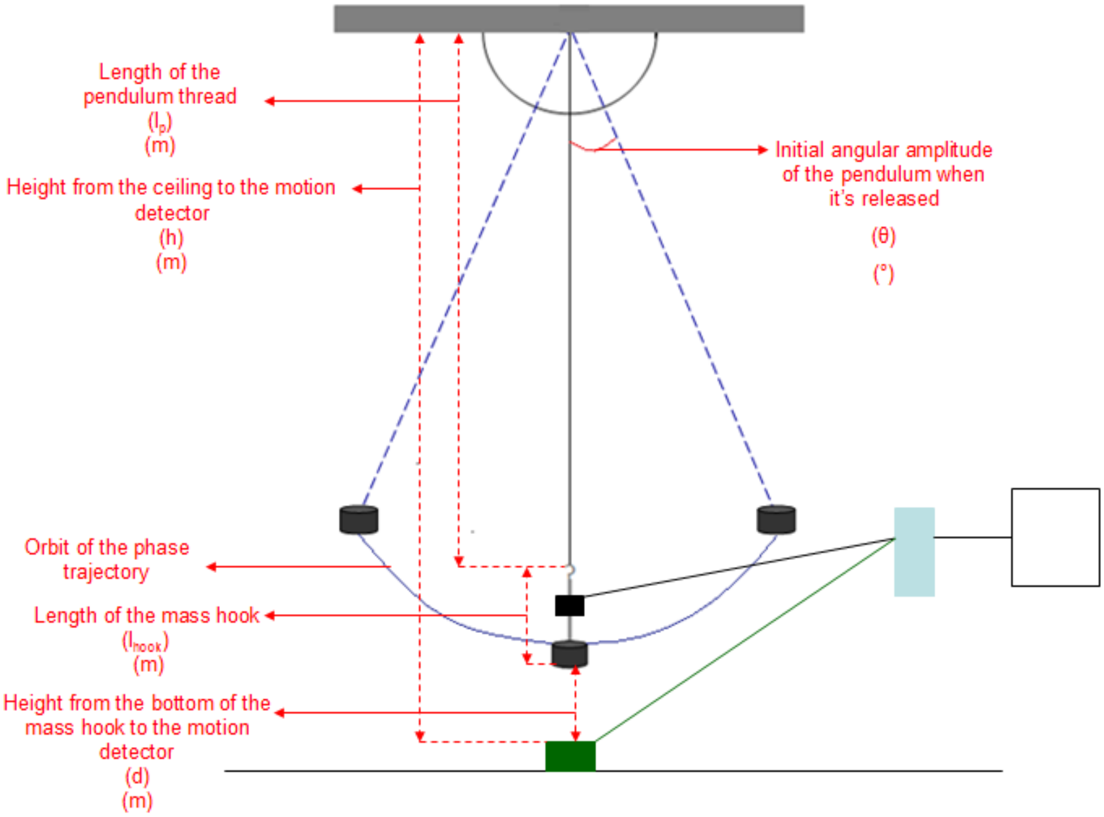


Figure 3 : The diagram of the pendulum with related terms that are used in data collecting

Although the Accelerometer records 100 samples per second, only the total time taken for 20 oscilations are indicated in Tables 1,2 and 3. The related acceleration versus time graphs are shown in Appendix IV.

APPENDIX II

EXPERIMENT A

Temperature of the room where the experiment takes place (°C) (±0.1)	**22.8**		
Length of the mass hook (cm) (±0.1)	**15.0**		
Height from the ceiling to the motion detector (m) (±0.001)	**2.134**		
Total mass of the cylindrical masses that are attached to the pendulum (g)	**250**		
Initial angular amplitude of the pendulum when it's released (°) (±1)	**10**		
TRIALS	**Height from the bottom of the mass hook to the motion detector when the masses are attached (m) (±0.001)**	**Length of the pendulum thread (cm) (±0.1)**	**Time taken for 20 oscilations (s) (±0.25)**
1.1			**49.63**
1.2	**0.444**	**153.6**	**49.99**
1.3			**49.87**
2.1			**45.67**
2.2	**0.702**	**127.5**	**43.39**
2.3			**46.23**
3.1			**41.07**
3.2	**0.992**	**98.0**	**41.23**
3.3			**41.39**
4.1			**51.71**
4.2	**0.304**	**159.0**	**51.71**
4.3			**50.79**
5.1			**38.39**
5.2	**1.145**	**82.7**	**38.47**
5.3			**37.83**

Table 1 : The measured length of the mass hook, height from the ceiling to the motion detector, total mass of the cylindrical masses, initial angular amplitude of the pendulum when it's released, height from the bottom of the mass hook to the motion detector, length of the pendulum thread, time taken for 20 oscilations of Experiment A

EXPERIMENT B

Temperature of the room where the experiment takes place (°C) (±0.1)	**22.8**	
Length of the mass hook (cm) (±0.1)	**15.0**	
Length of the pendulum thread (cm) (±0.1)	**176.3**	
Height from the ceiling to the motion detector (m) (±0.001)	**2.134**	
Height from the bottom of the mass hook to the motion detector when the masses are attached (m) (±0.001)	**0.210**	
Total mass of the cylindrical masses that are attached to the mass hook (g)	**250**	
TRIALS	**Initial angular amplitude of the pendulum when it's released (°) (±1)**	**Time taken for 20 oscilations (s) (±0.25)**
1.1		**53.19**
1.2	**10**	**52.55**
1.3		**53.07**
2.1		**52.55**
2.2	**20**	**53.19**
2.3		**52.99**
3.1		**53.39**
3.2	**30**	**53.39**
3.3		**53.39**
4.1		**53.71**
4.2	**40**	**53.95**
4.3		**53.83**
5.1		**53.83**
5.2	**50**	**53.99**
5.3		**54.03**

Table 2 : The measured length of the mass hook, height from the ceiling to the motion detector, total mass of the cylindrical masses, initial angular amplitude of the pendulum when it's released, height from the bottom of the mass hook to the motion detector, length of the pendulum thread, time taken for 20 oscilations of Experiment B

EXPERIMENT C

Temperature of the room where the experiment takes place (°C) (±0.1)	22.8		
Length of the pendulum thread (cm) (±0.1)	120.4		
Length of the mass hook (cm) (±0.1)	15.0		
Height from the ceiling to the motion detector (m) (±0.001)	2.134		
Initial angular amplitude of the pendulum when it's released (°) (±1)	10		
TRIALS	**Total mass of the cylindrical masses (g)**	**Height from the bottom of the mass hook to the motion detector (m) (±0.001)**	**Time taken for 20 oscilations (s) (±0.25)**
1.1	50	0.771	43.75
1.2			43.35
1.3			44.27
2.1	100	0.768	45.43
2.2			45.55
2.3			45.19
3.1	150	0.767	45.51
3.2			45.35
3.3			45.43
4.1	200	0.761	45.23
4.2			45.67
4.3			47.03
5.1	250	0.753	45.23
5.2			45.59
5.3			45.19

Table 3 : The measured length of the mass hook, height from the ceiling to the motion detector, total mass of the cylindrical masses, initial angular amplitude of the pendulum when it's released, height from the bottom of the mass hook to the motion detector, length of the pendulum thread, time taken for 20 oscilations of Experiment C

APPENDIX III

EXPERIMENT A

Temperature of the room where the experiment takes place (°C) (±0.1)	**22.8**
Length of the mass hook (m) (±0.001)	**0.150**
Height from the ceiling to the motion detector (m) (±0.001)	**2.134**
Total mass of the cylindrical masses (kg)	**0.250**
Initial angular amplitude of the pendulum when it's released (°) (±1)	**10**

TRIALS	Height from the bottom of the mass hook to the motion detector when the masses are attached (m) (±0.001)	Length of the pendulum thread (m) (±0.001)	Length of the pendulum when the thread elongates due to the mass (when the pendulum doesn't oscilate) (m) (±0.003)	Percentage of the error caused by the elongation of the thread (Pe) (%)	Time taken for an oscillation (T) (s) (±0.0125)	Average time taken for an oscillation (Tav) (s) (±0.0125)	Theoretical value of time taken for an oscillation (Tt) (±0.0005)	Percentage of the error of the average experimental period for each length (Plength) (%)
1.1					2.4815			
1.2	0.444	1.536	1.540	0.3	2.4995	2.4915	2.4862	0.2
1.3					2.4935			
2.1					2.2835			
2.2	0.702	1.275	1.282	0.5	2.1695	2.2548	2.2652	0.5
2.3					2.3115			
3.1					2.0535			
3.2	0.992	0.980	0.992	1.2	2.0615	2.0615	1.9859	3.8
3.3					2.0695			
4.1					2.5855			
4.2	0.304	1.590	1.680	5.7	2.5855	2.5702	2.5296	1.6
4.3					2.5395			
5.1					1.9195			
5.2	1.145	0.827	0.839	1.5	1.9235	1.9115	1.8243	4.8
5.3					1.8915			

Table 4 : The processed data of length of the mass hook, height from the ceiling to the motion detector, length of the pendulum thread, initial angular amplitude of the pendulum when it's released, total mass of the cylindrical masses, height from the bottom of the mass hook to the motion detector, length of the pendulum when the thread elongates due to the mass (when the pendulum doesn't oscilate), time taken for an oscillation (T), percentage of the error caused by the elongation of the thread, average time taken for an oscillation, percentage of the error of the average experimental period for each length in related SI units

EXPERIMENT B

Temperature of the room where the experiment takes place (°C) (±0.1)		**22.8**		
Length of the mass hook (m) (±0.001)		**0.150**		
Height from the ceiling to the motion detector (m) (±0.001)		**2.134**		
Length of the pendulum thread (m) (±0.001)		**1.763**		
Height from the bottom of the mass hook to the motion detector when the mass is attached (m) (±0.001)		**0.210**		
Length of the pendulum when the thread elongates due to the mass (when the pendulum doesn't oscilate) (m) (±0003)		**1.774**		
Percentage of the error caused by the elongation of the thread (Pe) (%)		**0.6**		
Total mass of the cylindrical masses that are attached to the mass hook (kg)		**0.250**		
Theoretical value of time taken for an oscilation (Tt) (±0.0005)		**2.6719**		
TRIALS	**Initial angular amplitude of the pendulum when it's released (°) (±1)**	**Time taken for an oscilation (T) (s) (±0.0125)**	**Average time taken for an oscilation (Tav) (s) (±0.0125)**	**Percentage of the error of the average experimental period for each angle (Pangle) (%)**
1.1		**2.6595**		
1.2	**10**	**2.6275**	**2.6468**	**0.9**
1.3		**2.6535**		
2.1		**2.6275**		
2.2	**20**	**2.6595**	**2.6455**	**1.0**
2.3		**2.6495**		
3.1		**2.6695**		
3.2	**30**	**2.6695**	**2.6695**	**0.1**
3.3		**2.6695**		
4.1		**2.6855**		
4.2	**40**	**2.6975**	**2.6915**	**0.7**
4.3		**2.6915**		
5.1		**2.6915**		
5.2	**50**	**2.6995**	**2.6975**	**1.0**
5.3		**2.7015**		
Average of time taken for an oscilation of all trials (Tall) (s) (±0,0125)		**2.6702**		
Percentage of the error of the overall average experimental period (Pav) (%)		**0.1**		

Table 5 : The processed data of length of the mass hook, height from the ceiling to the motion detector, length of the pendulum thread, initial angular amplitude of the pendulum when it's released, total mass of the cylindrical masses, height from the bottom of the mass hook to the motion detector, length of the pendulum when the thread elongates due to the mass (when the pendulum doesn't oscilate), time taken for an oscilation (T), percentage of the error caused by the elongation of the thread, average time taken for an oscilation, percentage of the error of the average experimental period for each angle, average of time taken for an oscilation of all trials, percentage of the error of the overall average experimental period in related SI units

EXPERIMENT C

Temperature of the room where the experiment takes place (°C) (±0.1)	22.8
Length of the mass hook (m) (±0.001)	0.150
Height from the ceiling to the motion detector (m) (±0.001)	2.134
Length of the pendulum thread (m) (±0.001)	1.204
Initial angular amplitude of the pendulum when it's released (°) (±1)	10
Theoretical value of time taken for an oscilation (Tt) (±0.0005)	2.2012

TRIALS	Total mass of the cylindrical masses (kg)	Height from the bottom of the mass hook to the motion detector (m) (±0.001)	Length of the pendulum when the thread elongates due to the mass (when the pendulum doesn't oscilate) (m) (±0.003)	Percentage of the error caused by the elongation of the thread (Pe) (%)	Time taken for an oscilation (T) (s) (±0.0125)	Average time taken for an oscilation (Tav) (s) (±0.0125)	Percentage of the error of the average experimental period for each mass (Pmass) (%)
1.1	0.050	0.771	1.213	0.7	2.1875	2.1895	0.5
1.2					2.1675		
1.3					2.2135		
2.1	0.100	0.768	1.216	1.0	2.2715	2.2695	3.1
2.2					2.2775		
2.3					2.2595		
3.1	0.150	0.767	1.217	1.1	2.2755	2.2715	3.2
3.2					2.2675		
3.3					2.2715		
4.1	0.200	0.761	1.223	1.6	2.2615	2.2988	4.4
4.2					2.2835		
4.3					2.3515		
5.1	0.250	0.753	1.231	2.2	2.2615	2.2668	3.0
5.2					2.2795		
5.3					2.2595		

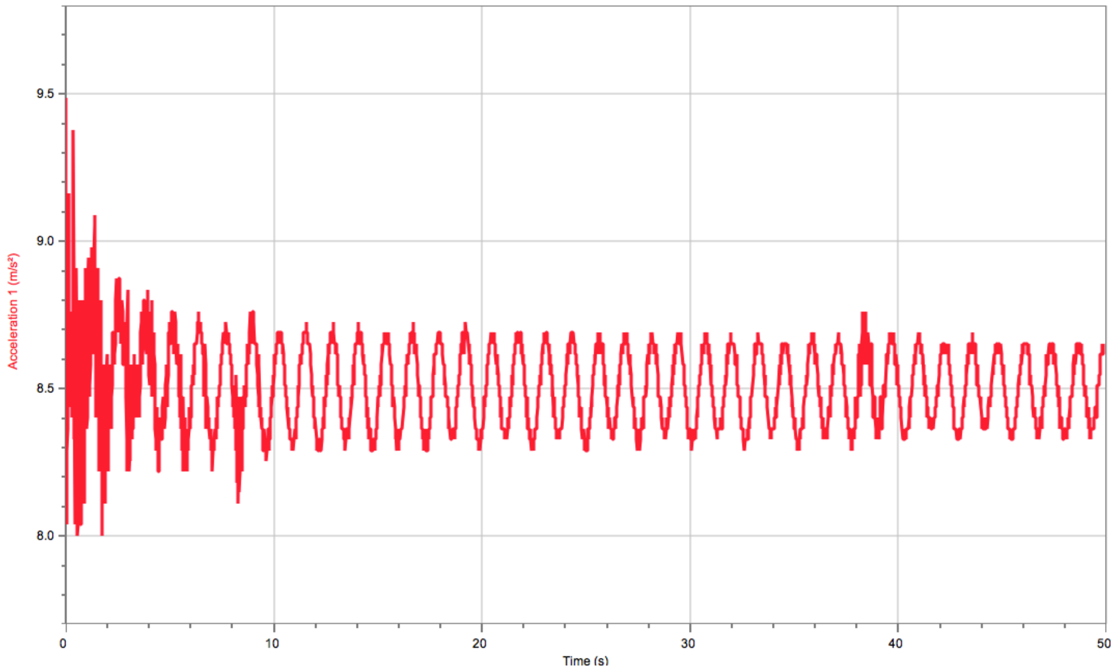
Average of time taken for an oscilation of all trials (Tall) (s) (±0,0125)	2.2592
Percentage of the error of the overall average experimental period (Pav) (%)	2.6

Table 6 : The processed data OF length of the mass hook, height from the ceiling to the motion detector, length of the pendulum thread, initial angular amplitude of the pendulum when it's released, total mass of the cylindrical masses, height from the bottom of the mass hook to the motion detector, length of the pendulum when the thread elongates due to the mass (when the pendulum doesn't oscilate), time taken for an oscilation (T), percentage of the error caused by the elongation of the thread, average time taken for an oscilation, percentage of the error of the average experimental period for each mass, average of time taken for an oscilation of all trials, percentage of the error of the overall average experimental period in related SI units

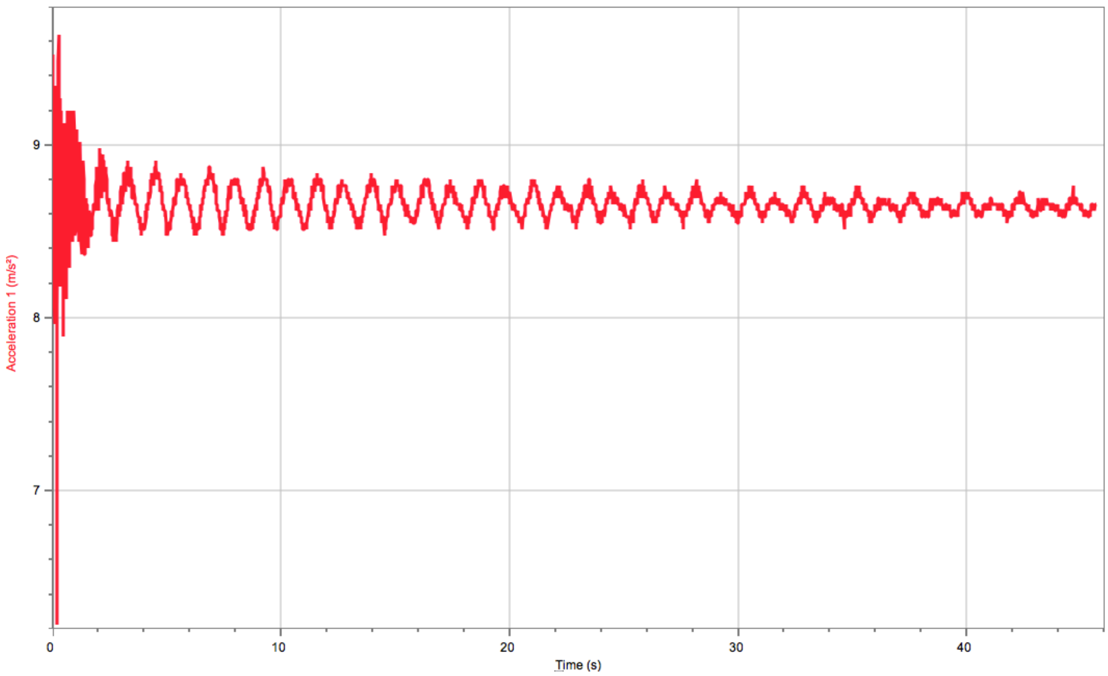
APPENDIX IV

Since 100 samples per second are collected via Vernier Accelerometer for each and every trial of the experiments, overall 200000 acceleration values are recorded (215664, to be precise.). Therefore, indicating all those values along with 15 graphs for each experiment (overall 45 graphs) would make the essay hard to be followed. For that reason, all the calculations were based on the collected data of the 20 oscilations (see Appendices II and III). What's more, for each independent value of the experiments, one trial –which has the closest value of 20T to the mean value- is presented below.

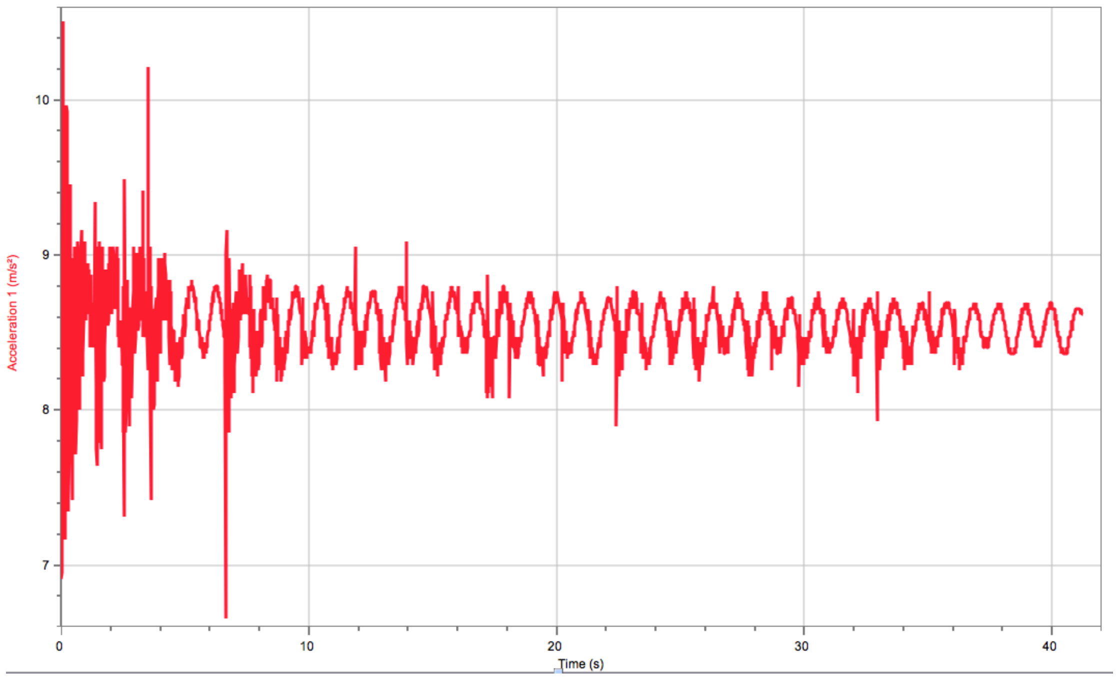
EXPERIMENT A



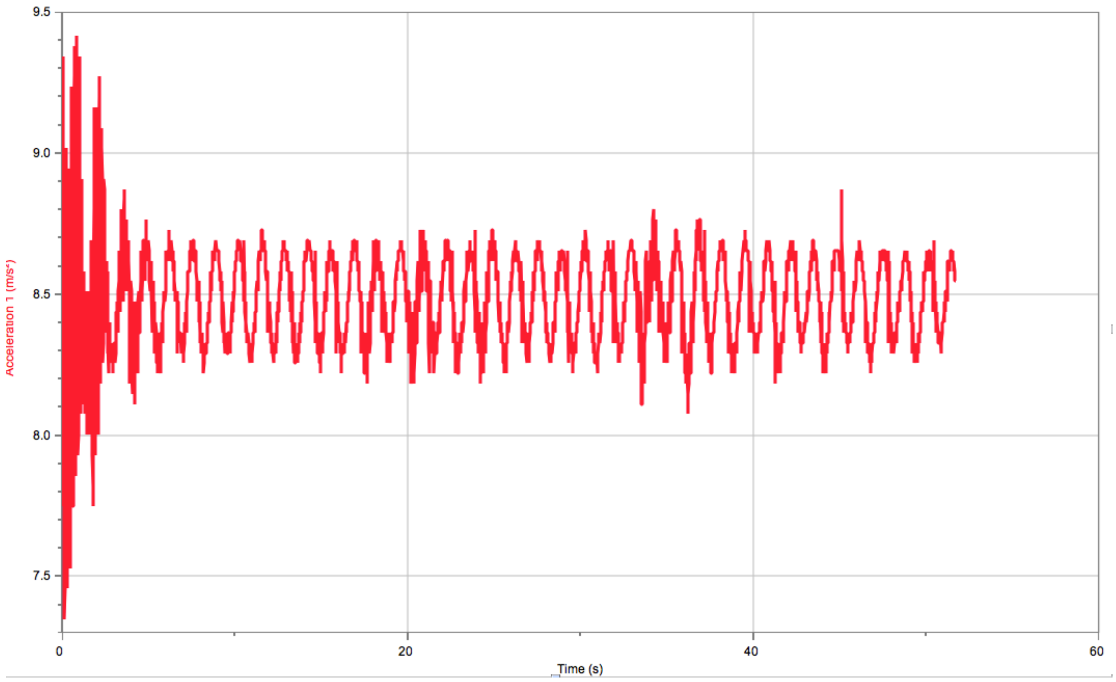
Graph 1 : Acceleration versus time graph for Trial 1.2 of Experiment A



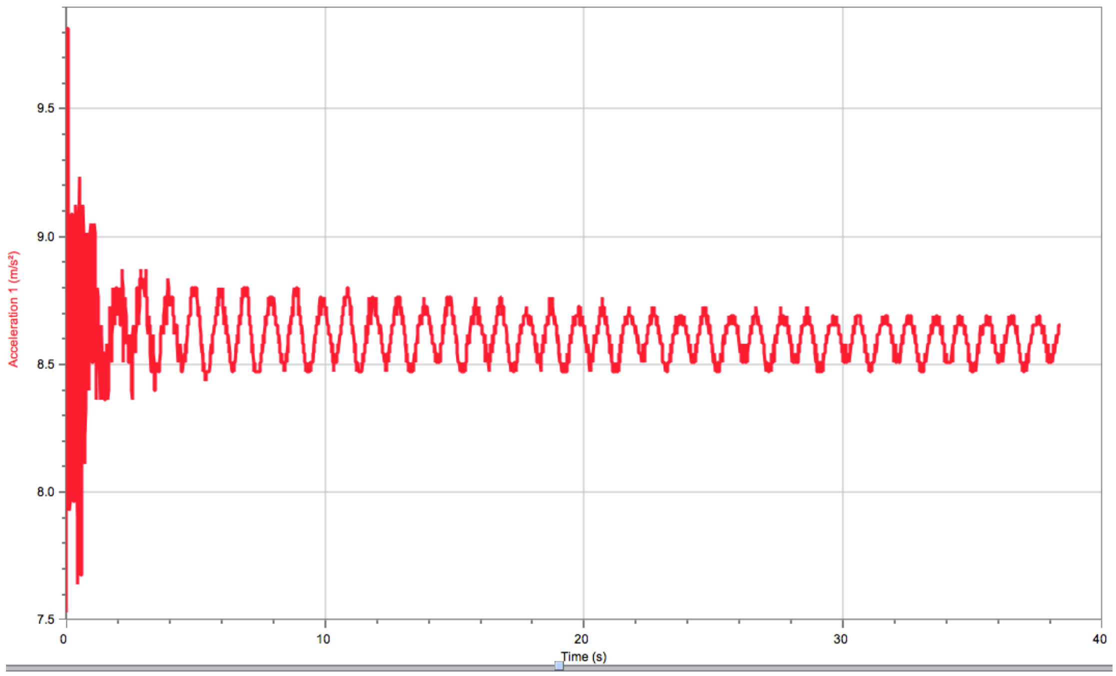
Graph 2 : Acceleration versus time graph for Trial 2.1 of Experiment A



Graph 3 : Acceleration versus time graph for Trial 3.2 of Experiment A

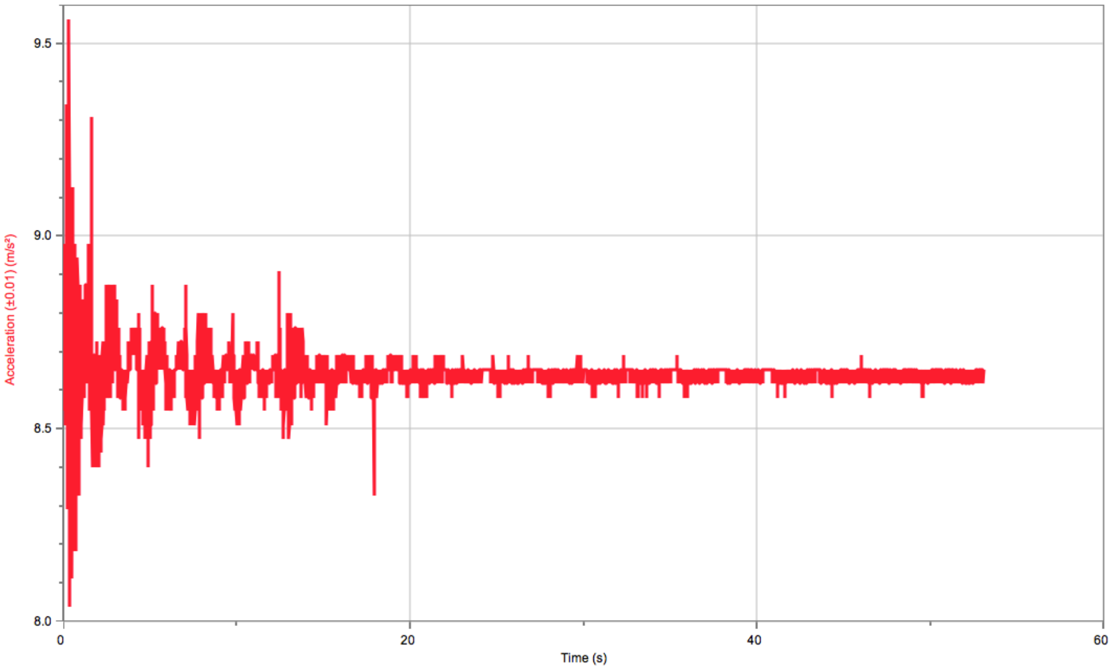


Graph 4 : Acceleration versus time graph for Trial 4.1 of Experiment A

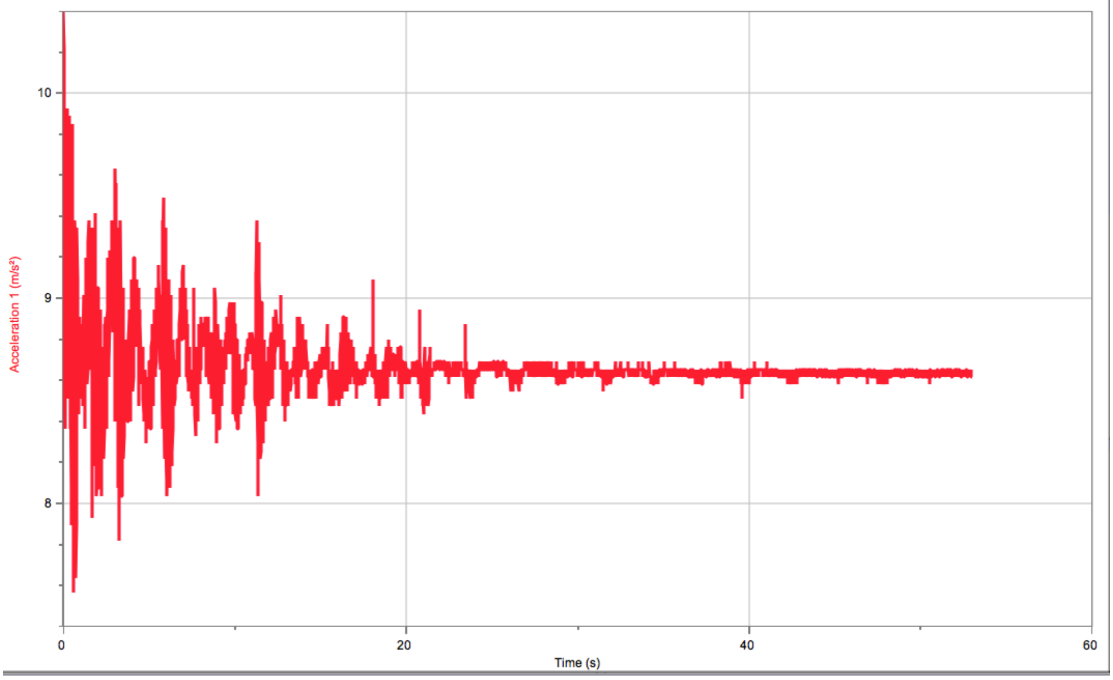


Graph 5 : Acceleration versus time graph for Trial 5.1 of Experiment A

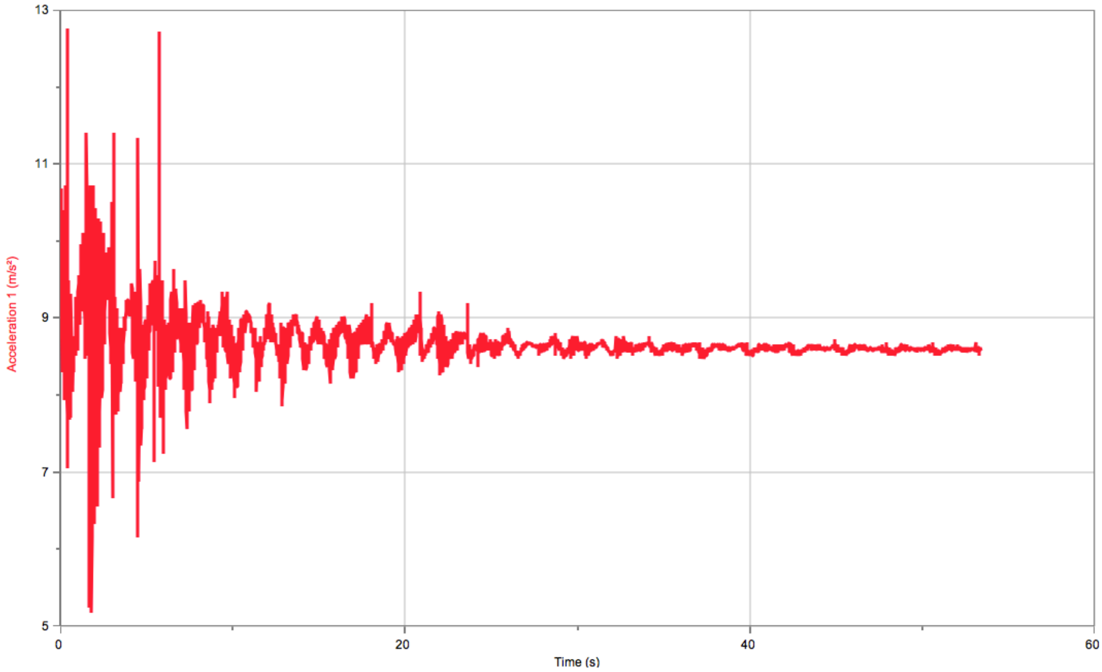
EXPERIMENT B



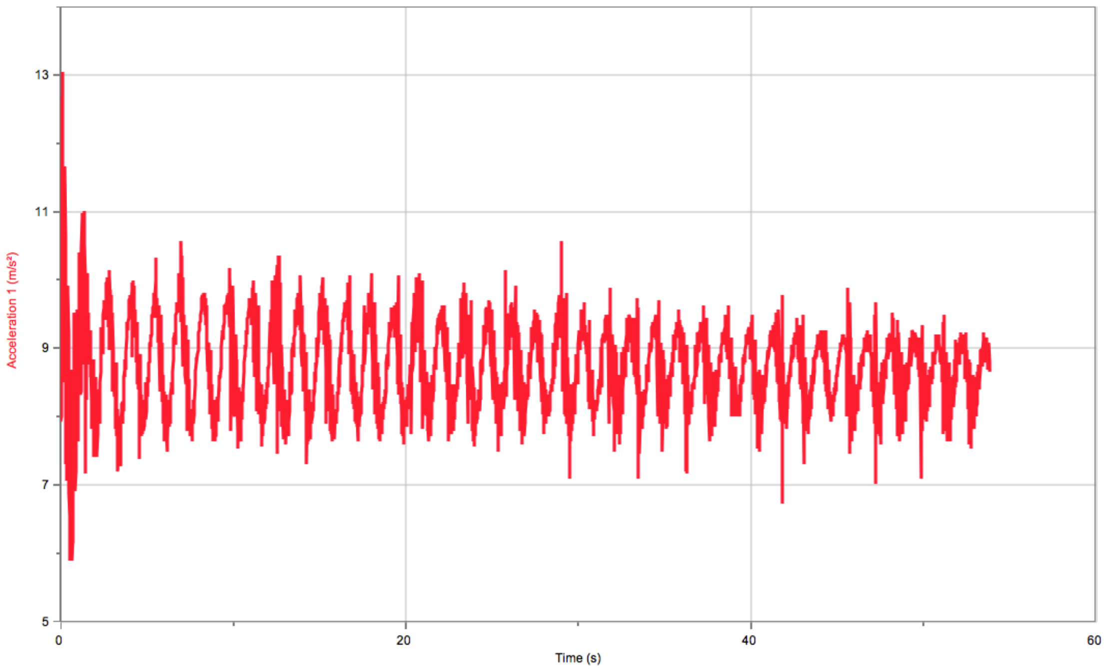
Graph 6 : Acceleration versus time graph for Trial 1.3 of Experiment B



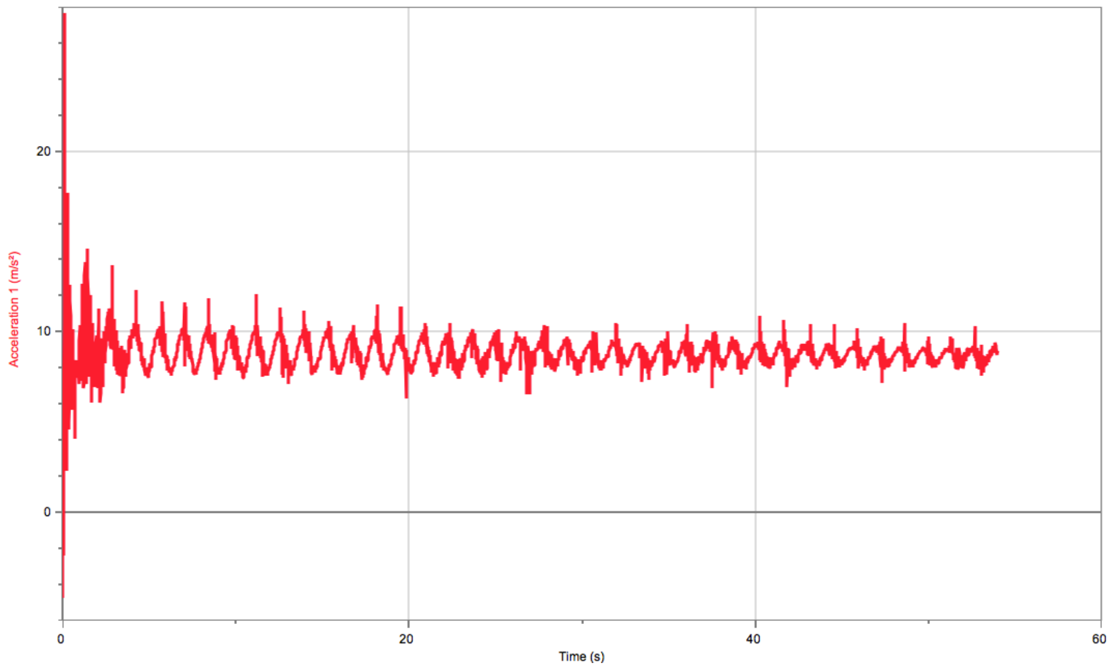
Graph 7 : Acceleration versus time graph for Trial 2.3 of Experiment B



Graph 8 : Acceleration versus time graph for Trial 3.1 of Experiment B

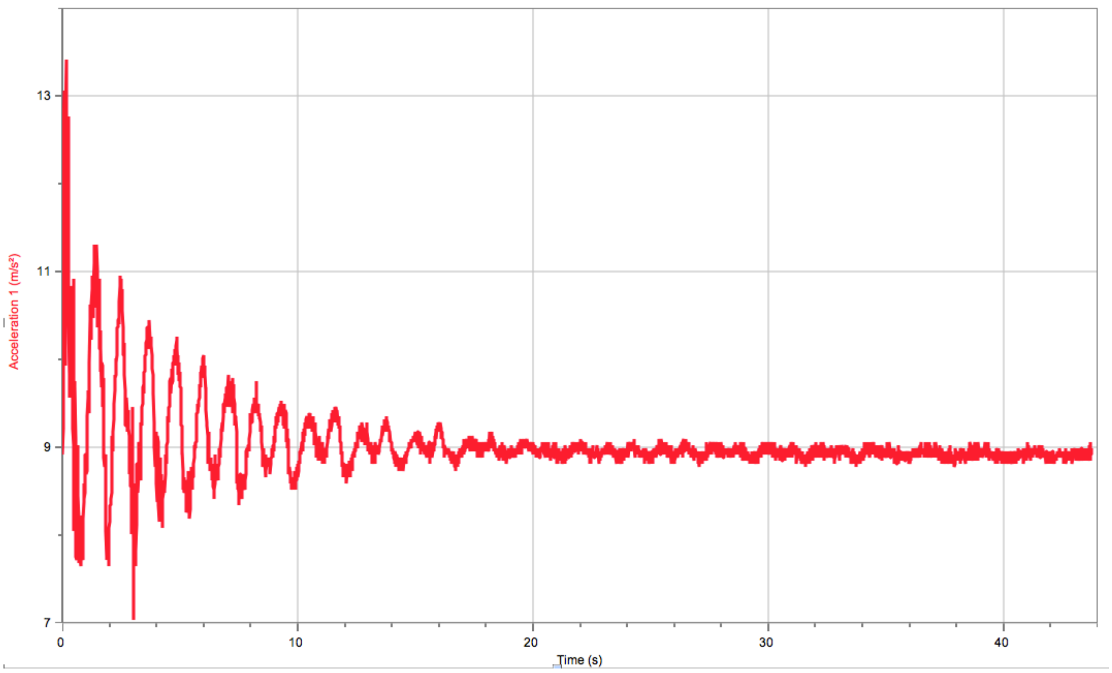


Graph 9 : Acceleration versus time graph for Trial 4.3 of Experiment B

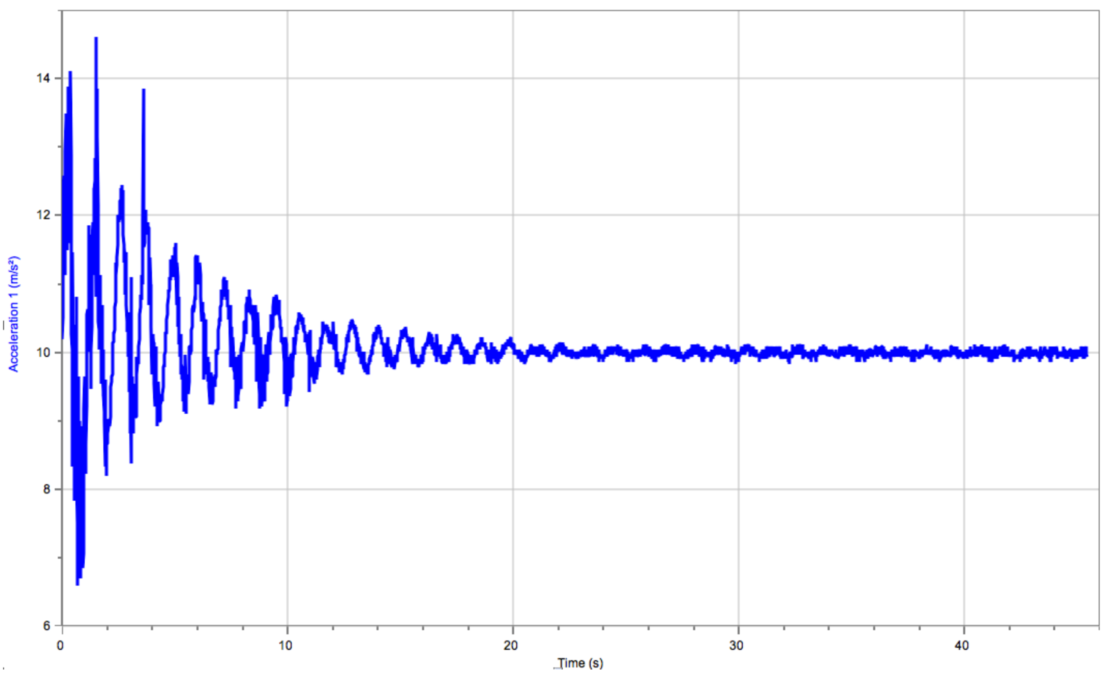


Graph 10 : Acceleration versus time graph for Trial 5.2 of Experiment B

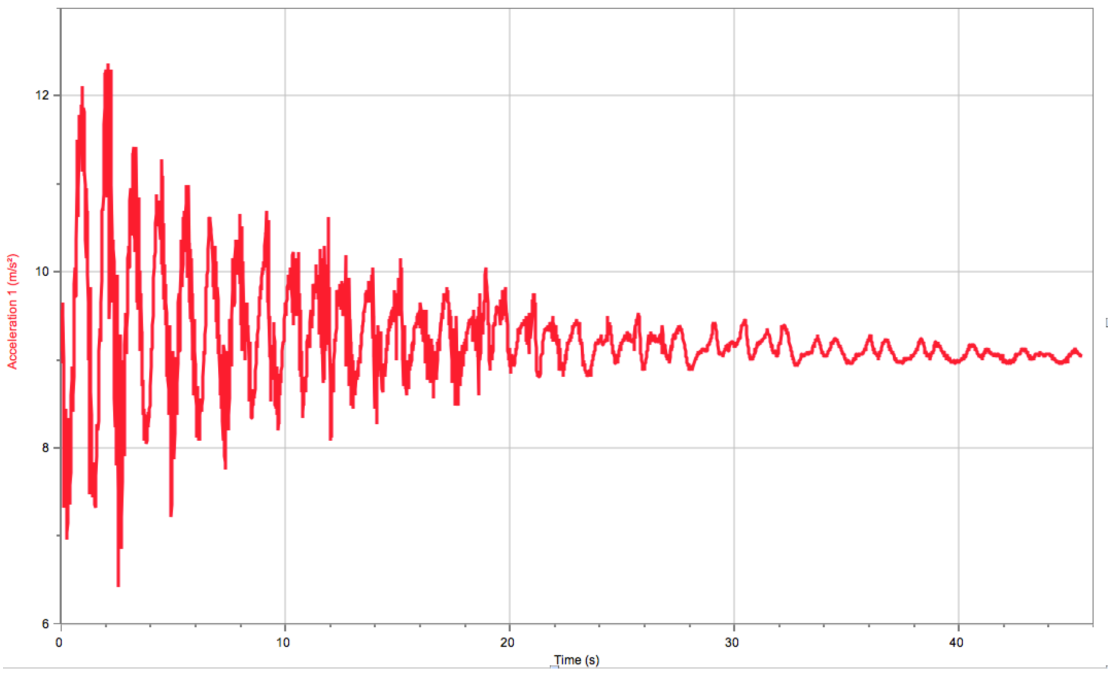
EXPERIMENT C



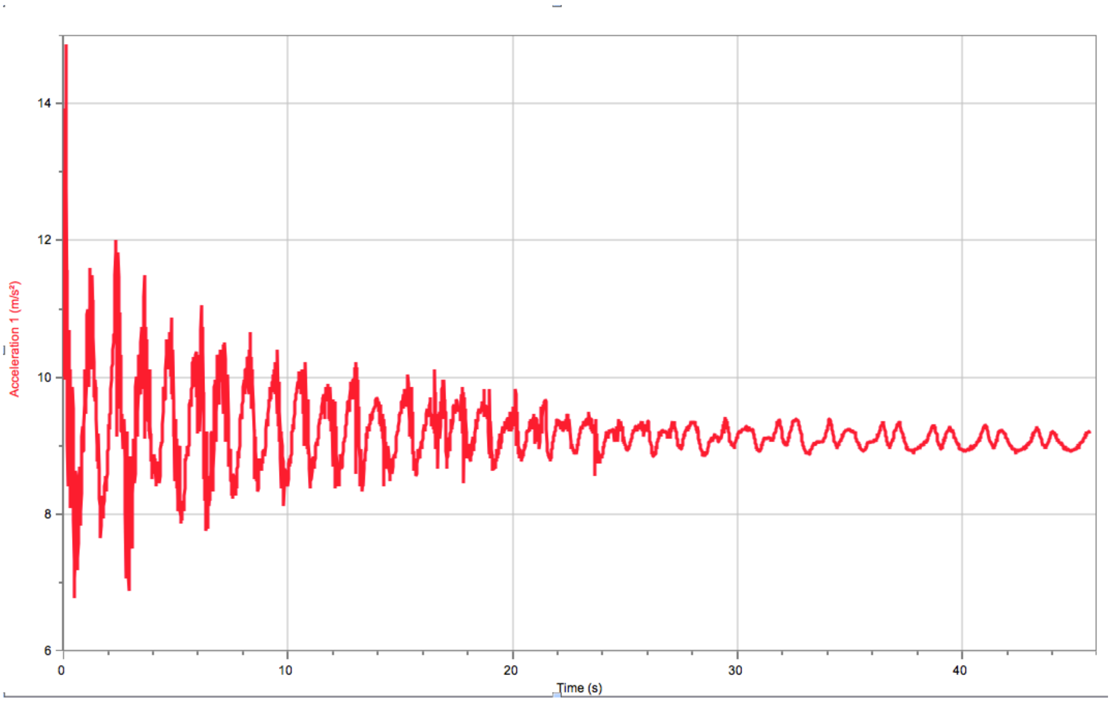
Graph 11 : Acceleration versus time graph for Trial 1.1 of Experiment C



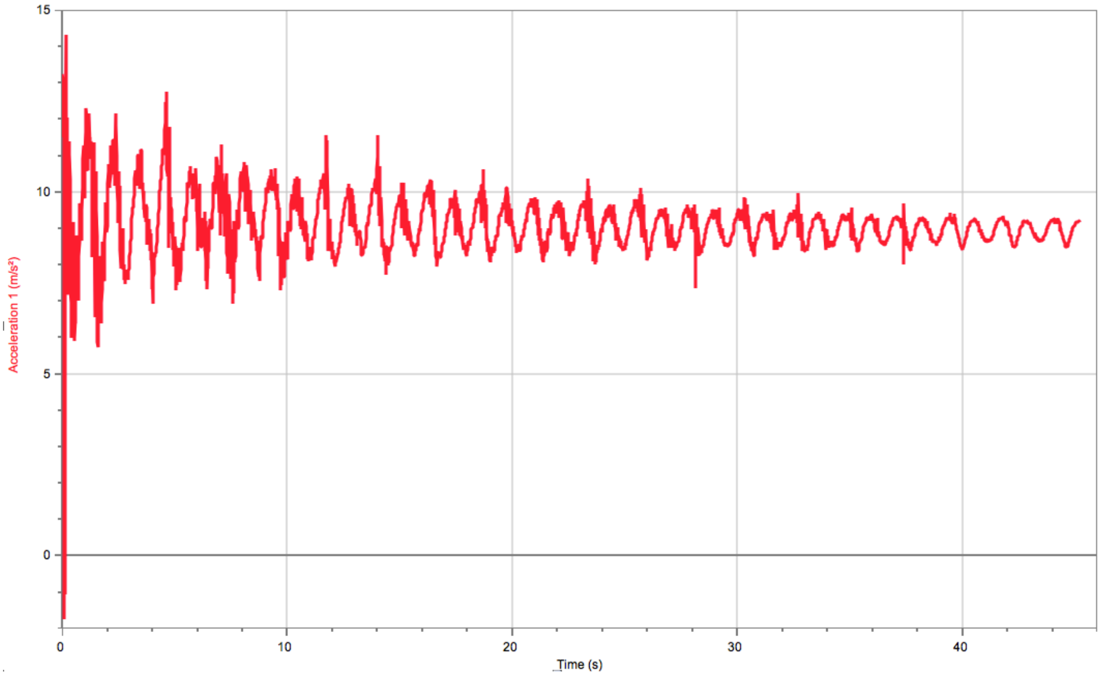
Graph 12 : Acceleration versus time graph for Trial 2.1 of Experiment C



Graph 13 : Acceleration versus time graph for Trial 3.3 of Experiment C



Graph 14 : Acceleration versus time graph for Trial 4.2 of Experiment C



Graph 15 : Acceleration versus time graph for Trial 5.1 of Experiment C

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