

## **IB HL Physics Extended Essay**

### **Title: Investigation of Terminal Velocity**

**How does Activation Energy of Viscous Flow (kJ/mol) of different fluids and the fluid temperature (K) effect the Vertical Terminal Velocity (m/s) of a metal marble in motion?**

**Word Count: 3988**

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## **1. Introduction**

Lubricants have a wide variety of usage, one of them being industrial applications and mechanical car parts. These lubricants allow the system to function with less friction and decrease the wear caused by the friction. Choosing the right lubricant is dire as high viscosity may cause a high viscosity lubricant increases energy consumption, and a low viscosity lubricant increases the wear on the machinery. The effectiveness of the lubricants depends on the viscosity of the fluid, and the viscosity of the changes as the temperature of the fluid increases. As the machine's temperature increases, the lubricants' temperature also increases. The change of viscosity as temperature increases is governed by the activation energy of viscous flow.

In this experiment the terminal velocity of a metal marble in various fluids at various temperatures will be used to determine the correlation between the terminal velocity of the marble and the temperature of the fluid. The activation energy of viscous flow of different fluids will govern the behavior of the fluid at different temperatures.

## **2. Background Information:**

### **2.1. Behavior of Sinking Objects in Fluids:**

An object that is sinking in a fluid is governed by the gravitational force, buoyant force, and the drag force. These three forces are the factors of determining how the object falls, when the object reaches terminal velocity and the terminal velocity. To understand the influence of these factors, buoyancy, viscosity, Reynolds number and terminal velocity must be investigated.

Gravitational Force ( $F_g$ ):

The gravitational force on a sinking object is given by Newton's Second Law.

$$F_G = mg$$

The value  $m$  is the mass (kg) of the object and the value  $g$  ( $\frac{m}{s^2}$ ) is the gravitational acceleration acting on the body. The gravitational force acts downwards.

Drag Forces:

As the body moves through the fluid a force opposes the force of direction. This force is called drag force. These drag forces are produced due to the relative motion of the fluid around the body.

$$F_d = \frac{1}{2} C_d \rho A v^2$$

$C_d$ : is the drag coefficient of the fluid (the value is dimensionless)

$\rho$ : is the density of the fluid. ( $kg/m^3$ )

$A$ : is the reference area of the body. ( $m^2$ )

$v$ : is the velocity of the object relative to the speed of the surrounding fluid. ( $m/s$ )

Drag is directly proportional to the square of velocity. Even though drag increases with speed, the acceleration of the body decreases. This is due to Newton's Second Law. Newton's Second Law is;

$$F_{net} = ma$$

$m$ : mass of the body (kg)

$a$ : acceleration of the body ( $m/s^2$ )

$F$ : the net force acting on the body (N)

When drag is applied the  $F_{net}$  value decreases, however the mass stays constant. The decrease in net force applied is because as the speed increases the drag force increases, decreasing the net force on the body. This decrease in net force applied on the body causes a decrease in acceleration. The acceleration in the formula is for the body's movement in the fluid, it should not be confused with the gravitational acceleration. The force that gravitational acceleration causes is opposed by the drag, this is why acceleration must be lower than the gravitational acceleration. There are no other forces that apply downwards.

## 2.2. Reynolds Number:

Reynolds Number is a quantity that predicts how flow patterns occur in different fluids. The Reynolds number is the ratio between inertial forces and viscous forces of fluid.

$$Re = \frac{\rho v L}{\mu}$$

$\rho$ : is the density of the fluid the object is moving in. (kg/m<sup>3</sup>)

$v$ : is the relative velocity of the object to the fluid (m/s)

$L$ : is the characteristic length -usually the volume of the system divided by its surface-of the object. (m)

$\mu$ : is the dynamic viscosity of the fluid (Pa·s)

In flows Reynolds Numbers are used to determine if a flow is either laminar or turbulent. For values under 2000 for Reynolds Number the flow is generally laminar, which is considered as smooth patterns. There are generally little to no mixing between layers of the fluid. However, when the Reynolds number is higher than 3000 the flow is identified as turbulent. Turbulent flow is more chaotic and there are more changes in flow velocity and pressure. There is disruption between the parallel

layers of the fluid.

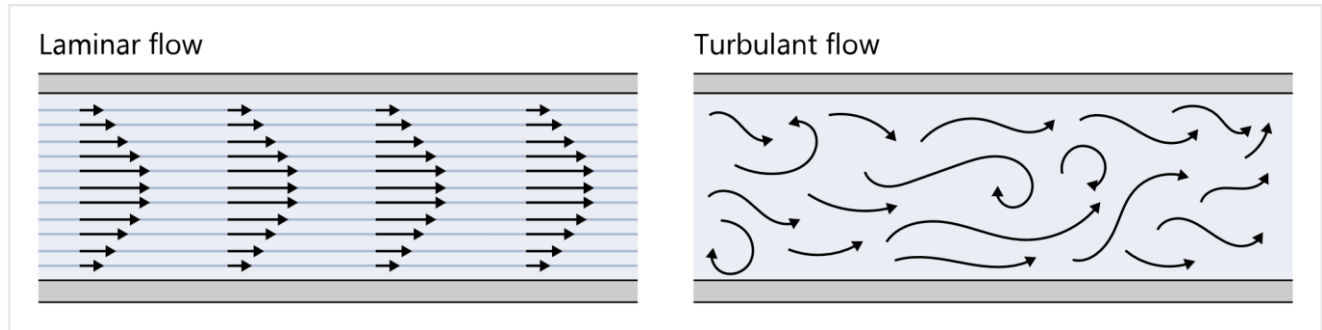


Image 1: Flow Patterns of Turbulent and Laminar Flow

Laminar Flow generally has near to constant velocity in a pipe. However turbulent flow tends to have constantly changed velocity. It can be stated that at lower Reynolds Numbers the flow can be predicted better, and the flow is highly resistant to disturbances.

### 2.3. Stoke's Law:

Stoke's Law applies to small spherical objects that are in motion in viscous fluids.

$$Fd = 6\pi\eta rv$$

$F_d$ : is the drag force

$\eta$ : is the viscosity of the fluid (Pa·s)

$v$ : is the speed of the object (m/s)

$r$ : is the radius of the spherical object (m)

The formula applies to objects that are in motion that are opposed by the viscous resistance. However, the conditions are laminar, and the Stoke's Law can only be used for flows with lower Reynolds Number then 1 ( $Re < 1$ ). The assumptions made by Stoke's Law also contain the knowledge that

the body in motion must be spherical, however in this case the only assumption that is not met is the Reynolds Number. Because the Stoke's Law does not apply, we must use drag force equation.

#### **2.4. Buoyancy:**

Buoyancy is the upward motion exerted by the fluid on the body. The force opposes the weight of the body. This force is caused by a pressure difference between the top and the bottom of the body. When the body is floating at a constant depth the magnitude of the buoyant force and weight is equal. If the buoyant forces are equal to the gravitational force the motion of the body is stopped (velocity is equal to zero), this is the case of a body being afloat. However, because the metal marble is denser than the fluids that are being submerged in the marble will sink. However buoyant forces still decrease the overall net force on the body by creating a reaction force upwards.

#### **2.5. Drag Force Equation:**

Because both the Archimedes Law (because the system is in motion) and the Stoke's Law does not apply we must use drag force coefficient;

$$F_d = \frac{1}{2} C_d \rho v^2 A$$

$F_d$ : Drag Force acting on the body

$C_d$ : Drag Coefficient (this value depends on shape, Reynold Numbers and the roughness of the surface of the body)

$\rho$ : fluid density (kg/m<sup>3</sup>)

$A$ : is the reference area of the body. (m<sup>2</sup>)

$v$ : is the velocity of the object relative to the speed of the surrounding fluid. (m/s)



This equation is preferred over the Stoke's Law as it takes inertial forces that viscous fluids have into account.

## **2.6. Viscosity and Motion:**

Viscosity is the measure of resistance of fluid in a flow. Viscosity affects the flow rate and the movement of a body in such fluids. Higher the viscosity slower the body moves, as the Drag coefficient increases. Highly viscous fluids significantly lower the terminal velocity.

## **2.7 Terminal Velocity:**

Terminal velocity is the maximum velocity that an object can reach for a falling body. Terminal velocity is reached when the resistance of the medium is maximum. When terminal velocity is reached, all net forces on the body are zero. The forces acting on the body are gravitational forces, buoyant forces and drag forces.

$$F_G = mg$$

$m$ : mass of the body (kg)

$g$ : gravitational acceleration ( $\sim 9.81 \frac{m^2}{s}$ )

The Buoyant Forces acting on the body in descent are:

$$F_B = \rho V g$$

$\rho$ : fluid density ( $\frac{kg}{m^3}$ )

$V$ : volume of the body ( $m^3$ )

$g$ : gravitational acceleration ( $\sim 9.81 \frac{m^2}{s}$ )

The last force acting on the body is the drag force;

$$F_d = \frac{1}{2} C_d \rho v^2 A$$

At terminal velocity all forces are balanced;

$$F_G = F_B + F_d$$

$$mg = \rho V g + \frac{1}{2} C D \rho v_t^2 A$$

$$v_t = \sqrt{\frac{2g(m - \rho_f V)}{(C_d \rho_f A)}}$$

In highly viscous fluids, the body will experience higher drag forces. A higher drag force will decrease the net force as drag forces oppose the direction of motion. A greater resistance means lower terminal velocity and a slower descent. Viscosity and terminal velocity are inversely proportional.

## 2.8. Effect of Temperature on Viscosity:

Temperature effects viscosity because as temperature increases the intermolecular forces of the fluid decrease, this reduces the resistance of the flow. The Arrhenius type equation below shows the temperatures dependence on viscosity.

$$\mu = \mu_0 e^{\frac{E_a}{RT}}$$

$\mu_0$ : reference viscosity at a standard temperature (mPa s)

$E_a$ : activation energy of viscous flow (J/mol)

$R$ : universal gas constant (J·mol<sup>-1</sup>·K<sup>-1</sup>)

$T$ : absolute temperature of the fluid (K)

From the equation it can be derived that as temperature increases the viscosity decreases, and from past formulas it is known that less resistance means higher terminal velocity.

## **2.9. Effect of Temperature and Activation Energy on Terminal Velocity:**

Activation energy of viscous flow is the measure of how resistant a fluid is against flow. It comes from intermolecular forces. As a fluid has higher intermolecular forces, the higher its activation energy of viscous flow becomes, and a higher energy is required to sustain flow. The activation energy of viscous flow refers to the minimum energy required for molecules to defeat these intermolecular forces, and move over each other.

As the Arrhenius type formula suggest there is a direct correlation between temperature and viscosity. This correlation is governed by the activation energy of viscous flow. A higher  $E_a$  suggests that viscosity is highly dependent and is sensitive to changes in temperature, therefore significantly changing terminal velocity reached.

$$v_t = \sqrt{\frac{2(mg - \rho_f Vg)}{(C_d \rho_f A)}}$$

$C_d$  is directly correlated with the Reynolds number.

$$Re = \frac{\rho v L}{\mu}$$

At high Reynolds numbers the correlation becomes;

$$C_d = k Re^{-n}$$

Now the new Reynolds number correlation can be used to substitute for  $C_d$  for the terminal velocity formula.

$$C_d = k \left( \frac{\rho v_t L}{\mu} \right)^{-n} = k \mu^n \rho^n L^{-n} v_t^{-n}$$

$$v_t = \sqrt{\frac{k \mu^n \rho^n L^{-n} v_t^{-n}}{k \mu^n \rho^n L^{-n} v_t^{-n} \rho A}}$$

The value of terminal velocity is re-written with viscosity as a variable in the formula. Now the viscosity can be replaced by Arrhenius-type equation;

$$\mu = \mu_0 e^{\frac{E_a}{RT}}$$

$$v_t = \sqrt{\frac{k \mu^n \rho^n L^{-n} v_t^{-n}}{k \mu_0^n e^{\frac{nE_a}{RT}} \rho^n L^{-n} v_t^{-n} \rho A}}$$

Therefore, the correlation between temperature of a fluid and the terminal velocity of the body on free-fall is;

$$v_t \propto \sqrt{e^{\frac{nE_a}{RT}}}$$

$$v_t \propto e^{\frac{nE_a}{2RT}}$$

$$\ln(v_t) \propto \frac{E_a}{R} \frac{1}{T}$$

As temperature increases the terminal velocity exponentially increases due to viscosity decreasing.

### **3. Experimentation and Procedure:**

#### **3.1. Aim:**

The aim of this research is to investigate the effect of temperature and the activation energy of fluids on terminal velocity reached by the metal marble dropped in it. Through investigating different fluids, the correlation between activation energy and temperature is recognized, and through different temperatures it is aimed to investigate the effect of temperature on the terminal velocity. Investigating both temperature and activation energy, all the independent variables that affect the change in viscosity are investigated. The effect of changed velocity is then investigated through the change in terminal velocity.

#### **3.2. Hypothesis:**

It is hypothesized that as the temperature of a fluid increases the viscosity will increase; therefore, a lower terminal velocity will be reached, at a shorter time frame. As viscosity increases the resistance will increase, causing higher drag forces and eventually reach the terminal velocity.

Additionally, as the activation energy of viscous flow increases the fluid will be more effected by changes in temperature.

### 3.3. Variables:

#### Dependent Variable:

The terminal velocity reached by the metal marble.

#### Independent Variables:

The activation energy of the fluid. (glycerin, vegetable oil, distilled water and vinegar)

Temperature of the fluid. (20°C,30°C,40°C,50°C, 60°C ,70°C). These temperatures are picked as the volumetric flask used has the maximum temperature marked as 80°C. These values are picked for safety and avoiding spills.

Controlled Variables	Method of Control	Reason of Control
Graduated Cylinder	The same graduated cylinder is used to precisely analyze the distance covered by the marble.	The same graduated cylinder is used to ensure a higher precision of distance measurement.
Measurement Tools	The camera, thermometer and the ruler are kept constant to minimize systematic errors.  Also, the camera angle and the distance from camera to	The same measurement tools are used to reduce systematic errors from occurring. Changin the angle or the distance of the camera would result in incorrect data analysis.

	volumetric flask is kept constant.	
Metal Marbles	3 identical metal marbles are used to carry out experiments	A change in diameter or an imperfection of the curvature changes the drag forces, changing the terminal velocity.

### 3.4. Apparatus:

Apparatus used to measure the values are.

1. Ruler ( $\pm 0.5$  mm): used to measure the length of the volumetric flask.
2. Digital Needle Probe Thermometer ( $\pm 0.1^\circ\text{C}$ ): Used to measure the temperature of the fluids.
3. High Fps (frame per second) Camera record the motion of the marble to then analyze.

Apparatus used to prepare the materials.

4. Metal Marble: Used as the test body for the experimentation.
5. Heat Source: used to apply heat to the fluids to achieve the aimed temperatures.
6. Beakers (500ml): Used to maintain the fluids.
7. Funnel: To transfer material from heat source to the volumetric flask.
8. Tongs: To drop the marbles into the fluids.

Apparatus used to analyze the experiment:

9. LoggerPro 3.16 Movie Analysis Software Program: All the recordings done on the high fps camera are then processed by the software to gain velocity values.

### **3.5. Materials:**

The different fluids that were used to conduct experiments are:

1. Vegetable oil (500ml)
2. Vinegar (500ml)
3. Glycerin (500ml)
4. Distilled Water (500ml)

### **3.6. Ethical Considerations and Risk Assessment:**

#### Environmental Considerations

1. Disposal of the Fluids: After using the fluids for experimentation the oil, vinegar, distilled water and glycerin are no longer food grade, so vinegar and water was safely disposed down the drain and the oil and glycerin was sealed in bags and given to proper authorities. (Local City Halls generally have disposal units)

#### Ethical Considerations:

2. No animal, living tissue or vegetation was used during the experimentation, therefore there are no ethical considerations to be made.

#### Health Considerations:

3. Preventing Spills: To prevent spills a funnel can be used. In the case of a spill, immediate cleanup should be done.

#### Safety Considerations:



4. Electrical Concerns: The camera is placed at a distance where no fluid spill can affect the hardware (at 0.5 meters).
5. Usage of Hot Fluids: Fluids are heated carefully and with a controlled source Heat resistant gloves are used, and it is made sure that the volumetric flask used can withstand the heat exerted from the fluids.

### 3.7. Experiment Setup:

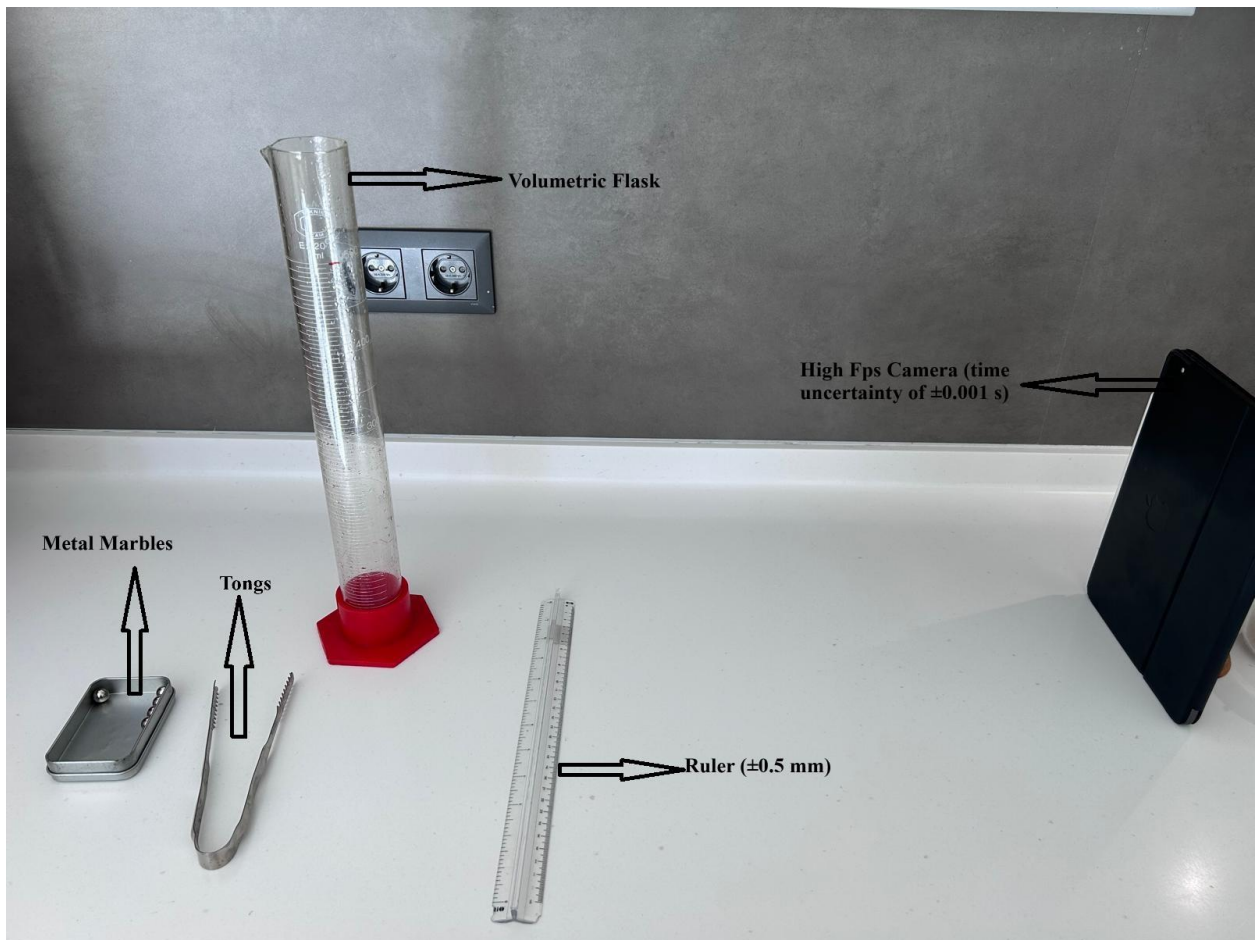


Image 2: Experimental Setup

### 3.8. Methodology:

#### Step 1 Preparation of The Fluids:

For each fluid use a heat source to increase the temperature of the aimed temperature (20°C, 30°C, 40°C, 50°C, 60°C, 70°C). To measure use a digital needle probe thermometer to ensure accurate results. Stir the fluids for even heat distribution. To make sure the heat loss does not affect the experiment make sure the temperature is a few degrees higher than the aimed temperature. Then by using a funnel and transferring the liquid to the graduated cylinder. Wait until the fluid cools down to the required temperature.

#### Step 2 Recording the Motion Body:

A high fps camera is positioned at perpendicular angle to the volumetric flask for accurate recordings. The ruler is used to create a reference length for software analysis. Then the video starts, marble is dropped, and the recordings are imported to LoggerPro.

#### Step 3 Dropping the Marble:

When the fluid is ready carefully position the marble just enough to break the surface tension. The marbles is again released from the same height to eliminate any potential energy differences. The marble should be dropped with no initial velocity or any force applied on it to ensure controlled experimentation.

#### Step 4: Analyzing the Experiment:

In the LoggerPro software the video is stopped frame by frame by the software and the starting point of the ball is selected for each of the measurements. The same reference point is selected to make sure only the change in position is considered. The software is then able to measure the instantaneous velocity of the marble. Then from the graph the terminal velocity is identified.

#### Step 5 Repetition:

To improve accuracy for every temperature of every fluid 3 trials are carried out. This ensures reliable results.

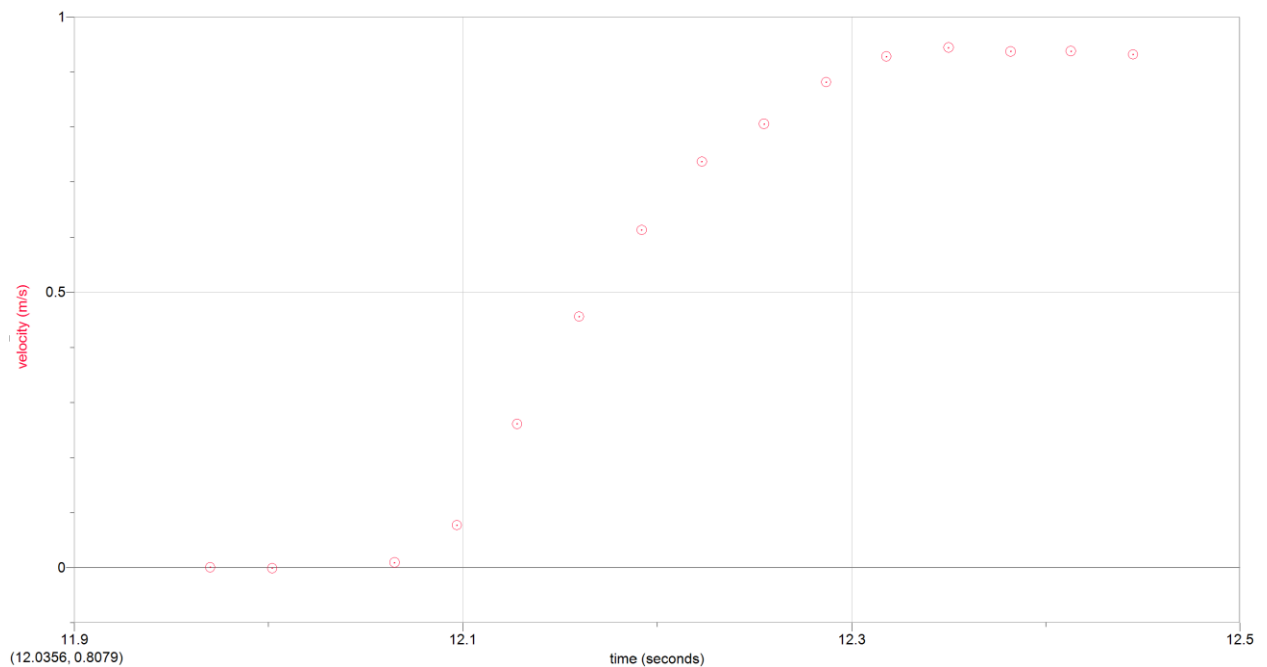
#### Step 6 Disposal:

After the experimentation is completed vegetable oil and glycerin are no longer safe to consume so they are sealed and given to the proper authorities whereas vinegar is drained.

### **4.Data Analysis:**

#### **4.1. Data Collection:**

Data is collected through the loggerpro analysis software, and the last 5 velocity values are picked as this is the point that generally terminal velocity is reached. A sample is given below:



#### 4.2. Raw Data Table:

Temperature	Instantaneous Velocity of Last 5 Frames				
	1	2	3	4	5
20°C± 0.1°C	0.9284	0.9442	0.9371	0.9228	0.9317
	0.9149	0.9295	0.9186	0.9231	0.9173
	0.9352	0.9214	0.9318	0.9198	0.9276
30°C± 0.1°C	1.3413	1.3248	1.3374	1.3291	1.3327
	1.3275	1.3415	1.3356	1.3242	1.3299
	1.3349	1.3252	1.3379	1.3305	1.3392
40°C± 0.1°C	2.4289	2.4391	2.4242	2.4312	2.4358
	2.4214	2.4325	2.4276	2.4389	2.4235
	2.4333	2.4208	2.4352	2.4265	2.4297
50°C± 0.1°C	4.9857	4.9741	4.9902	4.9785	4.9821
	4.9758	4.9893	4.9808	4.9751	4.9875
	4.9843	4.9769	4.9916	4.9793	4.9867
60°C± 0.1°C	7.9643	7.9489	7.9725	7.9581	7.9615
	7.9508	7.9702	7.9628	7.9491	7.9675
	7.9635	7.9562	7.9692	7.9586	7.9703
70°C± 0.1°C	12.1087	12.0951	12.1129	12.1012	12.0972
	12.0926	12.1096	12.0982	12.1052	12.1106
	12.1035	12.0981	12.1092	12.0956	12.1118

Chart 1: Raw Data Table of Temperature (K) against Terminal Velocity (m/s) for Glycerin.

Temperature	Instantaneous Velocity of Last 5 Frames				
	1	2	3	4	5
20°C± 0.1°C	1.3627	1.3489	1.3553	1.3662	1.3584
	1.3498	1.3645	1.3537	1.3594	1.3671
	1.3579	1.3526	1.3651	1.3613	1.3687
30°C± 0.1°C	2.2934	2.2789	2.2876	2.2961	2.2842
	2.2817	2.2948	2.2905	2.2779	2.2993
	2.2869	2.2804	2.2981	2.2916	2.2835
40°C± 0.1°C	3.9612	3.9448	3.9557	3.9683	3.9519
	3.9487	3.9651	3.9582	3.9476	3.9705
	3.9574	3.9492	3.9698	3.9541	3.9635
50°C± 0.1°C	5.2738	5.2594	5.2669	5.2785	5.2623
	5.2608	5.2751	5.2682	5.2576	5.2799
	5.2684	5.2637	5.2773	5.2649	5.2711
60°C± 0.1°C	9.8625	9.8471	9.8569	9.8732	9.8526
	9.8492	9.8685	9.8578	9.8459	9.8713
	9.8607	9.8538	9.8701	9.8504	9.8669
70°C± 0.1°C	13.2048	13.1895	13.1982	13.2127	13.1916
	13.1909	13.2091	13.1956	13.1864	13.2112
	13.2017	13.1938	13.2105	13.1887	13.2073

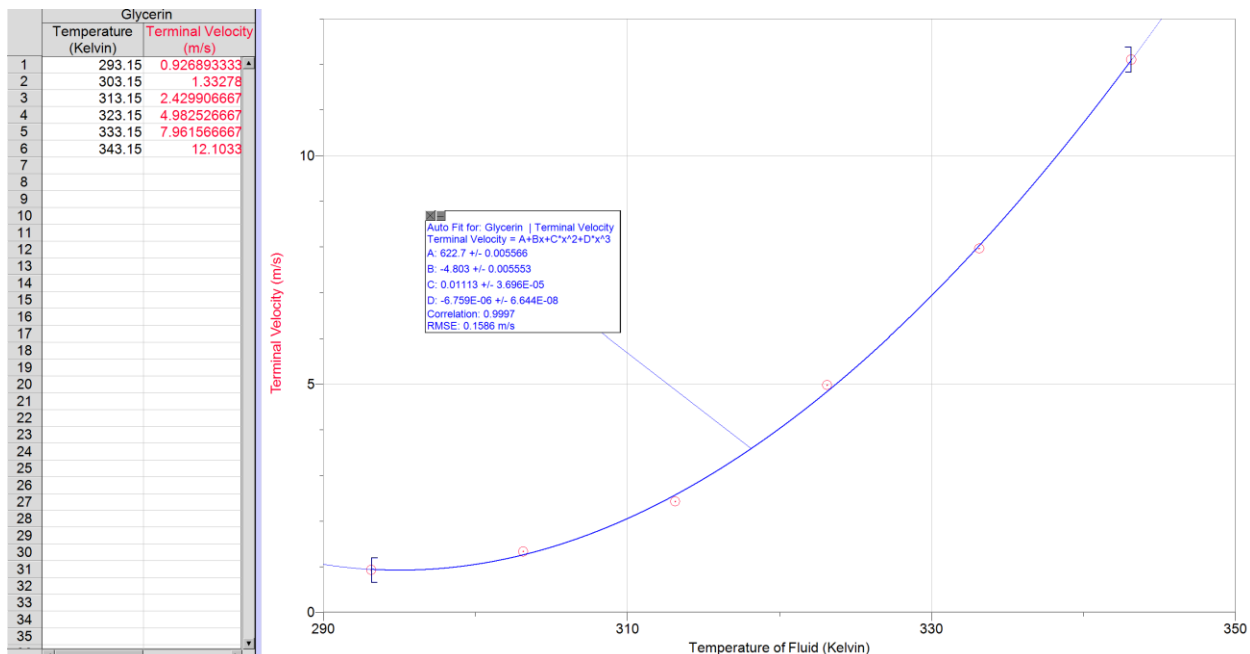
Chart 2: Raw Data Table of Temperature (K) against Terminal Velocity (m/s) for Vegetable oil.

Temperature	Instantaneous Velocity of Last 5 Frames				
	1	2	3	4	5
20°C ± 0.1°C	1.5268	1.7d889	1.6638	1.4436	1.5838
	1.7996	1.5772	1.6986	1.7092	1.5244
	1.806	1.5643	1.821	1.791	1.5275
30°C± 0.1°C	2.9148	2.8674	2.7394	2.807	2.7325
	2.7701	2.9234	2.8466	2.9398	2.8534
	2.9458	2.6345	2.6373	2.7776	2.8048
40°C± 0.1°C	4.1181	3.7357	4.0546	3.7482	3.8897
	3.9276	3.9381	3.8436	3.9172	4.0882
	3.8042	4.0372	3.753	3.7875	3.9579
50°C± 0.1°C	5.1195	5.2564	4.9833	5.1841	5.1205
	5.1927	5.1489	5.1718	5.2019	5.0619
	5.2456	5.1508	5.2394	5.1864	5.071
60°C± 0.1°C	9.098	8.9658	8.8028	8.7999	8.9794
	8.9843	8.8639	8.7941	9.1375	8.9688
	9.0738	8.9092	8.9395	8.9788	9.0171
70°C± 0.1°C	11.3394	11.1917	11.2335	11.4475	11.3368
	11.2204	11.294	11.4191	11.4117	11.2933
	11.471	11.4432	11.4561	11.3933	11.1793

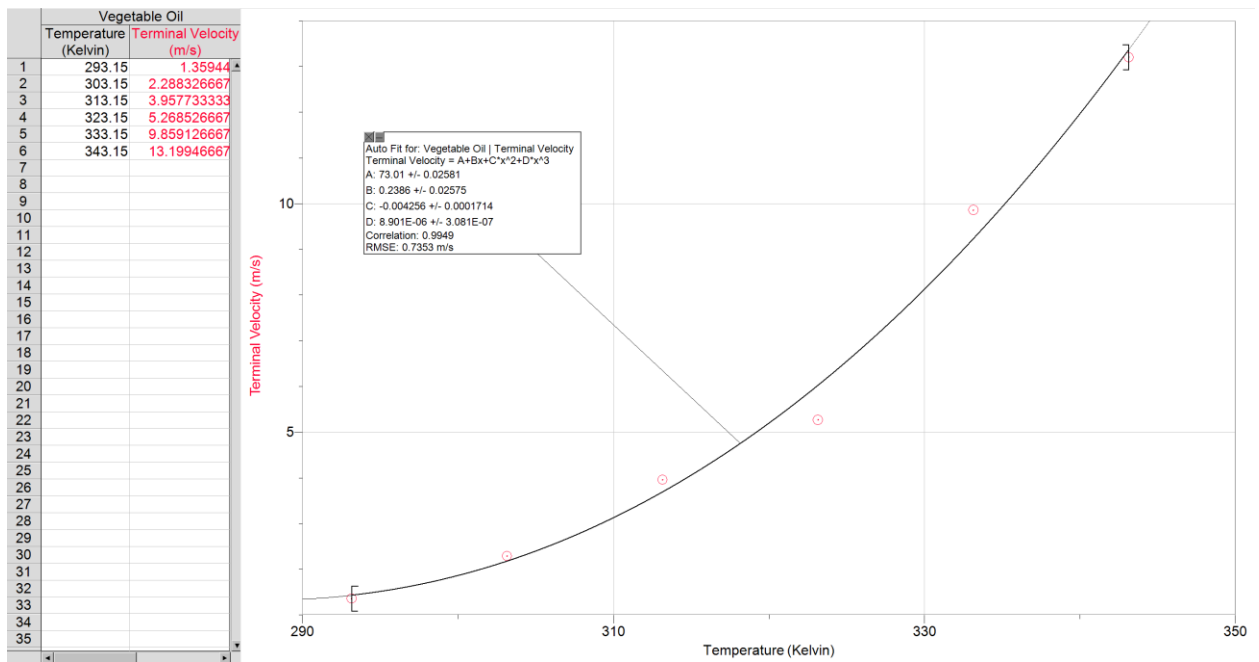
Chart 3: Raw Data Table of Temperature (K) against Terminal Velocity (m/s) for Vinegar.

Temperature	Instantaneous Velocity of Last 5 Frames				
	1	2	3	4	5
20°C ± 0.1°C	1.7074	1.8316	1.6556	1.7762	1.7503
	1.6652	1.6578	1.7314	1.5892	1.7966
	1.6065	1.8443	1.5729	1.6911	1.6322
30°C ± 0.1°C	2.344	2.3967	2.3236	2.1292	2.1894
	2.3342	2.3897	2.2806	2.3149	2.2451
	2.2826	2.2865	2.0743	2.3679	2.0621
40°C ± 0.1°C	2.9396	2.9596	2.743	2.6806	2.933
	2.6286	2.9099	2.6041	2.6823	2.8564
	2.9641	2.91	2.615	2.8262	2.7627
50°C ± 0.1°C	3.8177	3.4435	3.352	3.7894	3.3139
	3.3209	3.5383	3.7048	3.4994	3.6308
	3.7809	3.3406	3.5935	3.5784	3.6392
60°C ± 0.1°C	4.1032	4.5376	4.323	4.35	4.6735
	4.2449	4.0824	4.0831	4.6167	4.5978
	4.6779	4.4791	4.663	4.1169	4.1649
70°C ± 0.1°C	6.0301	6.2298	6.0289	5.8458	6.0232
	6.1288	5.7872	5.752	6.0162	5.9676
	6.0119	5.7167	5.7062	5.8297	6.1036

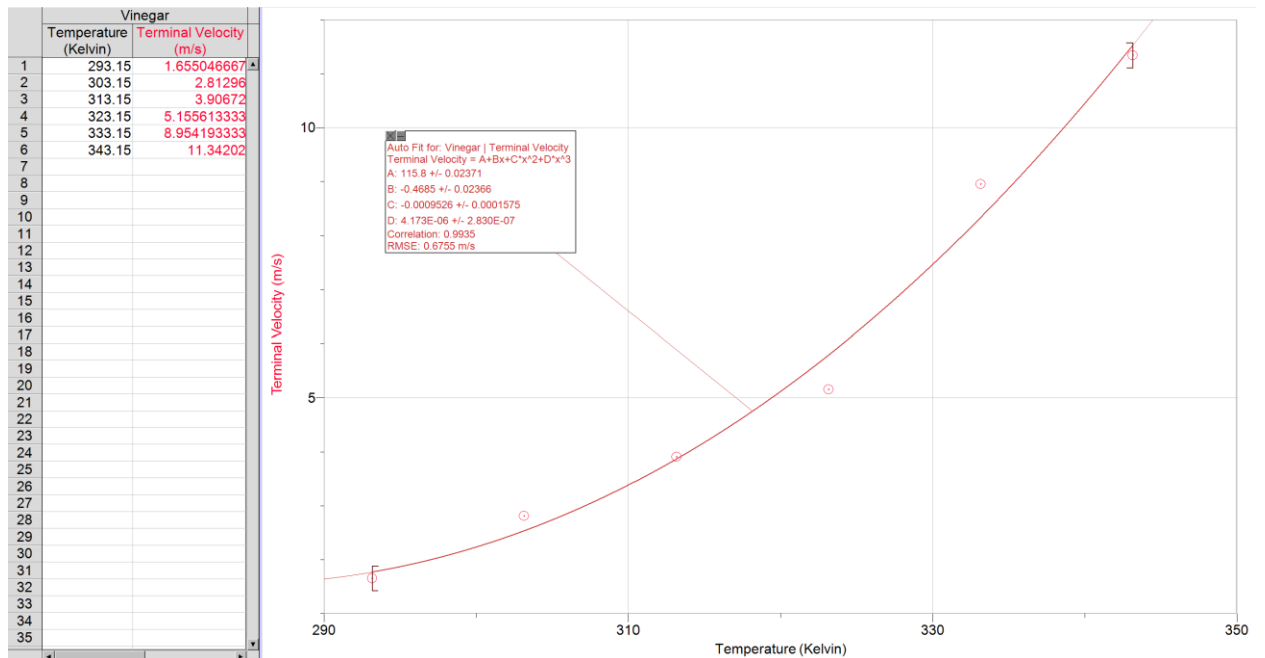
Chart 4: Raw Data Table of Temperature (K) against Terminal Velocity (m/s) for Water.



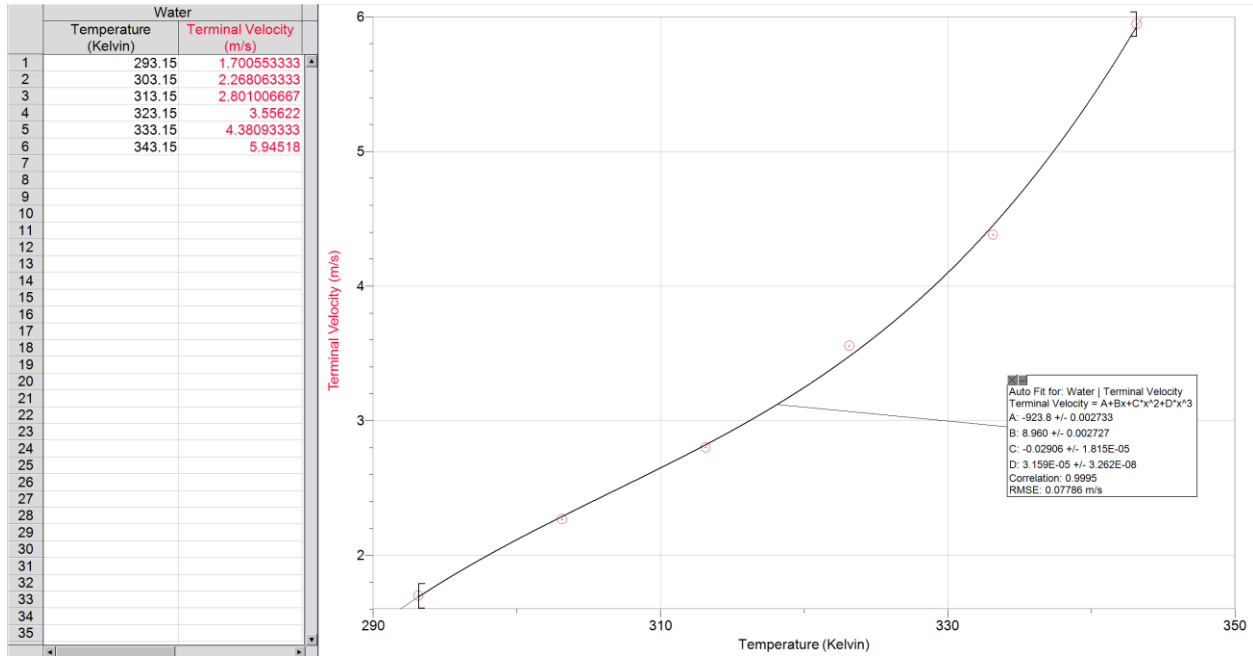
Graph 2: Graph of Temperature (K) against Average Terminal Velocity (m/s) for glycerin.



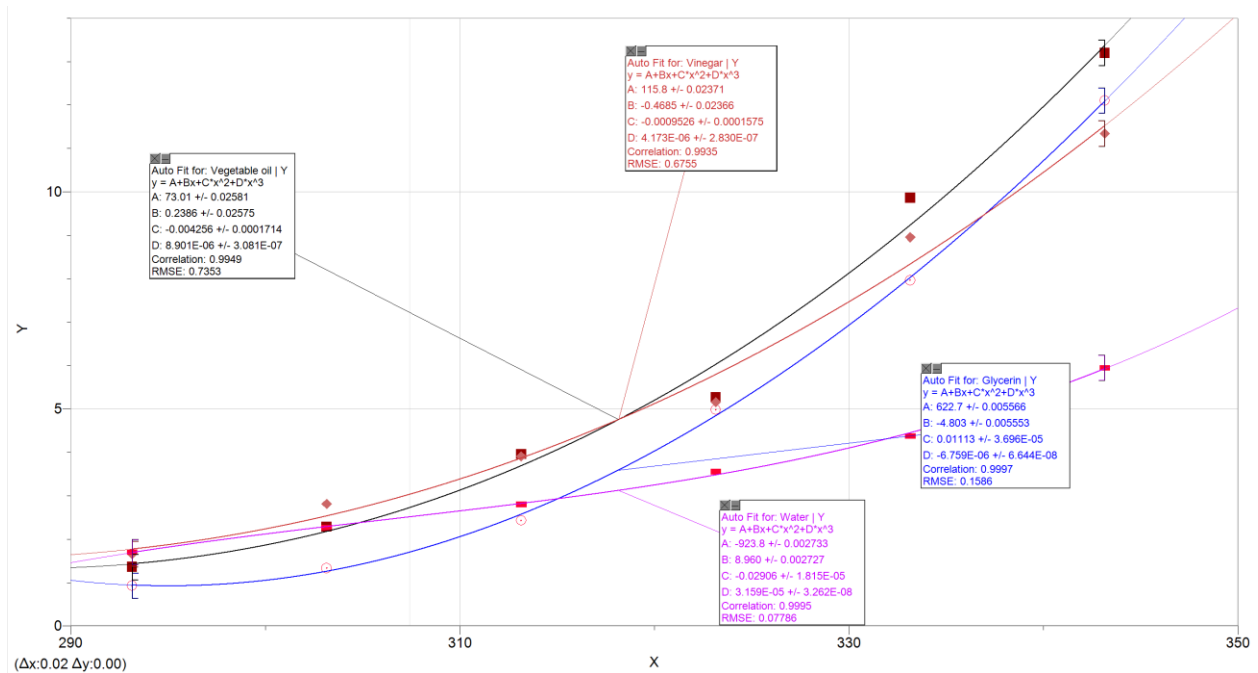
Graph 3: Graph of Temperature (K) against Average Terminal Velocity (m/s) for vegetable oil.



Graph 4: Graph of Temperature (K) against Average Terminal Velocity (m/s) for Vinegar.



Graph 5: Graph of Temperature (K) against Average Terminal Velocity (m/s) for Water



Graph 6: Graph of Temperature (K) against Average Terminal Velocity (m/s) for Glycerin, Vegetable oil and vinegar combined.



In graph 5 temperature and terminal velocity follow a cubic trend, and temperature increases the rate of change of terminal velocity increases. The reason for this non-linear behavior is that viscosity's dependence on temperature is exponential as well. As temperature decreases, viscosity decreases exponentially. So, the force acting on the body is reduced in a non-linear way.

The cubic correlation between temperature and terminal velocity implies that at lower temperatures the fluid is less affected by changes in temperature. But as temperature increases, changes in viscosity become more influential. This shows that at higher temperature the marble is affected by much lower drag forces. As viscosity and temperature follows an exponential curve, the resistance is reduced at higher temperatures. The resistance of the motion is not linear, so the body accelerates much more at higher temperatures, causing a cubic correlation. From this correlation and the data gained from the experiment it can be said that most viscous fluids have cubic correlations between the terminal velocity of a body in motion and the temperature of said fluid.

The trends between different fluids change, considerably the change of terminal velocity is sharper for glycerin than vegetable oil or vinegar. This suggests that glycerin is affected by changes in temperature at a much higher rate than other fluids that are tested. However, to prove this correlation the graphics must be linearized.

#### **4.3. Error Propagation:**

For error propagation, the standard deviation method is used. The formula is as follows;

$$\bar{x} = \frac{\sum x_i}{N}$$

$\bar{x}$ : The mean of the dataset

$N$ = number of trials conducted

$x_i$ : Individual values of velocity for each frame

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N - 1}}$$

$$\text{absolute uncertainty} = \frac{\sigma}{\sqrt{N}}$$

$\sigma$ : standard deviation

$N$ = number of trials conducted

Sample Calculation done for 20°C glycerin, trial 1:

$$\bar{v} = \frac{0.9284 + 0.9442 + 0.9371 + 0.9228 + 0.9317}{5}$$

$$\sigma = \sqrt{\frac{(0.9284 - 0.93284)^2 + (0.9442 - 0.93284)^2 + (0.9371 - 0.93284)^2 + (0.9228 - 0.93284)^2 + (0.9317 - 0.93284)^2}{5}}$$

$$\sigma = 0.00820$$

$$\text{Uncertainty} = \frac{\sigma}{\sqrt{N}}$$

$$\text{Uncertainty} = \frac{0.00820}{\sqrt{5}}$$

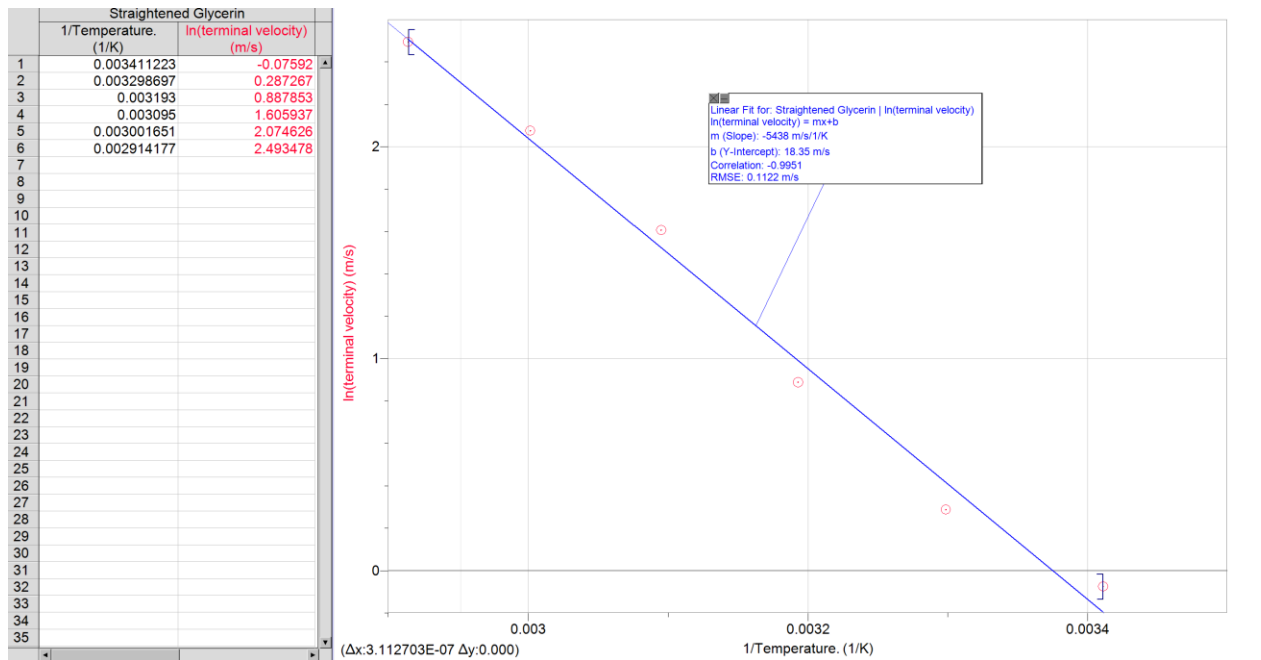
$$\text{Uncertainty} = 0.00367 \text{ m/s}$$

#### 4.4. Processed Data:

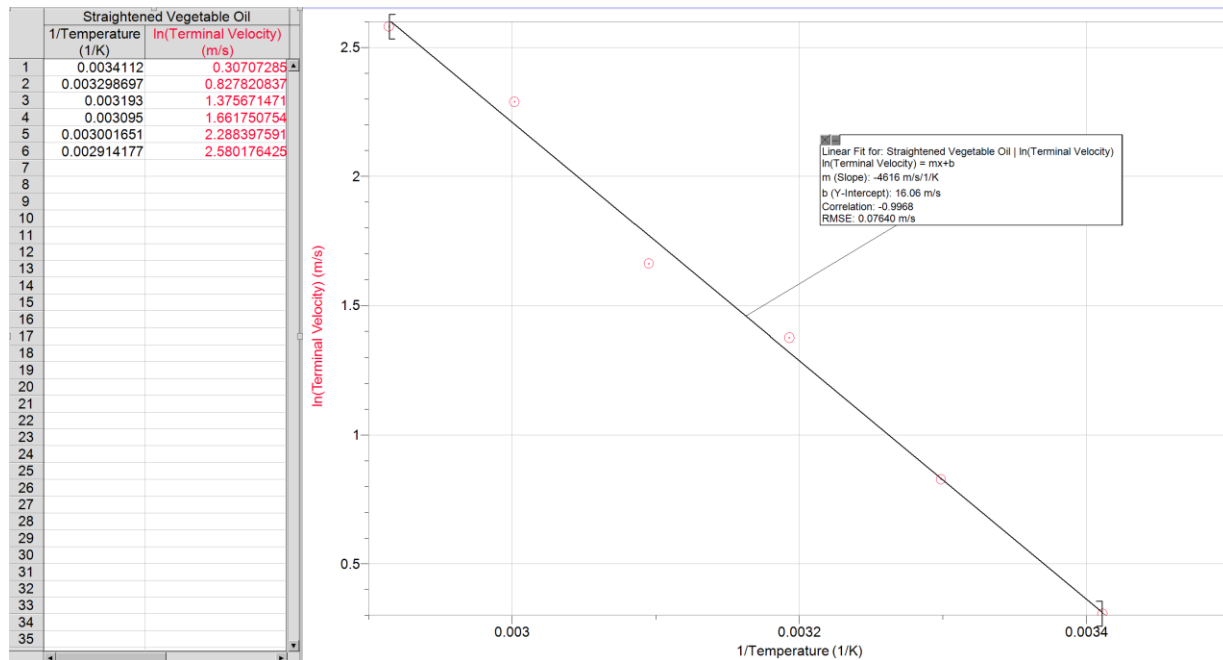
Fluids	Temperature				
		Average Terminal Velocity	standard deviation	absolute uncertainty	percentage uncertainty
Glycerin	20°C± 0.1°C	0.93284	0.008200793	0.003667506	0.393154878
		0.92068	0.005762118	0.002576897	0.279890663
		0.92716	0.006589992	0.002947134	0.317866841
	30°C± 0.1°C	1.33306	0.006533988	0.002922088	0.219201558
		1.33174	0.006862434	0.003068974	0.230448419
		1.33354	0.005737857	0.002566048	0.19242374
	40°C± 0.1°C	2.43184	0.00582692	0.002605878	0.107156637
		2.42878	0.007070856	0.003162183	0.130196345
		2.4291	0.005719703	0.002557929	0.105303563
	50°C± 0.1°C	4.98212	0.006235543	0.00278862	0.055972552
		4.9817	0.006530314	0.002920445	0.058623465
		4.98376	0.005859863	0.002620611	0.052583002
	60°C± 0.1°C	7.96106	0.008634118	0.003861295	0.048502274
		7.96008	0.00963779	0.004310151	0.054147079
		7.96356	0.006245238	0.002792955	0.035071694
	70°C± 0.1°C	12.10302	0.00757938	0.003389602	0.028006248
		12.10324	0.007698571	0.003442906	0.02844615
		12.10364	0.006943558	0.003105254	0.025655535
Vegetable Oil	20°C± 0.1°C	1.3583	0.00669216	0.002992825	0.220336064
		1.3589	0.007223226	0.003230325	0.237716172
		1.36112	0.00625076	0.002795425	0.20537681
	30°C± 0.1°C	2.28804	0.006931306	0.003099774	0.135477272
		2.28884	0.008921211	0.003989687	0.174310424
		2.2881	0.006966707	0.003115606	0.136165634
	40°C± 0.1°C	3.95638	0.008943545	0.003999675	0.101094308
		3.95802	0.010016836	0.004479665	0.113179447
		3.9588	0.008051397	0.003600694	0.090954188
	50°C± 0.1°C	5.26818	0.00792761	0.003545335	0.067297149
		5.26832	0.009374807	0.004192541	0.079580227
		5.26908	0.005443528	0.00243442	0.046201991
	60°C± 0.1°C	9.85846	0.009992647	0.004468848	0.045330079
		9.85854	0.011286851	0.005047633	0.051200615
		9.86038	0.008366421	0.003741577	0.037945568
	70°C± 0.1°C	13.19936	0.009569901	0.00427979	0.032424221

		13.19864	0.011024201	0.004930172	0.03735364
		13.2004	0.00910714	0.004072837	0.03085389
Vinegar	20°C ± 0.1°C	1.60138	0.132122829	0.059087126	3.689762923
		1.6618	0.110256247	0.049308093	2.967149635
		1.70196	0.143448189	0.06415198	3.769300129
	30°C± 0.1°C	2.81222	0.079455598	0.035533624	1.263543518
		2.86666	0.06795394	0.030389926	1.060116162
		2.76	0.130042282	0.058156676	2.107125953
	40°C± 0.1°C	3.90926	0.174057355	0.077840816	1.991190552
		3.94294	0.089289126	0.039931311	1.01272936
		3.86796	0.122969846	0.054993787	1.42177755
	50°C± 0.1°C	5.13276	0.100731167	0.045048347	0.877663233
		5.15544	0.056140431	0.025106764	0.486995563
		5.17864	0.0717584	0.032091332	0.619686485
	60°C± 0.1°C	8.92918	0.127523025	0.057030031	0.63869281
		8.94972	0.130770608	0.058482394	0.653455012
		8.98368	0.064733044	0.028949497	0.322245421
	70°C± 0.1°C	11.30978	0.100418808	0.044908656	0.39707807
		11.3277	0.085501901	0.038237612	0.337558484
		11.38858	0.120577514	0.053923904	0.473491022
Water	20°C ± 0.1°C	1.74422	0.066903229	0.029920033	1.715381857
		1.68804	0.078852444	0.035263885	2.089043222
		1.6694	0.106903461	0.047808681	2.863824202
	30°C± 0.1°C	2.27658	0.112368109	0.050252546	2.207370099
		2.3129	0.054763263	0.024490876	1.058881744
		2.21468	0.138053258	0.061739294	2.787729776
	40°C± 0.1°C	2.85116	0.129487405	0.057908528	2.031051497
		2.73626	0.138341147	0.061868042	2.261043974
		2.8156	0.136120112	0.060874765	2.162053022
	50°C± 0.1°C	3.5433	0.242404961	0.108406794	3.059486752
		3.53884	0.145819248	0.06521235	1.842760624
		3.58652	0.159020492	0.071116126	1.982872706
	60°C± 0.1°C	4.39746	0.218072804	0.097525123	2.217760319
		4.32498	0.26612737	0.119015778	2.75182262
		4.42036	0.267394461	0.119582439	2.705264696
	70°C± 0.1°C	6.03156	0.135908951	0.060780331	1.007704983
		5.93036	0.158462008	0.070866364	1.194975758
		5.87362	0.177900582	0.079559559	1.354523425

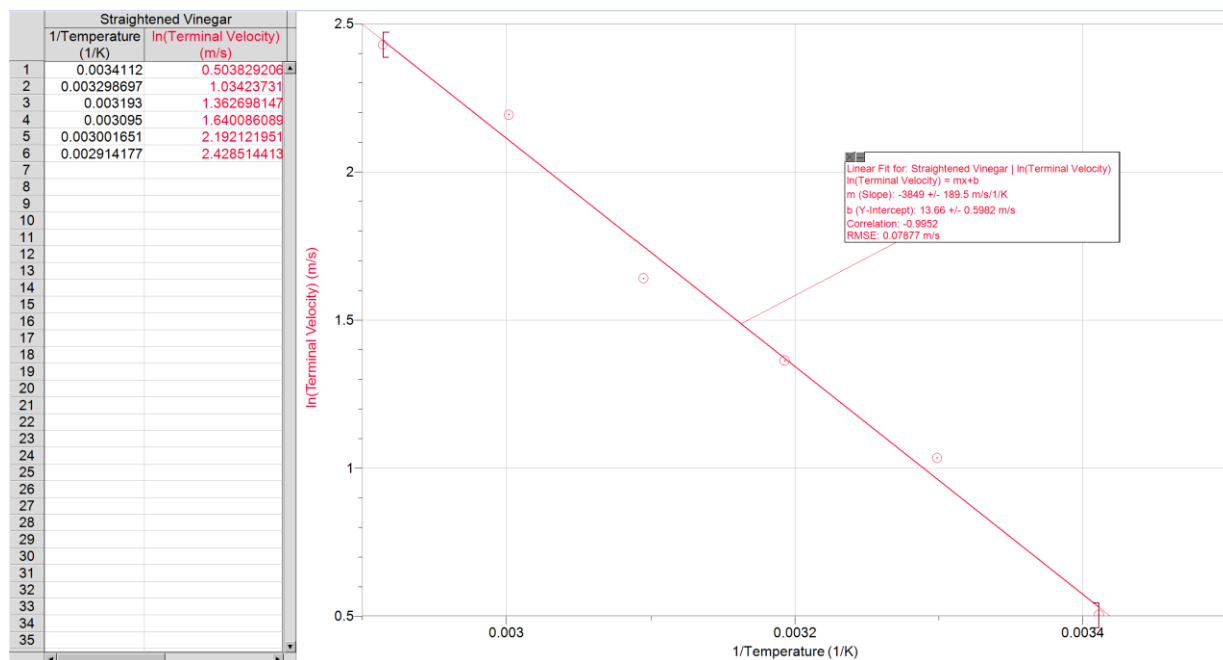
Chart 4: processed Data Table of all fluids at all temperatures (K).



Graph 7: Straightened Graph of 1/Temperature (K) against ln(Average Terminal Velocity(m/s)) for glycerin.

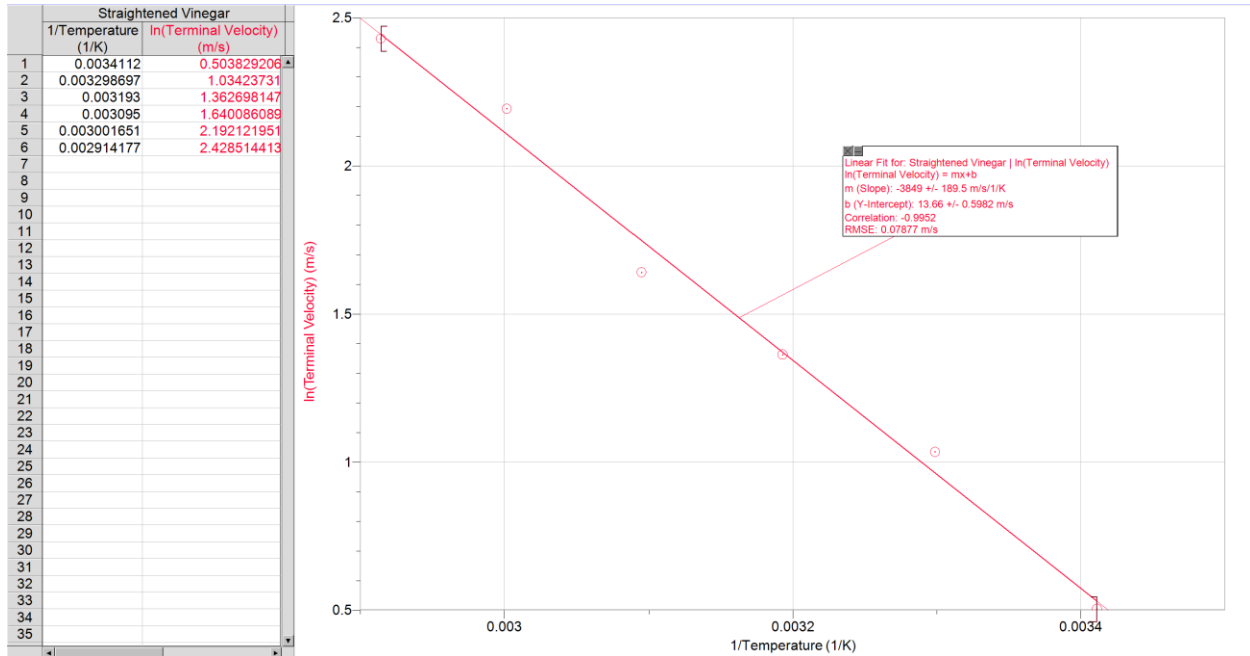


Graph 8: Straightened Graph of 1/Temperature (K) against ln(Average Terminal Velocity(m/s)) for vegetable oil.



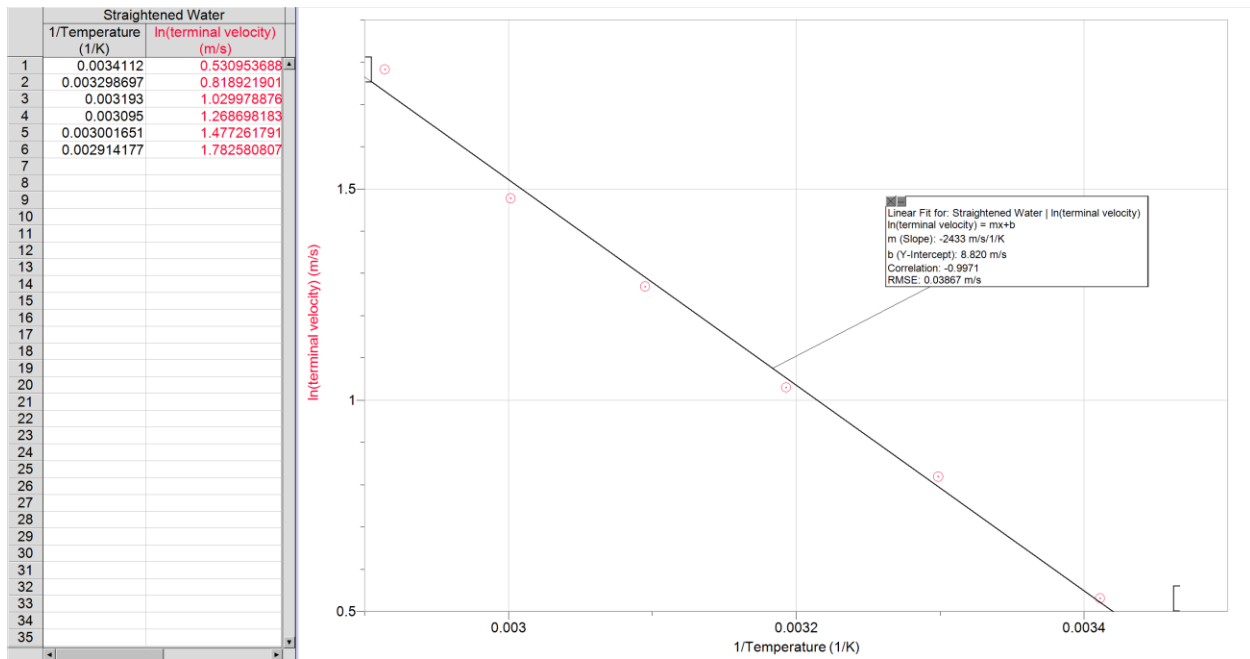
Graph 69: Straightened Graph of 1/Temperature (K) against ln(Average Terminal Velocity(m/s)) for

vegetable oil.



Graph 10: Straightened Graph of 1/Temperature (K) against ln(Average Terminal Velocity(m/s)) for

vinegar.



Graph 11: Straightened Graph of  $1/\text{Temperature (K)}$  against  $\ln(\text{Average Terminal Velocity(m/s)})$  for water.

The linearization of the graphs proves the Arrhenius type of behavior that was hypothesized beforehand. Because the values gained from the experiments were able to be linearized using the correlation that was derived at the background information part, we can use this data for activation energy of viscous flow calculations:

$$\ln(v_t) = -\frac{E_a}{R} \times \frac{1}{T} + C$$

Because temperature is an independent variable the slope of the graph is;

$$m = -\frac{E_A}{R}$$

From this value we can calculate the value of each  $E_a$  by multiplying the slope by  $-R$ .

$R$  is a constant as it is the universal gas constant and is equal to  $8.3145 \text{ J/mol}\cdot\text{K}$ . Because the universal gas constant has the SI unit of " $\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$ " and the final products SI unit is " $\text{kJ mol}^{-1}$ " the value is divided by 1000.

Sample Calculation for Glycerin:

$$m = -\frac{E_a}{R} = -5438$$

$$-\frac{E_a}{R} \times (-R) = -5438 \times (-R)$$

$$E_a = (-5438) \times (-8.3145) / 1000$$

$$E_a = 45.214251 \text{ kJ mol}^{-1}$$

The values of activation energy of other fluids are:



Fluid	Activation Energy of Viscous Flow
Glycerin	45.214251 kJ mol <sup>-1</sup>
Vegetable Oil	38.379732 kJ mol <sup>-1</sup>
Vinegar	32.002511 kJ mol <sup>-1</sup>
Water	20.2291785 kJ mol <sup>-1</sup>

Chart 5: Activation energy of viscous flow (J·mol<sup>-1</sup>·K<sup>-1</sup>) of different fluids tested

From the calculations it can be said that the fluid that is affected the most by temperature changes is glycerin, then vegetable oil and the one affected the least is vinegar. It can be mentioned that the fluid with the highest intermolecular forces is glycerin, then vegetable oil, then vinegar and the fluid with the least number of intermolecular forces is distilled water.

From a simple literature investigation, the value of activation energy of crude glycerin must be between 42 kJ mol<sup>-1</sup> and 90 kJ mol<sup>-1</sup>.<sup>1</sup> For water the value is 18.8<sup>2</sup> kJ mol<sup>-1</sup>.

## **5.Conclusion:**

The aim of the investigation was to find the correlations between temperature, activation energy of viscous flow and terminal velocity. The results align with theoretical expectations and derivations. The data collected demonstrates that terminal velocity increases exponentially with

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<sup>1</sup>

(n.d.).

Crnkovic, P. M. (September 2012). Determination of the activation energies of beef tallow and crude glycerin combustion using thermogravimetry. *Biomass and Bioenergy*, Pages 8-16.

<sup>2</sup> Horne, R. A., Courant, R. A., Johnson, D. S., & Margosian, F. F. (1965). The Activation Energy of Viscous Flow of Pure Water and Sea Water in the Temperature Region of Maximum Density. *The Journal of Physical Chemistry*, 69(11), 3988–3991.

temperature, this cubed correlation occurs on every tested fluid, validating the dependance. As temperature increases the terminal velocity increases, therefore It can be said that viscosity decreases.

The findings have direct implications for industrial use. It is now known that fluids with higher activation energies should be used rather than fluids with higher reference viscosity as it can be seen for vegetable oil and glycerin. Even though at 20 degrees Celsius both fluids had lower terminal velocities than water and vinegar, they both exceeded at 70 degrees Celsius outperforming both vinegar and water. Because fluids that have higher terminal velocities have lower resistive forces as established in background information, the experiment suggests that machinery that work at higher temperatures like car engines require fluids of those that have higher activation energies.

Water meets theoretical expectations as water weighs less per mol and glycerin has complex hydrogen bonds. It can be said that glycerin is the most temperature-sensitive fluid when it comes to viscosity.

## **6. Evaluation:**

The experiment was successful, and the hypothesis was proven, however a few key strengths and weaknesses can be mentioned.

Strength	Explanation
Data Collection	Loggerpro Data Analysis software is used for more accurate research. This yields a very low uncertainty.
Calculation of Activation Energy of Viscous Flow	Because the activation energy of viscous flow is calculated, the results can be compared against

	literature. The findings are similar, suggesting accurate research.
Controlled Conditions	The marble, graduated cylinder and all of the measurement devices are kept unchanged for better repeatability and minimized random errors.
Multiple Trials	Repeating the experiment multiple times allows for calculations to be more reliable.

Weakness	Explanation	Improvements in further research
Experimental Setup	Because the experimental setup was lacking in technology there was heat loss to surroundings.	More advanced temperature-controlled system. A bath can be used to minimize heat loss.
Lack of Fluids	Because the investigation surrounds industrial applications, more industrial	Industrial Lubricants could have been used.

	lubricants could have been used.	
Precision of Measurement	Video tracking is used for the experiment, which introduces small tracking errors.	A motion sensor or a higher Frame Per-Second camera could have been used.
Turbulent Flow	At high temperatures Reynolds number increase.  The assumption that the experiment is done with laminar flow is possibly broken.	A flow simulation can be used or a dye can be ejected to visualize the flow to determine the type of flow.

## 7. Resources:

- Smith, J., & Doe, A. (2020). The effect of viscosity on terminal velocity. *Journal of Fluid Mechanics*, 12(4), 455-467
- White, F. M. (2011). *Fluid Mechanics* (7th ed.). McGraw-Hill. Retrieved from [http://ftp.demec.ufpr.br/disciplinas/TM240/Marchi/Bibliografia/White\\_2011\\_7ed\\_Fluid-Mechanics.pdf](http://ftp.demec.ufpr.br/disciplinas/TM240/Marchi/Bibliografia/White_2011_7ed_Fluid-Mechanics.pdf)
- Crnkovic, P. M. (September 2012). Determination of the activation energies of beef tallow and crude glycerin combustion using thermogravimetry. *Biomass and Bioenergy*, Pages 8-16.

- <sup>1</sup> Horne, R. A., Courant, R. A., Johnson, D. S., & Margosian, F. F. (1965). The Activation Energy of Viscous Flow of Pure Water and Sea Water in the Temperature Region of Maximum Density. *The Journal of Physical Chemistry*, 69(11), 3988–3991.
- NASA Glenn Research Center. (n.d.). *Terminal velocity*. NASA. Retrieved from <https://www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/termv.html>
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- Hosseini, S. M., Ashrafizadeh, S. N., & Kharrat, R. (2014). *Viscosity activation energy*. ResearchGate. Retrieved from [https://www.researchgate.net/publication/263558765\\_Viscosity\\_activation\\_energy](https://www.researchgate.net/publication/263558765_Viscosity_activation_energy)