Physics High Level Extended Essay

Investigating the effect of changing the number of magnets on the oscillation period and the induced EMF in the solenoid.

Research Question:

"How does a number of magnet attached to a vertically oscillating spring affect its oscillation period in the presence of electromagnetic induction in the solenoid?"

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INTRODUCTION

Since I was doing my internship for a renewable energy company, I had the very rare opportunity to observe wind turbines closely. While most people see wind turbines as just machines for producing electricity, my view was far more technical: these long blades, nearly 50 meters long, spinning in high velocity, stopping them isn't an easy task as the ordinary friction brakes won't do because of the high amounts of heat and wear incurred (Zeng et al., 2018). Instead, they opted for electromagnetic braking systems, which enable smooth slow-downs due to no physical contact (Krause et al., 2013).

I saw a technician test one of these systems. It was while the turbine blades were rapidly spinning that the system was activated and the turbine slowly slowed down to a complete stop without any sound and contact. This, though, threw me into confusion: How can something stop without direct contact? The engineer explained that when the blades move past a stationary conductive plate, electromagnetic induction generates eddy currents in it (Halliday et al., 2013). These currents subsequently generate an opposing magnetic force which resists motion, thus slowing down the blades (Griffiths, 2017). That made me wonder if the same principle would also act on back-and-forth oscillatory motion, like in a mass-spring system.

The whole study will answer its research question: "*How does a number of magnet attached a vertically oscillating spring influence its oscillation period in the existence of electromagnetic induction in the solenoid*?"

The aim of the investigation has been to look specifically into how the number of magnets affects the period of oscillation of a spring-magnet system and the induced electromagnetic force (EMF) in the solenoid. In this case, a neodymium magnet attached to a low-stiffness spring oscillates where it is generating an induced EMF in the solenoid below. The galvanometer records the current variations, while the chronometer is employed to measure the oscillation period. The number of magnets will be increased by 0.015 kg to show the change in period of oscillation and induced EMF in the solenoid. Further, the trials shall be repeated 5 times in a bid to improve on the accuracy and the control of errors.

Hypothesis

As the number of magnets increases, the oscillation period is expected to increase, since adding a magnet in each trial increases the total mass of spring. Due to the increase in the period, the magnet will remain within the solenoid for a longer time, which will diminish the rate of change of magnetic flux, producing less induced EMF.

BACKGROUND INFORMATION

1- *Electromotive Force (emf) and Ohm's Law*

In an electrical circuit, the induced electromotive force (EMF) generates a current, which in turn creates a potential difference across the circuit's resistance, following Ohm's Law (Tsokos,2023):

$$\varepsilon = I \cdot R \quad [1]$$

Equation 1. Induced EMF-current relationship.

Where:

- ε = Induced electromotive force (V)
- I = Current measured by the galvanometer (A)
- R = Total circuit resistance, measured by the ohmmeter.

Because the galvanometer measures the current, we can determine the induced EMF in the system indirectly by this relationship. By analyzing the current response, we can see how electromagnetic induction affects the oscillatory system.

2- *Faraday's Law of Electromagnetic Induction*

Faraday's Law states that a changing magnetic field, *the region around a magnetic material or a moving electric charge within which the force of magnetism acts (Oxford University Press,* *2010)*, through a solenoid induces an electromotive force (EMF), *a type of physical interaction that occurs between electrically charged particles*, in the loop (Tsokos,2023). Mathematically, it is expressed as:

$$\varepsilon = -N\frac{d\Phi_B}{dt} \; [2]$$

Equation 2. Induced EMF Formula

Where:

- ε is the induced EMF (in volts, V),
- *N* is the number of loops in the conductor,
- $\frac{d\Phi_B}{dt}$ is the rate of change of magnetic flux, (in webers per second, Wb/s)
- The minus sign represents Lenz's Law, indicating that the induced EMF opposes the change in magnetic flux.

This law is fundamental to understanding electromagnetic induction and has been experimentally validated in numerous studies (Halliday et al., 2013; Griffiths, 2017).

Derivation od Faraday's Law

Step 1*- Magnetic Flux*

A change in magnetic flux over time induces an electromotive force (EMF) referred to as *Equation 1.*, magnetic flux (Φ) represents the total magnetic field passing through a given surface (Tsokos,2023). It is a scalar quantity and is defined as:

$$\Phi_B = B \cdot A \cdot \cos(\theta) \; [3]$$

Equation 3. Magnetic Flux Formula

Where:

- *B* is the magnetic field strength (T),
- *A* is the surface area of the loop (m^2),
- θ is the angle between the magnetic field and the normal to the loop's surface.

Step 2 - Change in Magnetic Flux

Taking the time derivative of both sides and substituting into *Equation 1*, the expression becomes:

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(B \cdot A \cdot \cos(\theta))$$

3- *Electric Field Induced by a Changing Magnetic Field in a Solenoid*

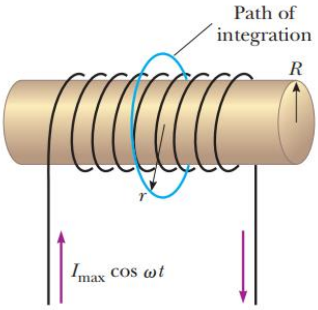


Fig.1 A long solenoid carrying a time-varying current given (Serway,2023)

The magnetic flux through a circular loop of radius r is given by:

$$A = \pi r^2 \quad [4]$$

Equation 4. Area Formula of Circle

Substituting this into *Equation 3.* Since the experiment is conducted with the loop perpendicular to the magnetic field, the angle θ between the field and the normal to the surface is 0^o, making $\cos\theta$ =1. Thus, the expression simplifies to:

$$\Phi_B = B\pi r^2$$

Substitute in the Faraday's Law:

$$\varepsilon = -N\,\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi r^2)$$

$$= -N\pi r^2 \frac{dB}{dt}$$

This result shows that the induced electromotive force (EMF) is proportional to the time rate of change of the magnetic field and scales with the area of the circular loop.

The experiment involves a solenoid with a time-varying magnetic field. The magnetic field (B) in the solenoid varies sinusoidally with time, as described by the equation (Serway,2023):

$$B(t) = B_{max} \sin\left(\frac{2\pi t}{T}\right) \quad [5]$$

Equation 5. Maximum Magnetic Field Formula

where:

- B_{max} is the maximum magnetic field strength,
- T is the oscillation period,
- t is the time.

The magnetic field changes from 0 to its maximum value (B_{max}) in one-quarter of the oscillation period (T/4). Therefore, the rate of change of the magnetic field $\left(\frac{dB}{dt}\right)$ is given by

$$\frac{dB}{dt} = \frac{B_{max}}{T/4} = \frac{4B_{max}}{T}$$

Substitute in the *Equation 5*:

$$\varepsilon = -N\pi r^2 \frac{4B_{max}}{T} \quad [6]$$

Equation 6. EMF of the Solenoid

This experiment illustrates Faraday's Law as the magnet hanging from the end of the iron rod attached to a spring goes down through the coil. In Figure 3, the change in the magnetic field flux represented by the red arrow points downward, induced EMF and current in the direction of the blue arrow inside the coil. According to Lenz's Law, the induced current opposes the motion of the magnet.

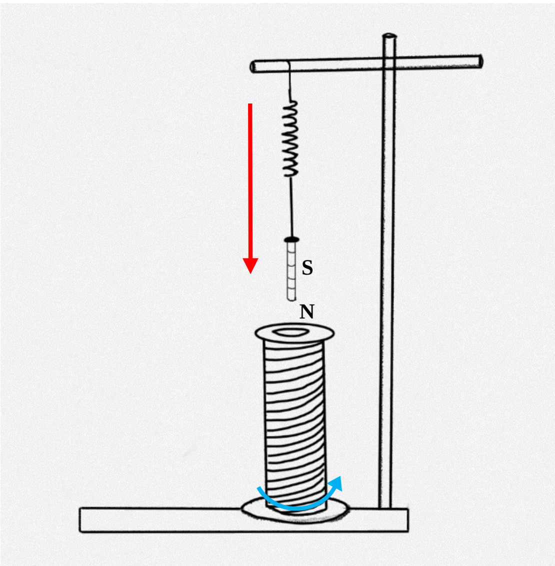


Fig.2 Diagram of Faraday's Law in an Oscillating Magnet-Spring System

4- *Lenz's Law of Electromagnetic Induction*

Lenz's Law states that the direction of the induced current in a circuit is such that it opposes the change in magnetic flux that produced it. (Tsokos,2023) This is represented by the minus sign in Faraday's Law:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Equation 2. Induced EMF Formula

The minus sign ensures that the induced EMF generates a current that opposes the change in magnetic flux, preventing energy amplification and conserving energy. For example:

- If the magnetic flux through a loop is increasing, the induced EMF will create a current whose magnetic field opposes the increase. (Tsokos,2023) *Figure 3* represents the scenario where the magnetic flux is increasing, and the induced current opposes the increase.

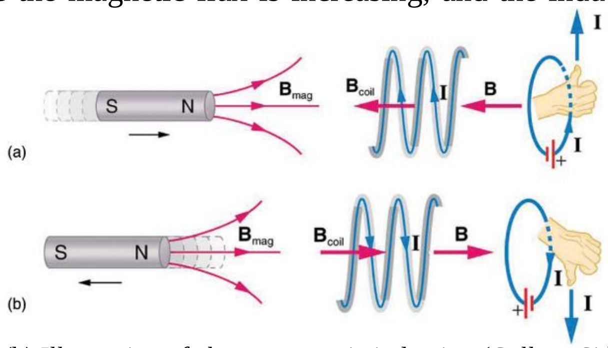


Fig.3 (a)-(b) Illustration of electromagnetic induction (College Sidekick,n.d.)

- If the magnetic flux is decreasing, the induced EMF will create a current whose magnetic field opposes the decrease. (Tsokos,2023) *Figure 2* represents the scenario where the magnetic flux is decreasing, and the induced current opposes the decrease

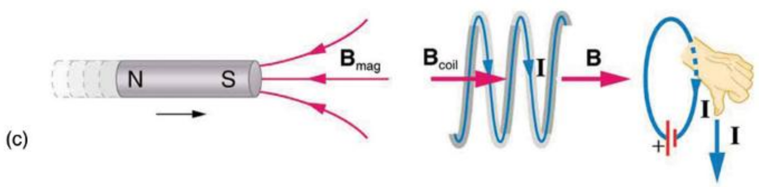


Fig.4 (c) Illustration of electromagnetic induction (College Sidekick,n.d.)

5- *Eddy Currents*

As a conductor moves through a changing magnetic field, Eddy currents are induced that develop an opposing magnetic field due to Lenz's Law (Serway, 2023). In this experiment, as the magnet oscillates through the solenoid, eddy currents are developed in the coil, yielding the force acting against the motion; hence, it creates a retardation effect to slow it down. In this case, as the oscillating magnet moves in the solenoid, eddy currents are induced in the coil, and they impact the oscillation period and induced EMF. Such effect involves the relationship between mass and electromagnetic induction in the spring-magnet system.

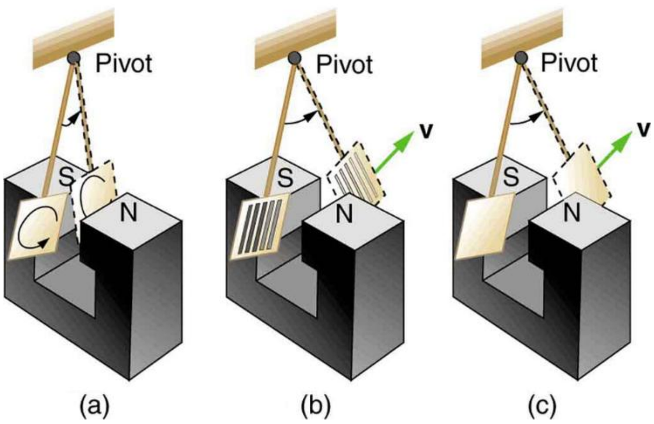


Fig.5 Formation of eddy currents in a conducting plate moving through a magnetic field. (Kelly, T., 2017)

6- *Oscillatory Motion*

Oscillatory motion, often referred to as simple harmonic motion (SHM)**,** occurs when an object moves back and forth about an equilibrium position. (Tsokos,2023) In this experiment spring-magnet system, the magnet attached to the spring exhibits oscillatory motion as it moves up and down through the coil.

The period of oscillation T is the time it takes for the magnet to complete one full cycle of movement. For a simple mass-spring system (neglecting air resistance and damping), the period of oscillation is given by the formula:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad [7]$$

Equation 7. Oscillatory Motion Formula

Where:

- T is the period of oscillation (in seconds, s),
- m is the mass attached to the spring (in kilograms, kg),
- k is the spring constant (in N/m).

Derivation of the Oscillation Period Formula:

Step 1 - Hooke's Law and Restoring Force:

For an object to experience Simple Harmonic Motion (SHM), it must experience a restoring force, which acts to bring the object back to its equilibrium position*, a point at which the spring force and the weight are equal in magnitude but opposite in direction (Blum, n.d.),* when displaced. This force is directed opposite to the displacement from the equilibrium position. The spring restores the system to equilibrium due to the applied force. According to Hooke's law, the force exerted by the spring is directly proportional to the displacement of the magnet. (Oxford University Press, 2010):

$$F = -k \cdot x \quad [8]$$

Equation 8. Hook’s Law Formula

Where:

- F is the restoring force (in Newtons, N),
- k is the spring constant (in Newtons per meter, N/m),
- x is the displacement from the equilibrium position (in meters, m).

This force is always opposite to the displacement of the mass, pulling the mass back toward the equilibrium position. This action creates oscillatory motion, as the mass moves back and forth around the equilibrium point due to the restoring force exerted by the spring.

***Step 2* - Newton’s Second Law**:

According to Newton’s Second Law, the force acting on an object is equal to its mass times acceleration: (Tsokos,2023)

$$F = m \cdot a \ \ [9]$$

Equation 9. Newton's Second Law Formula

In the context of Newton's second law, F is the force (in Newtons, N), m is the mass of the object (in kilograms, kg), and a is the acceleration (in meters per second squared, m/s²). Since the acceleration is the second derivative of the displacement with respect to time, we can write:

$$a = \frac{d^2x}{dt^2} \ \ [10]$$

Equation 10. Acceleration Formula

Thus, Newton's second law becomes:

$$m \cdot \frac{d^2x}{dt^2} = -k \cdot x$$

Rearranging:

$$\frac{d^2x}{dt^2} = -\frac{k}{m} \cdot x$$

***Step 3* – Solving the differential equation:**

The solution to this type of differential equation is a sinusoidal function of time. The general solution for SHM is:

$$x(t) = A \cdot cos(\omega t + \phi) \ \ [11]$$

Equation 11. the sinusoidal nature of SHM Formula

Where:

- *x(t)* is the displacement as a function of time,
- *A* is the amplitude (maximum displacement),
- *ω* is the angular frequency (in radians per second, rad/s),
- ϕ is the phase constant,
- *t* is time (in seconds).

Step 4 – Determining the Angular Frequency (ω)

By comparing the differential $\frac{d^2x}{dt^2} = -\frac{k}{m} \cdot x$ ith the general form of the solution $x(t) = A \cdot cos(\omega t + \phi)$, we can find the angular frequency ω.

When you take the second derivative of $x(t) = A \cdot cos(\omega t + \phi)$ with respect to time, you get:

$$\frac{d^2x}{dt^2} = -\omega^2 \cdot A \cdot cos(\omega t + \phi),$$

Since the original differential equation states that $\frac{d^2x}{dt^2} = -\frac{k}{m} \cdot x$ must be equal to $-\frac{k}{m} \cdot x$, we substitute $x = Acos(\omega t + \phi)$into the equation: we equate the coefficients of.

$$-\omega^2 Acos(\omega t + \phi) = -\frac{k}{m} Acos(\omega t + \phi)$$

For this equation to hold for all values of t, the coefficients of $Acos(\omega t + \phi)$ on both sides must be equal, giving:

$$\omega^2 = \frac{k}{m}$$

Solving for ω,we obtain:

$$\omega = \sqrt{\frac{k}{m}}$$

The period T of oscillation is the time it takes for the object to complete one full cycle. The period is related to the angular frequency ω by the formula:

$$T = \frac{2\pi}{\omega} \; [12]$$

Equation 12. Angular frequency and the Period Relation Formula

Substituting $\omega = \sqrt{\frac{k}{m}}$ into this equation:

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

Finally:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

METHODOLGY

All measurements were recorded using the units of the measuring instruments employed in the experiment. However, for consistency with SI units, values were converted where necessary**.**

Experimental Set up & Apparatus

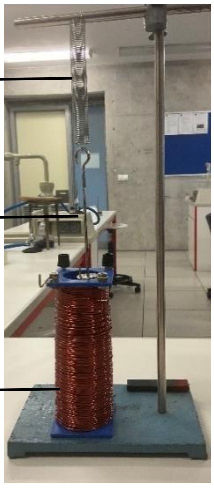


Fig.6 Experimental Set Up with Labeled Components

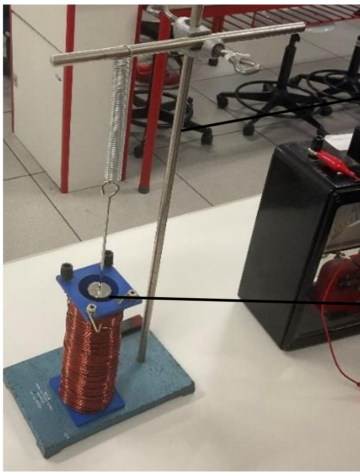


Fig.7 Experimental Set Up and with Labeled Components

Fig.8 Galvanometer that used in the experiment

Fig.9 Gaussmeter that used in the experiment

Item	***Purpose***
Spring (low-stiffness)	Provides restoring force for controlled oscillations.
Copper Wire (Solenoid) 150 turns	Induces eddy currents when the magnet moves through it.
Galvanometer(milliAmper±0.01 μA)	Measures the induced EMF.
Tripod Support	Stabilizes the system to prevent external interference.
*Neodymium Magnet (0.015kg× **5**)*	Provides magnetic field, mass adjustable for experimentation.
Ruler (0.3m ± 0.0005m)	Measures the displacement of the magnet during its oscillation.
Chronometer (Seconds ±0.005s)	Measures the oscillation period.
Gaussmeter (Tesla ±0.001 T)	Used to measure the magnets magnetic field strength
Digital Multimeter (Ω)	Used to measure the resonance

Table 1. Equipment and Purpose in the Experiment

Rationale for Selected Apparatus

- **Spring (low-stiffness):** The spring provides the enabling force of oscillations. In testing various options, a soft spring was chosen in preference to stiffer ones, which failed even to oscillate with very light masses, such as 0.015 kg, making it very difficult to obtain clear and reliable measurements.
- **Metal Rod**: The metal rod will have to be used to properly secure the neodymium magnets in placement since the magnetism of the spring cannot directly hold them. The rod guarantees that the magnets will remain attached to the spring while allowing them free oscillation. In that way, it prevents the detachment of the magnets during oscillation and will help secure the stability of the system all throughout the experiment.
- **Copper Wire:** The choice of 150 turns is based on available laboratory equipment and ensures the induced EMF is strong enough for accurate measurement.
- **Galvanometer**: A galvanometer was used because it was the most sensitive device in the laboratory for measuring EMF. Moreover, a galvanometer can show the direction of the current induced, which for Lenz's Law is vital in understanding the behavior of the system. I had first tried with a voltmeter but couldn't really get a reading as close as the galvanometer, nor did it show the direction of the current.
- **Tripod Support:** The tripod stabilizes the setup, keeping the spring, rod, and solenoid aligned while minimizing vibrations for accurate data.
- **Neodymium Magnets (0.015kg × 5)**: First, I tested 0.005kg magnets, and the results showed no significant difference when adding or removing magnets, but after I tested 0.015kg magnets I confirmed that 0.015 kg mass range is appropriate for studying the oscillation.
- **Ruler**: 0.3m ruler provides adequate measurement range that’s why a longer ruler was unnecessary for this experiment.
- **Chronometer:** A chronometer is used to get more concise measures in the oscillation period.

Variables

Type of Variable	Variable	Description and Importance	Method of Control
Independent Variable	Magnet Number	Mass is varied by adding 0.015kg magnets, affecting oscillation period and induced EMF.	Mass is controlled by stacking identical 0.015kg neodymium magnets.
Dependent Variables	Oscillation Period	The time taken for the system to complete one full oscillation, expected to vary with mass.	Measured using chronometer. Each mass will be tested 5 times to obtain averages.
	Induced Electromotive Force (emf)	The emf induced in a nearby coil due to changing magnetic flux from oscillating magnets.	Measured using a galvanometer to track current changes
Controlled Variables	Spring Constant (*k*)	Ensures a consistent restoring force across all trials, critical for valid oscillation period measurements.	The same spring is used throughout the experiment, with a measured spring constant of k=6.2N/m, determined using Hooke's Law (*Appendix A*).
	Magnetic Field Strength	A constant magnetic field is necessary for reliable EMF readings, as fluctuations affect magnetic flux.	Identical neodymium magnets are used for each trial and a Gaussmeter verified the magnetic field to keep it constant

Table.2 Variables Table

Procedure

1. Setting Up the Experiment

1. The tripod stand should be secured to a stable surface to minimize external vibrations.
2. Attach the low stiffness spring to the tripod stand.
3. Fix a metal rod at the lower end of the spring to hold the magnets in place.
4. A solenoid (copper wire with 150 turns) is placed directly beneath the oscillating magnet system so as to measure the induced EMF.
5. Connect a galvanometer to the solenoid to record current changes.
6. Place a ruler (0.3m) next to the setup to measure oscillation amplitude.

2. Conducting the Oscillations

7. Attach a single neodymium magnet (0.015kg) to the metal rod and ensure it is secured.

8. Allow the magnet to move downward naturally due to its weight and begin oscillations without applying any additional force.
9. Simultaneously, use the galvanometer to record the current during oscillations.
10. Use a chronometer to measure the time taken for 10 complete oscillations and then divide by 10 to get the average oscillation period (T) for each mass**.**
11. Ensure that the timing starts precisely when the mass passes the equilibrium position to maintain consistency.

3. Repeating for Different Masses

12. Repeat steps 7–11 with additional magnets, increasing the total mass in increments of 0.015 kg, 0.030 kg, 0.045 kg, 0.060 kg, and 0.075 kg by stacking magnets onto the rod.
13. Ensure the spring remains unchanged throughout all trials to maintain consistency in the restoring force.
14. Conduct five trials for each mass to minimize random errors.

Safety, Ethical, and Environmental Considerations

This involved careful handling of electrical equipment to avoid short circuits or overheating, though voltages were low during the experiment. Ethically, accurate data were recorded and reported. Environmentally, the impact was minimal; there were no hazardous materials involved.

DATA

Raw Data

1- Oscillatory Period Raw Data Table for 10 Complete Oscillations

Mass(Kg)	**Trail 1 Period** $(\pm 0.01s)$	**Trail 2 Period** $(\pm 0.01s)$	**Trail 3 Period** $(\pm 0.01s)$	**Trail 4 Period** $(\pm 0.01s)$	**Trail 5 Period** $(\pm 0.01s)$
0.015	4.42	4.70	4.16	4.66	4.81
0.030	6.17	6.55	6.79	6.36	6.70
0.045	7.29	7.24	7.57	7.98	7.58
0.060	8.84	8.40	8.45	8.64	8.87
0.075	9.50	9.43	9.34	9.37	9.29

Table.3 Oscillatory Period Raw Data Table

2- Current Raw Data Table

Mass(Kg)	**Trail 1 Current ($\pm 0.01 \times 10^{-7}$ A)**	**Trail 2 Current ($\pm 0.01 \times 10^{-7}$A)**	**Trail 3 Current ($\pm 0.01 \times 10^{-7}$A)**	**Trail 4 Current ($\pm 0.01 \times 10^{-7}$A)**	**Trail 5 Current ($\pm 0.01 \times 10^{-7}$A)**
0.015	5.1×10^{-6}	4.8×10^{-6}	5.2×10^{-6}	4.9×10^{-6}	5.0×10^{-6}
0.030	4.4×10^{-6}	4.4×10^{-6}	4.6×10^{-6}	4.7×10^{-6}	4.3×10^{-6}
0.045	4.0×10^{-6}	4.1×10^{-6}	3.9×10^{-6}	4.2×10^{-6}	3.8×10^{-6}
0.060	3.5×10^{-6}	3.6×10^{-6}	3.4×10^{-6}	3.7×10^{-6}	3.3×10^{-6}
0.075	3.2×10^{-6}	3.2×10^{-6}	3.0×10^{-6}	3.0×10^{-6}	3.1×10^{-6}

Table.4 Current Raw Data Table

Processed Data

The raw data collected from the experiment was processed to determine key parameters. Below, an example calculation for the 0.015 kg mass is provided, with the same procedure applied to all other mass values.

1- Oscillatory Period Processed Data

Mass(Kg)	**Avg. 10 Oscillation Time (s)**	**Avg. Period (s)**	**Standard Deviation (s)**	**Total Uncertainty($\pm$s)**
0.015	4.55	0.455	0.260	$\pm$0.116
0.030	6.51	0.651	0.252	$\pm$0.113
0.045	7.53	0.753	0.295	$\pm$0.132
0.060	8.64	0.864	0.216	$\pm$0.0971
0.075	9.39	0.939	0.0814	$\pm$0.0378

Table.5 Oscillatory Period Processed Data Table

1. **Average 10 Oscillation Time Calculation**

   The average oscillation time was obtained by calculating the arithmetic mean of five measured values from the raw data table. The formula used is:

$$\text{Average time} = \frac{\sum T_{measured}}{N}$$

where:

- $\sum T_{measured}$ measured represents the sum of the measured oscillation times,
- $N=5$ is the number of trials.

For 0.015 kg mass:

$$\text{Average time} = \frac{4.42 + 4.70 + 4.16 + 4.66 + 4.81}{5} = 4.55\ s$$

2. Average Period Calculation

Since the measured times correspond to 10 complete oscillations**,** the period of a single oscillation was determined using:

$$T_{avg} = \frac{Average\ Time}{10}$$

For 0.015 kg mass:

$$T_{avg} = \frac{4.55\ s}{10} = 0.455\ s$$

3. Standard Deviation Calculation

Standard Deviation (σ) **s**hows how spread out the raw measurements are**.** The standard deviation was calculated using the sample standard deviation formula**:**

$$\boldsymbol{\sigma} = \sqrt{\frac{\sum(\boldsymbol{x_i} - \overline{\boldsymbol{x}})^2}{\boldsymbol{N} - \mathbf{1}}}\ \ [\mathbf{13}]$$

Equation 13. Standard Deviation Formula

where:

- $\boldsymbol{x_i}$ are the individual measurements,
- $\overline{\boldsymbol{x}}$ is the mean of the measurements,
- N is the number of trials (5 in this case),
- The denominator $N-1$ is used because this is a sample, not a full population.

For example, for 0.015 kg mass, the five trials are:

4.42, 4.70, 4.16, 4.66, 4.81

- *Compute the mean:*

$$\bar{x} = \frac{4.42 + 4.70 + 4.16 + 4.66 + 4.81}{5} = 4.55 \text{ s}$$

- *Compute squared deviations from the mean:*

The squared deviations are:

$$(4.42 - 4.55)^2, (4.70 - 4.55)^2, (4.16 - 4.55)^2, (4.66 - 4.55)^2, (4.81 - 4.55)^2$$

Calculating each:

$$(\mathbf{-0.13})^{\mathbf{2}} = \mathbf{0.0169}$$

$$(\mathbf{0.15})^{\mathbf{2}} = \mathbf{0.0225}$$

$$(\mathbf{-0.39})^{\mathbf{2}} = \mathbf{0.1521}$$

$$(\mathbf{0.11})^{\mathbf{2}} = \mathbf{0.0121}$$

$$(\mathbf{0.26})^{\mathbf{2}} = \mathbf{0.0676}$$

- Compute the variance

$$\frac{0.0169 + 0.0225 + 0.1521 + 0.0121 + 0.0676}{4} = 0.0678$$

- Compute the standard deviation:

$$\sigma = \sqrt{0.0678} = 0.260$$

4. Total Uncertainty of the Collected Data

The total uncertainty is calculated by combining the experimental uncertainty (arising from the standard deviation) and the systematic uncertainty (arising from the precision of the chronometer).

$$\Delta \mathrm{T}_{\text{total}} = \sqrt{(\Delta \mathrm{T}_{\text{experimental}})^2 + (\Delta \mathrm{T}_{\text{systematic}})^2} \quad [14]$$

Equation 14. Total Uncertainty Formula

Where:

- $\Delta \boldsymbol{T_{experimental}}$: Uncertainty calculated from the standard deviation
- $\Delta \boldsymbol{T_{systematic}}$: Precision of the stopwatch ($\pm\ \mathbf{0.01}\ \boldsymbol{s}$)

Experimental uncertainty calculated as

$$T_{experimental} = \frac{\sigma}{\sqrt{N}}$$

Where:

- $\boldsymbol{\sigma}$**:** Standard deviation of the period measurements
- $\boldsymbol{N}$: Number of trails

For example, for 0.015 kg mass:

$$T_{experimental} = \frac{0.260}{\sqrt{5}} \approx 0.116\ s$$

$$T_{systematic} = 0.01\ s$$

$$\Delta T_{total} = \sqrt{(0.116)^2 + (0.01)^2} = \sqrt{0.013456 + 0.0001} = \sqrt{0.013556} \approx 0.116\ s$$

2- Oscillatory Period Theoretical Data

Mass(Kg)	Theoretical Period (s)	Uncertainty(±s)
0.015	0.309	±0.0054
0.030	0.437	±0.0072
0.045	0.535	±0.087
0.060	0.618	±0.010
0.075	0.691	±0.0112

Table.6 Oscillatory Period Theoretical Data Table

1. Theoretical Period Calculation

We can calculate the theoretical period by using the *Equation 6*, taking the spring constant 6.2 N/m (*Appendix A*).

Example calculation for 0.015 kg mass:

$$T_{theoretical} = 2\pi\sqrt{\frac{0.015}{6.2}} = 0.309$$

2. Uncertainty of the Theoretical Data

The uncertainty in T is calculated using error propagation. The uncertainty in the theoretical period (± s) is typically calculated by propagating the uncertainties in the measured quantities (mass *m*, spring constant *k*, or length *L*) through the formula for the period.

$$\Delta T = T\sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta k}{k}\right)^2} \quad [15]$$

Equation 15. Theoretical Uncertainty Formula

Substitute the values for the mass 0.015 kg:

$$\Delta T = 0.309\sqrt{\left(\frac{0.00010}{0.015}\right)^2 + \left(\frac{0.01}{6.2}\right)^2}$$

$$\Delta T \approx 0.00539s$$

3- Individual EMF Values for Each Trial

Mass(Kg)	**Trail 1 EMF (V)**	**Trail 2 EMF (V)**	**Trail 3 EMF (V)**	**Trail 4 EMF (V)**	**Trail 5 EMF (V)**	**Average EMF(V)**	**Uncertainty in EMF($\pm$V)**
0.015	3.16×10^{-5}	2.98×10^{-5}	3.22×10^{-5}	3.04×10^{-5}	3.10×10^{-5}	3.10×10^{-5}	8.77×10^{-7}
0.030	2.73×10^{-5}	2.73×10^{-5}	2.84×10^{-5}	2.91×10^{-5}	2.67×10^{-5}	2.78×10^{-5}	9.11×10^{-7}
0.045	2.48×10^{-5}	2.54×10^{-5}	2.42×10^{-5}	2.60×10^{-5}	2.36×10^{-5}	2.48×10^{-5}	8.77×10^{-7}
0.060	2.17×10^{-5}	2.23×10^{-5}	2.11×10^{-5}	2.29×10^{-5}	2.05×10^{-5}	2.17×10^{-5}	8.77×10^{-7}
0.075	1.98×10^{-5}	1.98×10^{-5}	1.86×10^{-5}	1.86×10^{-5}	1.92×10^{-5}	1.92×10^{-5}	5.54×10^{-7}

Table.7 Individual EMF Values for Each Trail Data Table

1- Individual EMF Calculation

By using the *Equation 1*, we can calculate the emf of the data's that collected. Resistance of this circuit is 6.20 ohm which is calculated by the Multimeter.

Example calculations for 0.015 kg mass:

$$\varepsilon = 5.0 \times 10^{-6} \cdot 6.20 = 3.10 \times 10^{-5}\, V$$

2- Average EMF

Example calculation for 0.015 kg mass:

$$\varepsilon_{avg} = \frac{3.16 + 2.98 + 3.22 + 3.04 + 3.10}{5} \times 10^{-5} = 3.10 \times 10^{-5}$$

3- Uncertainty in EMF

Example calculation for 0.015 kg mass:

$$\Delta\varepsilon = \sqrt{\frac{(3.16 - 3.10)^2 + (2.98 - 3.10)^2 + (3.22 - 3.10)^2 + (3.04 - 3.10)^2 + (3.10 - 3.10)^2}{5}} \times 10^{-5}$$

$$= 8.77 \times 10^{-7}\ \text{V}$$

4- EMF Theoretical Data

Mass(Kg)	**Trail 1 EMF (V)**	**Trail 2 EMF (V)**	**Trail 3 EMF (V)**	**Trail 4 EMF (V)**	**Trail 5 EMF (V)**	**Average EMF(V)**	**Uncertainty in EMF($\pm$V)**
0.015	-4.29×10^{-5}	-4.03×10^{-5}	-4.56×10^{-5}	-4.07×10^{-5}	-3.94×10^{-5}	-4.18×10^{-5}	2.23×10^{-6}
0.030	-3.07×10^{-5}	-2.89×10^{-5}	-2.79×10^{-5}	-2.98×10^{-5}	-2.83×10^{-5}	-2.91×10^{-5}	1.02×10^{-6}
0.045	-2.60×10^{-5}	-2.62×10^{-5}	-2.50×10^{-5}	-2.37×10^{-5}	-2.50×10^{-5}	-2.52×10^{-5}	8.09×10^{-7}
0.060	-2.14×10^{-5}	-2.25×10^{-5}	-2.24×10^{-5}	-2.19×10^{-5}	-2.13×10^{-5}	-2.19×10^{-5}	4.90×10^{-7}
0.075	-1.99×10^{-5}	-2.01×10^{-5}	-2.03×10^{-5}	-2.02×10^{-5}	-2.04×10^{-5}	-2.02×10^{-5}	1.70×10^{-7}

Table.8 Individual EMF Theoretical Data Table

To calculate the EMF of the solenoid, *Equation 7* is used. The solenoid has a radius of 0.02 m and an electromagnetic field of 2.51×10^{-5} T , measured using a Gaussmeter It consists of 150 turns of copper wire. The oscillation period for each trial was determined by measuring the time for ten complete oscillations and then dividing by ten to obtain the period per cycle.

Example calculations for 0.015kg:

1- **Theoretical EMF for each trial:**

Periods: T_1 = 0.442s, T_2= 0.470s, T_3= 0.416s, T_4 = 0.466s, T_5 = 0.481s

$$\varepsilon_1 = -150 \cdot \pi \cdot (0.02)^2 \cdot \frac{4 \cdot 2.51 \times 10^{-5}}{0.442}$$

$$\varepsilon_1 \approx -4.29 \times 10^{-5} \text{ V}$$

2- **Theoretical Average EMF:**

$$\varepsilon_{avg} = \frac{(-4.29) + (-4.03) + (-4.56) + (-4.07) + (-3.94)}{5} \times 10^{-5}$$

$$= -4.1810^{-5}$$

3- Theoretical EMF's Uncertainty

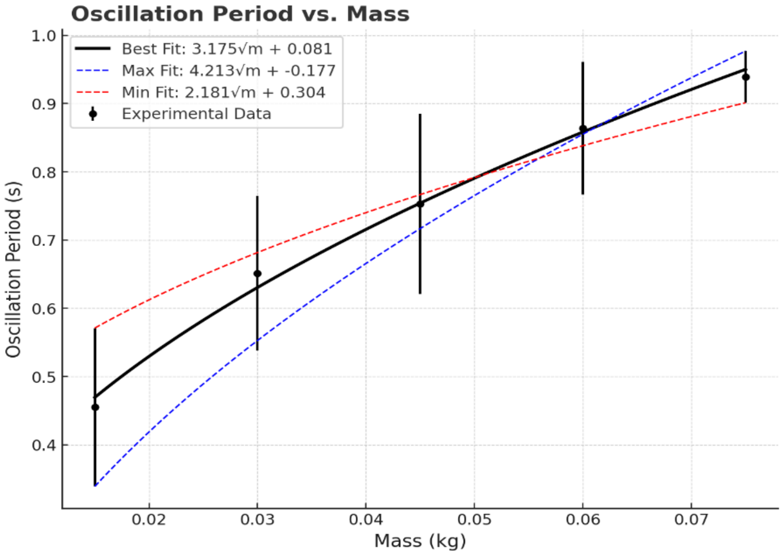
$$\sqrt{\frac{(-4.29+4.18)^2+(-4.03+4.18)^2+(-4.56+4.18)^2+(-4.07+4.18)^2+(-3.94+4.18)^2}{5}} \times 10^{-5}$$

$$\approx 2.23 \times 10^{-6}\ \mathrm{V}$$

ANALYSIS

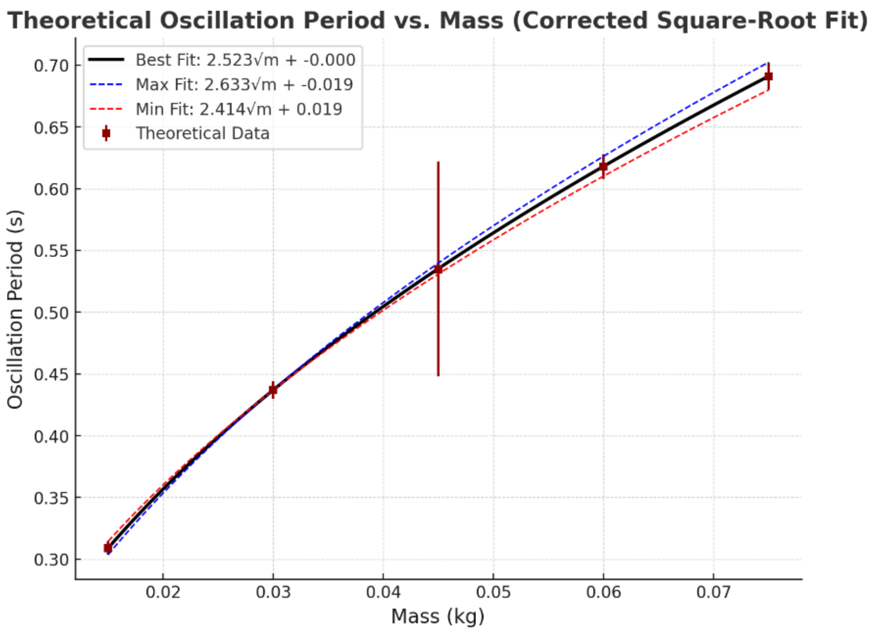
Graphs

1- Oscillation Period vs. Mass Graph



Graph 1. Oscillation Period vs. Mass Graph

2- Theoretical Oscillation Period vs. Mass



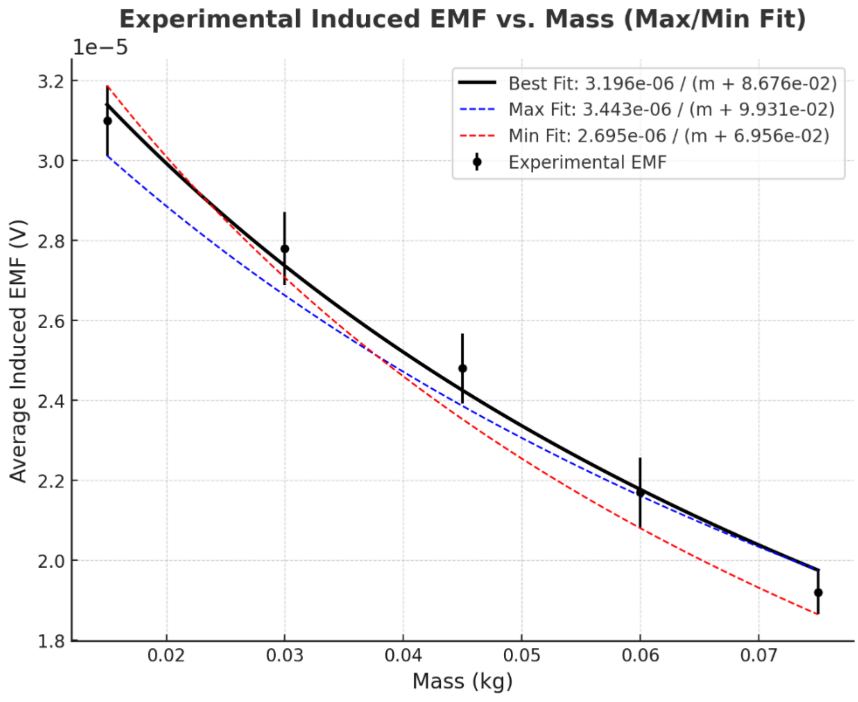
Graph 2. Theoretical Oscillation Period vs. Mass Graph

Trend Analysis & Comparison of the Graph 1 and Graph 2

The graphs both theoretical and experimental corroborate the square-root relationship between oscillation period and mass, as it is anticipated from SHM theory. The theoretical graph proposed the best-fit, as $T = a\sqrt{m} + b$, and the computed error bars cumulative over all mass ranges between ± 0.0054 s and ± 0.0112 s raised a bit in relative uncertainty to not more than 2% under all valve of mass values. Further, the line remained significantly less apart from both max and min fit lines, suggesting that theoretical predictions were sharper.

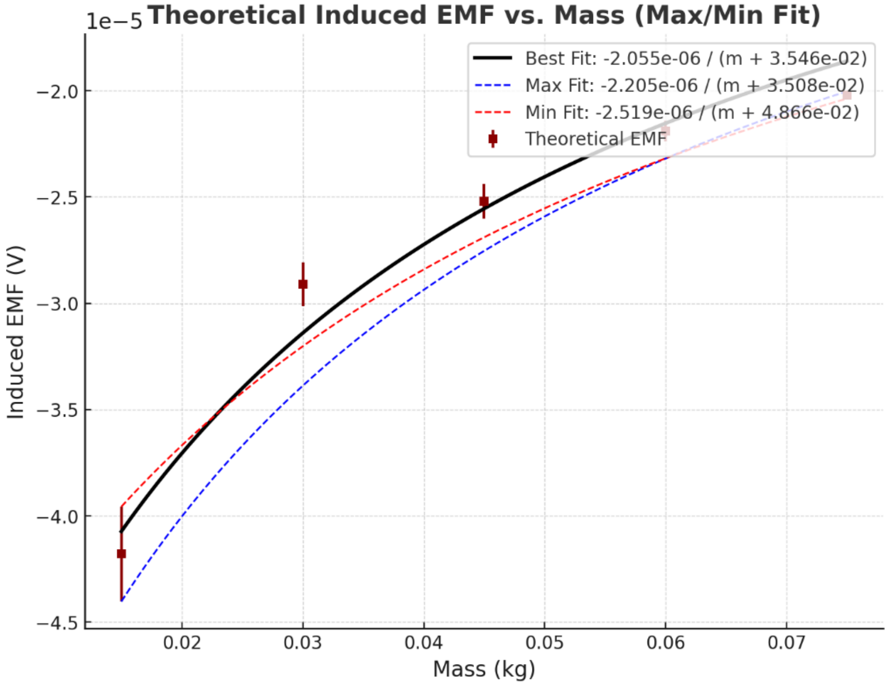
But whereas in experimental data, the maximum point still relates to the linear rising trend, the interval varies a lot. The error bars range from something like ±0.0971s at m=0.060m kg to ±0.132 s at m=0.045m kg, meaning relative uncertainties go past 10% here. Also, the lines which talk about fit max and min get farther apart, especially for higher masses where m=0.075 kg would show the keeping from standard of about 0.18s while the theoretical graph got only about 0.02s keeping apart. When they diverge, it just shows a wider measurement uncertainty and some other varying conditions acquired for the experiments. Despite these variations, both graphs maintain the expected mathematical relationship, confirming the theoretical model while also illustrating the impact of real-world measurement limitations.

3- Experimental Induced EMF vs. Mass



Graph 3. Theoretical Oscillation Period vs. Mass Graph

4- Theoretical Induced EMF vs. Mass



Graph 4. Theoretical Induced EMF vs. Mass

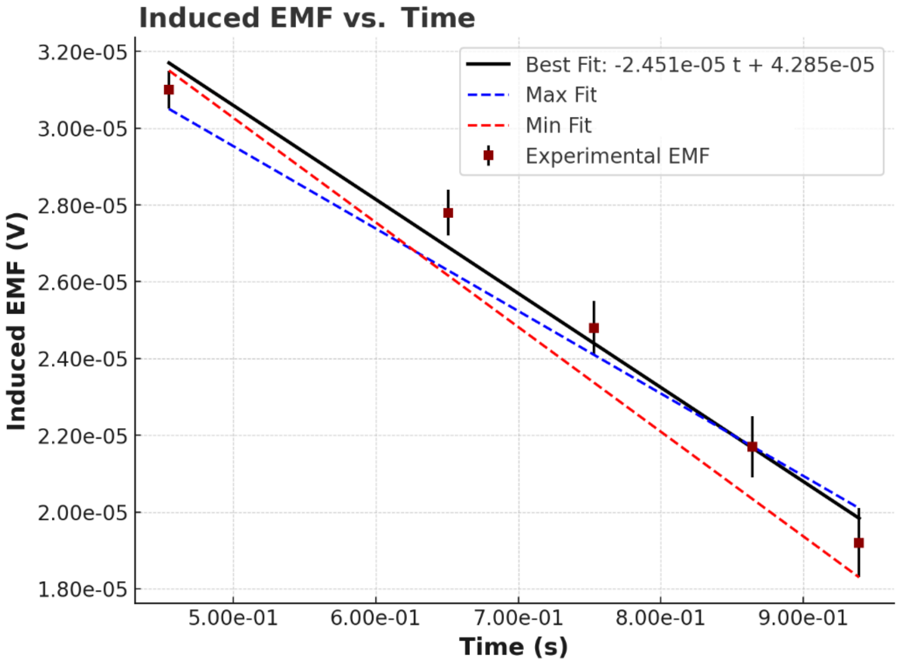
Trend Analysis & Comparison of the Graph 3 and Graph 4

Both the theoretical and experimental graphs confirm the expected inverse relationship between induced EMF and mass, aligning with Faraday's Law. In the theoretical graph, the best-fit curve follows a smooth downward trend, with small and consistent error bars ranging from $\pm 1.70 \times 10^{-7} V$ to $\pm 2.23 \times 10^{-7}$ V, indicating low uncertainty. Additionally, the max and min fit lines remain close together, suggesting high precision in theoretical predictions.

On the other hand, the experimental graph also follows the inverse trend but shows larger error bars, with uncertainties increasing from $\pm 5.54 \times 10^{-7}$ V at $m = 0.075$kg to $\pm 9.11 \times 10^{-7}$V at $m = 0.030\ kg$. The max and min fit lines diverge more significantly, especially at lower mass values, reflecting higher variability in real-world measurements.

The two graphs show a ratio of approximately -1.5; this further proves their almost symmetry with respect to the x-axis, abiding by the anticipated direction of expected EMF. From the analysis of the model, the negative sign in the theoretical calculations and graph is attributed to the fact that according to Lenz's Law, the voltage shows the direction of EMF, due to which the induced EMF opposes the change in magnetic flux; it does not denote absolute negative voltage.

5- Induced EMF vs. Time



Graph 5. Induced EMF vs. Time

Trend Analysis of Graph 5

The graph represents Induced EMF vs. Time, exhibiting a linear decrease in EMF as time progresses. The best-fit equation, $E = -2.451 \times 10^{-5}t + 4.285 \times 10^{-5}$, indicates a negative slope, signifying that the induced EMF declines over time. Experimental data points approached the best-fit line closely, and deviations lay within error bars. The maximum and minimum fit lines, blue and red dashed lines, exhibit uncertainties in the functions with very little differentiations within the slope and intercept. The initial EMF at $t = 0.5s$ is approximately $3.2 \times 10^{-5}V$, decreasing to about $1.8 \times 10^{-5}V$ at $t = 0.9s$. The correlation coefficient (R) is -0.998 in this case, which shows the near-perfect negative linear relation of induced EMF with time. This is a strongly supports the linear model used in the model.

EVALUATION

Weaknesses

1- Damping Effects

Damping effects, *the reduction of oscillation amplitude due to dissipative forces like eddy currents (Rao, 2011)*, beyond eddy currents were considered negligible due to the low flux and EMF values of the magnets used. The other major forms of damping, which include air resistance and internal friction, normally exist in larger systems; however, in that instance, they were neglected given the low energy scale involved. So, the absence of these damping mechanisms allowed for a detailed insight into magnetic induction but at the expense of practical applications. Had the investigation been carried out on a larger scale with stronger magnets, damping effects would become significant in affecting EMF readings and overall system behavior.

2- Measurement Uncertainty

Another weakness was the measurement uncertainty, as the period calculations were affected by the chronometer's precision and human reaction time. These measurements were conducted manually, with delays in human reaction introducing an added uncertainty, especially in a smaller-scale setup, where even a small delay of less than a second counts a lot in the recorded period.

Limitations

1. ***Small Scale Experimental Setup***

   The low flux and EMF values of the magnets limited the consideration of damping effects. While this simplification allowed for a more focused analysis of magnetic induction, it reduced real-world applicability.

2. ***Measurement Precision Constraints***

   The chronometer-based timing method introduced reaction time errors, affecting period measurements. Additionally, the sensitivity of the galvanometer limited the precision of EMF readings, making it difficult to verify values with absolute accuracy.

3. ***Instrumental and Setup Limitations***

   The experiment relied on manual data collection, where slight misalignments in the spring-magnet system and measurement tools could introduce inconsistencies

Ways to Improve the Investigation

To improve this experiment, one can expand the experimental parameters by using stronger magnets with greater flux density, thus making damping effects more pronounced and allowing more accurate EMF measurements. Replacing manual timing techniques with a high-speed motion sensor or laser timing system would essentially eliminate errors in human reaction time, thus allowing period measurements to be more accurately determined. In addition, using a digital oscilloscope instead of a galvanometer would provide higher-resolution EMF measurements, thus reducing uncertainty in voltage determinations. Performing the experiment in a controlled electromagnetic environment would help reduce interference from external fields, leading to more reliable data.

Strengths

1- ***Strong Agreement Between Theoretical and Experimental Results***
The results of the experiment agree with theoretical predictions, as most values remain within an acceptable range of deviation. This agreement validates the accuracy of the investigation.

2- ***Controlled Experimental Setup***
The design minimizes external changes, reducing variables which may influence outcomes. It enhances consistency.

3- ***Repetition of Measurements for Accuracy***
Each experiment was conducted five times, allowing for averaging out random errors and improving the reliability of the final results.

CONCLUSION

The experiment results supported the expected relationship between oscillation period and induced EMF, which had been hypothesized. Even though the magnets' number increased with the total mass of the apparatus, impacting the oscillation period, longer periods suggest that it keeps the magnet inside the solenoid for a longer time, and the rate of change of magnetic flux decreases based on the hypothesis, giving lower induced EMF. Analyzing the graphs and data obtained, especially *Graph 5*, we can see a negative linear relationship indicating EMF decreases as mass increases, proportional to $\frac{1}{\sqrt{mass}}$

The experiment results showed that the neodymium magnets have relatively weak magnetic flux, and the series arrangement of the magnets did not allow the total magnitudes of the field to double up proportionally. Even with an increase in the number of magnets, their use had never brought forth comparable increments of magnetic flux, and, of course, the associated increase in EMF was moderately limited.

This experiment is successful in demonstrating the relationship between changes in magnetic mass with oscillation period and induced EMF; however, some limitations would arise with respect to the measurement accuracy and the magnetic field. The data acquired by neodymium magnets shows clearly that their exhibition of the period of oscillation and of the induced EMF are close to those of the ideal theoretical model. This research is thereby important in explaining electromagnetic induction where oscillatory motion is concerned, and forms also a valid and strong link to explaining eddy current-type energy dissipation.

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APPENDIX

Appendix A: Calculation of the spring constant

Mass(kg)	**Force(N)**	**Displacement(m)**	**Spring Constant(k)**
0.015	0.15	0.024	6.25
0.030	0.30	0.048	6.25
0.045	0.45	0.073	6.16
0.060	0.60	0.097	6.18
0.075	0.75	0.121	6.19

Table 9. Appendix A Spring Constant Table

$F = k \cdot x$

$0.15 = k \cdot 0.024 \qquad k = 6.25$

$0.30 = k \cdot 0.048 \qquad k = 6.25$

$0.45 = k \cdot 0.073 \qquad k = 6.16$

$0.060 = k \cdot 0.097 \qquad k = 6.18$

$0.075 = k \cdot 0.121 \qquad k = 6.19$

$$\frac{6.25 + 6.25 + 6.16 + 6.18 + 6.19}{5} = 6.2\ N/m$$

As per Hooke's Law, k has assumedly been constant, minor deviations have always occurred due to experimental limitations and measurement noises. Taking the average value of k, given to be 6.2 N/m, ensures a more realistic approximation.