International Baccalaureate Diploma Program

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Subject: Physics HL

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Title: Measuring the effect of speed of a magnet and the magnetic flux in a solenoid system's effect on the induced current and EMF measured by galvanometer in a simple harmonic motion system controlled by an Arduino microcontroller

Research Question: How does the speed of a magnet and the magnetic flux in a solenoid system effect the induced current and EMF?

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Introduction

Electromagnetic induction is one of the most crucial phenomena that allows our modern world to function. It lays the foundation to the principle that allows AC generators to work and produce electricity that powers all our electronic devices and allows basic needs such as refrigerators, ovens, light bulbs and computers to work. As we were learning this topic in our HL course, I had a difficult time figuring how these two different topics; electricity and magnetism were connected but after learning about electromagnetic induction, something clicked in me, and everything started to make sense. I was already very into tinkering with electronics and computers, so this topic allowed me to better understand how these systems work so I decided to choose this topic to research.

I decided to make a easily replicate able experimental setup that would allow me to measure different factors determining the values of induction and how they change with varying variables. So my aim was to use my electronics and software knowledge to create a system that is a model of what Faraday used while he was discovering these principles using common and cheap components and use them to calculate different variables. I choose speed of the magnet moving in the system and the number of magnets as they are easier for everyone to change and see the effects of.

Background Information

Magnetism

Magnetic Flux is the expression that shows the total number of magnetic field lines passing through a surface. This gives the equation:

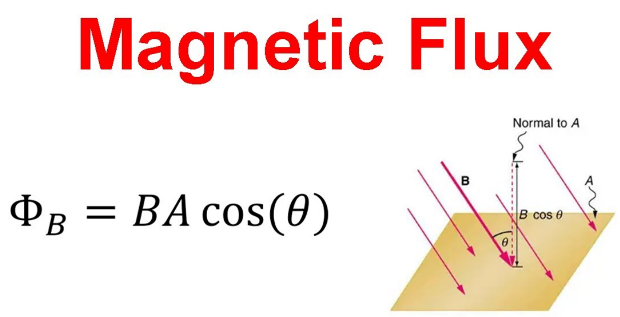

Figure 1:Magnetic Flux Representation, https://youtu.be/0UFaGcHuFT4?si=apih1VqMVQESqKdN

$$\Phi_B = B \cdot A \cdot \cos(\theta)$$

Where:

- B is the magnetic field strength
- A is the area of the surface that the lines pass through
- θ is the angle at which the magnetic field lines make with the normal of the surface

Now that we know how to find the magnetic field around a straight wire and know what magnetic flux is, let's talk about how we can find both the magnetic field and the current direction in a solenoid. As we know, solenoids are cylindrical objects with a conductive material (often a copper wire) tightly secured around them. In measuring the direction of magnetic field in solenoid, we again use the right-hand rule. In this case, we can clearly see that the current will be the variable that is rotating around the solenoid and the magnetic field is the straight one. So, curling our fingers around the solenoid in the current direction gives us the magnetic field direction in our thumb's direction. This information will be handy while calculating the induction caused by the magnets.

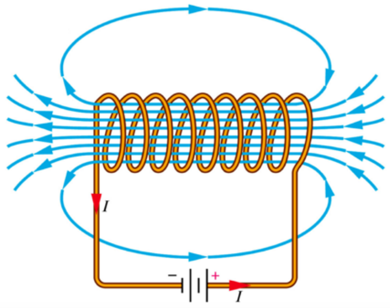

Figure 2 Solenoid Diagram

Induction:

Induction is the main topic that will be investigated in this paper. Electromagnetic induction refers to the phenomena that causes magnetic flux to be induced by a electric field, and an electric field causing a magnetic flux. Faraday first discovered this principle while he was working with a copper solenoid and a circuit that was connected to a galvanometer. He observed a change in the currents for an instant that he moved the magnet near the copper solenoid. This allowed the first ideas of electromagnetic induction to be born. Although Faraday realized the voltage of this circuit being equal to the change in magnetic flux, he couldn't figure out how and why the sign of this equation was negative. This was later investigated by Lenz who formed the Lenz's Law to explain this phenomenon (Tsokos 2023).

First we have to learn about EMF to understand the Faraday's Law, and that is electromagnetic force. This force is not force we know in the traditional sense; but this force refers to the energy per unit charge or the energy supplied to move charges through a circuit. EMF refers to the electromotive force that is done inside the circuit Faraday's Law states that electromagnetic force equals the rate of change of magnetic flux by time. Since this is essentially a rate of change, we can express the equation as:

$$\varepsilon = -\frac{\Delta\phi_B}{\Delta t}$$

Where:

- ε is the EMF
- ϕ_B is magnetic flux
- t is time

As aforementioned, the negative sign comes from the Lenz's Law, which states that a conductive material will induce a magnetic flux to oppose the rate of change of magnetic flux. This idea is also connected to Newton's Third Law, which states 'for every action (force) in nature there is an equal and opposite reaction'. As a conductive material enters inside a magnetic field, the material will experience an increase in the magnetic flux passing inside of it, so it will try to create an equal and opposite magnetic field to oppose this change, but to do this, it will also induce an electric current inside itself. With this principle, the system obeys the energyconservation law. A non-moving magnetic/electric field won't cause a flux therefore it won't induce a current. So, the EMF induced is also an expression of the force put into the system to move the magnet.

In this equation, there is an important element that helps to consider the system in a more practical manner and that is the inclusion of the number of turns of a coil. Although the Faraday's Law is the basis of the induction, a current induced in a single coil turn is not very effective so in practical uses like AC generators, thousands of turns of conductive material is used. Since there is a linear relationship, the final formulae we will has to be in a derivative form. To derive it, we have to use the Maxwell-Faraday equation.

The law states that the line integral of the electric field E around a closed loop C is equal to the negative rate of change of the magnetic flux Φ_B through the surface S bounded by that loop. Mathematically,

$$\oint_C E \cdot dl = -\frac{d}{dt}\int_S B \cdot dA$$

The magnetic flux Φ_B is given by the surface integral of the magnetic field B over the area S :

$$\Phi_B = \int_S B \cdot dA$$

The left side of the Maxwell–Faraday equation, $\oint_C E \cdot dl$, represents the electromotive force (emf) $\mathcal{E}$ induced in the loop. That is,

$$\mathcal{E} = \oint_C E \cdot dl$$

Substitute the definition of magnetic flux into the Maxwell–Faraday equation:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

The negative sign indicates that the induced emf produces a current whose magnetic field opposes the change in the original magnetic flux. This is known as Lenz's law. If the coil has N turns, the total induced emf is:

$$\mathcal{E} = -N\,\frac{d\Phi_B}{dt}$$

(Kinsler 2017)

Motor and Arduino:

Arduino UNO is a microcontroller. Microcontrollers are small computers on chips that allow the user to interact with electrical components with their digital pins. The most used pins are PWM (Pulse Width Modulation) pins, and they allow the microcontroller to send signals to electrical devices as pulses. These are sine waves that the microcontroller produces that get turned into binary waves. These signals are then computed by their width (or time, usually in milliseconds) by the electrical components and made into action (Arduino 2025). These widths are also considered in a unit called the duty cycle. The duty cycle is the ratio of the pulse width (open signal) over the period the system considers one cycle. In our case, one period refers to 20000µsecond (Cadence 2024).

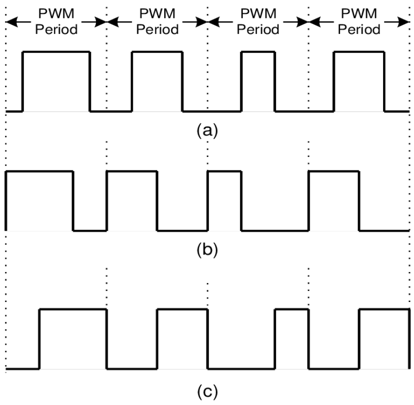

Figure 3: Pulse Width Wave Signal Representation

Experiment Setup

To explore this phenomenon, there should be a constant and measurable change in the magnetic flux. As I searched for experimental setup for this formula, I couldn't find anything useful. So, I decided to use a harmonic motion system to control a magnet's position. I decided to use my interest in electronics, so I connected a servo motor to my Arduino Uno and wrote Java code to continuously rotate the servo motos head 360 degrees. This way, I could easily push the magnet in and out of the system and measure the induced current. This also essentially creates an AC circuit, so it became easier to measure.

Variables

Variable	How It will be Changed/Measured	Effects on the Experiment
Independent Variable		
Speed of Magnets in the System	Controlled using an Arduino-powered servo motor with different PWM values	Affects the rate of change of magnetic flux, influencing the induced EMF.
Number of Magnets	2 types of magnets will be used. While one is constant, the weaker magnet will be increased by discrete values of 5.	More magnets will lead to an increase in magnetic flux, leading to a higher induced EMF and higher rate of change of magnetic flux.
Dependent Variable		
Induced Electromotive Force	Calculated Using the Faraday's Law	Determines how the changing magnetic flux affects the induced EMF
Induced Current	Measured using a galvanometer	Due to the changing EMF, a current will be induced.
Controlled Variables		
Resistance of the system	Same copper wire and circuit will be used in all trials	Ensures consistence resistance, preventing variations in current readings.

Magnetic Field Strength of the System	Same type of magnets will be used.	Ensures that only the number of magnets change, providing a standard interval in the readings
Range of motion of the Magnet	The servo motor will move the magnet through a fixed part and length	Keeps the distance travelled by the magnet consistent across trials (keeping the change in flux the same)
Uncontrolled Variables		
Stutters of the Servo Motor	It cannot be controlled and measured	May introduce minor errors n measurement, due to an increasing or decreasing rate of change due to the stutters

Table 1:Variables Table

Equipment Used

Equipment	How it will be used
Microcontroller (Arduino Uno)	Used to control the servo motor's head rotation speed and to power it.
Copper Solenoid	Used to displace magnets through it to measure the induced current
Galvanometer(±0.05mA)	Used to read the change in current caused by the magnet
Servo Motor (Without Potentiometer)	Used to create a harmonic motion system to make the magnet go in and out of the solenoid. A 360-degree rotating motor has been preferred due to the system requiring a harmonic motion without the need for a potentiometer
Neodymium magnet	Used to cause electromagnetic induction in the copper. Neodymium magnet was used since it is stronger than other magnets compared in size, making it more effective in creating induction therefore making it easier to see accurate results in the galvanometer.
Thin cotton robe	Used to connect the magnet and the servo motor's head. Cotton was picked since it is not a conductive material and won't cause a change in magnetic field.
Scissor	Used to cut the duct tape and the cotton robe
Duct tape	Used to make the servo motor and solenoid stable
Jumper Wires (Male to Female)	Used to connect servo motors pins to the Arduino Board
USB A to C Converter	Used to be able to connect the Arduino to my PC
USB A Cable	Used to connect the board to the USB A to C converter

Digital Gauss Meter (±0.001)	Used to measure the magnetic flux of the magnets

Table 2: Equipment Table

Ethics and Risk Assessment

It is the researcher's responsibility to use accurate data and not lie on it since the whole integrity of this experiment lies on the trustworthiness of the data.

This experiment contains risks on human health and the environment the experiment is conducted on. The main risk arises from the copper wire where the current is induced being in the open without any protection like rubber insulation. This can potentially shock the researcher if the experiment is not conducted carefully, but since the current inside the wire is low, it doesn't carry a lethal risk (Khan 2025).

Since there is a lot of adjusting done with the duct tapes and cotton robes, it will create a lot of waste product. These must be carefully collected and recycled to not cause any harm to the environment. Also, the use of scissors has to be carefully done in order to not damage anyone.

Neodymium magnets are very strong and can snap forcefully with any other metallic or magnetic surface, potentially damaging the surface and making a loud noise.

Procedure:

1. Connect the servo motor's pins into the corresponding pins on the Arduino board. The red cable on the servo connects to the 5 Volt pin, the brown cable connects to the GND (ground) pin and the yellow cable connects to the 9th Digital PWM pin (any PWM pin is OK for this experiment).
2. Connect a thin cotton robe to the end of the servo motor's rotating head holes. Connect it to the head with the largest diameter and the furthest hole for more displacement during rotation.
3. Download the compatible Arduino IDE for your computer.
4. After opening the app, download Arduino Uno Board package through the board's menu
5. Select the port that your Arduino is connected to
6. Write a suitable rotation code for your motor. In this case I used the servo.h library and only wrote code for the speed (usually, most servo motors would require a position parameter, but I decided to use one without a potentiometer, so it isn't included)
7. Write in the PWM signal width into your code for the first 5 trials (ones used here were 1600, 1700, 1800, 1900, 2000 milliseconds)
8. Incorporate a delay function to the code (3 seconds in this case, this is done for easier recordings of the data)

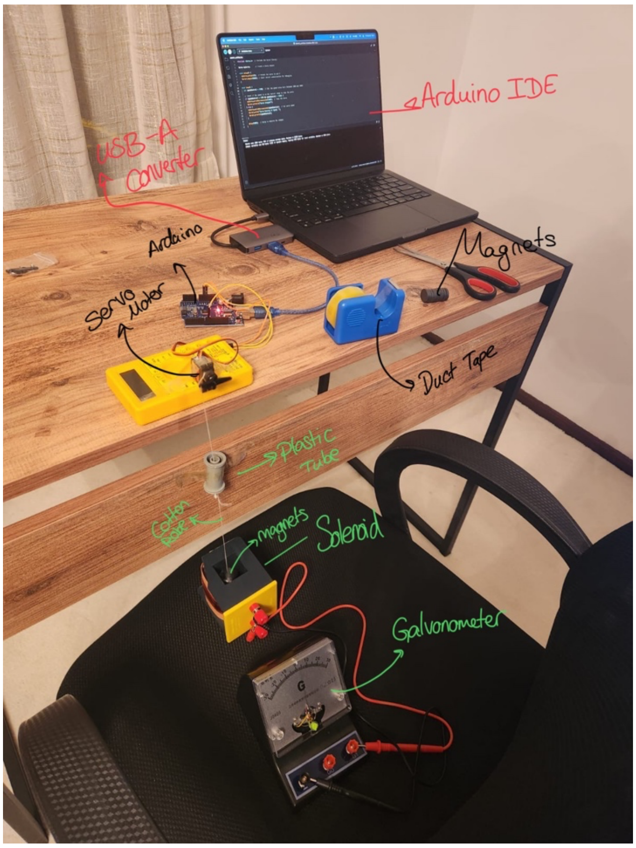

Figure 4: Experiment Setup with Captions

9. Connect a tube to a suitable vertical position on a corner to limit the horizontal axis movement
10. Let the cotton robe pass through the tube and connect the desired magnets to the end of the robe and stabilize it with a duct tape
11. Place the solenoid where the magnets can easily go through most of the height of it.
12. Connect the solenoid to a galvanometer
13. Connect the Arduino board to your PC through a USB A port.
14. Click the Verify and Upload button inside the IDE
15. Record down the values read from the galvanometer
16. Open the Duct tape and replace the magnets
17. *Repeat steps 7 to 14 5 times until 25 data have been recorded.*

Raw Data

Magnet Number (1 refers to the extra strong neodymium magnet)	Induced Current (milliamperes) (±0.05mA)					
	PWM Signal (µs)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
5+1	1600	2,60	2,50	2,50	2,30	2,40
	1700	2,92	3,01	3,05	2,97	3,04
	1800	3,95	4.02	3,98	4,01	4,04
	1900	4,48	4,52	4,49	4,47	4,51
	2000	4,95	5,02	5,00	4,98	5,03
10+1	1600	4.90	4,85	4,95	4,92	4,88
	1700	6,00	6,02	6,00	5,98	5.97
	1800	7.80	7.75	7.50	7.45	7,63
	1900	8.40	8,35	8,42	8,40	8,45
	2000	9.50	9,45	9,52	9.40	9,30
15+1	1600	7,35	7,28	7,40	7,32	7,36
	1700	8,75	8,68	8,80	8,72	8,78
	1800	10,15	10,10	10,20	10,12	10,18
	1900	11,28	11,22	11,30	11,25	11,29
	2000	12,65	12,60	12,70	12,62	12,68
20+1	1600	9,80	9,75	9,85	9,78	9,82
	1700	11,5	11,45	11,55	11,48	11,52
	1800	13,10	13,05	13,15	13,08	13,12
	1900	14,20	14,15	14,25	14,18	14,22
	2000	15,80	15,75	15,85	15,78	15,82
25+1	1600	12,25	12,18	12,30	12,22	12,28
	1700	14,40	14,35	14,45	14,38	14,42
	1800	16,30	16,25	16,35	16,28	16,32
	1900	17,80	17,75	17,85	17,78	17,82
	2000	19,70	19,65	19,75	19,68	19,72

Table 3: Raw Data Table of Induced current depending on the number and types of magnets used

Processed Data

To derive any data from the raw ones, we must process gather the speed values from the PWM signals as well as find the speed through the voltage of the servo motor. This way we can find how the magnetic flux changed and find the induced current.

Calculating the Magnetic Flux of Magnets:

As magnetic field reduces with distance, when calculating the magnetic flux, it is essential to use the distance used in the experiment to calculate the magnetic flux. This meant that I had to measure the magnets strength from a 2.00±0.05 distance as the radius of the square hole of the solenoid was 4.00±0.05cm. This yielded the results:

- One small black magnet: 0.31mT
- Strong neodymium magnet: 7.8mT

According to this, I will use the magnetic flux of these magnets in the processed table.

<u>Calculating the Speed of Rotation:</u>

Although speed of rotation from the data sheet calculations could've been used, I preferred to do a manual speed control because the speed of rotation can change when some resistance (in this case weights) is being applied on the motor and as this method is much more accurate. For this, I recorded the servo motor at different PWM speeds with a slow-motion video camera. I used to formula to calculate the period:

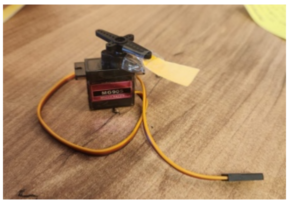

Figure 5: Servo Motor with One Head Marked, Used to Identify the Head to Measure the Period

$$Period = \frac{Frames\ per\ Rotation}{Frames\ per\ Second(240)}$$

PWM	Frames per Rotation	Period (seconds)
1600	796	3.32
1700	502	2.09
1800	380	1.58
1900	318	1.33
2000	274	1.08

Table 4: PWM Signal to Speed Conversion table via Video Measurement

**An uncertainty propagation wasn't included in this step as PWM signals have no uncertainty and the FPS of the camera includes a small uncertainty (4.17x10^{-3} seconds equating to %0.4 error) and it is mostly accurate so the general uncertainty of this step will be taken as %0.4.*

Processed Data

To process the data, we need to calculate the mean of each current value. We can do this with the formula for taking the average:

$$Average\ Current = \frac{Trial\ 1 + Trial\ 2 + Trial\ 3 + Trial\ 4 + Trial\ 5}{5}$$

<u>Uncertainty Calculations</u>

Calculating the uncertainties for frequencies: As mentioned beforehand, the uncertainty of the frequencies is %0.4.

Uncertainty of EMF: Let's take 9.35 magnetic flux with 0,30 frequency for the sample calculation

$$\frac{\Delta \Phi}{\Phi} = \frac{0.01}{9.35} \approx 0.00107mT$$

$$\Delta t = \frac{T}{4} = \frac{3.32}{4} = 0.83\text{ s } and\ \Delta(T/4) = 0.0025\text{ s so } \frac{\Delta(T/4)}{(T/4)} = \frac{0.0025}{0.83} \approx 0.00301\text{s}$$

$$\varepsilon = \frac{\Phi}{T/4} = \frac{9.35}{0.83} \approx 11.27\,\text{V}$$

Combined relative uncertainty:

$$\frac{\Delta\varepsilon}{\varepsilon} = \sqrt{(0.00107)^2 + (0.00301)^2} \approx \sqrt{1.15 \times 10^{-6} + 9.06 \times 10^{-6}} = 0.00320\,Volts$$

Magnetic Flux (mT)(Φ), ±0,01mT	Period of Rotation (± 0,01s)	Frequency (Hz)	Period/4 (Equals to the change in time form 0 to max Φ)	EMF (V)	% Uncertainty of EMF	Average Current Induced (I) ±0,05mA
9,35	3,32	0,30	0,83	11,27	0.31	2,46
	2,09	0,48	0,52	17,90	0.53	3,00
	1,58	0,63	0,39	23,67	0.40	4,00
	1,33	0,75	0,33	28,12	0.34	4,49
	1,08	0,93	0,27	34,63	0.28	5,00
10,90	3,32	0,30	0,83	13,13	0.31	4,90
	2,09	0,48	0,52	20,86	0.52	5,99
	1,58	0,63	0,39	27,60	0.40	7,63
	1,33	0,75	0,33	32,78	0.34	8,41
	1,08	0,93	0,27	40,37	0.28	9,42
12,45	3,32	0,30	0,83	15,00	0.31	7,34
	2,09	0,48	0,52	23,83	0.52	8,74
	1,58	0,63	0,39	31,52	0.40	10,14
	1,33	0,75	0,33	37,44	0.34	11,26
	1,08	0,93	0,27	46,11	0.27	12,64
14,00	3,32	0,30	0,83	16,87	0.31	9,80
	2,09	0,48	0,52	26,79	0.52	11,50
	1,58	0,63	0,39	35,44	0.40	13,10
	1,33	0,75	0,33	42,11	0.34	14,20
	1,08	0,93	0,27	51,85	0.28	15,80
15,55	3,32	0,30	0,83	18,73	0.30	12,24
	2,09	0,48	0,52	29,76	0.52	14,40
	1,58	0,63	0,39	39,37	0.40	16,30
	1,33	0,75	0,33	46,77	0.34	17,80
	1,08	0,93	0,27	57,59	0.28	19,70

Table 5: Processed Data Table

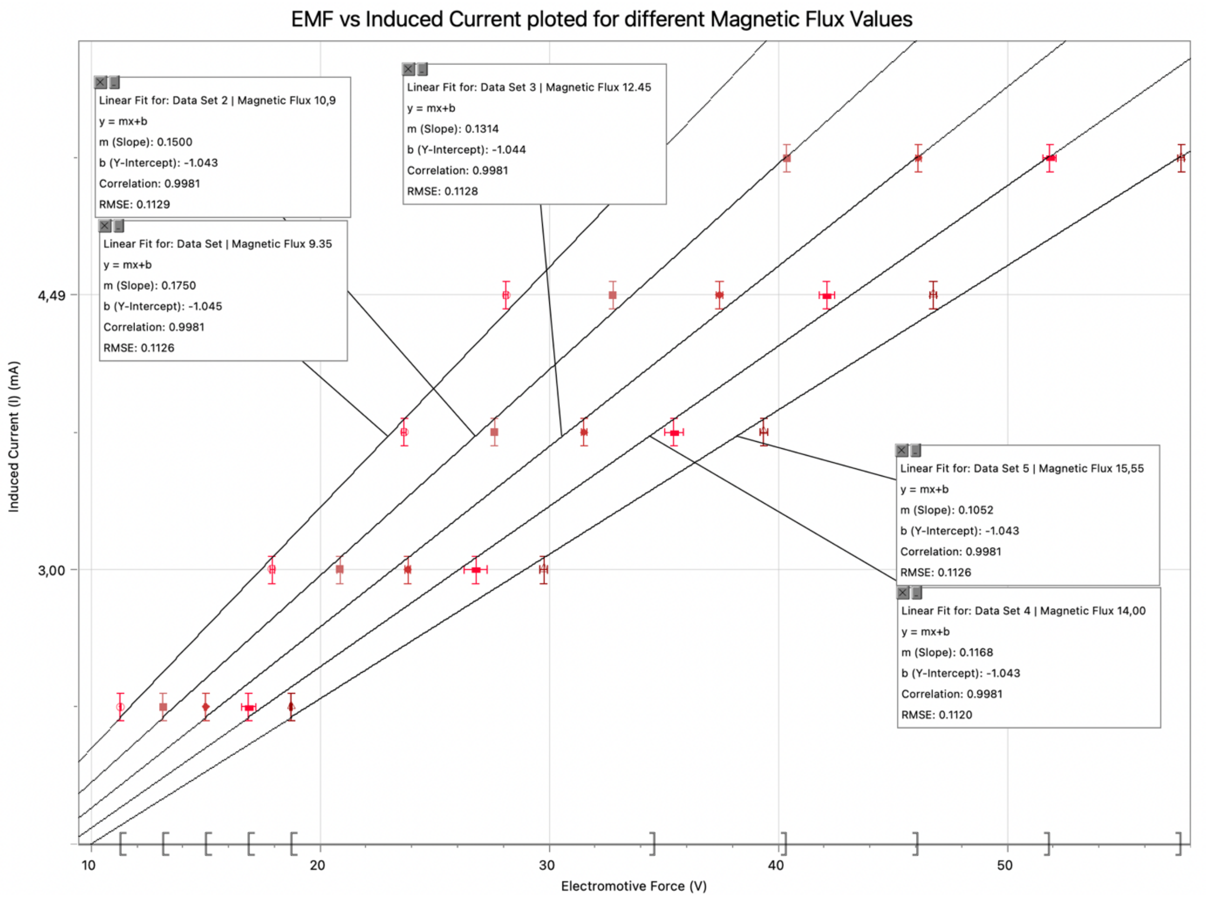

Figure 6: EMF vs Induced Current graph plotted with different Magnetic Flux Values

This graph shows a strong linear trend. This was to be expected as Ohm's Law suggest that $EMF(V) = I \times R$, so as the current increases, the EMF is to be increased with the slope being the resistance of the system. In this case, while the slope is changing, it's in a small range of values. The reasons for this will be better analyzed in the evaluation section.

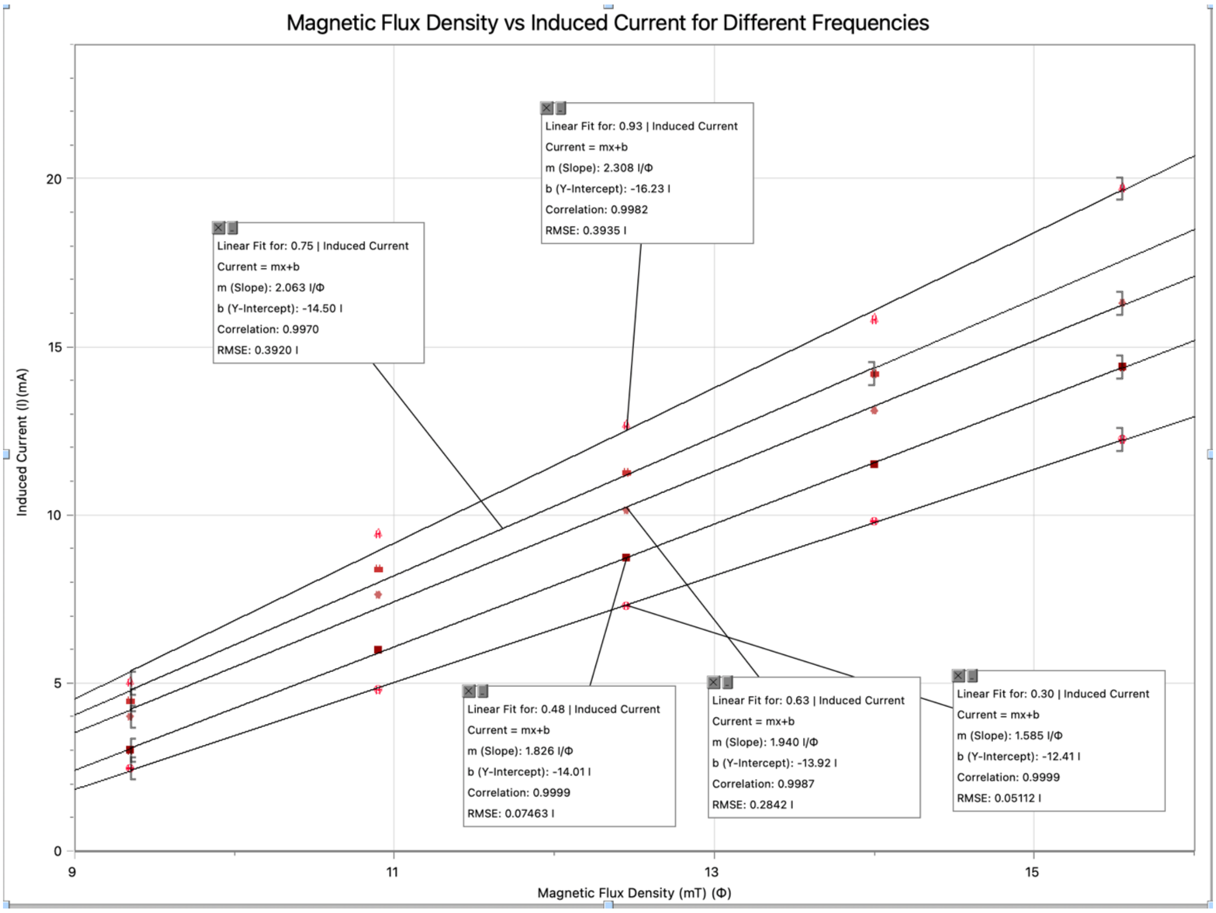

Figure 7: Magnetic Flux Density vs Induced Current graph plotted for Different Frequencies

In this graph, we would expect the graph to have the same slope (as the resistance is the same), and as the frequency increases, the y-intercept should increase. From our findings, we observe that although the y-intercept increases, the slope also increases slightly. This may be caused by the servo stutters. Since for higher magnetic flux values we have to use more magnets, the weight the servo motor should carry also increases. This creates stutters as servo motor is trying to keep in line with its internal period especially as the frequency is increased, sometimes causing small jumps. This can cause a higher rate of magnetic flux as the change is the same with a smaller time value. In this case, as frequency increased, we see a lower correlation. If these stutters were lower, we would expect to see the same slope.

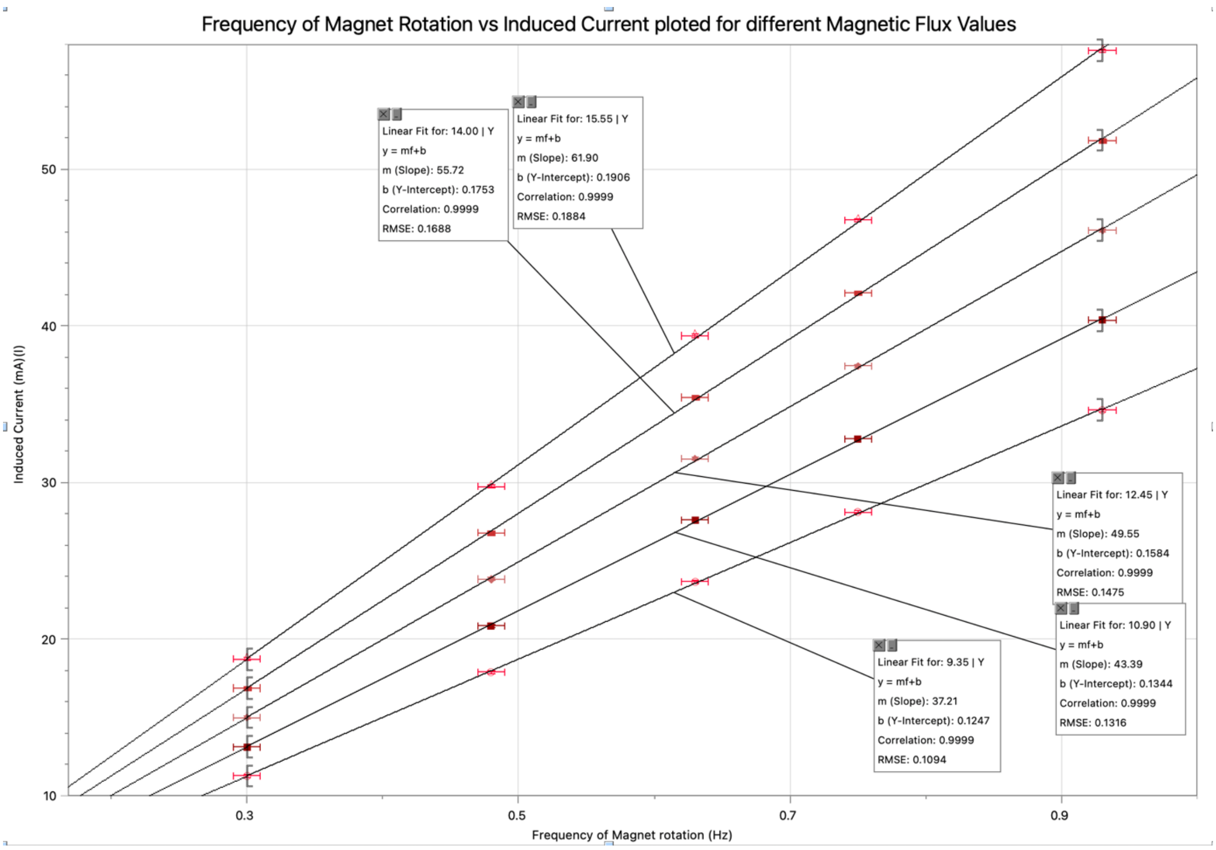

Figure 8: Frequency of Magnet Rotation vs Induced Current plotted for different Magnetic Flux Values

This graph is a direct result of the equation, so it gives a R^2 value of 1. As magnetic flux increases, the EMF induced will also increase linearly. As the frequency of rotation increase so did the EMF induced as time got smaller values. We would expect this graph to go through the origin, but the y-intercepts are all above the x -axis. This is likely due to the magnets getting not too far away from the coil to make a zero magnetic flux. This would cause the values we used for the experiment to have a small offset, as the predicted zero to maximum magnetic flux is not actually from zero, so it does not contain the full values of the flux we measured. This will be discussed more in-depth in the conclusion.

Conclusion

This investigation set out to answer the question of to what extent does the speed of a magnet and the magnetic flux in a system effected the induced current. The results proved our hypothesis, that an increase in both the variables lead to an increase in the induced current.

The Figure 6 exhibits a strong linear relationship as predicted by Ohm's Law. The slope of the graph corresponds to the resistance of the system. We would have expected to see the same slope across every magnetic flux value, but there were minor fluctuations. The y-intercepts were also expected to be at zero (as no EMF = no I) but the best fit lines were a bit above the y-axis. This could be due to the magnetic flux not being zero at the furthest part of the magnets' path. The graph gave a high correlation coefficient so although the problems are acknowledged, the relationship is correct.

Figure 7 shows a linear relationship between magnetic flux density and the induced current. This was expected as magnetic flux increase directly effects the EMF induced and it therefore increases the current induced. Within the same magnetic flux values, increase in the frequency shows a higher induced current as the period is shortened and therefore the rate is increased. Within each line, the slope is mostly constant. The lines show a positive y-axis intercept. This is likely due to the reasons mentioned beforehand (the magnetic flux not necessarily being zero when at its lowest value). One important feature of this graph is that the correlation shows a decrease as the frequencies increase. This is probably because as the weight and the frequency expected of the servo motor increased, it started to jitter and stutter. This therefore caused a nonlinear change in the rate of magnetic flux and effected the results. The other possible effects of this will be analyzed more in-depth in the evaluation.

Figure 8 was a direct graph resulting from the equation we used (Faraday's Law) so it shows a R^2 value of nearly 1. This graph was drawn to see the direct effect of changing the magnetic flux value on induced current and as the equation suggests, it showed an increase. The other importance of this graph is that the y intercepts are not zero, confirming our suspicions that the magnetic flux was never truly zero in the system.

Evaluation

The experiment was mostly accurate. The main problems about the experiment were the lack of range in the movement of the solenoid, and the power servo motor could produce. The narrow range caused the magnetic flux not to be zero causing the y-axis intercepts other than zero. This was the biggest systematic error in the system. The other problem being the torque the servo motor could produce. As the magnet number increased to ≥ 15 the servo motor began producing some stutter. This is caused by the reason that the servo motor is trying to be in sync with it's own time calibration and PWM signals but the weight (especially in the higher frequencies) caused the servo motor to get out of order of its signal and therefore produce higher amounts of power to account for the time lost. This in order creates a larger magnetic flux as the rate of change is suddenly shortened. This stutter is unpredictable and is the main reason the correlation got lower in Figure 7. This problem could've been avoided using a stronger servo motor but this was the most power the microcontroller could give to a servo in its output limit (5 Volts). Another problem was as the EMF induced got higher, the magnet started to shake inside of the solenoid as if it was being pulled to the corners. This was a result of Lenz's law and it also increased the stutters of the servo motor, as it opposed the change in magnetic flux. This was unaccounted for before the experiment, since I didn't realize these values of magnetic flux would be enough to trigger this from happening, yet it did, although it had a small effect. For this not to be present, a larger solenoid could've been used to decrease the effect of it or a narrower tube could have been used to limit the x-axis movement of the magnets.

For this experiment to be more accurate, a longer head for the servo motor could've been used to make sure the magnetic flux got zero outside the solenoid. This could've been done with a normal servo by calibrating its y-axis movement and measuring its magnetic flux values in the extremes of its motion so that the change in flux was more apparent, even though it would've made measuring trials much harder.

The strengths of the experiment were that it was done relatively cheaply and easily, with little coding knowledge required. It also gave very visual results, being optimal for teaching students about the subject. The items used were easy to find material in any high school laboratory. It showed deep insight into the workings of an AC generator and how it's efficiency can be increased and how to easily create one. The graphs showed accurate results although they weren't really precise. Further research can be done by changing the variables, for example turning the copper solenoid around a magnet to replicate a DC motor. Another experiment can be conducted by playing with the range of values of magnetic flux to see the limits of current that can be induced. This experiment would give insight into how electrons move and behave. To see the effect of the magnets being moved inside the solenoid, an experiment can be conducted to see how much force the magnetic field of the solenoid produces by dropping magnets inside long coper wires.

Overall, the experiment has been successfully conduced and replicated the findings of Faraday while confirming the equation he put out.

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