Title: The Effect of Temperature on Young's Modulus of Copper Wire

Research Question: How does the temperature effect Young's modulus of a copper wire with diameter of 0.6 mm and length of 1.5 m.

Subject: Physics

Word count: 3454

1. Contents

2. Introduction

Elasticity of materials has always interested me, since it is an important characteristic that allows materials to return to their original shape after being distorted. The study of elasticity is an important understanding for many applications such as clothes or construction. In the IB physics education the basic concepts of elasticity are explained but the complex relationship between material properties and environmental variables is mostly ignored. So, I thought it would be interesting to do an investigation that explores this topic.

The research question that this investigation aims to answer is: " How does the temperature affect Young’s modulus of a copper wire with diameter of 0.6 mm and length of 1.5 m?" The topic of this essay is to examine the correlation of temperature and Young's modulus of copper wire to present the behavior of elastic properties of copper with varying temperatures. The choice of copper as the material of this investigation is because of its major role in everyday technology and the possible implications that will help us to develop copper’s current applications. The goal of this essay might be useful for improving the performance of devices and systems that use copper components, especially in places with large temperature changes.

To understand the effects of temperature on Young’s modulus of a copper wire, a series of experiments will be performed under controlled environmental conditions. Within a predefined range, a sample of copper wire will be exposed to varying temperatures (20°C, 40°C, 60°C, 80°C, 100°C, 120°C, 140°C, 160°C, 180°C, 200°C and 220°C). Careful measurements will be made on the elongation of copper wire and Young's modulus will be calculated. By performing the calculations at each temperature, and then carefully analyzing the data, I will determine the relationship between both temperature and mechanical properties of copper wire. This approach will allow a detailed understanding of how temperature affects the stiffness of copper wire and provide a basis for further exploration into the copper's properties and potential applications.

2.1 Background Knowledge

In exploring how temperature affects the Young's Modulus of materials like copper wire, we need to understand the concepts of stress and strain of a material. These concepts are significant for understanding the behavior of materials under external forces and determining Young's modulus.

Stress (σ) is a measure of the internal forces that particles of a material exert on each other when subjected to an external force. It's calculated as the force (F) applied to the material divided by the cross-sectional area (A) of the material through which the force is acting[1]:

$$\sigma = \frac{F}{A}$$

Equation 1: Stress formula

Where:

- $\boldsymbol{F}$ is the force applied to the material (measured in newtons, N).
- $\boldsymbol{A}$ is the cross-sectional area of the material (measured in square meters, m^2).
- $\boldsymbol{\sigma}$ is the stress in the material (measured in pascals, Pa).

Stress essentially quantifies how much force is exerted per unit area of a material, indicating how strongly the material resists deformation.

[1] Wikipedia contributors. (2024f, February 14). Stress (mechanics). Wikipedia. https://en.wikipedia.org/wiki/Stress_(mechanics)

Strain (ε), on the other hand, describes the deformation of the material because of the applied stress. It is defined as the ratio of the change in length (ΔL) of the material to its original length (L_0)[2]:

$$\varepsilon = \frac{\Delta L}{L_0}$$

Equation 2: Strain formula

Where:

- ΔL is the change in length of the material (measured in meters, m).
- L_0 is the original length of the material (measured in meters, m).
- ε is the strain, a dimensionless quantity.

Strain measures how much a material stretches or compresses relative to its original length, providing a scale to understand the extent of deformation.

With the stress and strain defined, we can calculate Young's modulus (E), a critical parameter in the study of material properties. Young's modulus is the ratio of stress (σ) to strain (ε) and represents the stiffness of a material[3]:

$$E = \frac{\sigma}{\varepsilon}$$

Equation 3: Young's modulus formula

Where:

- E is Young's modulus (measured in pascals, Pa).

[2] Wikipedia contributors. (2024f, January 29). Strain (mechanics). Wikipedia. https://en.wikipedia.org/wiki/Strain_(mechanics)
[3] Wikipedia contributors. (2024g, February 2). Young’s modulus. Wikipedia. https://en.wikipedia.org/wiki/Young%27s_modulus

- σ is the stress applied to the material (measured in pascals, Pa), which is the force per unit area.
- ε is the strain, representing the material's deformation in response to the applied stress (dimensionless).

In the context of my experiment, Young's modulus will help determine how the stiffness of the copper wire changes with temperature.

It can be assumed that the relationship between stress-strain graph will have a linear graph, as there are no exponential variables in equations, where the slope of this linear part corresponds to Young's modulus.

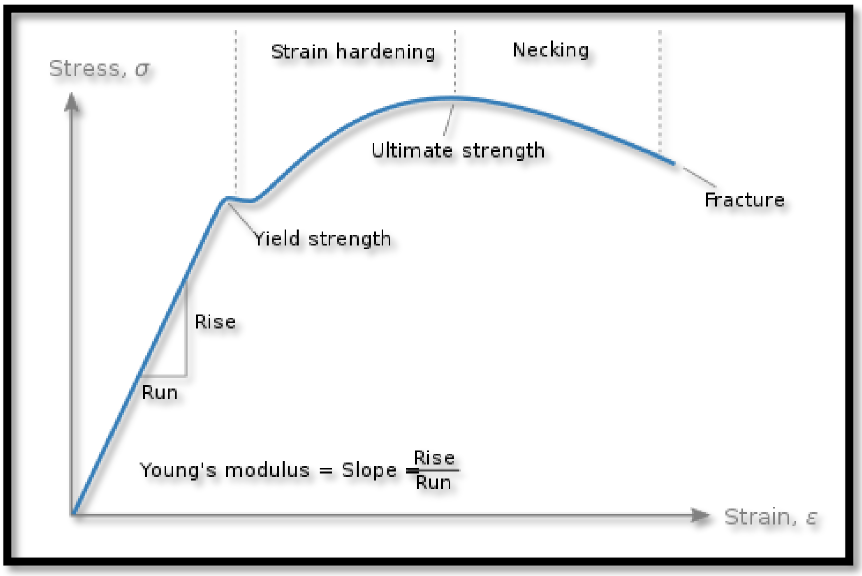


Figure 1: Stress-Strain Graph[4]

[4] Wikipedia contributors. (2024e, January 25). Stress–strain curve. Wikipedia. https://en.wikipedia.org/wiki/Stress%E2%80%93strain_curve

At first, as strain increases in a material such as copper, the stress follows a linearly proportional path as indicated by the straight line on the graph. This initial linear section indicates the material is behaving elastically, meaning it will return to its original shape when the stress is removed.

However, after a point as shown in Figure 1 as the strain gets larger, the curve starts to bend and moves away from linear. This is the transition from elastic to plastic deformation. In this region, the material starts to deform permanently which means it will not return to the shape it was when the force was removed. This nonlinear behavior is the result of changes at the microscopic scale in the material's structure.

In my case, I chose a mass of 0.200 kg to ensure the applied stress will be within the elastic limit of the copper wire, and the experiment will be restricted to the linear portion of the stress-strain curve. This means that Young's modulus can be determined accurately. This step is essential because it will ensure that the deformations I observe, and measure are purely elastic, reversible, and characteristic of the material's true elastic properties. Working within this linear zone will allow me to see how temperature variations affect the elasticity of the copper wire, without the complications of plastic deformation.

2.2 Hypothesis

The Young's modulus of copper wire decreases as its temperature increases.

3. Investigation

3.1 Variables

Independent Variable:

- **Temperature of Copper Wire (T):** This is the variable that will vary over the experiment. I will use the temperatures of 20°C, 40°C, 60°C, 80°C, 100°C, 120°C, 140°C, 160°C, 180°C, 200°C and 220°C to determine Young's modulus of copper wire.

Dependent Variables:

- **Young's Modulus (E):** This is the stiffness of the copper wire. As the temperature changes, Young's Modulus is expected to change, showing how the wire's stiffness is affected by temperature.

Controlled Variables:

- **Original Length of Wire (L_0):** To get the correct length for wire in tests, I will use only one length of copper wire (150 cm) that I will measure and keep constant. During trials, I will measure the wire every time with the tape measure.

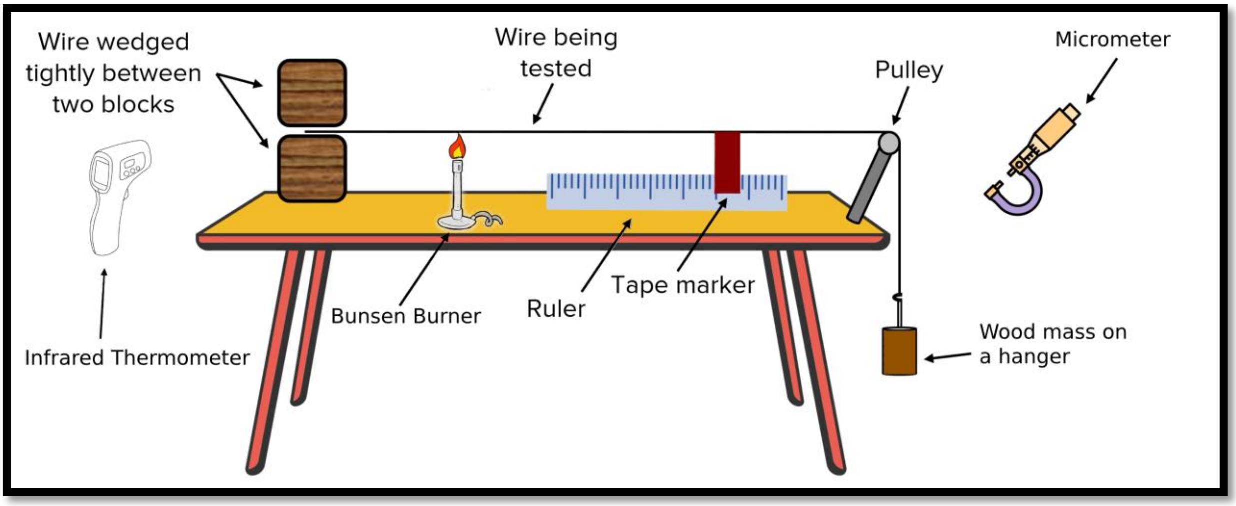
- **Cross-Sectional Area (A):** Stress calculations depend heavily on the cross-sectional area of the wire. Therefore, I will use a wire with the same diameter for all my experiments. I will use a micrometer to measure the diameter accurately.

- **Force (F):** To observe the wire's deformation under consistent conditions, I will apply the same force in each experiment. I will achieve this by using the same hanging mass (0.200 kg) across each trial.

- **Room temperature and Humidity**: Temperature and humidity changes might affect the mechanical properties of the copper wire; I will conduct all my trials in a controlled environment. I will use air conditioning to keep these values constant.

3.2 Materials

- Copper Wire (length of 7.5 m) (diameter of 0.6 mm)
- Bunsen Burner
- Infrared Thermometer
- Wooden block (0.200 kg)
- 30 cm Ruler (cm)
- Pen and Notebook
- Tape Mark
- 2 Heavy blocks (for tightening and stabilizing wire)
- Pulley
- Micrometer (mm)
- Wire Cutter
- Heat Resistant Gloves
- Tape Measure
- Digital Weigh Scale



3.1 Young's Modulus Experiment Diagram[5]

3.3 Methodology

1. Connect a pulley to one edge of the table.
2. Measure the mass of the wooden block.
3. Measure and cut a 1.5 m length of copper wire using tape measure and wire cutters, ensuring a clean cut without fraying.
4. Use the micrometer to measure the diameter of the copper wire at 5 points to calculate the average cross-sectional area.
5. Secure the copper wire between two heavy blocks on a table, ensuring it's stretched tight.
6. Attach a 0.200 kg wooden block to the free end of the copper wire over the pulley, using a simple hook mechanism, ensuring the mass hangs freely and applies tension to the wire.

[5] MME Revise. (2022, December 28). *Young's Modulus Questions and Revision | MME*. MME. https://mmerevise.co.uk/a-level-physics-revision/youngs-modulus/

7. Place a ruler on the table directly beneath the copper wire and place a tape mark directly above the middle of the ruler (which is 15 cm) to indicate the wire's initial position on the ruler.
8. Measure and record the initial length of the copper wire between the tape mark using the ruler. Note this as the starting point before heating.
9. Position Bunsen Burner behind the ruler and tape mark.
10. Turn on the Bunsen Burner and adjust the flame to gently heat the copper wire. Avoid direct contact between the flame and the wire.
11. Use the infrared thermometer to measure the temperature of the copper wire at regular intervals as it heats up.
12. In every 20°C (20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220) increase in copper turn off the Bunsen Burner, then observe and record the displacement of the tape mark from its initial position on the ruler.
13. Continue heating, recording the temperature and displacement until you reach a temperature 200°C more than the starting temperature.
14. Once the maximum temperature is reached, turn off the Bunsen burner and allow the wire to cool down to room temperature before removing the setup and conducting another trial.
15. Repeat these steps (3 to 13) 5 times to reduce random errors.
16. Analyze data by plotting a graph and looking for general trends.

4. Safety Guidelines

Before I began the experiment, I followed all standard safety precautions to ensure a safe workspace when the open flame was involved. Because the procedure requires heat, I wore heat-resistant gloves to prevent burns when working on the Bunsen burner and adjusting the copper wire.

I also worked in an organized area free of flammable items to reduce the risk of fire hazards, with the Bunsen burner on a stable, heat-resistant surface, and I kept a close eye on it throughout the experiment to prevent accidents.

During the entire experiment, I was in an area with enough ventilation to clear any smoke or fumes and reduce the possibility of someone having difficulty breathing or creating a flammable atmosphere.

Before the experiment started, I verified that all equipment was in proper working order and safe. This involved verifying that the wire had no cuts and that the measuring devices were undamaged and accurate. Using faulty equipment not only could compromise the experiment's results but also could bring a huge risk to my safety.

5. Results

5.1 Raw Data

Measured Mass (kg) ± 0.0001 (kg)
0.2034

Table 1: Measured Mass of the Wooden Block with Uncertainty

Digital weigh scale can measure up to 4th decimal places, so I took the uncertainty last decimal place, ± 0.0001 kg.

Trial Number	Measured Length (cm) ± 0.05 cm
1	150.00
2	149.00
3	151.00
4	150.00
5	149.00

Table 2: Initial Length Measurements of Copper Wire with Uncertainty

Tape measure can measure at least 0.1 cm, so I took the uncertainty half of that, ± 0.05 cm.

Trial	Measurement 1 (mm) ± 0.01 mm	Measurement 2 (mm) ± 0.01 mm	Measurement 3 (mm) ± 0.01 mm	Measurement 4 (mm) ± 0.01 mm	Measurement 5 (mm) ± 0.01 mm
1	0.60	0.60	0.61	0.60	0.60
2	0.61	0.61	0.60	0.61	0.61
3	0.59	0.60	0.60	0.60	0.60
4	0.60	0.61	0.61	0.60	0.61
5	0.60	0.60	0.61	0.60	0.60

Table 3: Diameter Measurements of Copper Wire with Uncertainty

Digital micrometer can measure up to 2^{nd} decimal places, so I took the uncertainty last decimal place, ± 0.01 mm.

Temperature (°C)	Tape distance (cm) ± 0.05 cm
20	15.00
40	15.00
60	15.00
80	15.10
100	15.10
120	15.20
140	15.20
160	15.30
180	15.40
200	15.50
220	15.60

Table 4: Trial 1 Tape Distance Measurements with Uncertainty

Infrared Thermometer can measure up to 1st decimal places, but since it is a small number compared to the temperature range in the experiment ignored it. Ruler (30 cm) can measure at least 0.1 cm, so I took the uncertainty half of that, ± 0.05 cm.

Temperature (°C)	Tape distance (cm) ± 0.05 cm
20	15.00
40	15.00
60	15.10
80	15.10
100	15.20
120	15.20
140	15.30
160	15.30
180	15.40
200	15.50
220	15.60

Table 5: Trial 2 Tape Distance Measurements with Uncertainty

Temperature (°C)	Tape distance (cm) ± 0.05 cm
20	15.00
40	15.00
60	15.10
80	15.10
100	15.20
120	15.20
140	15.30
160	15.30
180	15.40
200	15.50
220	15.60

Table 6: Trial 3 Tape Distance Measurements with Uncertainty

Temperature (°C)	Tape distance (cm) ± 0.05 cm
20	15.00
40	15.00
60	15.10
80	15.20
100	15.20
120	15.30
140	15.30
160	15.40
180	15.50
200	15.50
220	15.60

Table 7: Trial 4 Tape Distance Measurements with Uncertainty

Temperature (°C)	Tape distance (cm) ± 0.05 cm
20	15.00
40	15.00
60	15.00
80	15.10
100	15.10
120	15.20
140	15.20
160	15.30
180	15.40
200	15.50
220	15.60

Table 8: Trial 5 Tape Distance Measurements with Uncertainty

5.2 Calculations

I will first calculate the stress on the copper wire. To do this accurately, I need to determine the mean diameter of the wire from the measurements which I have made. By doing this, I can properly consider any slight variations in the width of the wire. Therefore, I can calculate the cross-sectional area of the copper wire more accurately. I will convert all measurements to meters. For consistency, the units for stress and strain calculations should be coherent within the metric system.

$$\bar{x} = \frac{\sum x}{N}$$

Equation 4: Mean value formula

Where:

- $\bar{x}$ is the average of terms.
- $\sum x$ is the sum of terms.
- N is the number of terms.

Example calculation for mean diameter of copper wire in trial 1:

$$\frac{(0.60 \times 10^{-3}) + (0.60 \times 10^{-3}) + (0.61 \times 10^{-3}) + (0.60 \times 10^{-3}) + (0.60 \times 10^{-3})}{5} = 0.000602$$

$$= 0.00060 \text{ (2SF to not change precision)}$$

$$\Delta x = \frac{x_{max} - x_{min}}{2}$$

Equation 5: Uncertainty estimation formula

Where:

- Δx is the uncertainty.
- x_{max} is the maximum value in the group.
- x_{min} is the minimum value in the group.

Example calculation for mean diameter uncertainty of copper wire in trial 1:

$$\frac{(0.61 \times 10^{-3}) - (0.60 \times 10^{-3})}{2} = 0.000005 = 0.00001 \text{ (rounded to match the last decimal of mean value)}$$

Trial	Mean Diameter (m) ± 0.00001 m
1	0.00060
2	0.00061
3	0.00060
4	0.00061
5	0.00060

Table 9: Mean Copper Wire Diameter and Uncertainty for Each Trial

With the mean diameter, I can use the formula for the area of a circle to calculate the cross-sectional area (A):

$$A = \pi(\frac{R}{2})^2$$

Equation 6: Area of circle formula

Where:

- R is the diameter of the wire (m).
- A is the area of the wire's cross-section (m^2).

Example calculation for cross-sectional area of copper wire in trial 1:

$$\pi(\frac{0.00060}{2})^2 = 2.8274333882 \times 10^{-7} = 2.827 \times 10^{-7} \text{ (4SF)}$$

I chose 4SF to use the value without changing too much. I will write with proper SF number at the end of calculations.

$$\Delta y = y \times \sqrt{\left(\frac{\Delta x_1}{x_1}\right)^2 + \left(\frac{\Delta x_2}{x_2}\right)^2 + \cdots + \left(\frac{\Delta x_n}{x_n}\right)^2}$$

Equation 7: General Propagation of Uncertainty Formula

Where:

- Δy is the uncertainty in the result y.
- $x_1, x_2, \cdots, x_n$ are the measured values.
- $\Delta x_1, \Delta x_2, \cdots, \Delta x_n$ are the uncertainties in the measured variables $x_1, x_2, \cdots, x_n$ respectively.
- y is the result of the operation on $x_1, x_2, \cdots, x_n$.

Example uncertainty calculation for cross-sectional area of copper wire in trial 1:

$$2.827 \times 10^{-7} \times \sqrt{\left(\frac{0.00001}{0.00060}\right)^2} = 9.425 \times 10^{-9}$$

Trial	Cross-sectional Area (m^2) ± Uncertainty (m^2)
1	$2.827\times10^{-7} \pm 9.425\times10^{-9}$
2	$2.922\times10^{-7} \pm 9.582\times10^{-9}$
3	$2.827\times10^{-7} \pm 9.425\times10^{-9}$
4	$2.922\times10^{-7} \pm 9.582\times10^{-9}$
5	$2.827\times10^{-7} \pm 9.425\times10^{-9}$

Table 10: Calculated Cross-sectional Areas and Uncertainties for Each Trial

To calculate the force applied to the copper wire by the hanging mass, we must consider the mass's weight, which is the force exerted by gravity. This force (F) can be calculated using Newton's second law of motion:

$$F = ma$$

Equation 8: Newton's second law of motion

Where:

- F is the force due to gravity (measured in newtons, N).
- m is the mass of the object (measured in kilograms, kg).
- a is the acceleration due to gravity (measured in meters per second squared, m/s^2).

For this experiment, the mass (m) of the wooden block is 0.2034 kg, and the acceleration due to gravity (a) is approximately 9.81 m/s^2. So, the force (F) would be:

$$0.2034 \times 9.81 = 1.995354 = 1.995 \text{ N (4SF)}$$

with the uncertainty of ± 0.001 N.

I can now determine the stress (***σ)*** on the copper wire by dividing the Force (F) by the Cross-sectional Area (A), as specified in *Equation 1*.

Example calculation for stress of copper wire in trial 1:

$$\frac{1.995}{2.827 \times 10^{-7}} = 7056950.83126 = 7.057 \times 10^{6} \text{ (4SF)}$$

Example uncertainty calculation for stress of copper wire in trial 1:

$$7.057 \times 10^6 \times \sqrt{\left(\frac{9.425 \times 10^{-9}}{2.827 \times 10^{-7}}\right)^2 + \left(\frac{0.001}{1.995}\right)^2} = 2.353 \times 10^5$$

Trial	Stress (Pa)	± Uncertainty (Pa)
1	7.057×10^6	2.353×10^5
2	6.828×10^6	2.239×10^5
3	7.057×10^6	2.353×10^5
4	6.828×10^6	2.239×10^5
5	7.057×10^6	2.353×10^5

Table 11: Calculated Stress and Uncertainties for Each Trial

Moving forward in my calculations, I will now calculate the strain experienced by the copper wire. To accomplish this, I need the initial length of the wire (L_0) and the change in the length (ΔL). I measured the original length of the copper wire on *Table 2* and the tape distance of every trial in *Table 4, Table 5, Table 6, Table 7*, and *Table 8*. So, I will calculate the distance changed from starting point (15 cm) and turn centimeter (cm) values to meter (m):

Example calculation for initial length of copper wire in trial 1:

$$150 \times 10^{-2} = 1.5000 \text{ (4 decimals to match uncertainty)}$$

Example uncertainty calculation for initial length of copper wire in trial 1:

$$0.05 \times 10^{-2} = 0.0005$$

Trial Number	Measured Length (m) ± 0.0005 m
1	1.5000
2	1.4900
3	1.5100
4	1.5000
5	1.4900

Table 12: Initial Length of Copper Wire in Meters with Uncertainty

Example calculation for tape displacement in trial 1 temperature 60°C :

$$0.1510 - 0.1500 = 0.0010$$

Temperature (°C)	Tape displacement for Trial 1 (m) ± 0.0005 (m)	Tape displacement for Trial 2 (m) ± 0.0005 (m)	Tape displacement for Trial 3 (m) ± 0.0005 (m)	Tape displacement for Trial 4 (m) ± 0.0005 (m)	Tape displacement for Trial 5 (m) ± 0.0005 (m)
20	0.0000	0.0000	0.0000	0.0000	0.0000
40	0.0000	0.0000	0.0000	0.0000	0.0000
60	0.0000	0.0010	0.0010	0.0010	0.0000
80	0.0010	0.0010	0.0010	0.0020	0.0010
100	0.0010	0.0020	0.0020	0.0020	0.0010
120	0.0020	0.0020	0.0020	0.0020	0.0020
140	0.0020	0.0030	0.0030	0.0030	0.0020
160	0.0030	0.0030	0.0030	0.0030	0.0030
180	0.0040	0.0040	0.0040	0.0040	0.0040
200	0.0050	0.0050	0.0050	0.0050	0.0050
220	0.0060	0.0060	0.0060	0.0060	0.0060

Table 13: Calculated Tape Mark Displacement at Various Temperatures with Uncertainties

With change in the length (ΔL) for each trial's temperatures are calculated, I can now divide change in the length (ΔL) to initial length (L_0) to calculate strain (ε):

Example calculation for strain in trial 1 temperature 80°C:

$$\frac{0.0010}{0.1500} = 0.006666666 = 6.67 \times 10^{-4}$$

Example uncertainty calculation for strain in trial 1 temperature 80°C:

$$6.67 \times 10^{-4} \times \sqrt{\left(\frac{0.0005}{0.1500}\right)^2 + \left(\frac{0.0005}{0.0010}\right)^2} = 0.0003333333 = 3.33 \times 10^{-4}$$

Temperature (°C)	Trial 1 Strain (ε)	Trial 2 Strain (ε)	Trial 3 Strain (ε)	Trial 4 Strain (ε)	Trial 5 Strain (ε)
20	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
40	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
60	0.00 ± 0.00	$6.71x10^{-4} ± 3.4x10^{-4}$	$6.62x10^{-4} ± 3.3x10^{-4}$	$6.67x10^{-4} ± 3.3x10^{-4}$	0.00 ± 0.00
80	$6.67x10^{-4} ± 3.3x10^{-4}$	$6.71x10^{-4} ± 3.4x10^{-4}$	$6.62x10^{-4} ± 3.3x10^{-4}$	$1.33x10^{-3} ± 3.3x10^{-4}$	$6.71x10^{-4} ± 3.4x10^{-4}$
100	$6.67x10^{-4} ± 3.3x10^{-4}$	$1.34x10^{-3} ± 3.4x10^{-4}$	$1.32x10^{-3} ± 3.3x10^{-4}$	$1.33x10^{-3} ± 3.3x10^{-4}$	$6.71x10^{-4} ± 3.4x10^{-4}$
120	$1.33x10^{-3} ± 3.3x10^{-4}$	$1.34x10^{-3} ± 3.4x10^{-4}$	$1.32x10^{-3} ± 3.3x10^{-4}$	$1.33x10^{-3} ± 3.3x10^{-4}$	$1.34x10^{-3} ± 3.4x10^{-4}$
140	$1.33x10^{-3} ± 3.3x10^{-4}$	$2.01x10^{-3} ± 3.4x10^{-4}$	$1.99x10^{-3} ± 3.3x10^{-4}$	$2.00x10^{-3} ± 3.3x10^{-4}$	$1.34x10^{-3} ± 3.4x10^{-4}$
160	$2.00x10^{-3} ± 3.3x10^{-4}$	$2.01x10^{-3} ± 3.4x10^{-4}$	$1.99x10^{-3} ± 3.3x10^{-4}$	$2.00x10^{-3} ± 3.3x10^{-4}$	$2.01x10^{-3} ± 3.4x10^{-4}$
180	$2.67x10^{-3} ± 3.3x10^{-4}$	$2.68x10^{-3} ± 3.4x10^{-4}$	$2.65x10^{-3} ± 3.3x10^{-4}$	$2.67x10^{-3} ± 3.3x10^{-4}$	$2.68x10^{-3} ± 3.4x10^{-4}$
200	$3.33x10^{-3} ± 3.3x10^{-4}$	$3.36x10^{-3} ± 3.4x10^{-4}$	$3.31x10^{-3} ± 3.3x10^{-4}$	$3.33x10^{-3} ± 3.3x10^{-4}$	$3.36x10^{-3} ± 3.4x10^{-4}$
220	$4.00x10^{-3} ± 3.3x10^{-4}$	$4.03x10^{-3} ± 3.4x10^{-4}$	$3.97x10^{-3} ± 3.3x10^{-4}$	$4.00x10^{-3} ± 3.3x10^{-4}$	$4.03x10^{-3} ± 3.4x10^{-4}$

Table 14: Calculated Strain at Fixed Temperatures with Uncertainties

After calculating the values of stress (σ) and strain (ε), I can now find the Young's modulus (E):

Example calculation for Young's modulus (E) in trial 1 temperature 80°C:

$$\frac{7.057 \times 10^6}{6.67 \times 10^{-4}} = 10580209895.0524 = 1.06 \times 10^{10}(3SF)$$

Example uncertainty calculation for Young's modulus (E) in trial 1 temperature 80°C:

$$1.06 \times 10^{10} \times \sqrt{\left(\frac{2.353 \times 10^5}{7.057 \times 10^6}\right)^2 + \left(\frac{3.33 \times 10^{-4}}{6.67 \times 10^{-4}}\right)^2} = 5.25 \times 10^9$$

Temperature (°C)	Trial 1 Young's Modulus (Pa ± Pa)	Trial 2 Young's Modulus (Pa ± Pa)	Trial 3 Young's Modulus (Pa ± Pa)	Trial 4 Young's Modulus (Pa ± Pa)	Trial 5 Young's Modulus (Pa ± Pa)
20	NaN	NaN	NaN	NaN	NaN
40	NaN	NaN	NaN	NaN	NaN
60	NaN	$1.02{\times}10^{10} \pm 5.17{\times}10^{9}$	$1.07{\times}10^{10} \pm 5.33{\times}10^{9}$	$1.02{\times}10^{10} \pm 5.08{\times}10^{9}$	NaN
80	$1.06{\times}10^{10} \pm 5.25{\times}10^{9}$	$1.02{\times}10^{10} \pm 5.17{\times}10^{9}$	$1.07{\times}10^{10} \pm 5.33{\times}10^{9}$	$5.13{\times}10^{9} \pm 1.29{\times}10^{9}$	$1.05{\times}10^{10} \pm 5.34{\times}10^{9}$
100	$1.06{\times}10^{10} \pm 5.25{\times}10^{9}$	$5.10{\times}10^{9} \pm 1.30{\times}10^{9}$	$5.35{\times}10^{9} \pm 1.35{\times}10^{9}$	$5.13{\times}10^{9} \pm 1.29{\times}10^{9}$	$1.05{\times}10^{10} \pm 5.34{\times}10^{9}$
120	$5.31{\times}10^{9} \pm 1.33{\times}10^{9}$	$5.10{\times}10^{9} \pm 1.30{\times}10^{9}$	$5.35{\times}10^{9} \pm 1.35{\times}10^{9}$	$5.13{\times}10^{9} \pm 1.29{\times}10^{9}$	$5.27{\times}10^{9} \pm 1.35{\times}10^{9}$
140	$5.31{\times}10^{9} \pm 1.33{\times}10^{9}$	$3.40{\times}10^{9} \pm 5.85{\times}10^{8}$	$3.55{\times}10^{9} \pm 6.00{\times}10^{8}$	$3.41{\times}10^{9} \pm 5.74{\times}10^{8}$	$5.27{\times}10^{9} \pm 1.35{\times}10^{9}$
160	$3.53{\times}10^{9} \pm 5.94{\times}10^{8}$	$3.40{\times}10^{9} \pm 5.85{\times}10^{8}$	$3.55{\times}10^{9} \pm 6.00{\times}10^{8}$	$3.41{\times}10^{9} \pm 5.74{\times}10^{8}$	$3.51{\times}10^{9} \pm 6.05{\times}10^{8}$
180	$2.64{\times}10^{9} \pm 3.38{\times}10^{8}$	$2.55{\times}10^{9} \pm 3.34{\times}10^{8}$	$2.66{\times}10^{9} \pm 3.43{\times}10^{8}$	$2.56{\times}10^{9} \pm 3.27{\times}10^{8}$	$2.63{\times}10^{9} \pm 3.45{\times}10^{8}$
200	$2.12{\times}10^{9} \pm 2.22{\times}10^{8}$	$2.03{\times}10^{9} \pm 2.16{\times}10^{8}$	$2.13{\times}10^{9} \pm 2.24{\times}10^{8}$	$2.05{\times}10^{9} \pm 2.14{\times}10^{8}$	$2.10{\times}10^{9} \pm 2.24{\times}10^{8}$
220	$1.76{\times}10^{9} \pm 1.57{\times}10^{8}$	$1.69{\times}10^{9} \pm 1.53{\times}10^{8}$	$1.78{\times}10^{9} \pm 1.59{\times}10^{8}$	$1.71{\times}10^{9} \pm 1.52{\times}10^{8}$	$1.75{\times}10^{9} \pm 1.59{\times}10^{8}$

Table 15: Calculated Young’s Modulus at Fixed Temperatures with Uncertainties

In some cells of the table, the value NaN (Not a Number) appears, indicating that data is not available for those specific entries. Since Young's Modulus is defined as stress divided by strain, if the strain is zero, this results in a division by zero scenario, which is mathematically undefined.

I will next calculate the mean Young's Modulus for each temperature point where valid measurements are available. By averaging the values from the individual trials and excluding the NaN I will obtain a more reliable estimate of the copper's elastic behavior under different thermal conditions. This average will help to reduce any experimental inconsistencies and provide a singular, more accurate Young's Modulus value at each given temperature.

Example calculation for mean Young's modulus (E) temperature 80°C:

$$\frac{(1.06\times10^{10})+(1.02\times10^{10})+(1.07\times10^{10})+(5.13\times10^{9})+(1.05\times10^{10})}{5}$$

$$= 9.426\times10^{9} = 9.43\times10^{9}\ (3SF)$$

Example uncertainty calculation for mean Young's modulus (E) temperature 80°C:

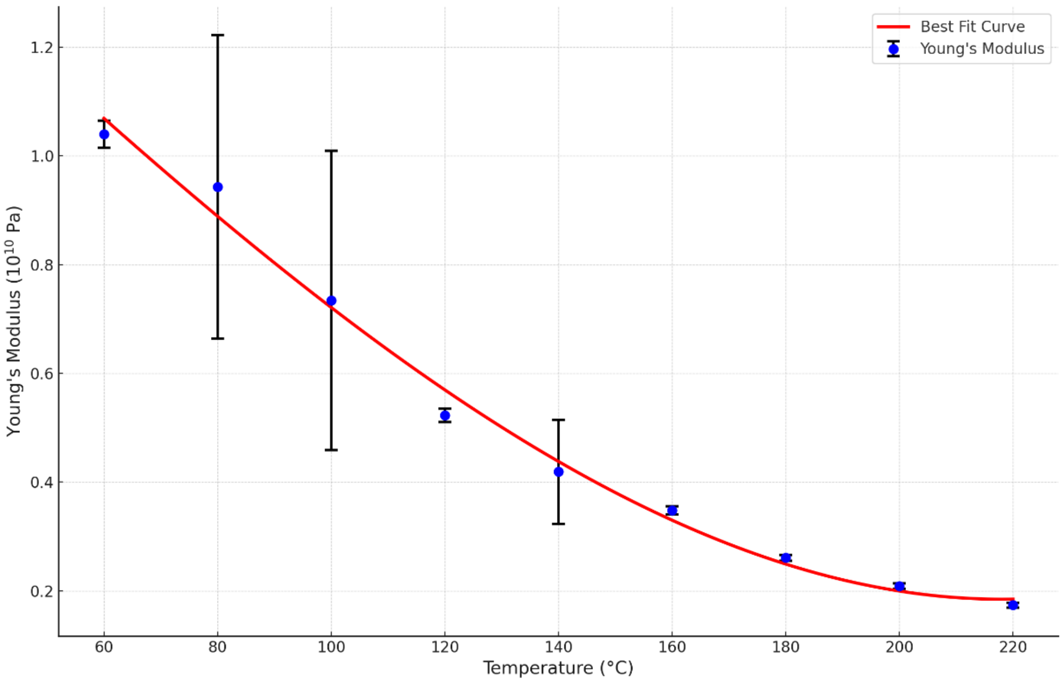
$$\frac{(1.07 \times 10^{10}) - (5.13 \times 10^{9})}{2} = 2.785 \times 10^{9} = 2.79 \times 10^{9} =$$

Temperature (°C)	Mean Young's Modulus (Pa) ± Uncertainty (Pa)
20	NaN
40	NaN
60	$1.04\times10^{10} \pm 2.50\times10^{8}$
80	$9.43\times10^{9} \pm 2.79\times10^{9}$
100	$7.34\times10^{9} \pm 2.75\times10^{9}$
120	$5.23\times10^{9} \pm 1.25\times10^{8}$
140	$4.19\times10^{9} \pm 9.55\times10^{8}$
160	$3.48\times10^{9} \pm 7.50\times10^{7}$
180	$2.61\times10^{9} \pm 5.50\times10^{7}$
200	$2.09\times10^{9} \pm 5.00\times10^{7}$
220	$1.74\times10^{9} \pm 4.50\times10^{7}$

Table 16: Mean Young's Modulus and Uncertainty for Each Fixed Temperature

6. Analysis

I will start by plotting the calculated mean Young's modulus for each temperature range against the respective temperature to have a visual understanding of how the material changes its behavior under different thermal conditions. By analyzing the graph trends, I will understand how the material elasticity changes with temperature.

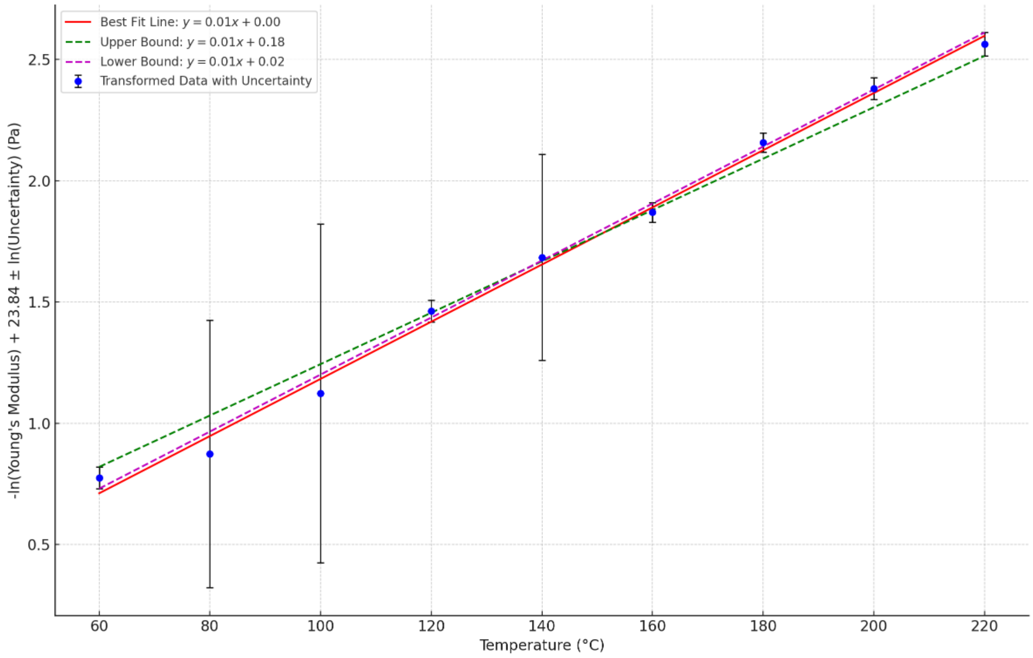


Graph 1: Young's Modulus against Temperature Graph (Made with Matplotlib)

The general trend of the graph shows that the Young's modulus is inversely proportional to temperature, meaning as the temperature increases, the Young's modulus decreases. This supports my hypothesis that Young's modulus of copper wire decreases with increasing temperature. The best fit curve clearly illustrates this negative correlation and inverse proportionality.

The R^2 value for this data set is 0.943, which is close to 1. This high R^2 value suggests a high degree of correlation between temperature and Young's modulus for the copper wire, indicating that temperature is a strong influencer of the modulus in this context.

To gain a better understanding of the relationship between temperature and Young's modulus, I will transform the data to present a linear graph. This will allow me to analyze the trend more effectively and make the relationship clearer. By doing so, I can more accurately determine how changes in temperature affect Young's modulus.



Graph 2: Transformed Young's Modulus against Temperature Graph (Made with Matplotlib)

The data demonstrates a positive linear relationship between temperature and the negative natural logarithm of Young's modulus. I made this by transforming Young's Modulus with the natural logarithm function due to the data's exponential nature. The best-fit line, which passes through the origin (0,0), indicates a direct proportionality between temperature and the transformed Young's modulus, with an offset of 23.84. This transformation enables a more interpretable relationship between temperature and Young's modulus, suggesting that as temperature increases, the negative natural logarithm of Young's modulus changes proportionally, with an additional offset of 23.84.

The R^2 value for the best fit line is approximately 0.994, indicating a very high degree of correlation between temperature and the natural logarithm of Young's Modulus. This suggests that the linear model provides a good fit to the transformed data.

6.1 Calculating Slope Uncertainty

Slope = 0.01 Pa

Next, I will calculate slope uncertainty to see the uncertainty in the graph:

$$Slope\ Uncertainty = \frac{Max\ Slope - Min\ Slope}{2}$$

$$\frac{0.01 - 0.01}{2} = 0.00$$

Slope and slope uncertainty:

0.01 ± 0.00 Pa

The low slope and zero uncertainty findings are because of the limited measurement sensitivity of the 30 cm ruler used in the experiment when measuring tape displacement. This measurement method had insufficient precision to accurately capture minimal changes in Young's modulus.

7. Conclusion

This essay aimed to investigate the effect of temperature on Young's modulus of copper wire, hypothesizing that Young's modulus decreases as temperature increases. This was explored through an experimental setup involving the manipulation of temperature of copper wire, while measuring the elongation under a constant applied force on certain points. The data collected provided insights into the copper's elastic behavior under varying thermal conditions.

The results strongly support my initial hypothesis. There was an inverse relationship between temperature and Young's modulus, plotted by a decreasing trend in *Graph 1*. The high R^2 value obtained from the graph (0.9913) proves the reliability of this trend, showing a significant correlation between temperature and the Young’s modulus.

The analysis was further enhanced through a transformation that presented a linear relationship between temperature and the negative natural logarithm of Young's modulus. This approach confirmed the inverse proportionality and determined the relationship.

In conclusion, this experiment validates the hypothesis that Young’s modulus of copper wire decreases as its temperature increases. This conclusion came from both the direct analysis of Young's modulus against temperature and the transformed linear relationship. This experiment illustrates the importance of considering thermal effects in the application of copper and similar materials in engineering and design, where temperature variations could impact material performance.

7.1 Improvements

1. The measurement sensitivity of the 30 cm ruler used for measuring tape displacement impacted the precision of measurements. Using a more precise measuring instrument, such as a digital caliper could significantly improve the accuracy of displacement measurements, thereby improving the reliability of the calculated strain and Young's modulus values.

2. The experiment relied on manual observation and recording of temperature and displacement at each fixed temperature, which could introduce human error. Implementing automated data systems, such as digital temperature sensors and displacement transducers connected to a computer, would allow for continuous and more accurate data collection, reducing potential observational errors.

3. The thermal distribution along the copper wire might not have been uniform due to the localized application of heat from the Bunsen burner. A more controlled heating method, such as an electrically heated wire or a temperature-controlled oven, could ensure a uniform temperature distribution across the wire, leading to more consistent and reliable measurements.

7.2 Strengths

1. The experiment's design effectively isolates the effect of temperature on Young's modulus by controlling other variables such as the original length of the wire, cross-sectional area, and the force applied. This control ensures that any observed changes in Young's modulus can be attributed to temperature variations.

2. Using a wide range of temperature points (from 20°C to 220°C) provides a detailed view of how Young's modulus changes across a temperature spectrum. This extensive data set allows for a detailed analysis of the thermal sensitivity of copper's mechanical properties.

8. Bibliography

- Wikipedia contributors. (2024f, February 14). Stress (mechanics). Wikipedia. https://en.wikipedia.org/wiki/Stress_(mechanics)
- Wikipedia contributors. (2024f, January 29). Strain (mechanics). Wikipedia. https://en.wikipedia.org/wiki/Strain_(mechanics)
- Wikipedia contributors. (2024g, February 2). Young's modulus. Wikipedia. https://en.wikipedia.org/wiki/Young%27s_modulus
- Wikipedia contributors. (2024e, January 25). Stress–strain curve. Wikipedia. https://en.wikipedia.org/wiki/Stress%E2%80%93strain_curve
- MME Revise. (2022, December 28). *Young's Modulus Questions and Revision | MME*. MME. https://mmerevise.co.uk/a-level-physics-revision/youngs-modulus/
- J. D. Hunter, "Matplotlib: A 2D Graphics Environment", Computing in Science & Engineering, vol. 9, no. 3, pp. 90-95, 2007.