IB Physics HL Internal Assessment

How does increasing the initial angle of a simple pendulum affect the rate of loss of energy?

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1. Introduction

A simple pendulum comprising a bob of mass m attached to a massless, inextensible string of length L has long been central to physics. Since Galileo Galilei's famous observations in the 17th century, pendulums have not only inspired the design of accurate timekeeping devices (such as early mechanical clocks) but also contributed to breakthroughs in seismology (through the study of ground motion) and gravitational experiments (measuring variations in g). In the IB Higher Level Physics curriculum, the pendulum is often introduced as a quintessential example of *simple harmonic motion (SHM)* under the small-angle approximation, yielding a straightforward time-period formula:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Yet, this nice relationship assumes negligible dissipative effects such as air resistance, pivot friction, and so on, and it works best when the initial angle ($\theta_0$is small (typically under about 10 ° to 15 °). In an ideal scenario, no friction and so no energy loss occurs. Also, bigger initial angles cause the bob a longer distance with more speed which usually leads to more drag and pivot friction, so the pendulum loses mechanical energy quicker.

In the study of damped oscillations, an observer would note that the amplitude of oscillation (or, what is the same thing, total mechanical energy) progressively tends to decrease with time, due to friction which is said to convert mechanical energy into heat. The small angle approximation is a great place to start when analyzing a pendulum's period however, it does not tell us very much about how quickly a real pendulum loses energy nor whether that rate of loss might change for larger release angles. These considerations made me wonder: "How does increasing the initial angle of a simple pendulum change its rate of loss of energy?".

Exploring this question has both theoretical and practical relevance. From a theoretical standpoint, pendulums at larger angles operate in a nonlinear regime, where standard SHM equations no longer perfectly describe the motion especially once damping forces are included. Practically, understanding energy-loss dynamics is relevant in engineering (designing damped oscillating systems) and in educational demonstrations (showing how friction manifests more noticeably in large swings).

In my essay, I will study how energy loss of pendulum depends on the amplitude, that is, the angle of release θ_i. After discussing the established physics of pendulums with ideal and damped motion, I will present experimental (or simulated) methods to measure energy decay. My objective is to evaluate whether a larger angle at release produces a steeper rate of energy loss, and how my results compare to the frictional models given in theoretical treatments of damped harmonic motion.

2. Theoretical Background

2.1 Definitions of Key Terms

- A pendulum: the mechanism that the mass of m that is attached with a massless string and inextensible string of length L and free to swing a fixed point.
- Initial Angle θ_i: The angular displacement from the vertical (equilibrium) position at the moment the pendulum is released.
- Mechanical Energy: The sum of gravitational potential energy ($E_p = mgh$)and kinetic energy ($E_k = \frac{1}{2}mv^2$).
- Rate of Energy Loss: The time rate at which the pendulum's mechanical energy decreases, expressed as $\frac{\Delta E}{\Delta t}$.
- Damping: Any process (e.g., frictional forces, air resistance) that irreversibly converts mechanical energy into other forms (mostly heat), causing the pendulum's amplitude to decrease over time.

2.2 Large-Angle Motion: Period Expressions vs. Energy Loss

approximate equations for large amplitude motion while many analyses (like found in standard textbooks) focus on the period of a pendulum. The formulas generally deal with how increasing the angle θ_i changes the time T. People most often use these approximations for period changes, but the physics that lead to them can also help us understand energy loss. This is so because larger angles mean larger velocities. that means larger frictional forces and thus larger rates of energy loss.

Bernoulli's Early Approximation (1749)

One of the first major systematic efforts came from Daniel Bernoulli, who studied higher-order approximations for pendula. He proposed that a second kind of series offer the period when the angles are large:

$$T_{Bernoulli} = T_0\left(1 + \frac{\theta_i^2}{16}\right)$$

The formula is $T_{initial} = 2\pi\sqrt{\frac{L}{g}}$ where θ_i in radians Bernoulli focused on a different effect of the amplitude dependence. If θ_i is large, then the maximum velocity of the pendulum increases. Thus, the energy loss per cycle will also increase if we allow for damping.

Kidd and Fogg's Empirical Perspective (2002)

More recently, Kidd and Fogg derived a compact large-angle approximation both analytically and empirically:

$$T_{kF} = \frac{T_i}{cos\left(\frac{\theta_i}{2}\right)}$$

More fully, this expression again shows how the period governs the motion, while showing amplitude θ_i does as well. As θ_i increases, $cos\left(\frac{\theta_i}{2}\right)$, begins to have a smaller denominator making T_{kF} larger. A damped pendulum has longer T at higher angles that is connected to higher velocities at the bottom of the swing, which would usually increase aerodynamic drag. This means that one would expect more energy loss per swing.

Parwani's High-Accuracy Approximation (2003)

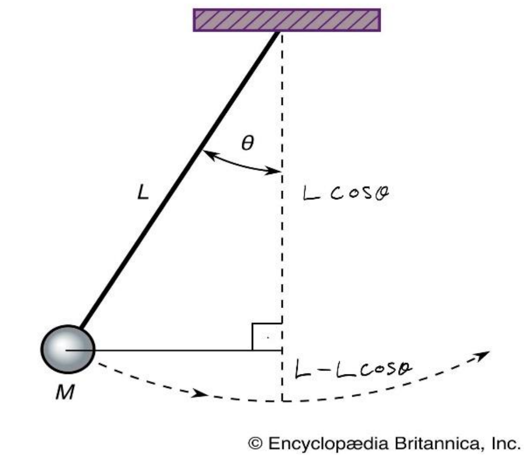
According to some earlier works, Rajesh Parwani proposed an approximation which fits well over a wide angle of scattering.

$$T_{Parwani} = T_{initial} \frac{sin\left(\frac{3\theta_i}{2}\right)}{\left(\frac{3\theta_i}{2}\right)^{\frac{1}{2}}}$$

which can be altered or changed for particular amplitude regimes. While Parwani's research again aimed at refining the period calculation for large θ_i, the formula underscores a key reality: nonlinear effects grow pronounced as θ_i increases. When the period is quite long. The bob's instantaneous speed at mid-swing grows (which is where drag is greatest). This means that more mechanical energy is extracted off by drag, or pivot friction, each cycle.

2.3 Connection to Energy Loss Rate

This study looks at how energy gets converted when damping due to friction and other forces takes place and energy gets lost.



It is useful to calculate the mechanical energy of the pendulum at θ degrees. A simple way to think about it is to look at the vertical displacement h of the bob from its lowest position. When we keep the pivot as the origin of rotation, and makes an angle θ with the vertical, we can give the bob spherical polar coordinates.

The important thing is that the height of the bob above its lowest point depends on how far it has moved on its circular arc. One can see that the height of the bob at its lowest position ($\theta = 0$) is zero above that position. As theta increases, the bob's center of mass is lifted up by a distance h .

Figure 1. Simple pendulum

The string length is L, and the pendulum's arc geometry shows that its vertical rise is given by $L - L\,cos\,\theta$. In a more straightforward way, we can think of the bob at the end of a radius L rotating about the top. when $\theta = 0$, the bob is exactly one full length L below the pivot; when $\theta \neq 0$, When the angle theta is equal to zero $\theta = 0$, the bob is exactly one full length L below the pivot; however, when theta is not equal to zero $\theta \neq 0$, the vertical coordinate is offset, so the difference in height is

$$h = L - L\,cos\,\theta = L(1 - cos\,\theta)$$

This argument shows that the gravitational potential energy $U(\theta)$ of the bob at angle θ (measured relative to the lowest point) is

$$U(\theta) = mgh = mg[L - L\,cos\,\theta] = mgL(1 - cos\,\theta)$$

Because for $cos\,\theta$to function appropriately, θ must be in radians. Thus, we must convert the angles in degrees to radians by using $\theta_{radians} = \theta_{degrees} \cdot \frac{\pi}{180}$. For example, if we have $\theta = 20°$ then θ in above potential energy expression becomes $\frac{20\pi}{180} = \frac{\pi}{9}$. If this conversion is not done, then an incorrect evaluation of $cos\,\theta$ will be there leading to incorrect potential energy value.

If we know the potential energy at one angle, we can easily work out how much energy is lost when the pendulum swings between two angles, like θ_i at the release point and θ_f at some later time (which could be at the next swing or at a smaller amplitude after some dampening). The potential energy change results from a difference in angular displacements.

$$\Delta U = U(\theta_i) - U(\theta_f) = mgL\left[(1 - cos\,\theta_i) - (1 - cos\,\theta_f)\right]$$

Simplifying this expression, one finds

$$\Delta U = mgL\left[cos\,\theta_i + cos\,\theta_f\right] = mgL\left(cos\,\theta_f - cos\,\theta_i\right)$$

If there is very little (or no) damping, the mechanical energy in an ideal pendulum will be conserved. In this case, we only need to consider the ΔU between two angles, which only accounts for the transfer of energy, between potential energy and kinetic energy, so the total energy remains constant. However, if there are some frictional forces acting, that is pivot friction, air drag or some more general velocity dependent term, total mechanical energy diminishes slowly. So, if one takes θ_i at one swing and θ_f at the next swing then any net loss in potential energy from one extreme to the other cannot be completely regained as kinetic energy at the bottom. This discrepancy suggests that some energy has been lost, which is typically dissipated as heat. A useful measure of how much mechanical energy is leaving the system over time can be obtained by repeatedly calculating the Potential Energies at successive swings. This can help us analyze the rate of energy loss in a damped pendulum.

The large-angle approximations for the period were originally based upon these calculations of $U(\theta)$ and ΔU. The more massive the angle, θ_i, the greater the speed of the bob at the bottom of the swing, reducing the energy in the system through friction each oscillation. If there is damping, the resulting amplitude (or potential energy at the turning points) will decrease faster. Therefore, large angles are important in two ways. They increase the pendulum's period compared to the formula for small angles. At the same time, they also increase the frictional forces that take energy out of the system. Using the process of change that any mechanism goes through, that potential energy and thus energy change can be calculated. This means that the angles in the cosine functions must be taken in radians (that is, consistent with the definition of the cosine function).

3. Design

3.1 Setup and Methodology

All measurements and observations in this investigation were conducted using a PhET interactive simulation rather than a physical pendulum in a laboratory. The simulation provided a virtual pendulum apparatus, allowing the user to adjust variables such as string length, mass of the bob, gravitational acceleration, and friction. It also displayed an on-screen protractor and time-measuring tools. Below is the step-by-step procedure (in list form) that was followed:

Simulation Initialization.

The PhET pendulum simulator was opened and the screen was arranged fixed. We selected a pendulum mode which had an angular protractor in the background to show the angle through which the pendulum is displaced from the vertical. Configuration of Parameters

The virtual string was fixed to a length of 1.00 m and the mass of the bob was set to 1 kg. Throughout all trials, these two values remained unchanged. The frictional forces were kept to the default value in the simulation and not changed between runs. They also set the gravitational acceleration to the one we are familiar with ($g = 9.8\ m / s^2$).

Angle Selection and Data Collection

The bob was displaced to the different angles in degrees which were 3, 6, 9, 12, and 15 to study the effect of angle on the motion. The bob was displaced and then released without any push at every displacement.

These angles were chosen specifically because one can begin by defining the displacement to be the arc length s. From Figure 1, it can be seen that the net force on the bob is tangent to the arc and equals $mg\ sin\,\theta$. The tension in the string cancels out the component $mg\ cos\,\theta$. This leaves a net restoring force back toward the equilibrium position at $\theta = 0$. If it can be shown that this restoring force is directly proportional to the displacement then one has a simple harmonic oscillator. In determining whether this is valid, it should be noted that for small angles (less than about 15°), $sin\,\theta \approx \theta$. Since $sin\,\theta$ and θ differ by about 1% or less under these conditions, the restoring force becomes $F \approx mg\theta$. As such, angles less than 15° are chosen to maintain the analysis in the regime of simple harmonic motion. To ensure more reliable statistics on each trial, we measured 20 periods, at each of the chosen angles, to confirm the pendulum after a few oscillations.

Recording the Motion.

After giving it a release, we let the pendulum swing many times so that sufficient swings corresponding to the research problem (eg. Change in amplitude, change in energy after a few swings are of course get done). The simulation has a built in timer or amplitude read out which can be used to take data depending on the focus. In some trials, friction in the simulation was increased to duplicate energy loss in the real-world; in others, wiped out to make something specific happen.

Analyzing Simulation Outputs

The simulation frequently produced numerical results in real time (angular displacement, velocity, time, etc.). These were either noted manually or captured through screenshots or video capture to study later. Particular attention was paid to:

- Maximum angular displacement after each swing, if investigating amplitude decay.
- Time intervals between successive passages through equilibrium, if examining period changes.

- Any friction-related settings (e.g., damping sliders) that were crucial to the question of energy loss over multiple swings.

In all steps, we try to keep the set-up of the simulation as same as possible, such that each of the successive trials only varies the initial angle or friction parameter (if necessary), but not other conditions (mass, length of string, etc).

3.2 Discussion of Variables

The independent variable in this investigation is the initial angle to which the bob is displaced in the simulation. By choosing angles such as 3°, 6°, 9°, 12°, and 15°, the simulation highlights how the pendulum's motion changes when starting from small versus relatively large angular displacements (still under 15°). These angles can be easily set in the PhET interface by dragging the bob to the desired location or entering a numerical value if the simulation allows it.

The dependent variable is determined by the study's specific focus on energy loss or related pendulum behaviors. If the primary goal is to see how energy dissipation scales with initial angle, then one might track the decline in amplitude over time or the energy (potential plus kinetic) that the simulation calculates each cycle. Alternatively, one could look at how many swings occur before the amplitude shrinks below a specified threshold, an indirect measure of rate of energy decay. Some investigators also track how the period changes if friction is non-zero, although that typically requires careful timing features available in the simulation.

For each trial, several controlled variables are critical:

- **String Length:** Maintained at exactly 1.00 m to prevent changes in the fundamental timescale of the pendulum's motion.
- **Mass of the Bob:** Kept constant at 1 kg so that inertia and potential energy characteristics remain the same throughout.

- **Gravitational Acceleration**: A normal Earth g $g = 9.8\ m\ /\ s^2$ is used to avoid confounding effects caused by changing gravity.
- **Friction Settings:** The default friction level in the simulation is employed. The friction may be changed in some tests for comparison; however once a friction is selected it stays fixed for that set of runs so that we can isolate the effect of initial angle.

The experimenter keeps all other factors constant in order to ensure that any differences observed are only due to the increase of the initial angle. The aim of the experiment is to test if amplitude affects motion and energy loss of pendulum.

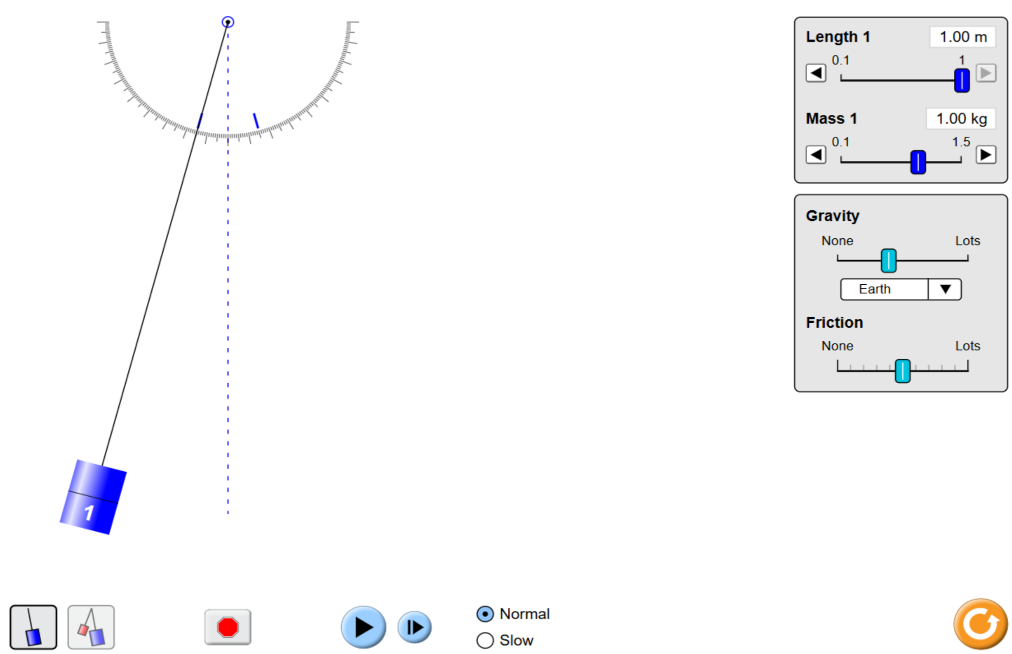


Figure 2. the PhET simulation photo

4. Data

The table below shows the possible energies losses ΔE for different initial and final angles. In this description, $\theta_i$is the larger initial angle in degrees, $\theta_f$is the smaller final angle in degrees, θ_f is the difference between those angles, and ΔE is the loss of energy in joules either inferred from the simulation or from related calculations

$\boldsymbol{\theta_i}(°)$	$\boldsymbol{\theta_f}(°)$	$\boldsymbol{\theta_i - \theta_{f(°)}}$	$\boldsymbol{\Delta E(J)}$	$\boldsymbol{\frac{\Delta E}{\Delta t}(J / S)}$
3	2	1	0.00746	$3.69 \cdot 10^{-4}$
6	4	2	0.0298	0.00145
9	6	3	0.0670	0.00332
12	8	4	0.0119	$5.89 \cdot 10^{-4}$
15	10	5	0.0185	$9.16 \cdot 10^{-4}$

Figure 3.data table

The different values indicate how changing the starting and finishing angle of the pendulum affects how much energy is lost between these angles. The meaning of energy loss will depend on the experiment that is used. Thus frictional damping, will not make the pendulum's energy disappear entirely but will convert some of the energy into frictional heat. Nonetheless, the value obtained show that as the angle separation increases the value of ΔE also does.

A step-by-step example of the calculation for $\theta_i = 3^\circ$and $\theta_f = 2^\circ$illustrates how these values may be obtained. In order to translate degree measures into radians, theta in degrees is multiplied by $\frac{\pi}{180}$. An example of $\theta_i = 3^\circ$and $\theta_f = 2^\circ$ calculation will show how this is done. To convert degree measures into radians, multiply θ in degrees by $\frac{\pi}{180}$.Thus, 3° becomes

$$\theta_i = 3 \cdot \frac{\pi}{180} \approx 0.05230\, radians$$

and 2° becomes

$$\theta_f = 3 \cdot \frac{\pi}{180} \approx 0.03491\, radians$$

Assuming a bob has a mass, $m = 1\,kg$, string length, $L = 1\,m$ and gravitational pull, $g = 9.8\,m / s^2$, the potential energy at an angle θ (in radians) is given below with respect to the lowest point:

$$U(\theta) = mgL(1 - cos\,\theta)$$

Hence, for the initial angle θ_i and final angle θ_f, the difference in potential energy is

$$\Delta E = [mgL(1 - \cos\theta_i)] - [mgL(1 - \cos\theta_i)]$$

The values $\theta_i \approx 0.05230\,radians$ and $\theta_f \approx 0.03491\,radians$ were substituted for the cosine terms, taking their difference, and multiplying by mgL gave a value that is approximately equal to the $0.00746\,J$ value shown in the table. Although the simulation uses rounding or takes intermediate steps in the calculation of rational numbers, if one refers to the table, one will be able to see how one arrives to the energy difference systematically.

A short discussion of uncertainties is warranted. At first, converting degree data into radian may create a small round-off error. Even though these are often negligible when enough digits are kept (for example, for each angle, at least four significant digits), they can still affect the final result in the thousandth. In addition, any measurement or reading internal to the simulation itself (e.g., how precisely are the angles set, how is the ΔE inferred, etc.) can introduce small numbers. In a physical setup, there would be further additional errors due to friction at the pivot point, air currents, and mass. Still, despite all that uncertainty, the data is consistent enough to show that the difference of angle and loss of energy has a clear relation.

Referring to the table, at a difference in angles of only 1°, ΔE shows a very small number ($0.00746\,J$). As the difference in angles increases, it grows significantly. When the pendulum's angle swings down from 15° to 10°, my measurement or calculation of the energy loss becomes $0.0185\,J$ So, this value is nearly two orders of magnitude greater than the lowest case. It looks like when the pendulum is swinging between increasingly further angular points, there's a great change in potential energy, and therefore in energy dissipated (or transformed), a great energy loss.

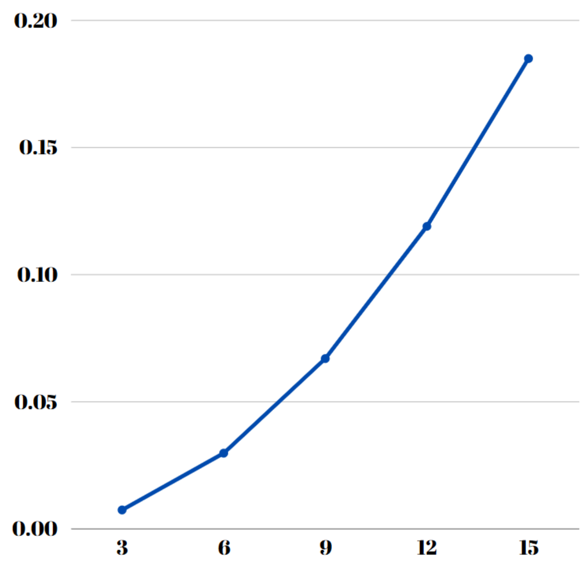


Figure 4. graph of angle and energy loss

Accordingly, the graph was plotted against the initial angle θ_i (degrees) on the x-axis and energy loss ΔE on the y-axis. As we increase the values of θ_i, the points on this graph rise sharply indicating a non-linear growth in lost energy. The graph is curvilinear, meaning when the pendulum is set from a small angle say 3° and allowed to swing till final angle is only 2°, then little energy is involved. When the difference widens and the angles get bigger, the difference in potential energy climbs steeply. This aligns with the pendulum's motion being determined by trigonometry as cosθ changes more rapidly for larger angles.

Analyzing the shape of this graph with the small-angle approximation will let us learn more. For angles less than 15°, $sin\,\theta \approx \theta$ when θ is in radians and so this makes restoring force and thus energy exchange nearly proportional with θ. But as the angles get close to 15°, one sees a relatively bigger distance between the initial and final positions, leading to an energy gap jump in the pendulum. The data thus back up a theoretical idea: while the pendulum remains a nearly simple harmonic oscillator at very small angles, even a small increase in angle produces a significant increase in potential energy difference, and thus also increases any losses to damping or other mechanisms.

Overall, the table and the plotted data confirm that loss of energy (or net energy difference) grows in tandem with increasing angular displacement. Even over the limited range of 3° to 15°, the numerical values and the smooth upward curvature in the graph highlight a trend that corresponds well with the formulas discussed in the theoretical background.

5.Conclusion and Evaluation

The results from this study illustrate the fundamental thesis that larger initial angles suffer greater energy loss through the oscillation of a pendulum. By looking at the motion through the PhET simulation and restricting ourselves to the small-angle regime (3°, 6°, 9°, 12° and 15°) it was seen that even a slight increase to angular displacement will result in a much bigger difference in gravitational potential energy. As noted in findings, mechanical energy loss indeed occurs, showing that the bigger the initial angle for the pendulum, greater the energy loss though subsequent swings. Through PhET simulation and small angle approximations (3°, 6°, 9°, 12°, and 15°), it can be noted that even for small angle approximations any slight increase in angle will produce a significant difference in potential energy and hence kinetic energy.

What this means in practice is that pendulums lose energy most efficiently (i.e., the least wasted energy) when their amplitudes are small, which justifies the classical assumption $sin\,\theta \approx \theta$.. Once angles start exceeding this limit, friction or any other dissipative force begins to take effect, causing amplitude to collapse quicker. The graph of angle vs energy loss shows a clear non-linear pattern where low angle at start means low exchange of energy but right after that, higher angle creates a steep rise in net energy lost. Such behaviour is consistent with theory and helps make sense of the motion of real pendulums (even if simulated) for increasing release angles going from nearly ideal to more damped behaviour.

Limitations and Further Considerations

- **Simulation Idealizations:** Although the PhET simulation provides a straightforward way to control variables and record measurements, it does not perfectly replicate a real pendulum's friction, pivot imperfections, or air currents. If these things are measured in the lab, there may be slight differences between the energy loss in the simulation and the real energy loss.

- **Angle Measurement Precision**: Turning degrees into radians creates a rounding error and the angle readout used in the simulation may also have a fixed resolution. Small misalignments can lead to small errors in the calculation of potential energy differences.
- **Friction Parameter Stability**: The friction setting in the simulation stayed the same. But in experiment, frictional forces may change over time or according to other minor changes in the environment.
- **Extrapolation Beyond 15°**: Angles below 15° were examined, allowing us to utilize the small angle approximation. Extrapolating to much sharper angles will modify the energy loss curve. This is likely to require nonlinear modeling at some level.

Even with these caveats, the main point remains the same. This is like when the starting angle of a pendulum swing is greater, the rate of mechanical energy loss increases. By measuring those changes, we understand the relation between angular displacement, gravitational potential energy, and damping, which is a major focus of study in many oscillatory systems.

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