HL PHYSICS EXTENDED ESSAY

THE EFFECT OF LENGTH AND HEAD MASS OF A SPECIFIC GOLF CLUB (DRIVER) ON THE FINAL DISTANCE OF A GOLF BALL

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INTRODUCTION

The effect of shaft length on the club speed and club head mass is investigated to obtain a longer distance of a golf shot. Clubs labelled parts shown in Figure 1. The purpose of the golf game is to finish the hole in as few shots as possible. It is essential to increase distance to reduce effort of getting closer to the pin. As being a national golf player and playing the game professional for three years, I have come to conclusion that whilst choosing clubs, longer/shorter or lighter/heavier shafted clubs may help increase my golfing performance. Especially my first shot as it will affect my consecutive shots depending on its range. Hence investigating whether length and mass influences distance. Also being a high-level physics course student, allows me to apply my knowledge of physics on this investigation with the topics motion and momentum. Irons are the most frequently used clubs in a golf course which have different angles on the club head. Driver, a club with the longest shaft and distance is used for starting and long-ranged distances. For this investigation, the club which I use mostly on the tee box (starting points) on a golf course is selected, which is driver, the most distanced club in the bag. The launch angle (Figure 1.1) of a driver typically ranges from 8 to 15, which is a controlled variable being 10.5 for this study, applied for all clubs that have different lengths and mass.

The investigation is analysed from the moment of backswing (Figure 1.2), where player's wrists, (the player is me), are 90 degrees with their leading (left) arm, to the point of uncocking (releasing) is 180 degrees, the impact. The distance taken by the club head is calculated theoretically to find club head speed, which will eventually lead to ball distance, and be used for comparison with real life and sensor-measured values with "GarminGolf" measurement gear. Models are drawn by hand as in Figure 1.3 and calculations are done on Excel.

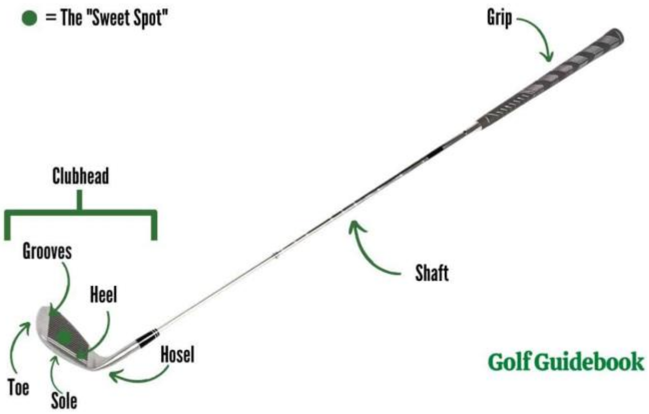


Figure 1: Club labelled parts [1]

Figure 1.1: Launch Angle: The vertical angle relative to the horizon of the golf ball's center of gravity movement immediately after leaving the club face (2)

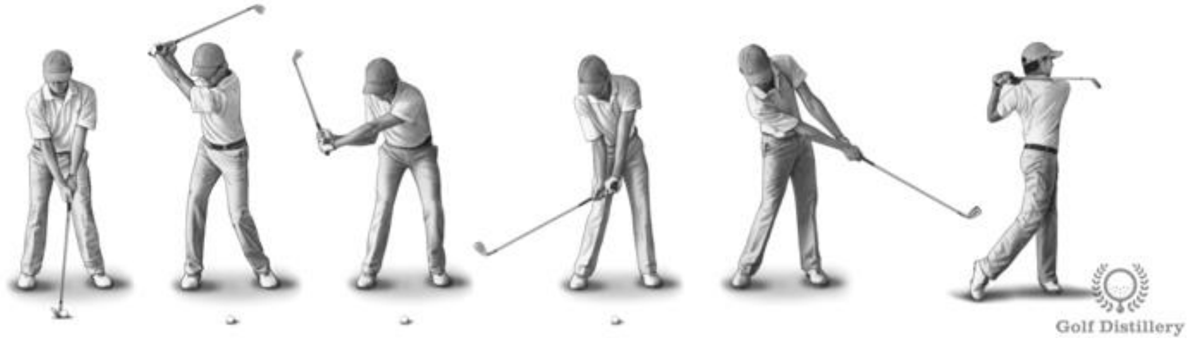


Figure 1.2: Golf swing sequence (2nd image: backswing, 3rd & 4th image: downswing) (3)

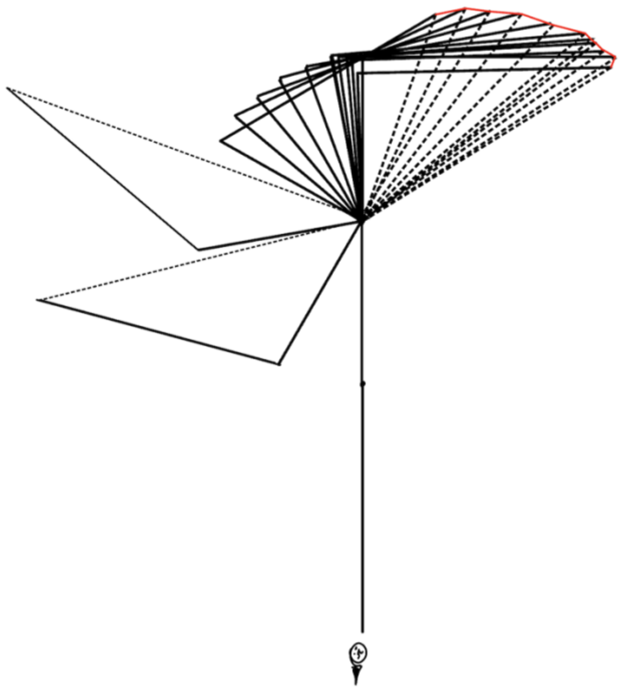


Figure 1.3: Drawn model by hand

<u>RESEARCH QUESTION:</u> **How do shaft length and head mass of different drivers (0.82-0.1806, 0. 91-0.1922, 0.92-0.1954, 0.93-0.1994, 0.96-0.2055, 0.98-0.2101, 0.99-0.2126, 1.00-0.2134, 1.05-0.2183, 1.09-0.2207, 1.16-0.2251 meters and kilograms respectively) effect the final distance of a golf ball?**

<u>HYPOTHESIS</u>

It is expected that longer shafted club will increase the club head's acceleration and final velocity before impact. With its head mass, using conservation of momentum, more weighed head mass will create a higher horizontal launch velocity of the ball. Consequently, distance taken by the ball will increase; $Range\ = Horizontal\, Velocity\ \ x\, Time$

BACKGROUND INFORMATION

Golf is a complex sport which is originated in 15th century Scotland. [4] Various clubs and a single ball are used in a game of 18 holes. The complexity comes from every shot being hit from different positions and conditions as golf is played in an open field. There are 40.000 golf courses around the world and each of them is unique and has a different design which requires different game strategies. The aim of the game is to finish it as few strokes as possible. Therefore, every shot is very important and must be accurate. The swing that the player is doing must be accurate according to the type of shot they are planning to make. A straight-going high ball is not always the ideal shot when it comes to windy or rainy conditions. Also, in order to finish the game in the least strokes possible, the distance of the ball must be long to conquer the course and make less shots. This is why every player works on their swing's technical aspects, such as the ideal angle of backswing, hand placement, grip, stance, body rotation and gear. Gear must be ideal to the player to maximize their distance and potential. Neither too heavy or long nor too short or light. For this investigation I will be finding my ideal club length and mass of Driver to maximize my performance. In a research paper released by Mizoguchi, Masato & Hashiba, Toshinao & Yoneyama, Takeshi. (2005) "Effect of Shaft Length on the Club Head Speed in Golf, they discuss the effect of the shaft length on the head speed and the impact thereof on increased distance generated off the driver. [5] It is concluded that length increases speed, hence the distance. According to research done in Mississippi University Institute of Golf and published at the 9th Conference of the International Sports Engineering Association (ISEA), it is found that the lightest and longest club had the greatest spin rates, launch angles and ball distance. [6] However, both researches were made with different professional male players, which do not correlate entirely with my performance as I am firstly a female player with a different swing motion, so the optimum Driver chosen for them will not fit me and maximize my distance. Moreover, this investigation does a personal club fitting for me. However, as the spin of the ball on its own axis in neglected for this investigation, which is stated at the end of this essay, it is suggested to be furtherly examined in another investigation.

METHODOLOGY

Variables

Independent Variables
The length and head mass of Driver
(0.82-0.1806, 0. 91-0.1922, 0.92-0.1954, 0.93-0.1994, 0.96-0.2055, 0.98-0.2101, 0.99-0.2126, 1.00-0.2134, 1.05-0.2183, 1.09-0.2207, 1.16-0.2251 meters and kilograms respectively)

Dependent Variables
Final distance of the ball
(found in steps of club head speed, ball speed and final distance)

Controlled Variables

Table 1 Controlled variables

Variable	Significance	Value
Air conditions	As it is an outdoor sport, a day with nice weather with no windy or rainy conditions and consistent pressure level is chosen for data collection.	Air density (g/L): 1.03 Temperature (deg C): 22.2 Air Pressure (kPa): 87.8 Relative humidity (%): 43
Body speed	As a rotation happens during the swing, my body also has a constant speed to be able to rotate. However, only the club head's speed will be investigated.	
Ball weight	As ball weight could also be a dependent on changing the final distance, it is held constant	45.6 g ±0.1
Driver launch angle	Launch angle will be used to find the horizontal component of ball velocity. All drivers are set standard.	10.5 degrees
Arm length	As the investigation is held upon me, no player hence arm length was changed.	0.65 m
Time spent from backswing to impact	Time is held constant as it is a factor which affects both velocity and distance of the golf club and creates additional independent variable unless held constant. It is measured with an application called "Swing Tune-up" which uses time-lapse.	0.316 s

Procedure

Club head mass information was gathered from brand websites, or the information written underneath the club head. Length was measured with a measuring tape for each club from the grip to the ending of the club head. My arm length was also measured with the same tape, which is a controlled variable. Identical balls used for shots were weighed on a digital balance. Air conditions were measured with GarminR10 sensor gear. It also measured 5 shots of 11 different clubs' speed before and after impact, ball speed, launch angle and final distance. 5 shots were made, and an average was found to reduce random errors for each club. Time spent of flight of the ball is also measured 5 times with a chronometer and the average value was used for all range calculations.

Calculation of club head speed before impact, dependent to the club length, was done with forming an excel table containing multiple equations. Excel table and calculations are explained with modules of the movement. Theoretical speed and sensor measured values were compared and deviations were found. The independent variable, club length, has determined the change in speed before impact. Club length to speed before impact graph was created using the program LoggerPro for both theoretical and real-life values.

With the information obtained from theoretical club speed, ball speed was calculated theoretically with the law of conservation of momentum, knowing both ball and head mass information. The independent variable, club head mass, has determined the launch speed of the ball. Theoretical ball speed and sensor measured values were compared and deviations were again found. Club head mass to ball's launch speed graph was created using the program LoggerPro for both theoretical and real-life values.

The theoretical ball speed is then used to find a theoretical ball distance. These values were also compared with sensor measured ball range values.

As two independent values (club head mass and length) effect one dependent value (range of the ball), multiple linear regression was created with a code written on R console, which evaluated the effect of these two independent variables on the range of the ball.

<u>Experimental Set-up</u>


Photo 1 Drivers used Photo 2 GarminGolf Measurement Gear

Results

Club Information

The mass and length information for each club is given. As mass and length increases, sample number increases.

Table 2 Club information

Sample no -Club Brand/Name	**Driver Head Mass (kg)± 0.0005 (written on scale)**	**Length (m) ±0.01**
1-Top Flite XJ	0.1806	0.82
2- Callaway Great Big Bertha	0.1922	0.91
3- JazzOmni	0.1954	0.92
4- Wilson Uniflex	0.1994	0.93
5- Taylormade R15	0.2055	0.96
6- Sim Max 2	0.2101	0.98
7- Callaway Epic Speed AV	0.2126	0.99
8- Taylormade R7	0.2134	1.00
9- Ping G400 Max	0.2183	1.05
10- Callaway Epic Flash	0.2207	1.09
11- Callaway Epic Speed Even Flow	0.2251	1.16

CALCULATIONS BEFORE IMPACT

Theoretical Calculation and Comparison of Sensor Measured Values (Club Head Speed)

Theoretical Calculation of Club Head Speed

In this part, final speed of club head before impact will be calculated theoretically. At the top position of the backswing, club head speed becomes zero and starts to accelerate on the downswing. That is why the starting point for calculation starts from backswing, follows with downswing, and ends at impact, where final speed is found. It is found from calculating the distance taken (m) by the club head. It is found from the small "d" values shown in Figure 2, which are the distance traveled between every index, and summing them to find the total distance traveled. From that information and knowing the time spent, acceleration is found, which then led me to the final speed before impact. The swing is divided to 90 moments as the wrists open 90 degrees in 0.316 seconds, which creates 90 different "d" values.

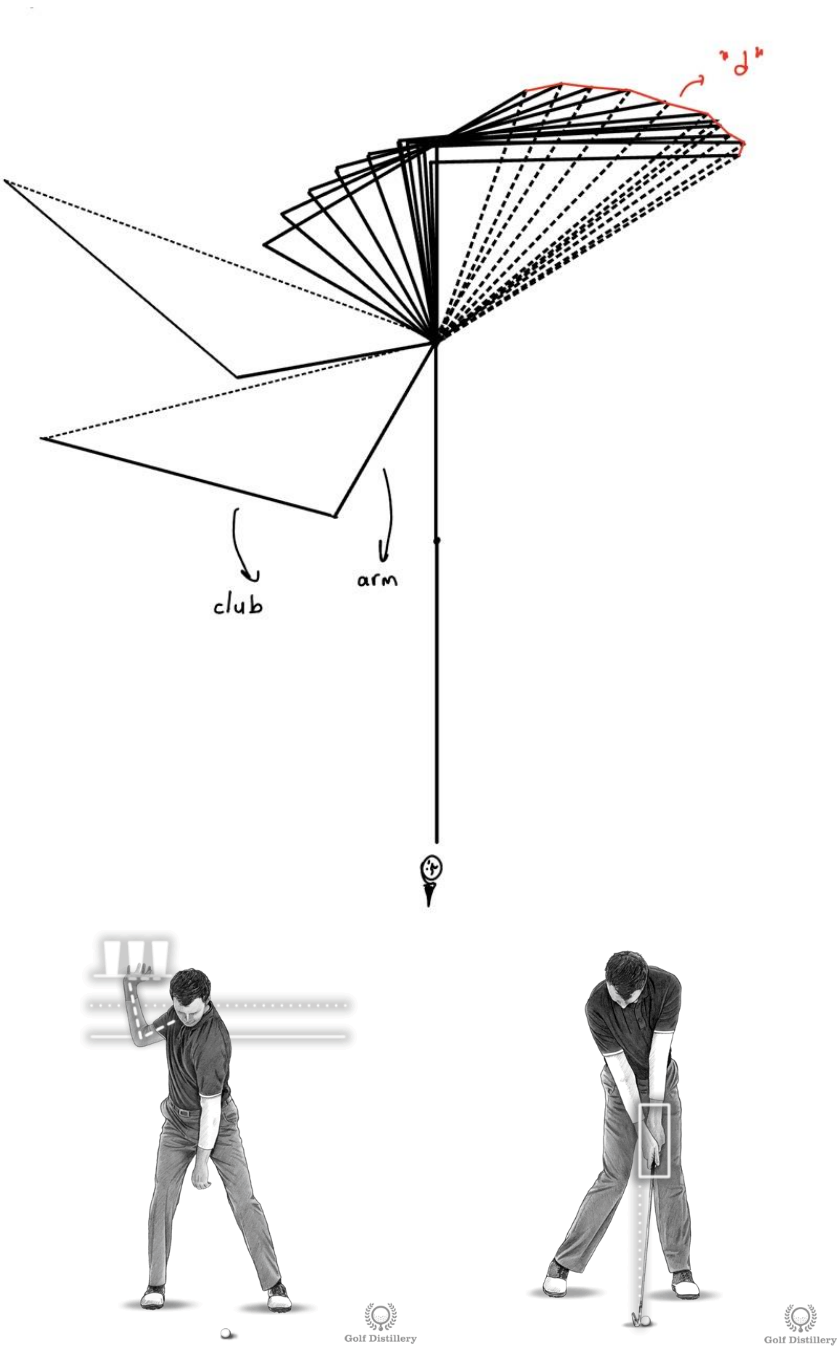


Figure 2: A comparison of real-life image of backswing and moment before impact and the module [7]

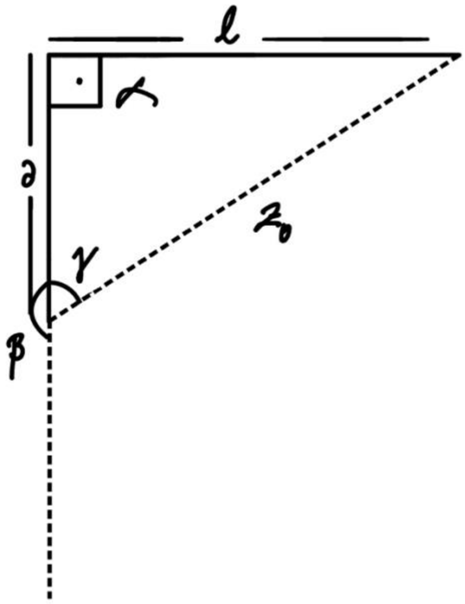


Figure 3 Initial position (top of backswing) and notation

The arm length, which is notated with *"a"* is 0.65 meters. The club length is one of the independent variables of this investigation and is notated with the letter *"l" or "L"*. The distance from pivot point (shoulder) to the head of the club is named as *"z"*, which is the hypotenuse for this position. It is named as z_0 to indicate the initial position.

"α" is the angle between the wrists and club, which is 90 degrees initially. "γ" is named after the angle between *"a"* and *"z"* lines. "β" is the angle between the arm and vertical plane, which is initially 0 degrees.

"γ" is found as $\gamma = \tan^{-1}\frac{L}{a}$

All *"z"* values are found from the cosine theorem [8], which is $a^2 = b^2 + c^2 - 2bc\cos\alpha$. For this notation, it is applied as $z^2 = a^2 + l^2 - 2al\cos\alpha$.

$$z = \sqrt{a^2 + L^2 - 2al\cos\alpha}$$

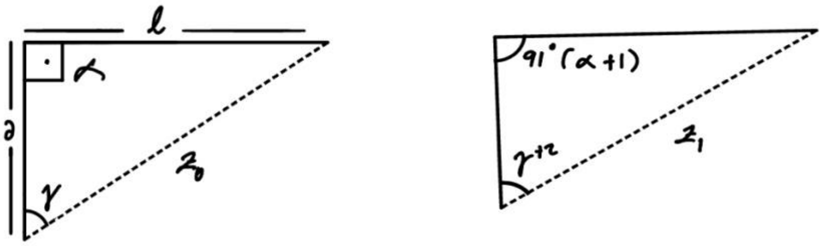


Figure 4 1st and 2nd moment of backswing

As both arm and wrist angle will become vertically parallel at impact position being 180 degrees at the same time, at each investigated position/moment of the swing, "α" opens up 1 degree while "γ" and "β" opens up 2 degrees.

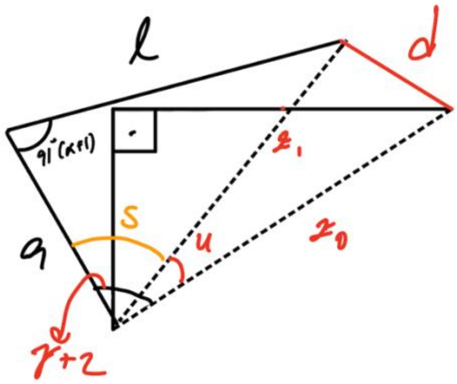


Figure 5 Notations of angles and longitudes on 1st and 2nd moments of backswing

Initial and first moment is investigated to show every calculation applied for every moment. The ultimate goal of finding the distance, notated as "d", is found as in the equation as follows using cosine theorem,

$$d = \sqrt{{z_0}^2 + {z_1}^2 - 2 z_0 z_1 \cos u}$$

To find the angle "u":

$$\gamma + 2 - s = u$$

Angle "u" is dependent to $\gamma + \ 2n$ relative to "z_0". Hence, for new "u" values (in this case, u'), sum of previous u values are subtracted for every remaining moment as shown below by investigating 1st, 2nd and 3rd moments.

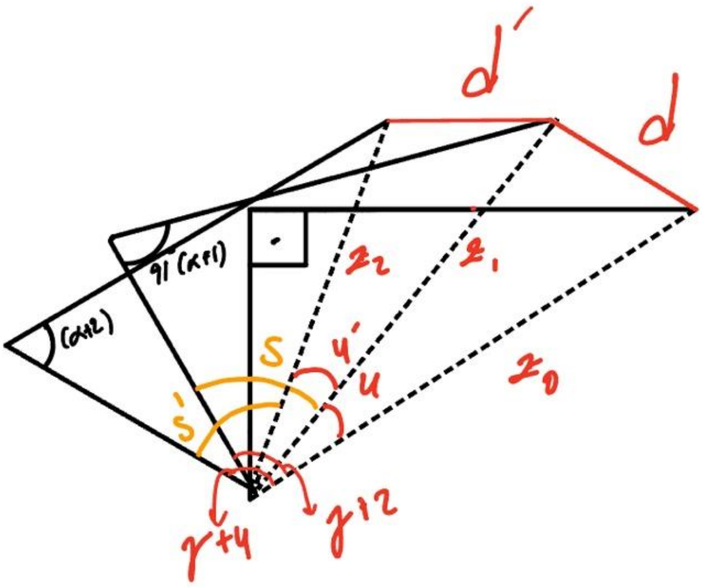


Figure 6 Notations of angles and longitudes on 1st, 2nd and 3rd moments of backswing

$$\gamma + 4 - s' = u' + u$$
$$u' = \gamma + 4 - s' - u$$

To find the angle "s" sinus theorem was used, as follows according to Figure 5:

$$\frac{\sin(\alpha + 1)}{z_1} = \frac{\sin(s)}{L}$$

$$s = \sin^{-1}\frac{(\sin(\alpha+1)\times L)}{(z_1)}$$

Sample calculation to find the initial "d", d_1 ,value according to Sample 8 where L=1 meters:

-Finding γ from the equation $\gamma = \tan^{-1}\frac{L}{a}$:

$$\gamma = \tan^{-1}\frac{1}{0.65}$$

$$\gamma \approx 56.98$$

-Finding z_0 and z_1 from the equation $z = \sqrt{a^2 + L^2 - 2al\cos\alpha}$

$$z_0 = \sqrt{0.65^2 + 1^2 - 2\times 0.65\times 1\times \cos 90}$$

$$z_0 \approx 1.1927\ m$$

$$z_1 = \sqrt{0.65^2 + 1^2 - 2\times 0.65\times 1\times \cos 91}$$

$$z_1 \approx 1.2022\ m$$

-Finding the angle "s" from the equation $s = \sin^{-1}\frac{(\sin(\alpha+1)\times L)}{(z_1)}$

$$s = \sin^{-1}\frac{(\sin(91)\times 1)}{(1.2022)}$$

$$s \approx 56.27$$

- Finding the angle "u" from the equation $\gamma + 2 - s = u$

$$56.98 + 2 - 56.27 = u$$

$$u \approx 2.70$$

-Finding d_1 from the equation $d = \sqrt{{z_0}^2 + {z_1}^2 - 2z_0z_1\cos u}$

$$d = \sqrt{1.1927^2 + 1.2022^2 - 2\times 1.1927\times 1.2022\ \cos 2.70}$$

$$d \approx 0.0572\ m$$

90 different d values using the method above, was found and the total sum equalled to the total distance travelled by club head, which is stated at the end of the following table. Equations were defined in Excel and results for each index are shared. Time, arm length (a) and club length (L) were written in Table 3 and other values were calculated accordingly. Table 3 is on Appendix.

Sum of "d" values = 6.1954 m

90 different ‘d’ values for each index are summed up to find the total distance traveled by the club head. In this case, where “L” is 1 meter, sum of “d” is 6.1954 meters as stated above.

To obtain the final speed before impact, following calculations were done, using the equations $d_{total} = v_0 t + \frac{1}{2}at^2$ then, $v_f^2 = V_0^2 + 2ax$,where $a = acceleration$ and $v_0, v_f = initial, final\ velocity\ respectively.$

Sample Calculation for “L” =1 m (Sample no:8)

$$6.1954 = 0 + \frac{1}{2}a0.316^2$$

$$a = 124.09\ ms^{-2}$$

$$v_f^2 = 0 + 2 \times 124.09 \times 6.1954$$

$$\boldsymbol{v_f = 39.21\ ms^{-1}}$$

Final theoretical speed before impact of every sample is given in the following table:

Table 4 Theoretical club speed before impact

Sample:	1	2	3	4	5	6	7	8	9	10	11
v_f (m/s)	34.04	36.62	36.91	37.19	38.06	38.63	38.92	39.21	40.66	41.82	43.85

<u>Sensor Measured Club Head Speed Values (m/s)</u>

5 shots were done to reduce random errors for each golf club. Table 5 suggests every data and their averages. As sensors were highly sensitive and stated 15 decimal places and uncertainty was too small, it was ignored.

Table 5 sensor measured club speed before impact

Sample:	1	2	3	4	5	6	7	8	9	10	11
Shot 1	33.23	37.43	37.09	37.01	38.27	37.74	39.51	39.41	41.80	41.29	44.09
Shot 2	32.53	36.09	36.91	37.04	38.31	38.34	38.35	39.48	40.32	42.07	43.52
Shot 3	33.30	37.35	36.83	37.09	38.31	38.43	39.19	38.22	39.98	42.65	43.33
Shot 4	32.41	36.14	37.19	37.18	37.87	39.24	38.50	38.86	41.34	41.33	44.41
Shot 5	32.49	36.88	37.20	37.57	37.52	39.42	38.32	40.31	39.65	41.69	43.90
Average	**32.79**	**36.78**	**37.04**	**37.18**	**38.06**	**38.63**	**38.77**	**39.26**	**40.62**	**41.81**	**43.85**

CALCULATIONS AFTER IMPACT

Theoretical Calculation of Ball Speed After Impact

In this part, the launch speed of the ball is calculated theoretically using the theoretical values of club head speed which were found earlier. With the law of conservation of momentum, elastic collision is applied to calculate ball speed after impact. At each shot, the place of impact on the club, the face of the club, is bent inside which affects the momentum. However, as it does not affect the results as much this factor was negligible. Assuming no energy was lost. For the theoretical calculation of ball speed, the measure of club head speed after impact is used from data from sensors.

Table 6 Club head speed after impact (gathered from sensors)

Sample:	1	2	3	4	5	6	7	8	9	10	11
v_f (m/s)	22.36	24.78	25.19	25.20	26.31	27.09	27.32	27.52	29.21	30.25	32.47

V initial of club head before impact (theoretically calculated before) (Table 4)

Sample:	1	2	3	4	5	6	7	8	9	10	11
v_i (m/s)	34.04	36.62	36.91	37.19	38.06	38.63	38.92	39.21	40.66	41.82	43.85

As the momentum is conserved and the collision is supposed to be fully elastic, the following equation for each club is used:

$$m_1 \times v_{1i} + m_2 \times v_{2i} = m_1 \times v_{1f} + m_2 \times v_{2f}$$

Sample 1 calculation:
m club head: 0.1806 kg
m ball: 0.0456 kg
v club initial: 34.04 m/s
v ball initial: 0 m/s
v club final: 22.36 m/s

$$0.1806 \times 34.04 + 0.0456 \times 0 = 0.1806 \times 22.36 + 0.0456 \times v_{ball\ f}$$
$$v_{ball\ f} = 46.26\ ms^{-1}$$

Same calculations are applied for each sample/club follows in the Table 7 below;

Table 7 Launch speed of the golf ball after impact

Sample:	1	2	3	4	5	6	7	8	9	10	11
v_{launch} (m/s)	46.26	49.90	50.22	52.35	52.95	53.17	54.08	54.70	54.81	56.00	56.18

Sensor Measured Ball Speed Values (m/s)

5 shots were done to reduce random errors for each golf club. Table 8 suggests every data and their averages. As sensors were highly sensitive and stated 15 decimal places and uncertainty was too small, it was ignored.

Table 8 Sensor measured ball speed

Sample:	1	2	3	4	5	6	7	8	9	10	11
Shot 1	46.47	50.14	50.86	52.40	52.99	53.78	53.82	54.77	55.17	56.35	56.01
Shot 2	47.97	50.49	50.37	53.12	53.18	53.43	54.14	53.68	54.89	55.65	56.43
Shot 3	47.82	51.73	51.86	52.85	52.49	53.15	54.38	54.16	55.66	55.32	55.97

Shot 4	47.05	50.63	51.24	51.32	53.25	52.68	54.23	55.95	54.97	56.12	57.06
Shot 5	45.84	49.31	51.01	53.32	53.36	54.02	54.17	54.44	55.45	55.89	56.88
Average	**47.03**	**50.46**	**51.07**	**52.60**	**53.05**	**53.41**	**54.15**	**54.60**	**55.23**	**55.87**	**56.47**

CALCULATIONS OF FINAL BALL DISTANCE

Theoretical Calculation of Final Ball Distance

As stated in the controlled variables table, launch angle of the ball is taken as constant, which is 10.5 degrees. It is the standard launch angle for every brand in golf. The launch velocity of the ball's horizontal vector, will be indicating its final range from the equation $Range = v_{horizontal} \times time$

Calculation of horizontal speed referring to Table 7 is found by $v_{launch} \times \cos(10.5)$

Sample 1 calculation using the equation $v_{launch} \times \cos(10.5)$:

$$46.26 \times \cos(10.5) = \mathbf{45.49}\ ms^{-1}$$

Table 9 Horizontal speed of golf ball

Sample:	1	2	3	4	5	6	7	8	9	10	11
$v_{horizontal}$ (m/s)	45.49	49.06	49.37	51.47	52.06	52.28	53.17	53.78	53.89	55.06	55.24

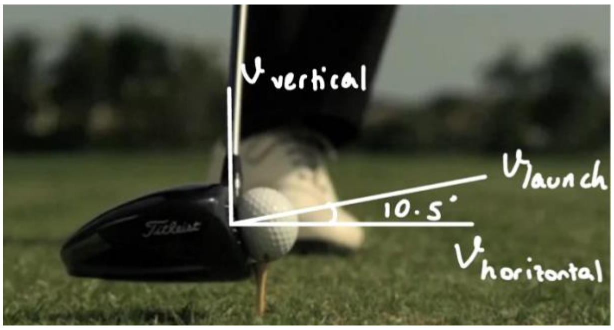


Figure 7 Vector drawing of ball velocity on driver

Calculation of range/ final distance travelled by the ball is found from the equation $Range = v_{horizontal} \times time$. Time taken for each sample is measured with a chronometer 5 times to reduce random errors as shown in Table 10.

Table 10 Time of flight

Sample:	1	2	3	4	5	6	7	8	9	10	11
Shot 1	3.52	3.53	3.52	3.53	3.76	3.81	3.85	3.90	3.87	3.95	3.99
Shot 2	3.50	3.51	3.53	3.55	3.79	3.83	3.87	3.87	3.90	3.90	4.07
Shot 3	3.52	3.53	3.54	3.55	3.78	3.84	3.88	3.85	3.88	3.89	4.01
Shot 4	3.50	3.50	3.56	3.56	3.77	3.82	3.86	3.89	3.90	3.91	4.00
Shot 5	3.51	3.53	3.53	3.54	3.79	3.84	3.88	3.86	3.89	3.93	4.06
Average	**3.51**	**3.52**	**3.54**	**3.55**	**3.78**	**3.83**	**3.87**	**3.88**	**3.89**	**3.92**	**4.03**

Sample 1 calculation:

$$Range = 45.49 \times\ 3.51$$
$$Range = 159.67\ m$$

Table 11 Final distance of golf ball theoretically

Sample:	1	2	3	4	5	6	7	8	9	10	11
Final distance	**159.67**	**172.69**	**174.77**	**182.72**	**196.79**	**200.23**	**205.77**	**208.67**	**209.63**	**215.84**	**222.62**

Sensor Measured Final Distance of Ball

5 shots were done to reduce random errors for each golf club. Table 12 suggests every data and their averages. As sensors were highly sensitive and stated 15 decimal places and uncertainty was too small, hence it is ignored.

Table 12 Sensor measured final ball distance

Sample:	1	2	3	4	5	6	7	8	9	10	11
Shot 1	135.27	148.97	150.83	149.38	178.73	174.66	180.57	182.01	188.85	186.22	192.37
Shot 2	142.77	150.83	153.95	163.41	172.30	169.64	176.49	183.74	178.87	186.42	205.84
Shot 3	140.91	153.76	156.17	163.60	174.70	175.23	179.20	180.42	185.31	190.58	201.39
Shot 4	139.87	153.95	148.97	152.12	165.56	173.85	181.33	186.44	187.70	183.95	197.93
Shot 5	139.51	152.24	153.76	160.29	165.11	181.05	180.01	177.30	181.02	191.09	199.99
Average	**139.67**	**151.95**	**152.74**	**157.76**	**171.28**	**174.89**	**179.52**	**181.98**	**184.35**	**187.65**	**199.50**

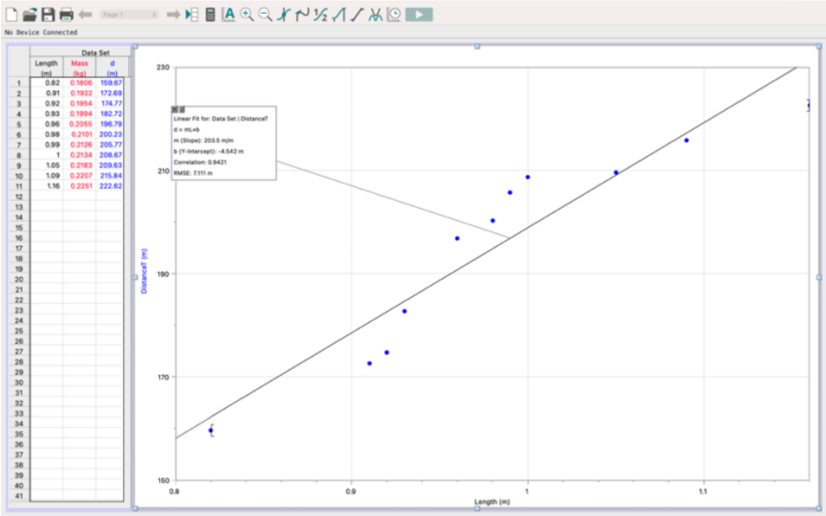
*Theoretical final distance was found greater than sensor measured distance values as air resistance creates an opposite force and decreases velocity, as well as distance.

MATHEMATICAL ANALYSIS

Vernier LoggerPro was used to find the correlation between all independent values (length and mass of the club) and dependent value (ball distance) with separate graphs. As there are two independent variables, their abundance of affect to distance was deduced with a statistical approach, multiple linear regression. R console was used to find statistical results by writing a regression code.

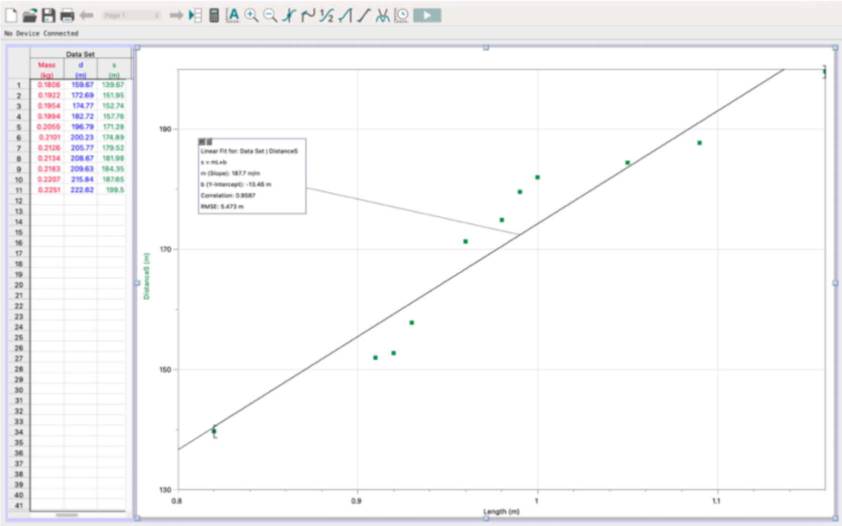
Vernier LoggerPro

Correlation between length and distance based on both theoretical and sensor measured values



Graph 1 Length to theoretical distance

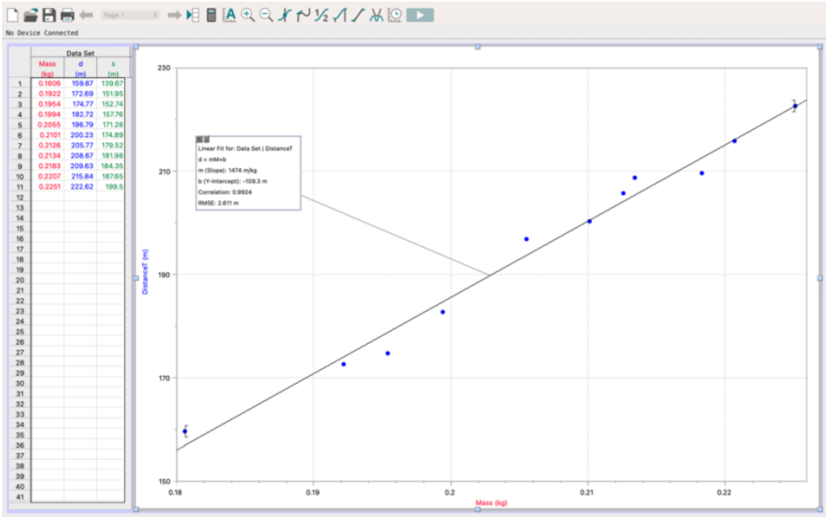
The graph having a linear fit with correlation, 0.9421, close to 1, it is understood that length and theoretical distance are directly proportional, and data is accurate.



Graph 2 Length to sensor measured distance

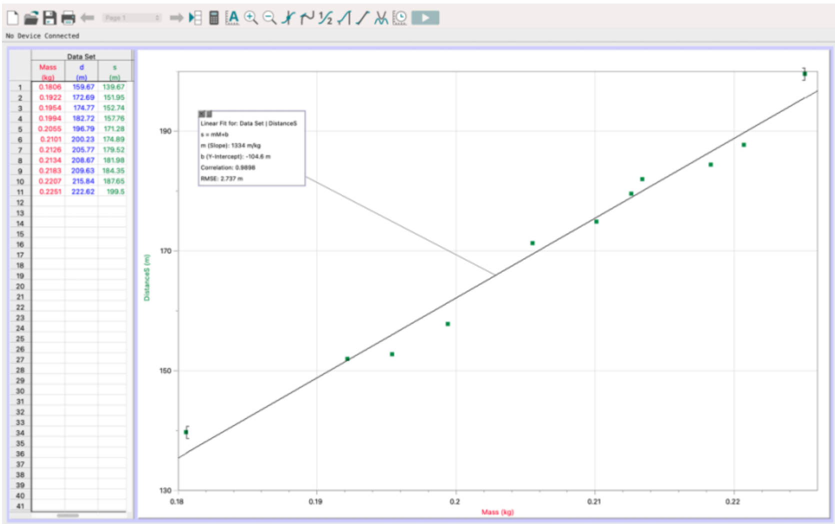
The graph having a linear fit with correlation, 0.9587, close to 1, it is understood that length and sensor-measured distance are directly proportional, and data is accurate.

<u>Correlation between mass and distance based on both theoretical and sensor measured values</u>



Graph 3 Mass to theoretical distance

The graph having a linear fit with correlation, 0.9924, close to 1, it is understood that mass and theoretical distance are directly proportional, and data is accurate.



Graph 4 Mass to sensor measured distance

The graph having a linear fit with correlation, 0.9898, close to 1, it is understood that mass and sensor-measured distance are directly proportional, and data is accurate. [9]

<u>R Console</u>

Codes written to create regressions for all variables are on Appendix. For both theoretical and sensor-measured distance values were related separately to weight and length variables to find their separate significance. It was found that both weight and length are significant variables to find final distance. As both independent variables are used together to find the final distance, multiple linear regression was conducted.

Multiple linear regression on length and mass on theoretically calculated distance

Table 13 MLR results theoretical

Independent variables	Estimate Std **(Coefficients)**	Error	T value	Pr(>|t|)
(Intercept)	**-116.698**	36.295	-3.215	0.01233 *
Weight	**1477.369**	428.938	3.444	0.00877 **
Length	**5.942**	59.603	0.100	0.92304
Residuals standard error: 4.787 on 8 degrees of freedom Multiple R-squared: 0.9547 Adjusted R-squared: 0.9434 F-statistic: 84.36 on 2 and 8 DF p-value: 4.199e-06				

Multiple linear regression on length and mass on sensor measured distance

Table 14 MLR results sensor-measured

Independent variables	Estimate Std **(Coefficients)**	Error	T value	Pr(>|t|)
(Intercept)	**-94.86**	30.14	-3.147	0.0137 *
Weight	**1072.21**	356.24	3.010	0.0168 *
Length	**44.38**	49.50	0.897	0.3961
Residuals standard error: 3.975 on 8 degrees of freedom Multiple R-squared: 0.962 Adjusted R-squared: 0.9525 F-statistic: 101.3 on 2 and 8 DF p-value: 2.079e-06				

In summary, according to both regressions, p-values are small indicating the data are accurate. It is deduced that weight variable is stated more significant than length variable as stated with the sign "*" and p-value being smaller. [10]

CONCULUSION

In this study, the effect of length and head mass of driver (a golf club) on the final distance of a golf ball was investigated. Eleven different clubs, having different masses and lengths were used and final distance was found both theoretically and with sensors. It was observed that theoretical values and sensor-measured values are coherent with each other from tables 4-5, 7-8 and 11-12. As length and mass increases from sample 1 to 11, distance of ball increased, hence both independent variables are directly proportional to the dependent variable and observed with appropriate graphs. The effect abundance between two variables were found with multiple linear regression and found that the variable weight was more significant than the variable length on affecting distance. As clubs are produced in a way of both length and mass increase/decrease proportionally for every brand, it was impossible to investigate this study with only one independent variable. Adding masses to equate every sample's mass was a thought solution, however it effected the player's swing and accurate shots could not be reached. Hence, investigating the ball distance according to length and mass broadened the scope of this study, while conducting multiple linear regression. As measurements were taken from my personal shots, the investigation has indicated which club was best fit for me by using my maths and physics knowledge.

EVALUATION

Strengths

Experimenting with 11 samples of different lengths and masses increased the validity as it is a large scale of data. To reduce random errors at every sensor measured values, five trials for each sample were made. As all the materials were calibrated, no systematic error was obtained. Two different mathematical analysis were done to deduce the relationship between independent variables and dependent variable accurately. Being a national athlete increased the validity and accuracy of data as every shot was made with very close swing movement.

Weaknesses and Limitations

As the club head's mass increases, it was observed that the player (myself) was struggling with swinging, so body speed decreased which resulted in decreasing the final distance. However, the heaviest club was still found to result in highest distance. Factors such as, ball's spin on its own axis, air resistance, uncertainty in sensors were neglected, which limits the scope of calculations. Also, it was assumed that every shot had a launch angle of 10.5 degrees as it is standardized for all drivers which were used, and it was merely impossible to measure the launch angle in the moment of impact.

Further Research

Velocity of ball is not solely upon factors of length and mass. The player's physical properties have a crucial role on final distance of ball and energy transferred. Hence, extending this research to a biophysical scope could result on more personalized and precise values. As the ball's spin axis, air resistance were neglected factors, adding them to calculations could also develop the results of final distance of ball and make them more accurate. One independent variable could be held controlled by fabricating new clubs accordingly.

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APPENDIX

<u>Calculation of “d” values on Excel</u>

Table 3 (Sample 8’s calculations are shown for example, L=1.00 m)

Index/moments	time (s)	a (m)	L (m)	z (m)	α	β	γ	s	u	sum of previous u values	Sin (s)	$d \times 10^{-2}$ (m)
0	0.000	0.65	1.00	1.1927	90	0	56.98					
1	0.004	0.65	1.00	1.2022	91	2	58.98	56.27	2.70	2.70	0.83	5.72
2	0.007	0.65	1.00	1.2116	92	4	60.98	55.58	2.70	5.40	0.82	5.76
3	0.011	0.65	1.00	1.2209	93	6	62.98	54.88	2.70	8.09	0.82	5.80
4	0.014	0.65	1.00	1.2301	94	8	64.98	54.19	2.69	10.79	0.81	5.83

5	0.018	0.65	1.00	1.2393	95	10	66.98	53.50	2.69	13.48	0.80	5.87
6	0.021	0.65	1.00	1.2484	96	12	68.98	52.81	2.69	16.16	0.80	5.90
7	0.025	0.65	1.00	1.2574	97	14	70.98	52.13	2.68	18.85	0.79	5.94
8	0.028	0.65	1.00	1.2663	98	16	72.98	51.45	2.68	21.53	0.78	5.97
9	0.032	0.65	1.00	1.2751	99	18	74.98	50.77	2.68	24.21	0.77	6.01
10	0.035	0.65	1.00	1.2838	100	20	76.98	50.09	2.68	26.88	0.77	6.04
11	0.039	0.65	1.00	1.2925	101	22	78.98	49.42	2.67	29.56	0.76	6.07
12	0.042	0.65	1.00	1.3011	102	24	80.98	48.75	2.67	32.23	0.75	6.11
13	0.046	0.65	1.00	1.3096	103	26	82.98	48.08	2.67	34.90	0.74	6.14
14	0.049	0.65	1.00	1.3180	104	28	84.98	47.41	2.67	37.57	0.74	6.17
15	0.053	0.65	1.00	1.3263	105	30	86.98	46.74	2.67	40.23	0.73	6.21
16	0.056	0.65	1.00	1.3345	106	32	88.98	46.08	2.66	42.89	0.72	6.24
17	0.060	0.65	1.00	1.3426	107	34	90.98	45.42	2.66	45.56	0.71	6.27
18	0.063	0.65	1.00	1.3506	108	36	92.98	44.76	2.66	48.21	0.70	6.30
19	0.067	0.65	1.00	1.3586	109	38	94.98	44.10	2.66	50.87	0.70	6.33
20	0.070	0.65	1.00	1.3664	110	40	96.98	43.45	2.66	53.53	0.69	6.36

21	0.074	0.65	1.00	1.3742	111	42	98.98	42.79	2.65	56.18	0.68	6.39
22	0.077	0.65	1.00	1.3818	112	44	100.98	42.14	2.65	58.83	0.67	6.42
23	0.081	0.65	1.00	1.3894	113	46	102.98	41.49	2.65	61.48	0.66	6.45
24	0.084	0.65	1.00	1.3969	114	48	104.98	40.84	2.65	64.13	0.65	6.48
25	0.088	0.65	1.00	1.4042	115	50	106.98	40.20	2.65	66.78	0.65	6.51
26	0.091	0.65	1.00	1.4115	116	52	108.98	39.55	2.65	69.43	0.64	6.54
27	0.095	0.65	1.00	1.4187	117	54	110.98	38.91	2.64	72.07	0.63	6.57
28	0.098	0.65	1.00	1.4258	118	56	112.98	38.26	2.64	74.71	0.62	6.60
29	0.102	0.65	1.00	1.4327	119	58	114.98	37.62	2.64	77.35	0.61	6.63
30	0.105	0.65	1.00	1.4396	120	60	116.98	36.98	2.64	79.99	0.60	6.65
31	0.109	0.65	1.00	1.4464	121	62	118.98	36.34	2.64	82.63	0.59	6.68
32	0.112	0.65	1.00	1.4531	122	64	120.98	35.71	2.64	85.27	0.58	6.71
33	0.116	0.65	1.00	1.4596	123	66	122.98	35.07	2.64	87.91	0.57	6.73
34	0.119	0.65	1.00	1.4661	124	68	124.98	34.44	2.63	90.54	0.57	6.76
35	0.123	0.65	1.00	1.4725	125	70	126.98	33.80	2.63	93.17	0.56	6.78
36	0.126	0.65	1.00	1.4787	126	72	128.98	33.17	2.63	95.81	0.55	6.81

37	0.130	0.65	1.00	1.4849	127	74	130.98	32.54	2.63	98.44	0.54	6.83
38	0.133	0.65	1.00	1.4909	128	76	132.98	31.91	2.63	101.07	0.53	6.86
39	0.137	0.65	1.00	1.4969	129	78	134.98	31.28	2.63	103.70	0.52	6.88
40	0.140	0.65	1.00	1.5027	130	80	136.98	30.65	2.63	106.33	0.51	6.90
41	0.144	0.65	1.00	1.5084	131	82	138.98	30.02	2.63	108.95	0.50	6.93
42	0.147	0.65	1.00	1.5141	132	84	140.98	29.40	2.63	111.58	0.49	6.95
43	0.151	0.65	1.00	1.5196	133	86	142.98	28.77	2.63	114.21	0.48	6.97
44	0.154	0.65	1.00	1.5250	134	88	144.98	28.15	2.62	116.83	0.47	6.99
45	0.158	0.65	1.00	1.5303	135	90	146.98	27.52	2.62	119.45	0.46	7.01
46	0.162	0.65	1.00	1.5355	136	92	148.98	26.90	2.62	122.08	0.45	7.04
47	0.165	0.65	1.00	1.5405	137	94	150.98	26.28	2.62	124.70	0.44	7.06
48	0.169	0.65	1.00	1.5455	138	96	152.98	25.66	2.62	127.32	0.43	7.08
49	0.172	0.65	1.00	1.5504	139	98	154.98	25.03	2.62	129.94	0.42	7.10
50	0.176	0.65	1.00	1.5551	140	100	156.98	24.41	2.62	132.56	0.41	7.11
51	0.179	0.65	1.00	1.5597	141	102	158.98	23.80	2.62	135.18	0.40	7.13
52	0.183	0.65	1.00	1.5643	142	104	160.98	23.18	2.62	137.80	0.39	7.15

53	0.186	0.65	1.00	1.5687	143	106	162.98	22.56	2.62	140.42	0.38	7.17
54	0.190	0.65	1.00	1.5730	144	108	164.98	21.94	2.62	143.03	0.37	7.19
55	0.193	0.65	1.00	1.5771	145	110	166.98	21.33	2.62	145.65	0.36	7.20
56	0.197	0.65	1.00	1.5812	146	112	168.98	20.71	2.62	148.27	0.35	7.22
57	0.200	0.65	1.00	1.5852	147	114	170.98	20.10	2.62	150.88	0.34	7.24
58	0.204	0.65	1.00	1.5890	148	116	172.98	19.48	2.61	153.50	0.33	7.25
59	0.207	0.65	1.00	1.5927	149	118	174.98	18.87	2.61	156.11	0.32	7.27
60	0.211	0.65	1.00	1.5963	150	120	176.98	18.25	2.61	158.72	0.31	7.28
61	0.214	0.65	1.00	1.5998	151	122	178.98	17.64	2.61	161.34	0.30	7.30
62	0.218	0.65	1.00	1.6032	152	124	180.98	17.03	2.61	163.95	0.29	7.31
63	0.221	0.65	1.00	1.6065	153	126	182.98	16.42	2.61	166.56	0.28	7.32
64	0.225	0.65	1.00	1.6096	154	128	184.98	15.80	2.61	169.17	0.27	7.34
65	0.228	0.65	1.00	1.6127	155	130	186.98	15.19	2.61	171.78	0.26	7.35
66	0.232	0.65	1.00	1.6156	156	132	188.98	14.58	2.61	174.39	0.25	7.36
67	0.235	0.65	1.00	1.6184	157	134	190.98	13.97	2.61	177.00	0.24	7.37
68	0.239	0.65	1.00	1.6211	158	136	192.98	13.36	2.61	179.62	0.23	7.38

69	0.242	0.65	1.00	1.6236	159	138	194.98	12.75	2.61	182.22	0.22	7.39
70	0.246	0.65	1.00	1.6261	160	140	196.98	12.14	2.61	184.83	0.21	7.40
71	0.249	0.65	1.00	1.6284	161	142	198.98	11.53	2.61	187.44	0.20	7.41
72	0.253	0.65	1.00	1.6306	162	144	200.98	10.92	2.61	190.05	0.19	7.42
73	0.256	0.65	1.00	1.6327	163	146	202.98	10.32	2.61	192.66	0.18	7.43
74	0.260	0.65	1.00	1.6347	164	148	204.98	9.71	2.61	195.27	0.17	7.44
75	0.263	0.65	1.00	1.6365	165	150	206.98	9.10	2.61	197.88	0.16	7.45
76	0.267	0.65	1.00	1.6383	166	152	208.98	8.49	2.61	200.48	0.15	7.45
77	0.270	0.65	1.00	1.6399	167	154	210.98	7.88	2.61	203.09	0.14	7.46
78	0.274	0.65	1.00	1.6414	168	156	212.98	7.28	2.61	205.70	0.13	7.47
79	0.277	0.65	1.00	1.6427	169	158	214.98	6.67	2.61	208.31	0.12	7.47
80	0.281	0.65	1.00	1.6440	170	160	216.98	6.06	2.61	210.91	0.11	7.48
81	0.284	0.65	1.00	1.6451	171	162	218.98	5.46	2.61	213.52	0.10	7.48
82	0.288	0.65	1.00	1.6462	172	164	220.98	4.85	2.61	216.13	0.08	7.49
83	0.291	0.65	1.00	1.6471	173	166	222.98	4.24	2.61	218.73	0.07	7.49
84	0.295	0.65	1.00	1.6478	174	168	224.98	3.64	2.61	221.34	0.06	7.49

85	0.298	0.65	1.00	1.6485	175	170	226.98	3.03	2.61	223.95	0.05	7.50
86	0.302	0.65	1.00	1.6490	176	172	228.98	2.42	2.61	226.55	0.04	7.50
87	0.305	0.65	1.00	1.6495	177	174	230.98	1.82	2.61	229.16	0.03	7.50
88	0.309	0.65	1.00	1.6498	178	176	232.98	1.21	2.61	231.76	0.02	7.50
89	0.312	0.65	1.00	1.6499	179	178	234.98	0.61	2.61	234.37	0.01	7.50
90	0.316	0.65	1.00	1.6500	180	180	236.98	0.00	2.61	236.98	0.00	7.50

R Console: Regression between length and distance; mass and distance for both theoretical and sensor measured distance values.

Theoretical distance to weight:

```
Call:
lm(formula = distance ~ weight, data = data)

Residuals:
    Min       1Q   Median       3Q      Max
-10.4100  -1.5787   0.1905   3.1645   4.8143

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   -119.4       22.9  -5.213 0.000555 ***
weight        1518.5      110.3  13.768 2.37e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.516 on 9 degrees of freedom
Multiple R-squared:  0.9547,  Adjusted R-squared:  0.9496
F-statistic: 189.6 on 1 and 9 DF,  p-value: 2.369e-07
```

Theoretical distance to length:

```
Call:
lm(formula = distance ~ length, data = data)

Residuals:
   Min     1Q Median     3Q    Max
-8.847 -5.245 -1.385  5.699  9.756

Coefficients:
```

```
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   -4.542     23.813  -0.191    0.853
length       203.456     24.133   8.431 1.45e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 7.111 on 9 degrees of freedom
Multiple R-squared:  0.8876,  Adjusted R-squared:  0.8751
F-statistic: 71.07 on 1 and 9 DF,  p-value: 1.453e-05
```

Sensor measured distance to weight:

```
Call:
lm(formula = distance ~ weight, data = data)

Residuals:
   Min     1Q Median     3Q    Max
-7.902 -1.866  0.001  2.638  5.477

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -114.95      19.94  -5.764 0.000271 ***
weight       1379.52      96.03  14.366 1.64e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.932 on 9 degrees of freedom
Multiple R-squared:  0.9582,  Adjusted R-squared:  0.9536
F-statistic: 206.4 on 1 and 9 DF,  p-value: 1.641e-07
```

Sensor measured distance to length:

```
Call:
lm(formula = distance ~ length, data = data)

Residuals:
    Min      1Q  Median      3Q     Max
-6.5106 -4.1601 -0.8077  4.4479  7.7110

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   -13.46      18.33  -0.734    0.481
length        187.73      18.57  10.107 3.27e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.473 on 9 degrees of freedom
Multiple R-squared:  0.919,    Adjusted R-squared:   0.91
F-statistic: 102.1 on 1 and 9 DF,  p-value: 3.275e-06
```

<u>Codes written for R console with theoretical distance values</u>

```
# Diagnostic plots
> par(mfrow=c(2,2))  # Create a 2x2 grid for diagnostic plots
> plot(model)  # Residuals vs Fitted, Normal Q-Q plot, Scale-Location, Residuals vs Leverage
>
> # Reset the plotting settings
> par(mfrow=c(1,1))
>
> # Sample data
> weight <- c(0.1806, 0.1992, 0.1954, 0.1994, 0.2055, 0.2101, 0.2126, 0.2134, 0.2183, 0.2207, 0.2251)
> length <- c(0.82, 0.91, 0.92, 0.93, 0.96, 0.98, 0.99, 1.00, 1.05, 1.09, 1.16)
> distance <- c(159.67, 172.69, 174.77, 182.72, 196.79, 200.23, 205.77, 208.67, 209.63, 215.84, 222.62)
>
> # Create a data frame
> data <- data.frame(weight, length, distance)
>
> # Perform multiple linear regression
> model <- lm(distance ~ weight + length, data=data)
>
> # Summary of the regression model
> summary(model)
```

Codes written for R console with sensor-measured distance values

```
# Sample data
> weight <- c(0.1806, 0.1992, 0.1954, 0.1994, 0.2055, 0.2101, 0.2126, 0.2134, 0.2183, 0.2207, 0.2251)
> length <- c(0.82, 0.91, 0.92, 0.93, 0.96, 0.98, 0.99, 1.00, 1.05, 1.09, 1.16)
> distance <- c(139.67, 151.95, 152.74, 157.76, 171.28, 174.89, 179.52, 181.98, 184.35, 187.65, 199.50)
>
> # Create a data frame
> data <- data.frame(weight, length, distance)
>
> # Perform multiple linear regression
> model <- lm(distance ~ weight + length, data=data)
>
> # Summary of the regression model
> summary(model)
```