Title:

Investigating the Effect of Magnets' Rotating Period on the Electromotive Force Of an Electric Generator System

Research Question:

How does the period of the magnets' rotation affect the magnitude of emf in an electric generator system?

Word Count: 3830

Subject: Physics

Introduction

Electromagnetism is an interesting topic that can be looked at in from many different perspectives. When I first learned about generators, it amazed me how these systems work and combine different laws of physics in order to create a power so big that it lightens hundreds of houses and helps in other everyday problems. I have always been interested in the mechanical field of physics and when I was thinking of a topic for my extended essay in physics, I remembered the time when the electricity went out throughout the city and while the lights of other houses did not come back, the generator in our house instantly turned on and the electricity was restored. I thought it was the perfect opportunity for me to combine the mechanical and electrical aspects of physics, considering the working principle of generators. Thus, I also realized I would come across the subject of capacitors and electromagnetism in the IB curriculum, so I decided to do some research upon the subject. My further research pushed me to learn more about the working principle of electric generators and made me realize these mesmerizing machines can work in various ways, either taking its energy from wind or from water systems. I thought that these different working principles would provide me with a wide range of resources throughout my investigation, and I decided to move forward in this direction. This investigation will not only allow me to work with the two core fields of physics, but it will also help me to learn more about electrical principles of physics , about which I don't have much prior knowledge.

Background Knowledge

1.Electric Generators

An electric generator is an equipment capable of converting mechanical energy into electrical energy, to be used for power distribution and transmission by power lines to users such as residential, commercial and industrial. Generators also power the electricity of vehicles, airplanes, ships, and trains.

Mechanical power needed for an electric generator is usually supplied by a rotating shaft, and the electrical power output is the product of shaft torque and rotational speed. This mechanical energy may be derived from several sources, including hydraulic turbines located at dam or waterfall, wind turbines, and steam turbines which produce steam by using natural gas or nuclear fission. A direct ignition of the gas by the gas turbine and ignition of gasoline or diesel engines are also the primary sources. The design and the speed of operation of a generator can differ greatly, based on the properties of its mechanical energy source.

Almost all generators connected to electrical grids generate alternating current (AC), and alternating polarity at a constant frequency (e.g., 50°/60 cycles/s, double reversal/s). Because more than one generator is coupled in a power network, they are required to operate at the same frequency, so that they may be operated synchronously. These generators are called synchronous generators, or, in some contexts, alternators.

Devices for electromagnetic energy conversion, generators, etc., work on the principle of electromagnetic induction.

It is so, and according to this principle, in a changing magnetic field around a conductor an electric current is generated in the circuit. When a coil rotates under the influence of a magnetic field, a current is induced in the coil with a polarity determined by Fleming's right-hand rule.

Here are a few methods for altering the strength of a magnetic field in a loop:

-Rotating the magnet in relation to the coil
-Moving the coil in and out of the field of magnetism
-Changing the area of the coil with respect to the field
-Moving the magnet closer to or further from the coil
-By changing the speed of rotating magnets.

In this investigation the value of t will be changed to measure the change in emf (t_0 will be 0 seconds.)

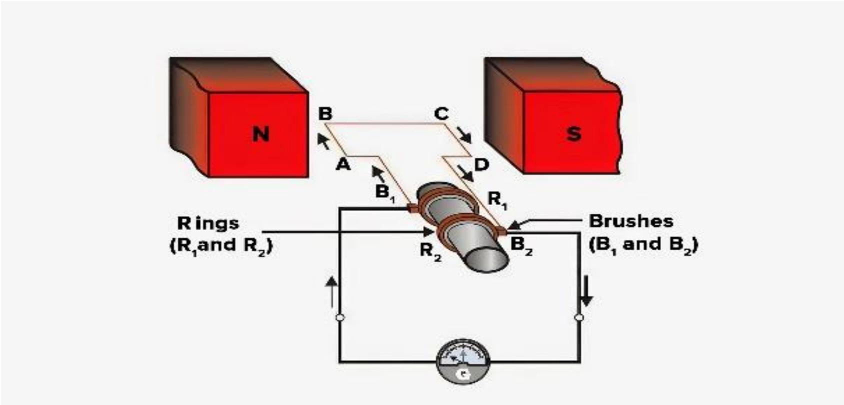


Figure 1: An illustrative diagram of an electric generator

2.Electromagnetic Induction

Faraday's law of electromagnetic induction, commonly referred to as Faraday's law, is a fundamental principle of electromagnetism that explains how a magnetic field interacts with an electric circuit to produce an electromotive force (EMF), a phenomenon known as electromagnetic induction. The understanding of electromagnetic induction is derived from extensive experiments conducted by Michael Faraday and Joseph Henry. Based on his measurement, Faraday inferred that an EMF is induced in a given coil when the magnetic flux through it is varied in time.

Accordingly, the electromagnetic induction of currents in a conductor is described by Faraday's first law of electrical induction, in which when such a conductor is in motion within a changing magnetic field, an electromotive force is induced. If the circuit is closed, this EMF induces a current, known as the induced current. Faraday envisioned a magnetic field consisting of lines of induction which a small magnetic compass would tend to orient itself upon. The amount of these lines that cross a given area is called magnetic flux. Faraday attributed electrical phenomena to alterations in this magnetic flux.

Years later, James Clerk Maxwell, a Scottish physicist, expanded upon this concept, proposing that a changing magnetic flux generates an electric field not only within a conductor (where it moves electric charge) but also in empty space, even in the absence of electric charges. Maxwell's derived a mathematical formula quantifying the change in magnetic flux and related induced electromotive force (E or EMF) change. This equation, known as Faraday's law of induction (distinct from his laws of electrolysis), states that the magnitude of the induced EMF in a circuit is directly proportional to the rate of change of the magnetic flux (Φ) over time (t):

$$emf = -\frac{d\Phi}{dt}$$

Equation 1: emf formula

If the rate of change of magnetic flux is expressed in units of Weber's per second, the induced *emf* has units of volts. Faraday's law is one of the four Maxwell Equations that define electromagnetic theory. The magnetic flux in the equation is:

$$\Phi_B = B \times A = BA\cos\theta$$

Equation 2: magnetic flux formula

Where,

-B is the magnitude of the magnetic field (having the unit of Tesla, T),

- A is the area of the surface (m^2),

- θ is the angle between the magnetic field lines and the normal (perpendicular) to A.

If we increase the time of rotation, while keeping the change in magnetic flux ($\Delta\Phi$) constant, the rate of change of magnetic flux decreases. This is because the EMF is inversely proportional to the time interval over which the change occurs. Therefore, as the time of rotation increases, the induced EMF must decrease.

The derivative of the rate of change of magnetic flux with time can be written as:

$$-\frac{d\Phi}{dt} = -\frac{\Delta\Phi}{\Delta t}$$

In our experiment, we aim to keep the magnetic field perpendicular to the surface, so we assume ($\theta = 0$), so $\cos\theta = 1$. Thus:

$$\Phi = B \times A$$

In our experiment, the magnetic field (B) and the surface area (A) will be kept constant since the aim is to observe only the changing period of the emf. When we substitute $\Phi = B \times A$,

$$-\frac{d\Phi}{dt} = -B \times \frac{A}{\Delta t}$$

If we ignore the negative sign which only indicates the direction of the induced emf due to Lenz' Law, we get:

$$emf = \frac{B \times A}{\Delta t}$$

These steps provided us the emf formula which indicates that the rotating period is inversely proportional to the electromotive force.

$$emf \propto \frac{1}{\Delta t}$$

Hypothesis: if the rotating period (time of rotation) increases, the electromotive force (EMF) should decrease, assuming all other factors remain constant. That is, because, as seen in the formula emf is proportional with $\frac{1}{\Delta t}$.

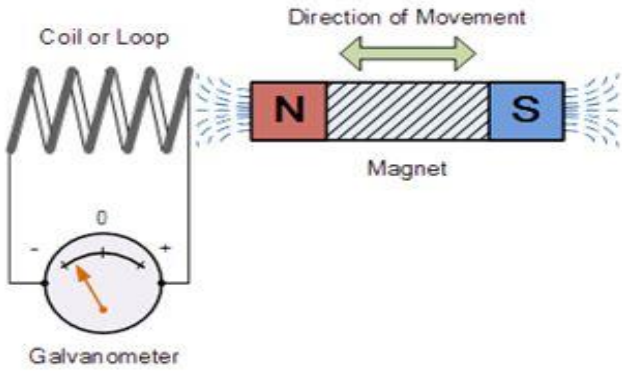


Figure 2: Magnetic field intensity in a closed loop

Contrary to this figure, in our system, instead of a bar magnet neodymium magnets attached to the turbine using wind energy will create the magnetic field.

Equipment

To carry out the experiment we are going to need various materials:

-Voltmeter (voltage will be the emf value.) ($\pm$0.5 volts)

-A wind turbine

-A copper wire of 8.00 centimeters of radius. ($\pm$0.05 centimeters)

-8 neodium magnets (which will be connected to the turbine.)

-Fan (55 watts) ($\pm$5 watts)

-Gaussmeter (To measure the magnitude of magnetic field)

-Wires (10 centimeters each)

-Stopwatch ($\pm$0.01 seconds)

Variables

Dependent Variable: Potential difference (emf)**:** Will be measured with a voltmeter:

-Connect the red probe (positive) to the positive terminal.

-Connect the black probe (negative) to the negative terminal.

-The multimeter will display the voltage (potential difference) between the two points.

Independent Variable**:** Amount of time magnets are allowed to rotate (5 seconds, 10 seconds, 15 seconds, 20 seconds, 25 seconds, 30 seconds, 35 seconds, 40 seconds)

Controlled Variables:

-Distance between the magnets and the wire (2 centimeters): By keeping the distance constant, we ensure that the changes in EMF are due to the rotating period rather than variations in magnetic field strength due to changes in distance.

-Number of neodium magnets (8): More or less magnets lead to a stronger magnetic field and thus a greater induced EMF. The magnitude of the magnetic field is dependent on the 8 magnets and we know that in the formula $B \times A = \emptyset$, a change in flux B will directly change the value of the flux, therefore affecting the emf value.

-Linear speed of the fan (5.23 m/s): Variations in speed can alter the frequency of changes in magnetic flux. If the speed of the fan changes, frequency, and therefore the period will change. The formula is:

$$freq. = \frac{\omega}{2\pi}$$

-Area of the loop of coil (0.10 m^2): A larger area would intercept more magnetic field lines, resulting in greater EMF. Keeping area constant ensures consistent flux linkage across trials.

-Magnitude of magnetic field (0.50 T): According to Faraday's Law, the strength of the magnetic field directly influences the emf.

Methodology

1. Get together the wind turbine using plastic spoons. (to capture the wind more effectively
2. Stick the magnets to the 4 of the blades of the turbine. (2 for each)
3. Measure the magnetic field caused by the magnets.
4. Connect the wires to the copper wire and the other ends to the voltmeter.
5. Place the fan the turbine and the coil of wire accurately
6. Start the fan
7. Record the value in the voltmeter when the time is stopped.
8. Repeat steps 5, 6, 7 for each of the 8 periods of rotation.

Picture 1. The hand-made wind turbine with the neodium magnets being attached

Picture 2. The system being constructed with the coiled wire, multimeter, the turbine and the wires

The coiled wire will be held in a constant position with the use of a stand and the turbine will be stuck to a carton body, because if it is held by human hands, the distance between the coil and the turbine, which we need to keep constant, may change and this will directly affect the output values.

After we record the data we obtained, we will insert the values of B, A, an period to see how accurate the voltage we found is compared to the values coming from the formula.

Raw Data Table

	Electromotive Force (Millivolts)		
Trials/Seconds $(\pm\mathbf{0.05})$	**1st Trial**	**2nd Trial**	**3rd Trial**
5.00 seconds $(\pm\mathbf{0.15})$	7.80	8.00	8.10
10.00 seconds $(\pm\mathbf{0.15})$	6.70	6.40	6.50
15.00 seconds $(\pm\mathbf{0.20})$	4.80	4.40	4.60
20.00 seconds $(\pm\mathbf{0.05})$	3.20	3.30	3.30
25.00 seconds $(\pm\mathbf{0.10})$	2.50	2.60	2.40
30.00 seconds $(\pm\mathbf{0.25})$	1.60	1.70	2.10
35.00 seconds $(\pm\mathbf{0.25})$	1.10	1.50	1.00
40.00 seconds $(\pm\mathbf{0.20})$	0.8	1.1	0.7

Table 1. The recorded emf values for each time interval

The maximum uncertainty calculation for each period of rotation is found by:

$$\frac{Volt_{max.} - Volt_{min.}}{2}$$

Example calculation for 5.00 seconds is:

$$\frac{8.1 - 7.8}{2} = 0.15$$

Maximum uncertainty value for 5.00 seconds is ± 0.15 volts.

Calculations For the Processed Data Table

First, we need to calculate the averages for each interval:

$$Avg._{5\ sec.} = \frac{7.8 + 8.0 + 8.1}{3} = 7.97\ millivolts\ (rounded\ to\ 3\ s.f.)$$

After we found the average value, we can convert it to volts:

$$1\text{ volt} = 1000\text{ millivolts}$$

So, to find the volt equivalent of 7.97 millivolts, we must divide it by the factor of 1000:

$$\frac{7.97}{1000} = 0.00797\ volts$$

Now this is the experimental value of the electromotive force, we follow the same steps for the rest of the 7 time intervals, we will have the experimental values.

Previously, we measured the values for the area of the coil (0.10 m^2) and the magnitude of magnetic field (0.50 T). So that we know all the values in the formula, we can now use the formula to come up with theoretical values:

$$emf = \frac{B \times A}{\Delta t}$$

-B is 0.50 T

-A is 0.10 m^2

-Δt is 5 seconds.

So, the theoretical value of electromotive force for 5 seconds time interval is;

$$emf = \frac{0.50 \times 0.10}{5} = 0.01000\ volts$$

The theoretical emf value for 5 seconds is 0.1000 volts.

Error Calculations

Measurement plays a significant role in scientific computation. Yet, getting perfectly correct measurement values is a matter of extreme rarity. In measurement of any parameter, small errors occur in general. There can be several sources of such errors, and therefore, values of measurement will vary in such cases. All such types of errors can be mathematically represented, and through them, one can calculate with even greater accuracy and formulate techniques for minimizing them. There are two types of errors in general, namely, absolute and relative errors.

Absolute error is a variation between a measured value and an actual value of a quantity. For instance, in a case of long distance, such as a highway, an error of a few centimeters is not significant. In a case of an electromotive force (EMF) of a generator, even a minor variation in millivolts can make a significant impact. In both cases, errors arise, but in a case of an EMF of a generator, an error is significant. The formula for absolute error is:

$$Absolute\ error = |Theoritical\ value - Experimental\ Value|$$

For our case with 5 seconds, it is:

$$|0.01000 - 0.00797| = 0.00203\ volts$$

Relative error is a ratio between a measured value and its absolute error. With determination of a relative error, one can assess a measurement's accuracy in terms of its actual value. Relative error can even allow one to make an approximation of an absolute error's value when its actual value is not determined. In such a scenario, one can utilize a measured value in its computation. Relative error, being a unitless value, possesses no unit and is frequently represented in terms of a percentage when multiplied with 100. The formula for percentage error is:

$$Relative\ error = \frac{|Theoritical\ value - Experimental\ value|}{Theoritical\ value} \times 100$$

For our specific case:

$$\frac{0.00203}{0.01000} \times 100 = \%20.3$$

The relative error for the 5 seconds rotating time is %20.3:

After we follow the same steps for the other 7 time intervals, The processed Data Table should look like this:

Processed Data Table

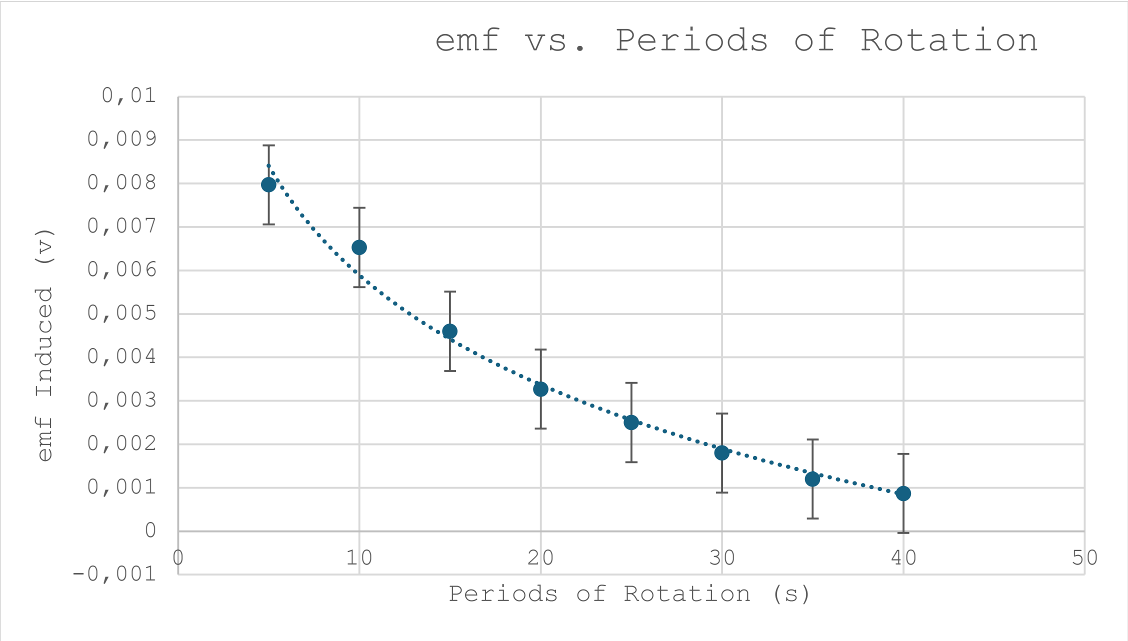
Time Intervals (Seconds) (± 0.05)	***Average experimental emf value (volts)*** (± 0.00050)	***Theoretical Value (volts)***	***Absolute Error (volts)***	***Relative error (%)***
5.00	0.00797	0.01000	0.00203	%20.3
10.00	0.00653	0.00500	0.00153	%30.7
15.00	0.00460	0.00333	0.00127	%38.1
20.00	0.00327	0.00250	0.00077	%30.8
25.00	0.00250	0.00200	0.00050	%25.0
30.00	0.00180	0.00167	0.00013	%8.0
35.00	0.00120	0.00143	0.00023	%16.0
40.00	0.00087	0.00125	0.00038	%30.7

Table 2. Experimental averages, theoretical values and error calculations

Graphing

To show the correlation between the time intervals and the decreasing emf as the intervals gets bigger, we plot the results in a best-fit line graph.

Experimental Findings Graph



Graph 1. Best-fit line of experimental values of emf varying with time.

We can see that the best-fit line for the graph is a logarithmic trendline and that is because the y-variable decreases with a decreasing rate as x-variable increases. In theory, this is the result of the inverse proportionality between emf and time. The absolute error findings are displayed on the graph as the error bars of y-axis.

Standard Deviation

Standard deviation is a statistical descriptor of how much the individual data points deviate from the mean of a set of data. The further apart data points are from the mean, the greater deviation of the data set. It is computed as the square root of variance.

Calculating standard deviation:

1-Calculate the mean of all data points. The mean is calculated by adding all the data points and dividing them by the number of data points.

So, $\frac{0.00797+0.00653+0.00460+0.00327+0.00250+0.00180+0.00120+0.00087}{8} = 0.0035925$

2-Calculate the variance for each data point. The variance for each data point is calculated by subtracting the mean from the value of the data point:

For 0.00797:

$$0.00797 - 0.0035925 = 0.0043775$$

3-Square the variance of each data point

$$0.0043775^2 = 1.92 \times 10^{-5}$$

4-Sum of squared variance values

When we subtract the mean value from each of the data points and square them, the result of their sum will be 4.6419892×10^{-5}.

5-Divide the sum of squared variance values (from Step 4) by the number of data points in the data set less 1.

$$\frac{4.6419892 \times 10^{-5}}{(8-1)} = 6.631413143 \times 10^{-6}$$

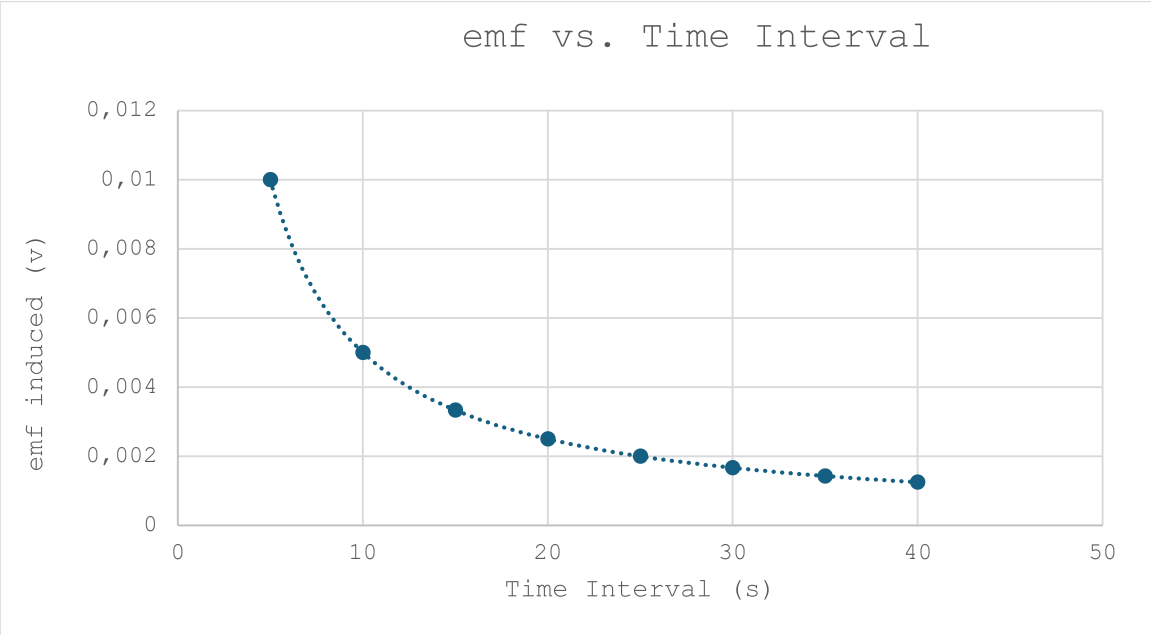
6-Take the square root of the quotient (from step 5).

$$\sqrt{6.631413143 \times 10^{-6}} = 0.002575153$$

We found that the standard deviation for our data set is 0.002575153.

To see the differences between the trends more clearly, we will plot a best-fit line for the theoretical values.

Theoretical Values Graph



Graph 2. Best-fit line of theoretical values of emf varying with time

We notice that the best-fit line for the theoretical values of emf vs. time graph is exponential, This suggests that the emf decreases in a manner proportional to its current value over time.

This type of behavior typically arises in systems governed by exponential decay or growth processes.

The theoretical emf vs. time graph follows an exponential trendline, so it has the form:

$$E(t) = E_0 e^{-kt}$$

Where,

-E(t): The value of the electromotive force at time t.

-E_0: The initial value of the emf at t=0.

-t: Time interval, the independent variable

-k: The decay constant, which determines the rate at which E(t) decreases. Larger values result in faster decay. (It is influenced by the rate of magnetic flux change)

This formula is the proof that emf decreases rapidly at first and then flattens out. The difference between the first two data points (0.005) being relatively greater than the differences between the other consecutive data points suggest that the emf decrease is greater between the first two data points of the graph.

In our experiment, we don't have an emf value at t=0, so we will work out the most accurate constants for both E_0 and decay constant k:

Given the exponential equation,

$$E(t) = E_0 e^{-kt}$$

To linearize the equation, we take the natural logarithm of both sides:

$$\ln\big(E(t)\big) = \ln(E_0) - kt$$

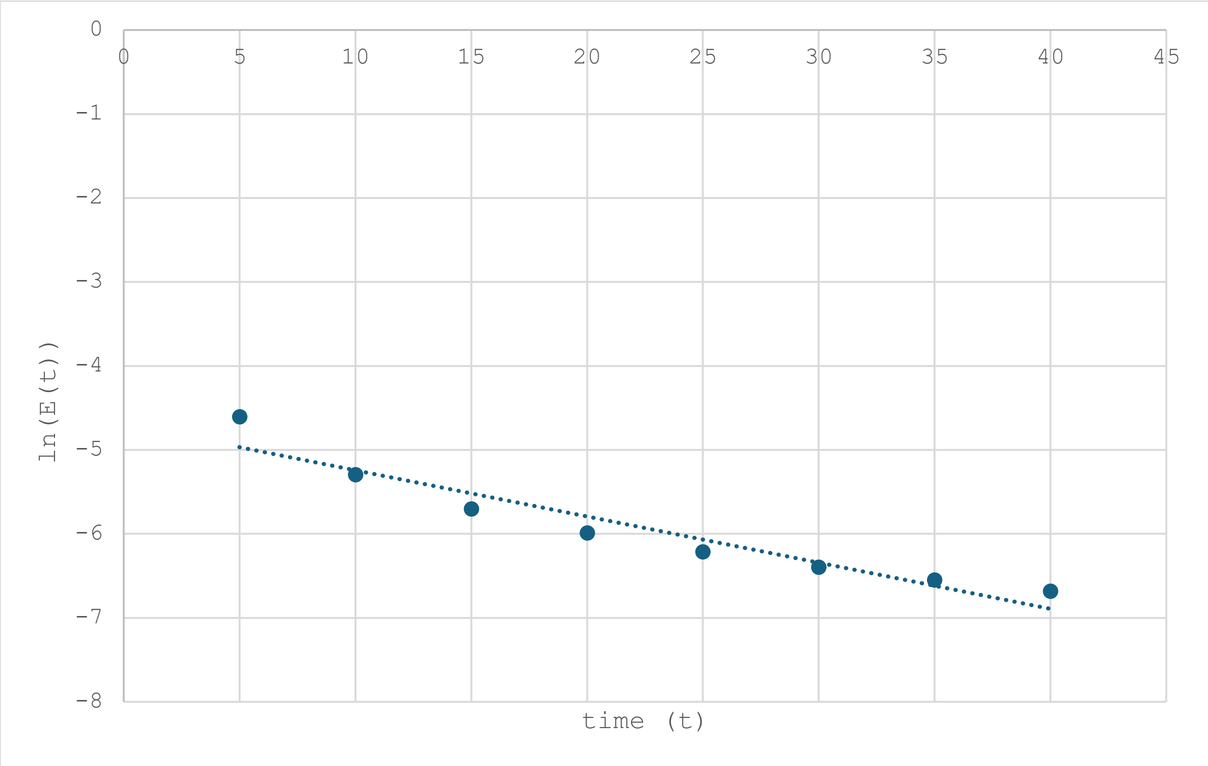
In this formula, -k becomes the slope.

We compute $\ln(E(t))$ for each t:

t (seconds)	$\mathbf{ln(E(t))}$
5	$\ln(0.01000) \approx -4.6052$
10	$\ln(0.00500) \approx -5.2983$
15	$\ln(0.00333) \approx -5.7038$
20	$\ln(0.00250) \approx -5.9915$
25	$\ln(0.00200) \approx -6.2146$
30	$\ln(0.00167) \approx -6.3969$
35	$\ln(0.00143) \approx -6.5491$
40	$\ln(0.00125) \approx -6.6846$

Table 3. Logarithmic transformations of each theoretical emf value

Now to find the slope -k, we plot t against $\ln(\mathrm{E(t)})$:



Graph 3. Periods of rotation versus $\boldsymbol{ln(E(t))}$

As seen from the graph, the data set is not exactly linearized, but we can still find -k using the data points.

-Point 1: (5, −4.6052)

-Point 2: (45, −6.6846)

The slope m is:

$$m = \frac{\ln(E(t_2)) - \ln(E(t_1))}{40} = \frac{-6.6846 - (-4.6052)}{40} = \frac{-2.0794}{40} \approx -0.051985$$

Since $m = -k$, we have:

$$k \approx 0.051985$$

The y-intercept b is:

$$b = \ln(E_0) = \ln(E(t)) + kt$$

Using point 1:

$$\ln(E_0) = -4.6052 + 0.051985 \times 5 \approx -4.6052 + 0.2599 \approx -4.3453$$

Therefore:

$$E_0 = e^{-4.3453} \approx 0.0129\ volts$$

Substituting k and $\ln(E_0)$ back to the linearized equation:

$$\ln\big(E(t)\big) = -0.051985t - 4.3453$$

To express the original exponential equation with the derived constants:

$$E(t) = 0.0129e^{-0.051985t}$$

Substituting t=5 into the formula gives approximately 0.00995 volts when the actual theoretical value is 0.01000. The differences between the values may be due to the formula representing an ideal exponential decay. In real world systems,

Conclusion

Our investigation started by the question: ''How does the period of the magnets' rotation affect the magnitude of emf in an electric generator system?'', our aim following the research question was to see whether the hypothesis which stated that there is an inverse proportionality was true or not. Based on our experiment and findings, we can say that there is a direct relationship that supports when the magnitude of period gets larger, the magnitude of electromotive force decreases.

The graphs showed that there are differences between the theoretical and experimental values of the emf vary with time. The relationship between the experimental emf and the rotating period was found to be logarithmic, supported by the best-fit line in the graph whereas the relationship between the theoretical emf values and time interval was exponential. This suggested that the experimental findings were rather constant percentage change, but the rate of change of the theoretical values changed over time (In our experiment, it started to flatten after the second data point.).

The relative error values for each variable allow us to comment on the accuracy of our measurements and experimental setup. Considering the relative errors vary from %8.0 to %38.1 we can classify these errors as low and high. The lowest relative error was found in time interval 30 seconds, indicating a relatively small discrepancy and suggesting that the experimental procedure performed best at 30 seconds. 30 seconds being the 3rd highest time interval, we can infer that as the time interval gets larger, the accuracy of the measurements compared to the theoretical values increases. However, at 40 seconds the relative error jumped to %30.7 from %16.0 which tells that there are still some points to improve the system regarding the consistency of our findings.

This discrepancy in the variables which had relative uncertainties higher than %30 could be attributed to factors such as measurement errors, resistance in the circuit, or inaccuracies in the rotational speed of the magnets, which were assumed to be constant in the theoretical model.

The standard deviation value, which happened to be 0.002575153, indicates reliable and repeatable results. The relatively small standard deviation showed that there are almost no inconsistencies in rotational speed, noise measurement, or in magnetic field.

Evaluation

Strengths

The experimental setup had its strengths and weaknesses. The copper coiled wire was structured to maximize surface area, which plays a critical role in improving performance. By coiling the wire, I increased the effective surface area, allowing for better interaction with the magnetic field and enhanced electromagnetic induction. This design reduces resistance and minimizes energy losses, resulting in a higher current output.

The blades of the turbine being plastic spoons with deep and large surface areas captured the wind more effectively than flat surfaced blades and allowed the turbine to rotate with enough rotational speed to create a magnetic field along the copper wire.

Weaknesses

There are couple of weaknesses regarding the experimental setup and the theoretical model. Our theoretical model made simplifying assumptions, such as neglecting resistance, assuming a perfectly uniform magnetic field, and idealized behavior of the generator and the wind source. Future work could include more detailed models that account for resistance, non-uniform magnetic fields, and real-world factors that could influence EMF generation, making the theoretical model more accurate.

We have tested a limited range of time intervals without exploring more extreme values. Experiment with a wider range of rotational speeds and periods to better understand the full relationship between the rotating period and emf.

Errors in timing, observation, or data recording can contribute to inconsistencies. To improve this, we can use automated data collection systems (e.g., a sensor to detect magnet position and an automated timer) to reduce human error.

Our data being small (millivolts) might result in this experiment being conducted with larger scales have different outcomes from ours. This can be solved by conducting the experiment in a fully equipped lab with more precise measuring tools and stronger energy sources rather than a modal wind turbine.

Citation

Libretexts. (2024, November 6). *15.6: Electric generators and Faraday's Law*. Physics LibreTexts.

How electricity is generated - U.S. Energy Information Administration (EIA). (n.d.).

Contributor, T. (2018, August 13). *electromagnetic induction*. WhatIs.

Bhuyan, S. (2023, February 2). *Faraday's Law: Statement, equation, and applications*. Science Facts.

The Editors of Encyclopaedia Britannica. (2024, October 26). *Faraday's law of induction | Definition, Formula, & Facts*. Encyclopedia Britannica.

Hogan, R. (2024, March 23). *How to Calculate Relative Uncertainty- Isobudgets*.

CalculatorSoup, L. (2023, August 17). *Percent error Calculator*. CalculatorSoup.

Vedantu. (n.d.). *Absolute and relative error*. VEDANTU.

Hargrave, M. (2024, August 5). *Standard Deviation Formula and Uses vs. Variance*. Investopedia.

Figure 1. https://byjus.com/question-answer/what-is-the-working-principle-of-an-electric-generator/

Figure 2. https://www.toppr.com/guides/physics/electromagnetic-induction/faradays-and-lenzs-law/