International Baccalaureate

Physics HL

Topic: Resonance

Research Question:

What is the effect of tension applied on a string to its resonance frequency?

Word Count: 3740

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1. Introduction

Personal Engagement

For unknown reasons that trouble scientists like my mother to this day, I have always been passionate STEM enthusiast, especially taking an interest in Astrophysics. A big concept that has always interested me in physics has been waves. As phenomena we learn at young ages, they seem like they should not create such ambiguities in people's minds. They have always troubled me, since at first glance do not seem like important phenomena. After learning about their importance in physics, they became a huge interest for me. From explaining the states of subatomic particles to having gravitational waves, waves really seem like a gift from existence. Standing waves are very interesting in their own ways. For things that require very special conditions to take place, they seem to play too big of a role in existence itself.

After deciding to investigate standing waves for my Extended Essay, I found that resonance can be quite an important thing in engineering because of its dual nature of both demolishing and building. I decided to investigate this topic further and then decided that this was the thing that interested me. Throughout this investigation, I did not only focus on learning how to write a scientific paper but also tried to follow my curiosity into the wonders of the universe.

2. Background Information

Principles of Waves

A disturbance that propagates through a medium that transfers energy without a net movement of the particles that are in it is called a wave. Mechanical waves that travel through mediums require the presence of a material medium, like a string. These mechanical waves can be classified into two main types:

Transverse waves, where the displacement of the particles is perpendicular to the direction of wave propagation such as waves that propagate in a string [1].

Longitudinal waves, where the displacement of the particles is parallel to wave propagation such as sound waves [2].

The fundamental properties of a wave can be expressed through the following equation:

$$v = f\lambda$$

Where:

- v is the wave's velocity,
- f is the wave's frequency,
- λ is the wave's wavelength[3].

The velocity of a mechanical wave is dependent on the medium's properties. In the case of a vibrating string, the speed is affected by two leading properties:

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[1] Halliday, Resnick, & Walker, Fundamentals of Physics, 2020.

[2] Tipler & Mosca, Physics for Scientists and Engineers, 2019.

[3] Serway & Jewett, Principles of Physics, 2018.

Tension (T) that is being applied to the string, which has an affect on the tightness of the particles that are connected.

Linear mass density (μ), which states the mass that is needed to move one unit length of the string[4].

In most cases, a higher tension applied to the string generally leads to faster wave propagation, while an increased linear mass density inversely affects the speed of the wave. The relationships of these properties to the behavior of a wave plays a vital role in determining the resonance frequency of a vibrating string.

Standing Waves and Their Properties on a String

A standing wave is a type of wave where two waves that possess the same properties (identical waves) interfere when they are traveling in opposite directions. This phenomenon creates a pattern of alternating points where constructive and destructive interference take place. Standing waves, unlike other waves, seem stationary because their nodes (points where amplitude is zero) and antinodes (points where their amplitude is at its maximum and minimum) remain fixed because of the interference pattern. For a vibrating string that is fixed on both ends, nodes form at the ends [5].

A string fixed at both ends can only be a medium for a stationary wave to form if the conditions provide for the following rule:

$$L = \frac{n\lambda}{2}$$

[4] Hecht, Optics, 2017.
[5] Young & Freedman, University Physics, 2019.

where:

- L is the length of the section of the string,
- n is the harmonic number $n = 1,2,3, \ldots$ where $n = 1$ represents the fundamental frequency)[6].

In harmonic representation, the simplest standing wave with one antinode at the middle and two nodes at both ends of the string, is called the first harmonic ($n = 1$). The vibration frequency of the first harmonic is called the natural frequency (f_n).

Higher harmonics appear as n increases ($n = 1,2,3, \ldots$.) where the nodes and antinodes increase in number This results in further division of the string into sections.

The frequency of each harmonic can be expressed with the equation:

$$f_n = nf_1$$

where:

- f_n is the frequency of the nth harmonic,
- f_1 is the fundamental frequency.

The observation of nodes and antinodes helps to identify the harmonics and by alternating the tension applied to the string and observing the changes in harmonic frequencies, the affect of tension on standing waves can be observed.

Mechanical Resonance and Tension

Resonance is the phenomena that happens when an external force drives a system at its natural frequency. Resonance for a vibrating string happens when an external force applies a

[6] Burnett, B., *Effects of Tension on Resonant Frequencies of Strings*, 2018.

vibration at the natural frequency of the string, maximizing the energy transferred to the string[7]. This allows a stable standing wave to form. Resonance can be induced by external sources like an electromagnetic driver.

In the following equation, the relation of tension applied to a vibrating string and its velocity can be determined by:

$$v = \sqrt{\frac{T}{\mu}}$$

Since frequency is established with the relation:

$$f = \frac{v}{\lambda}$$

An increase in tension should result in a higher wave velocity, which should increase the resonance frequency. It can also be observed that since velocity is related to the square root of tension and directly proportional to the frequency, the relationship of frequency against tension should follow a non-linear trend.

Derivation of the Relationship Between Tension and Resonance Frequency in a String

Derivation of the equation relating the fundamental frequency to the tension applied on the string starts with the general wave equation.

$$v = f\lambda$$

The boundaries of a standing wave to form on a string with two closed ends only allow waves with wavelengths that provide the equation:

[7] Nave, R., GSU HyperPhysics, 2019.

$$L = \frac{n\lambda}{2}$$

which can be rearranged to:

$$\lambda_n = \frac{2L}{n}$$

by multiplying both sides of the equation by $2/\,n.$ For a wave vibrating at its fundamental frequency, the wavelength is given by the equation:

$$\lambda_1 = 2L$$

By substituting $\lambda_1 = 2L$ into the wave equation $v = f\lambda$, the equation:

$$v = f_1(2L)$$

can be obtained. Solving for f_1:

$$f_1 = \frac{v}{2L}$$

The wave speed in a stretched string depends on the tension and linear mass density. From Newton's Second Law, the speed of a transverse wave on a stretched string is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

Substituting $v = \sqrt{\frac{T}{\mu}}$ for v in $f_1 = \frac{v}{2L}$ gives:

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

The given equation is the desired equation for the fundamental frequency. For higher harmonics, the equation becomes:

$$f_n = nf_1 = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

This equation highlights three important factors:

- String Length (L) – A longer string results in a lower frequency.
- Tension (T) – A higher tension increases the wave speed, leading to a higher resonance frequency.
- Linear Mass Density (μ) – A heavier string has a lower wave speed, resulting in a lower resonance frequency.

3. Planning and Prior Experimentation Information

Aim

The aim of the investigation is to determine how the variation to the tension applied to a string effect on its resonance frequency. The investigation does this by systematically changing the tension applied to the string and observing its corresponding resonance frequency. The study also aims to analyze if the existing mathematical models are effective in explaining the investigated phenomenon.

Hypothesis

It is hypothesized that an increase in the tension applied to the string will increase its resonance frequency. Based on the equation $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, the fundamental frequency of a wave should follow a non-linear trend against tension. Furthermore, the square of the frequency should be directly proportional to the tension, which should result in a linear relationship when f^2 is plotted and observed against tension.

Variables

Variable Type	*Variable*	*Description*
Dependent Variable	*Resonance Frequency (Hz)*	The resonance frequency of the string, measured at the point of maximum amplitude (point where the string resonates, antinode).
Independent Variable	*Tension on the String (N)*	Controlled by hanging different masses. Tension is calculated using the formula: $T = m \cdot g$ where m is the mass in kg and $g = 9.81$ m/s^2 (gravitational acceleration).
Controlled Variables	*String Length (m)*	The length of the string was kept constant at a value of 0.90 m (±0.01 m) between fixed supports to ensure a constant wavelength.
	Linear Mass Density	The same string was used for every trial to ensure the linear mass density does not change and effect the results of the experiment. The total string length (1.00 m ± 0.01 m) and mass (0.55 g ± 0.01 g) were measured to calculate μ.
	Mode of Vibration	The string was driven to resonate at its fundamental mode (or a harmonic with an antinode at the center) for consistency. The resonance was detected using the paper rider method, which ensures that only harmonics with a central antinode are observed.
	Environmental Factors	The experiment was conducted at standard conditions and air currents were blocked to prevent unwanted variations in string tension caused by thermal expansion or air damping.

Table 1. Variables – The different types of variables and their explanations and relations

Materials and Apparatus

- **Sonometer apparatus:** A hollow box with two fixed collums which are 0.90 meters apart, where the string is stretched. One end of the string is fixed to the sonometer and the other end is stretched over a pulley. This mechanism allows the string to stay at the same length, also allowing it to be stretched freely to adjust the tension on it.
- **String:** A string of length 1.10 meters. The nylon cord of desired flexibility was used as the medium for the wave to form. The string was also chosen for its uniform properties and its ability to hold under various tension without deforming or breaking.
- **Meter ruler (±0.005 m):** Used to measure and set the length of the string between the supports (0.90 m) and to verify the total length of string used.
- **Tuning forks:** A set of tuning forks with known frequencies were used to drive the string into resonance. Each tuning fork produces a fixed frequency; by trying different forks one can find which frequency causes the string to resonate at each tension.
- **Ruler:** A ruler was used to check the length of the string periodically to ensure that a consistent investigation was being conducted.

Safety Considerations

Precautions were taken during the experiment to ensure safety. Operations like adding the masses or stretching the string were done gently to avoid the string breaking and accidents happening. Also, anything sharp was handled with the necessary caution to avoid work accidents and any joints and knots were checked to ensure nothing broke and caused an accident.

Methodology

1. The sonometer was set up and fixed in place with clamps on a small table. One end of the string was connected to the sonometer, making sure there was enough length to go over both columns. The length between the two columns was adjusted to 0.90 meters and was not changed after. The other end of the string was stretched over the pulley and masses were attached to the end. Initially, no additional mass was added (only the 0.10 kg hanger provided a minimal tension to remove slack in the string). The string's total length was measured as 1.00 m, and its mass (0.00055 kg) was measured with an electric balance to determine the linear mass density:

$$\mu = \frac{m_{\text{string}}}{L_{\text{string}}} = \frac{5.5 \times 10^{-4}\ \text{kg}}{1.00\ \text{m}} = 5.5 \times 10^{-4}\ \text{kg/m}.$$

2. To achieve different tension values on the string, masses that meet the desired weight were added to the free end of the string. The total hanging mass $m = m_{\text{hanger}} + m_{\text{added}}$ was recorded. To create vibrations in the string, a tuning fork with a known frequency of vibration was struck to induce a vibration. It was then touched to the end of the sonometer box without the weights to induce a vibration in the string. This allowed the fork to drive the string at its own frequency. This process was repeated with different forks, each inducing a wave on the string. When the fork that induced the wave with the highest amplitude was found, it was tested multiple times against forks of similar frequencies to ensure that the fork in question gave the peak response. Also to observe more accurately, a slow-motion camera was used to observe with caution. When the fork was found, it was idealized and taken as as the fundamental resonance for the given tension.
3. When resonance was achieved for the investigated tension, the frequency of the driving tuning fork was recorded to be the resonance frequency. To ensure reliable

data analysis, the procedure was repeated three times for each weight set that was being investigated. Also, the strings were allowed to slowly dampen into rest after each trial to ensure that the data was not being influenced. The process was repeated for every tension (weight) that was investigated.

4. The experiment started with 0 kg added mass (only the 0.10 kg hanger, providing a small baseline tension). Then, for the test group, weights of 0.5 kg were added for each value of tension that was being recorded. This process was repeated until the total mass of the weights reached 3 kilograms, after which the string was observed to warp and risk breaking. This gave a range of 0.98 N (baseline ~0.10 kg) up to approximately 30 N (3.10 kg total).
5. Throughout the experiment, joints, the length of the string and the state of the string were constantly checked by eye or a ruler for the length to ensure a consistent experiment.

4. Data Analysis

Data Collection and Results

Hanging Mass (kg)	Resonance Frequency (Hz) - Trial 1	Resonance Frequency (Hz) - Trial 2	Resonance Frequency (Hz) - Trial 3
0.50	255.0	256.7	252.1
1.00	345.9	345.8	345.2
1.50	441.4	449.4	443.0
2.00	513.8	511.0	511.2
2.50	578.2	571.8	570.5
3.00	646.9	655.8	637.7

Table 2. Raw data – Resonance frequencies of the string for various applied masses (tensions).

Using the raw data table, average resonance frequencies for each mass was calculated. Also to compare, by using the theoretical formula for fundamental mode: $f_{\text{theory}} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ theoretical resonances were calculated. Percentage differences between the average of the measured data and the theoretical value were calculated to compare how close the experimental results were to the theory.

Hanging Mass (kg)	Tension (N)	Average Measured Frequency (Hz)	Calculated Frequency (Hz)	Percentage Error (%)
0.50	5.886	255.0	242.6	5.1
1.00	10.791	345.9	328.5	5.3
1.50	15.696	441.4	396.2	11.4
2.00	20.601	513.8	453.9	13.2
2.50	25.506	578.2	505.0	14.5
3.00	30.411	646.9	551.5	17.3

Table 3. Processed data – Average measured frequency vs calculated frequency (theoretical) for each tension, with percentage error.

The tension values are calculated with the 0.10-kilogram hanger. For example, if 0.50 kilograms is added, the total mass comes out to be 0.60 kilogram which results in the net tension on the string being calculated by $T = 0.60 \times 9.81 = 5.886$ N. The average frequencies are the mean values of the values calculated through three trials which are shown in table 1. The percentage error shows the deviation from the theoretical expectations.

Calculations and Derivations

To be able to effectively interpret the data, the calculations regarding the linear mass density (μ) of the string and the tension applied to the string were calculated. Also to observe and compare the experiment's results, the theoretical values of calculated theoretical resonance frequencies and the percentage errors were calculated. The linear mass density was calculated using the equation:

$$\mu = \frac{m}{L}$$

where the symbols represent:

- m is the total mass of the string,
- L represents the length of the vibrating portion of the string.

For this experiment, the string had a mass of 0.300 g $= 3.00 \times 10^{-4}$ and a length of 1.000 m, so:

$$\mu = \frac{3.00 \times 10^{-4}}{1.000} = 3.00 \times 10^{-4} \text{ kg/m}$$

To calculate the tension generated by the hanging masses, the following formula was used:

$$T = m_{\text{total}} g$$

where:

- m_{total} is the total mass attached to the string, including the 0.10 kg hanger,
- $g = 9.81$ is gravitational acceleration.

If a mass of 0.50 kg was taken as an example, the total mass would be:

$$m_{\text{total}} = 0.50 + 0.10 = 0.60 \text{ kg}$$

$$T = (0.60)(9.81) = 5.89 \text{ N}$$

(This process was repeated for all the other mass values to find each one.)

The resonance frequency of a standing wave on a string can be modeled by the equation:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Substituting the values of $L = 1.000$m and $\mu = 3.00 \times 10^{-4}$kg/m, the theoretical frequency for $T = 5.89$ can be calculated as follows:

$$f = \frac{1}{2(1.000)}\sqrt{\frac{5.89}{3.00 \times 10^{-4}}}$$

$$f = \frac{1}{2}\sqrt{1.963 \times 10^{4}}$$

$$f = \frac{1}{2}(140.1)$$

$$f = \frac{1}{2}(140.1)$$

(This calculation was repeated for all tension values.)

To find out the accuracy of the experiment, percentage errors were used. They were calculated with the method that follows as:

$$\text{Percentage Error} = \left(\frac{|\, f_{\text{measured}} - f_{\text{calculated}} \,|}{f_{\text{calculated}}}\right) \times 100\%$$

If the measured frequency at $T = 5.89$ was taken as an example, which is 255.0 Hz, and the theoretical frequency was 242.6 Hz, then:

$$\text{Percentage Error} = \left(\frac{|\ 255.0 - 242.6\ |}{242.6}\right) \times 100\%$$

$$= \left(\frac{12.4}{242.6}\right) \times 100\%$$

$$= 5.1\%$$

(This process was repeated for all findings to calculate their deviation from the theoretical model.)

Uncertainty

The measured data of resonance frequencies, tension and propagation are natural aspects that arise in calculations due to human and machine error or imprecision. Given that the frequency measuring instrument had a precision of ±1 Hz, the uncertainty in the measured frequency is:

$$\Delta f_{\text{measured}} = \pm 1 \text{ Hz}$$

To objectively impart meaning on the impact of these uncertainties, the maximum and minimum values of resonance for each trial is considered:

$$f_{\text{max}} = f_{\text{measured}} + \Delta f_{\text{measured}}, f_{\text{min}} = f_{\text{measured}} - \Delta f_{\text{measured}}$$

Hanging Mass (kg)	Measured Frequency (Hz)	Frequency Uncertainty (Hz)	Minimum Frequency (Hz)	Maximum Frequency (Hz)
0.50	255.0	±1	254.0	256.0
1.00	345.9	±1	344.9	346.9
1.50	441.4	±1	440.4	442.4
2.00	513.8	±1	512.8	514.8
2.50	578.2	±1	577.2	579.2

3.00	646.9	±1	645.9	647.9

Table 4. Uncertainty – Summery of uncertainties in measured frequencies.

Tension is calculated using the formula:

$$T = m_{\text{total}} g$$

where $g = 9.81$ and m_{total} is composed of the title weights and the 0.10-kilogram hanger. The uncertainties in mass and gravitational acceleration creates uncertainty in tension. The uncertainty in mass Δm is ±0.01 kg, and uncertainty in g is ±0.01 m/s². Using propagation of uncertainty:

$$\left(\frac{\Delta T}{T}\right) = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta g}{g}\right)^2}$$

Hanging Mass (kg)	Total Mass (kg)	Tension (N)	Tension Uncertainty (N)
0.50	0.60	5.89	±0.12
1.00	1.10	10.79	±0.22
1.50	1.60	15.69	±0.32
2.00	2.10	20.60	±0.42
2.50	2.60	25.51	±0.52
3.00	3.10	30.41	±0.62

Table 5. Uncertainty – Summery of uncertainties in measured tension values.

Using the resonance frequency equation:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

and using uncertainty propagation:

$$\left(\frac{\Delta f}{f}\right) = \frac{1}{2}\left(\frac{\Delta T}{T} + \frac{\Delta \mu}{\mu}\right)$$

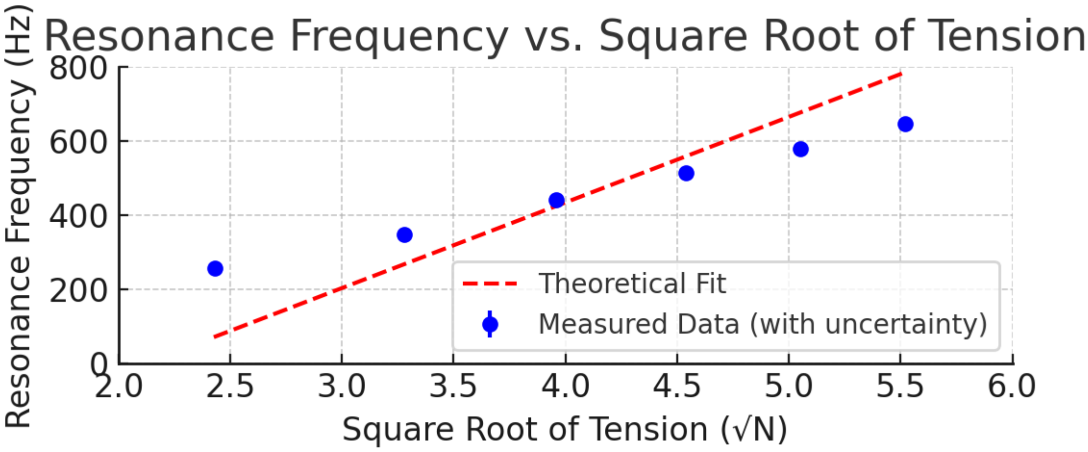
The propagated uncertainty in frequency for each measurement can be calculated by also incorporating the uncertainty in linear mass density $\mu = \pm 0.00005\ kg/m$.

Hanging Mass (kg)	**Measured Frequency (Hz)**	**Frequency Uncertainty (Hz)**
0.50	255.0	±2.8
1.00	345.9	±3.7
1.50	441.4	±4.5
2.00	513.8	±5.2
2.50	578.2	±5.9
3.00	646.9	±6.5

Table 6. Uncertainty – Summery of uncertainties in measured tension values.

These calculations tell us that the uncertainty in frequency measurement is systematic and instrument-dependent, remaining constant across all measurements. However, small changes in tension uncertainty result in larger variations in the final frequency uncertainty. Uncertainty is the main contributor to the resonance uncertainty as it has a square root term. Also, as the tension applied to the string increases, the absolute uncertainty in frequency also shows an increasing trend which suggests that measurement accuracy decreased at increased values of tension.

Graphical Analysis



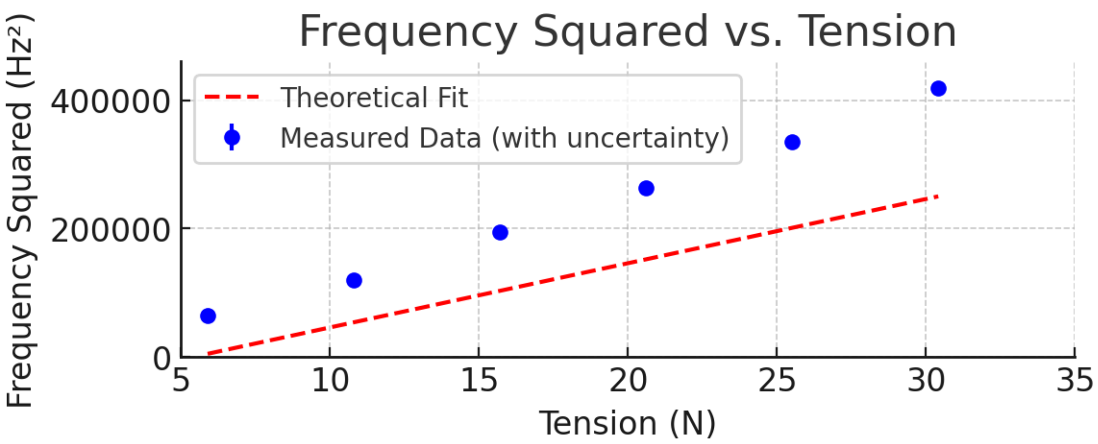
Graph 1. Graphical Analysis – Square root of tension (x-axis) plotted against the resonance frequency (y-axis).

The graph presents the relation between the square root of tension and resonance frequency.

The prediction by the theoretical model predicts the relation:

$$f \propto \sqrt{T}$$

In the graph, the theoretical fit loosely fits the measured data but its slope is greater than what would appear to be a very good fit for the graph.



Graph 2. Graphical Analysis – Tension (x-axis) plotted against the square of resonance frequency (y-axis).

The graph explores the relationship between frequency squared and tension follows a similar trend, but again the slope seems to not be the best fit for the data. Overall, both graphs show a matching relation but do not succeed at capturing the magnitude of the relationship best.

5. Evaluation and Conclusion

Strengths

Strengths	Explanation
Theoretical Accuracy	The experiment closely followed the mathematical model of standing waves, confirming the expected proportionality between frequency and the square root of tension.
Clear Trend in Data	Despite some deviations, the measured frequencies displayed a strong correlation with theoretical expectations.
Use of Multiple Trials	Three trials per measurement ensured reliability and helped mitigate random errors.
Graphical Analysis	The use of multiple graphs provided a detailed visualization of the results, reinforcing conclusions.

Uncertainty Considerations	Errors were calculated systematically, and uncertainties were accounted for in all measured values.

Table 7. Strengths – Strengths of the experiment and the investigation and their explanations.

Limitations and Improvements

Limitations	Suggested Improvements
Instrumental Precision	Use a higher-precision frequency measuring device to reduce uncertainty.
String Elasticity Variations	Using more consistent material with a uniform density to reduce systematic errors.
Environmental Factors	Conduct the experiment in a controlled environment to minimize air resistance and external vibrations.
Manual Error in Determining Resonance	Utilize an electronic signal analyzer to precisely detect resonance without human subjectivity.
High-Frequency Deviations	Extend the experiment to include lower and higher tensions to verify behavior across a broader range.

Table 8. Limitations and Improvements–The limitations and improvements of the investigation and the experiment.

Conclusion

The aim of the investigation was to investigate the relationship between the effect of tension on a string to its resonance frequency. The data collected and analyzed supported the existing theoretical models, showing that:

$$f \propto \sqrt{T} \text{ and } f^2 \propto T$$

The experimental findings did support the validity of the relation however, it did not succeed greatly at capturing the magnitude of the relationships and other elements that make them up. The likelihood of systematic errors shows the importance of using high precision instruments in experiments of magnitude. While the experiment provided an analysis of waves, it also provided a deeper relation between theory and experimental. The hypothesized claim of an increase in tension inducing an increase in the frequency of the waves came ou to be confirmed.

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