INTERNATIONAL BACCALAUREATE DIPLOMA PROGRAMME

PHYSICS EXTENDED ESSAY

INVESTIGATION OF MOVEMENT OF OBJECTS BY ANALYSING DISPLACEMENT

Word count: 3476 words

1. INTRODUCTION

In the everlasting existence of the universe, movement is the essence of everything. From giant galaxies moving through the dark abyss called space to microscopic particles moving around each other, motion lies behind every corner we look. To catch a glimpse of what is happening around us, it is mandatory to understand the dynamic phenomenon of motion. Displacement, the fundamental shift in position, helps us understand the core of movement, while its derivatives give us a option to widely examine the details of the main displacement.

This extended essay will be a dive into the world of kinematics, looking into the details of motion as far as we are able to comprehend. As acceleration is the highest derivative we use in equations, others are usually not taken into account, even though they are in every daily life example we can find. Let's consider a normal car for such an example, when it is used in a problem, the driver is said to give a constant acceleration to the car by holding his feet right in the place. But in our daily life, no one ever keeps their feet pushing constantly. What that means is that acceleration is also changing per unit time in such an example.

In this essay, my aim is to create a movement experiment that will involve an acceleration that is changing over time. For such experiment, a displacement that has two factors is needed. Because of this, falling of a chain from the edge of a table is the perfect example because as the chain is falling, its falling part gets larger which makes it speed up, and also the mass of its part moving on the table is decreasing which means the friction force acting on it is decreasing, thus making it speed up. As it speeds up at an increasing speed, we could say that its acceleration is increasing as well. This makes the perfect example to investigate as even though it is known that its acceleration is increasing over time, we do not know its rate, so the increase of acceleration might be increasing over time as well!

1.1. Research Question

How does change in the mass rate of the falling part of the chain affect the rate of change of acceleration under the conditions that the total mass and length of the chain are controlled?

1.2. Hypothesis

I believe that in this experiment, as we find the rate in which the displacement is occurring, we will be able to find that the acceleration is increasing exponentially over time, with this, we will be able to comprehend the motion fully, thus proving that higher derivatives of motion could be found anywhere, even in a basic falling movement, especially in the ones that friction takes part. The importance of this topic in frictional instances proves that this topic is necessary in real life as there is no “ignoring the frictional force” such as the ones used to study the topics of mechanic.

1.3. Personal Engagement

As a mathematics and physics higher level student, I have always loved to investigate, and even overlap, concepts from both subjects. When I learned about derivatives, the first thing that came to my mind was how velocity was the derivative of displacement and acceleration was the derivative of velocity. Then I asked myself if there was any further derivative of displacement, and I was right. To find out how they can be implemented into real life, I started researching for a suitable experiment, and then I found about a chain falling from the side of a table.

2. BACKGROUND INFORMATION

2.1. Kinematics and accelerated motion in one dimension

Kinematics is the study of the motion of mechanical points, bodies and systems without consideration of their associated physical properties and the forces acting on them. The study is often referred to as the geometry of motion, and it models these motions mathematically using algebra.[1]

Kinematics is very useful in the conceptual design of mechanical systems. Initial geometries and velocities of bodies are a part of the model. While kinematics can help determine whether a design is theoretically possible, there are more complexities when designing something for the real world. Without consideration of materials, and the forces acting upon them, many theoretically possible designs would be prone to failure.[1]

2.2. Free fall

An object that falls through a vacuum is subjected to only one external force, the gravitational force, expressed as the weight of the object. An object that is moving only because of the action of gravity is said to be free falling and its motion is described by Newton's second law of motion. With algebra we can solve for the acceleration of a free falling object. The acceleration is constant and equal to the gravitational acceleration g which is 9.8 meters per square second at sea level on the Earth. The weight, size, and shape of the object are not a factor in describing a free fall. In a vacuum, a beach ball falls with the same acceleration as an airliner.[2]

[1] *Contributor, TechTarget. "What Is Kinematics?: Definition from TechTarget." WhatIs, TechTarget, 4 May 2018,*

[2] *"Motion of Free Falling Object." NASA, NASA, 21 July 2022, www1.grc.nasa.gov/beginners-guide-to-aeronautics/motion-of-free-falling-object/*

2.3. Mathematical aspect of kinematics with regard to derivative

Derivative, in mathematics, the rate of change of a function with respect to a variable. Derivatives are fundamental to the solution of problems in calculus and differential equations. In general, scientists observe changing systems (dynamical systems) to obtain the rate of change of some variable of interest, incorporate this information into some differential equation, and use integration techniques to obtain a function that can be used to predict the behavior of the original system under diverse conditions.[3]

Geometrically, the derivative of a function can be interpreted as the slope of the graph of the function or, more precisely, as the slope of the tangent line at a point. Its calculation, in fact, derives from the slope formula for a straight line, except that a limiting process must be used for curves. The slope is often expressed as the "rise" over the "run," or, in Cartesian terms, the ratio of the change in y to the change in x. For the straight line shown in the figure, the formula for the slope is $(\frac{y_1-y_0}{x_1-x_0})$. Another way to express this formula is

$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$, if h is used for $x_1 - x_0$ and $f(x)$ for y. This change in notation is useful for advancing from the idea of the slope of a line to the more general concept of the derivative of a function.

[3] *"Derivative." Encyclopædia Britannica, Encyclopædia Britannica, inc., 1 Dec. 2023, www.britannica.com/science/derivative-mathematics.*

DERIVATIVES AND INTEGRALS OF POSITION

Symbol	Derivative	Name	SI unit
$\vec{x}$	x	displacement	m
$\vec{v}$	$\frac{\partial x}{\partial t}$	velocity	$m \cdot s^{-1}$
$\vec{a}$	$\frac{\partial^2 x}{\partial t^2}$	acceleration	$m \cdot s^{-2}$
$\vec{j}$	$\frac{\partial^3 x}{\partial t^3}$	jerk	$m \cdot s^{-3}$
$\vec{s}$	$\frac{\partial^4 x}{\partial t^4}$	snap/jounce	$m \cdot s^{-4}$
$\vec{c}$	$\frac{\partial^5 x}{\partial t^5}$	crackle	$m \cdot s^{-5}$
$\vec{p}$	$\frac{\partial^6 x}{\partial t^6}$	pop	$m \cdot s^{-6}$
$\vec{l}$	$\frac{\partial^7 x}{\partial t^7}$	lock	$m \cdot s^{-7}$
$\vec{d}$	$\frac{\partial^8 x}{\partial t^8}$	drop	$m \cdot s^{-8}$
$\vec{S}$	$\frac{\partial^9 x}{\partial t^9}$	shot	$m \cdot s^{-9}$
$\vec{P}$	$\frac{\partial^{10} x}{\partial t^{10}}$	put	$m \cdot s^{-10}$
$\vec{?}$	$\frac{\partial^{11} x}{\partial t^{11}}$	?	$m \cdot s^{-11}$

Symbol	Derivative	Name	SI unit
$\vec{x}$	x	displacement	m
A	$\int x \, \partial t$	absement	$m \cdot s$
B	$\iint x \, \partial t$	absity	$m \cdot s^2$
C	$\iiint x \, \partial t$	abseleration	$m \cdot s^3$
D	$\iiiint x \, \partial t$	abserk	$m \cdot s^4$
E	$\int \overset{\cdots}{5} \int x \, \partial t$	absnap	$m \cdot s^5$
F	$\int \overset{\cdots}{6} \int x \, \partial t$	absackle	$m \cdot s^6$
G	$\int \overset{\cdots}{7} \int x \, \partial t$	absock	$m \cdot s^7$
H	$\int \overset{\cdots}{8} \int x \, \partial t$	absrop	$m \cdot s^8$
I	$\int \overset{\cdots}{9} \int x \, \partial t$	absot	$m \cdot s^9$
J	$\int \overset{\cdots}{10} \int x \, \partial t$	absut	$m \cdot s^{10}$
K	$\int \overset{\cdots}{11} \int x \, \partial t$	abs?	$m \cdot s^{11}$

Figure 1: The derivatives and integrals of displacement

2.4. Explanation of the experiment with regard to similar, more basic counterparts

The equation of friction force and force of gravity is a basic concept of motion that is used for teaching forces, coefficient of dynamic friction and equilibrium. It is done as an experiment which is done similar to the one investigated in this essay, or by tethering two blocks together, letting one fall while the other one slows or stops it by its frictional force against gravitational force acting on the first block. Though falling chain seems no different, it adds the factor of mass change into the equation, resulting in change of velocity. This is used fairly common to teach the basic concepts of displacement with the omission of opposing forces such as frictional forces. When added, these add a huge amount of complexity to the equation. Even though it seems similar without any equation, when added, you realize that it is far more complex to calculate how it truly moves in detail. This experiment is also worked out just by the addition of sliding frictional force. Even though it is better to work out the equation with the addition of air resistance, it is not possible to go into that much detail is such an investigation.

3. METHODOLOGY

3.1. Variables

Table 1. Variable Table

Type of variable	**Variable identified**	**Method for changing/ observing/controlling**
Independent Variables	Rate of change in mass of the falling part of the chain	As the chain is falling, this variable changes over time. So as there is only 1 experiment repeated over for minimizing error, our independent variable is the mass at a given time. It is not possible to calculate the instantaneous rate of change, so the relation between falling and not falling parts of chain is found with acceleration instead.
Dependent Variables	Rate of change of vertical velocity of the falling part of the chain	As the mass of falling part of the chain increasing, mass of the part on the table is decreasing. These show us that the acceleration of the chain is both positive and also increasing over time.
Control Variables	Coefficient of dynamic friction between the chain and the table Total mass of the chain Gravitational acceleration	This variable will be calculated and pointed on: Exploration-Calculation of the force of friction between wooden table and metal chain Other control variables are all kept to minimize the error percentage.

3.2. Apparatus and Materials needed for the main experiment

- Metal chain
- Smooth horizontal surface (table)
- Tape measure ($\pm 0.05\ cm$)
- Digital camera capable of recording slow motion videos
- LoggerPro software
- Desmos graphing calculator

3.3. Labeled Diagram and Experimental Setup

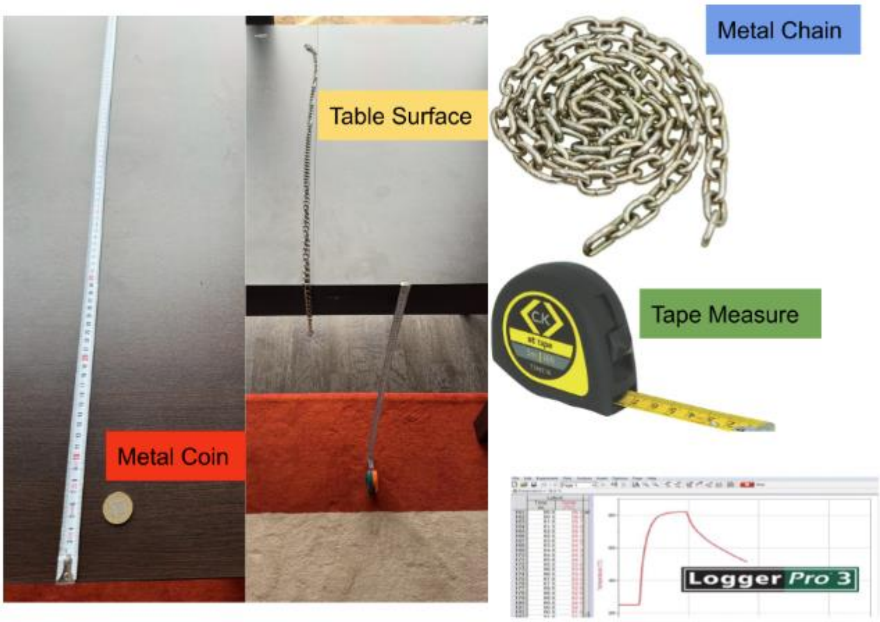


Figure 2: Labeled diagram of the experimental setup

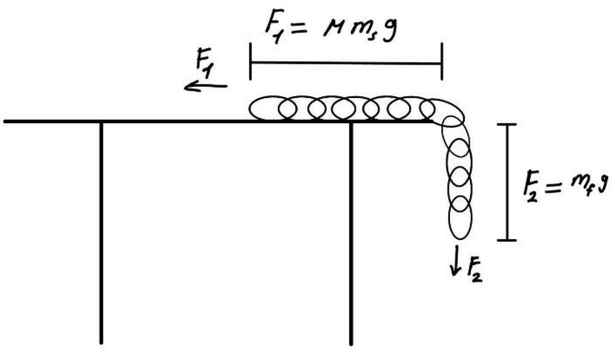


Figure 3: Scheme of the experiment

m_s → total mass of chain left on the surface of the table

m_f → total mass of chain free falling

$$Fnet = F2 - F1$$

$$(m_f + m_s)a = m_f g - \mu m_s g$$

$$\left(m_f + m_s\right)a = g(m_f - \mu m_s)$$

$m_{total} = m_f + m_s$ stays constant

$$a \propto m_f - \mu m_s$$

m_f is increasing in increasing order,

m_s is decreasing in increasing order,

So, a is increasing in increasing order.

Thus, we can theorize that a-t graph is either a quadratic or a higher polynomial function.

3.4. Experimental Procedure

3.4.1. Procedure for the Primary Investigation with the Metal Coin

As of starting the experiment, we will carry out a series of equations to calculate our graphs. But before the calculation, we will need to know the certain coefficient of dynamic friction. For this reason, before starting the main experiment, the calculation of coefficient of friction is done.

Calculation of the force of friction between wooden table and metal chain

This part is the primary experiment to find the coefficient of dynamic friction to be able to work out the equations on the main experiment.

In this experiment, we will find the coefficient of dynamic friction between the wooden table and metal chain that will be used. As the main part of this experiment is to look at the coefficient between wood and metal, we could use a coin of same metal type. The apparatus needed for this experiment are:

- A metal coin
- Table which will be used for the main experiment
- Chronometer
- Tape measure ($\pm 0.05\ cm$)

The procedure is as follows:

1. Put the tape measure down starting from the side of the table.
2. Put the coin near the tape measure where its middle point aligns with the 0 on the measure and is on the edge of the table.
3. Hit the coin through the table and have someone record the movement from the start (hitting the coin) until the end (stopping of the coin).
4. Repeat step 3 two more times.
5. Create velocity-time (v-t) graphs and calculate the average acceleration.
6. Work out force equations to calculate the coefficient of dynamic friction.

3.4.2. Experimentation

Figure 5: The falling chain experiment model.

1. Coefficient of dynamic friction between the table and the chain is found by a different experiment made with pushing the coin on the surface of the table. (The procedure for this calculation will be detailed in: Exploration-Calculation of the force of friction between wooden table and metal chain)
2. Main experiment is held by recording the chain falling from the side of a table with the ruler standing vertically to show the displacement according to time.
3. The step 2 is repeated 4 more times.
4. Using LoggerPro video analysis tool, instances were taken from each video with 10 frames of gap between and displacement-time graphs were created separately for one by pointing lower tip of the chain in each interval (instances were taken between the hand leaving the chain and the tip of the chain hitting the ground.)
5. The change in the length of chain falling per time graph (x-t graph) is drawn.
6. Velocity-time (v-t) graph taken from the LoggerPro.
7. Further derivatives of displacement are calculated by Desmos graph calculator until a horizontal line is reached.
8. The results are commented, stating why that number of derivatives were reached.

3.5. Qualitative observations

In this experiment to measure the dynamic friction coefficient of the table an experiment with metal coin is conducted by hitting the metal coin to measure its distance taken and time taken until it stops. During this measurement the subject of the study or in other saying the conductor should be aware to align the coin in a straight way to prevent possible deviations from a linear line until it stops. Moreover, in the other part of the experiment working with metal chain, the conductor should try to find the position of the chain to start its motion from

the table to reach the floor by observing its dynamics such as how it has curves, how it turns around.

3.6. Risk Assessment and Environmental Approach

This experiment was not a risky experiment because there was no usage of sharp, fragile, or risky materials. The metal chain used during the experimental procedure is chosen as a very light one to reduce the possible results of harming the floor after falling on it. When it comes to the environmental approach of the experiment, it was an environmentally friendly experiment due to no hazardous waste output and all the materials used in the experiment were reusable.

4. DATA COLLECTION AND PROCESSING

4.1. Raw Data for the Primary Investigation to Find Dynamic Friction Coefficient

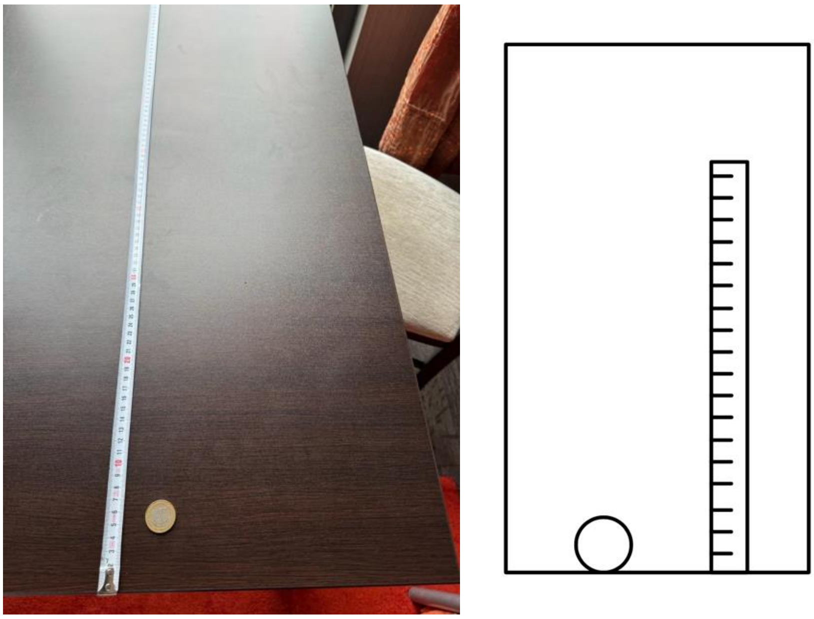


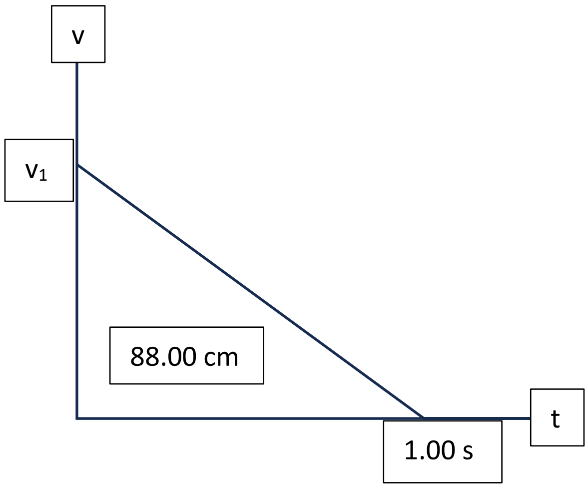
Figure 5: The coin toss experiment model.

Table 2. Raw data table 1

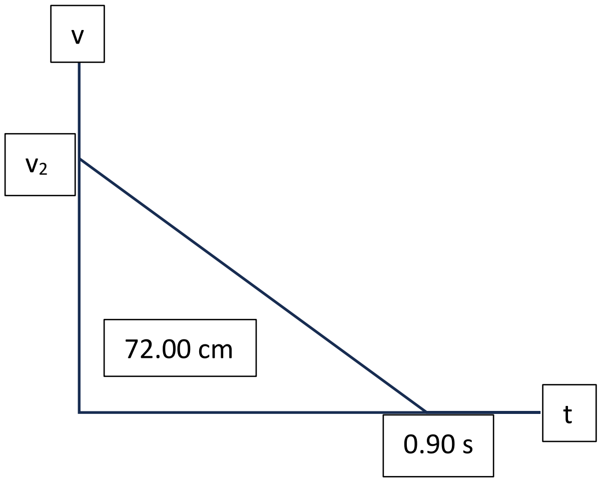
Time interval for the metal coin to stop after it is hit (sec) ($\pm 0.01\ s$)	Distance taken for the coin until it stops (cm) ($\pm 0.05\ cm$)
1.00	88.00
0.90	72.00
1.20	109.00

Then, the velocity-time (v-t) graphs are created.

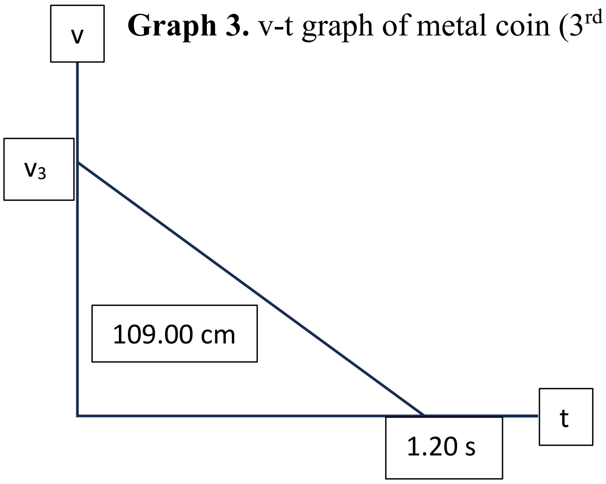
Graph 1. v-t graph of metal coin (1st trial)



Graph 2. v-t graph of metal coin (2nd trial)



Graph 3. v-t graph of metal coin (3rd trial)



4.1.1. Justification of the Uncertainty of the Raw Data

- For the length measurements a tape measure is used, and because of it is a manual reading the half of the smallest increment which is 0.05 cm taken as the absolute uncertainty.
- For the time measurements chronometer of a mobile phone is used, and the smallest reading which is 0.01 s taken as the absolute uncertainty.

4.2.Processed Data for the Primary investigation with Metal Coin

To find the coefficient of dynamic friction, we would need to find the force of friction and then equate it to the general force formula.

$$\frac{v_{initial}\, t}{2} = x$$

Table 3. Processed data table 1

Measured displacement of metal coin until it stops (cm) ($\pm 0.05\ cm$)	Measured time for the metal coin until it stops (s) ($\pm 0.01 s$)	Calculated initial velocity for the metal coin until it stops (m s^{-1}) ($\pm$%0.99)
x_1= 88.00 cm	t_1=1.00 s	v_1=1.76 ms^{-1}
x_2=72.00 cm	t_2=0.90 s	v_2=1.60 ms^{-1}
x_3=109 cm	t_3=1.20 s	v_3=1.82 ms^{-1}
x_{avg}= 89.67 cm ($\pm$%0.16)	t_{avg}= 1.03 s ($\pm$%2.91)	v_{avg}= 1.73 ms^{-1} ($\pm$%3.07)

From the value of the gradient, the experimental values of a1, a2 and a3 are calculated.

Table 4. Processed data table 2

From $\frac{v_1}{t_1}$	From $\frac{v_2}{t_2}$	From $\frac{v_3}{t_3}$	Average acceleration
a_1=1.76 ms^{-2}	a_2=1.77 ms^{-2}	a_3=1.51 ms^{-2}	a_{avg}=1.68 ms^{-2} (±%5.98)

Even though the acceleration is found directly through each trial, average values are used to find error percentages to minimize the uncertainty

Using newton's second law, we could find the constant between the object.

$$F_{net} = ma$$

$$F_{friction} = ma$$

$$\mu mg = ma$$

$$\mu g = a$$

$$(by\ using\ g = 9.81\ m\ s^{-2})$$

Finally, we could reach the coefficient of dynamic friction between our coin and the wooden table is:

$$9.81\mu = 1.68$$

$$\mu = 0.17$$

Using the uncertainty reached from acceleration uncertainty, we can find the uncertainty for μ.

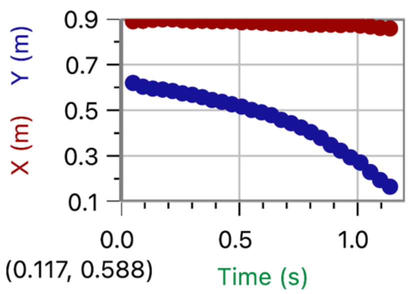
$$0.17 \times \frac{5.98}{100} = \pm 0.0102$$

4.3. Raw Data for the Main Experiment

During the main experiment, values for the chain's movement on both axes are given on the position-time tables. But as the x value is the horizontal movement of the chain which is irrelevant for the experiment, it is omitted during data analysis and only vertical movement (y value) is processed.

4.3.1. x and y positions and velocity on y-axis graphs of the metal chain obtained from Logger Pro 3 video analysis tool

Graph 4. x and y positions of the first data set (Trial 1)



Graph 5. Velocity on y axis vs. time graph (Trial 1)

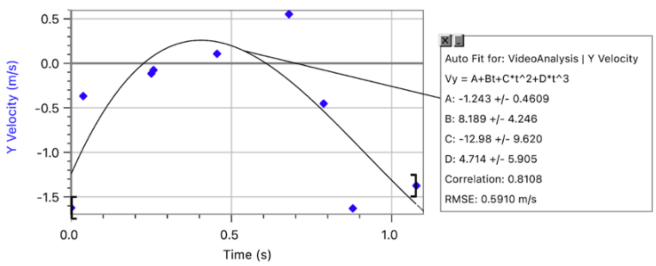
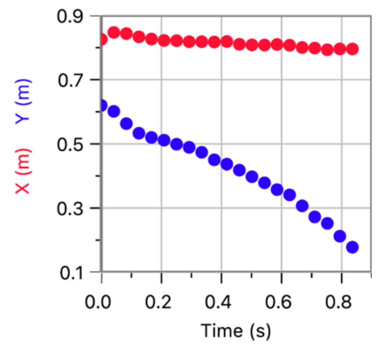


Table 5. Raw data table for 1st trial

	VideoAnalysis				
	Time (s)	X (m)	Y (m)	Vx (m/s)	Vy (m/s)
1	0	0.8720	0.6775	0.933	-1.623
2	0.03762	0.9399	0.5698	0.058	-0.368
3	0.2502	0.8855	0.5784	-0.098	-0.115
4	0.2569	0.8915	0.5861	0.037	-0.076
5	0.4550	0.8890	0.5375	0.139	0.107
6	0.6793	0.8737	0.4756	0.400	0.552
7	0.7874	1.167	1.021	-0.037	-0.453
8	0.8785	0.8685	0.3872	-0.559	-1.629
9	1.077	0.8569	0.2389	-0.394	-1.374

Graph 6. x and y positions of the first data set (Trial 2)



Graph 7. Velocity on y axis vs. time graph (Trial 2)

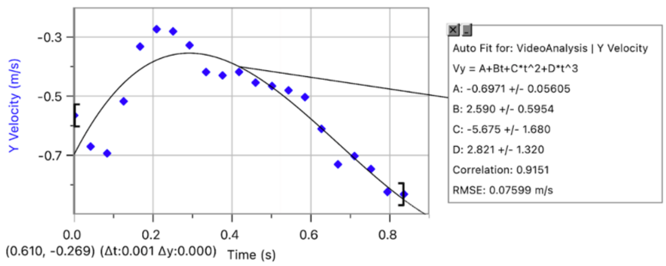
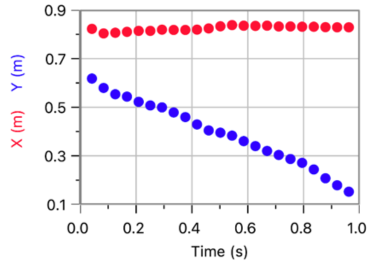


Table 6. Raw data table of 2nd trial

	VideoAnalysis 2				
	Time (s)	X (m)	Y (m)	Vx (m/s)	Vy (m/s)
1	0.0501(	0.8896	0.6197	0.061	-0.336
2	0.0919)	0.8917	0.6031	0.068	-0.261
3	0.1338	0.8958	0.5948	0.057	-0.181
4	0.1756	0.8979	0.5906	0.017	-0.154
5	0.2174	0.8979	0.5844	-0.037	-0.183
6	0.2592	0.8937	0.5740	-0.066	-0.199
7	0.3010	0.8896	0.5678	-0.034	-0.214
8	0.3428	0.8917	0.5574	-0.014	-0.250
9	0.3846	0.8896	0.5449	-0.018	-0.245
10	0.4264	0.8896	0.5366	-0.015	-0.239
11	0.4682	0.8896	0.5263	-0.039	-0.255
12	0.5100	0.8854	0.5159	-0.044	-0.284
13	0.5518	0.8854	0.5014	-0.040	-0.296
14	0.5936	0.8834	0.4910	-0.057	-0.308
15	0.6354	0.8792	0.4785	-0.041	-0.375
16	0.6772	0.8792	0.4578	-0.015	-0.408
17	0.7190	0.8792	0.4433	-0.014	-0.422
18	0.7608	0.8792	0.4246	-0.036	-0.480
19	0.8026	0.8751	0.4038	-0.037	-0.553
20	0.8444	0.8751	0.3789	-0.015	-0.637
21	0.8862	0.8751	0.3478	-0.018	-0.656
22	0.9280	0.8730	0.3229	-0.014	-0.654
23	0.9698	0.8751	0.2938	-0.039	-0.669
24	1.012	0.8709	0.2710	-0.085	-0.763
25	1.053	0.8668	0.2295	-0.107	-0.841
26	1.095	0.8605	0.1942	-0.099	-0.794
27	1.137	0.8585	0.1652	-0.074	-0.745
28					
29					

Graph 8. x and y positions of the first data set (Trial 3)



Graph 9. Velocity on y axis vs. time graph (Trial 3)

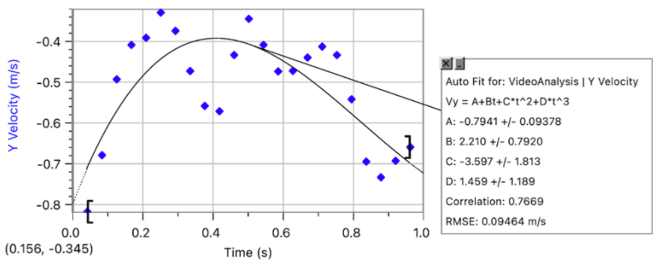
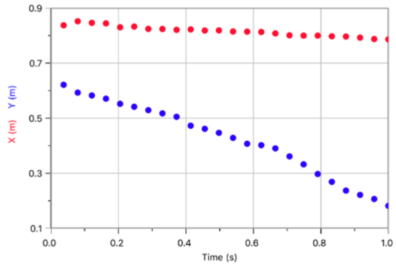


Table 7. Raw data table of 3rd trial

	VideoAnalysis				
	Time (s)	X (m)	Y (m)	Vx (m/s)	Vy (m/s)
1	0	0.8258	0.6200	0.337	-0.565
2	0.04178	0.8472	0.6013	0.128	-0.670
3	0.08356	0.8436	0.5631	-0.083	-0.694
4	0.1253	0.8335	0.5332	-0.154	-0.518
5	0.1671	0.8267	0.5200	-0.130	-0.332
6	0.2089	0.8223	0.5115	-0.077	-0.273
7	0.2507	0.8217	0.4989	-0.050	-0.281
8	0.2925	0.8184	0.4892	-0.034	-0.328
9	0.3343	0.8184	0.4738	-0.019	-0.419
10	0.3760	0.8174	0.4503	-0.022	-0.430
11	0.4178	0.8189	0.4369	-0.069	-0.418
12	0.4596	0.8103	0.4181	-0.088	-0.454
13	0.5014	0.8084	0.3977	-0.039	-0.466
14	0.5432	0.8080	0.3788	-0.011	-0.481
15	0.5849	0.8096	0.3575	-0.028	-0.503
16	0.6267	0.8073	0.3411	-0.085	-0.611
17	0.6685	0.8003	0.3070	-0.093	-0.731
18	0.7103	0.7985	0.2725	-0.074	-0.703
19	0.7521	0.7933	0.2524	-0.034	-0.747
20	0.7939	0.7959	0.2124	0.003	-0.824
21	0.8356	0.7958	0.1781	0.005	-0.832
22					

Graph 10. x and y positions of the first data set (Trial 4)



Graph 11. Velocity on y axis vs. time graph (Trial 4)

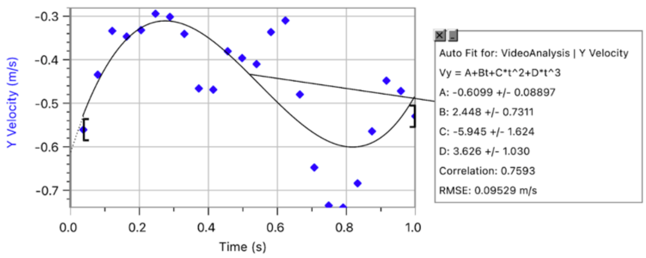
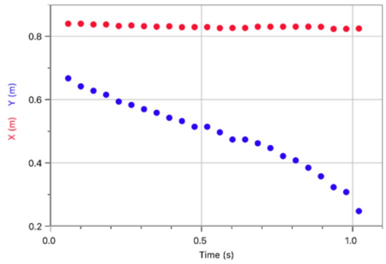


Table 8. Raw data table of 4th trial

	VideoAnalysis				
	Time (s)	X (m)	Y (m)	Vx (m/s)	Vy (m/s)
1	0.03764	0.8374	0.6213	0.220	−0.561
2	0.07946	0.8520	0.5929	0.064	−0.434
3	0.1213	0.8462	0.5826	−0.064	−0.334
4	0.1631	0.8444	0.5708	−0.143	−0.347
5	0.2049	0.8302	0.5520	−0.132	−0.332
6	0.2468	0.8327	0.5414	−0.093	−0.294
7	0.2886	0.8243	0.5291	−0.087	−0.302
8	0.3304	0.8238	0.5175	−0.049	−0.341
9	0.3722	0.8209	0.5051	−0.030	−0.466
10	0.4140	0.8224	0.4726	−0.031	−0.469
11	0.4559	0.8184	0.4614	−0.039	−0.381
12	0.4977	0.8189	0.4468	−0.042	−0.396
13	0.5395	0.8149	0.4284	−0.047	−0.410
14	0.5813	0.8144	0.4071	−0.044	−0.337
15	0.6231	0.8131	0.4020	−0.079	−0.310
16	0.6650	0.8079	0.3904	−0.112	−0.480
17	0.7068	0.8009	0.3613	−0.085	−0.648
18	0.7486	0.7999	0.3324	−0.038	−0.735
19	0.7904	0.7999	0.2972	−0.034	−0.740
20	0.8323	0.7974	0.2689	−0.044	−0.684
21	0.8741	0.7965	0.2369	−0.063	−0.564
22	0.9159	0.7927	0.2214	−0.088	−0.448
23	0.9577	0.7874	0.2065	−0.076	−0.472
24	0.9995	0.7863	0.1816	−0.052	−0.530
25					

Graph 12. x and y positions of the first data set (Trial 5)



Graph 13. Velocity on y axis vs. time graph (Trial 5)

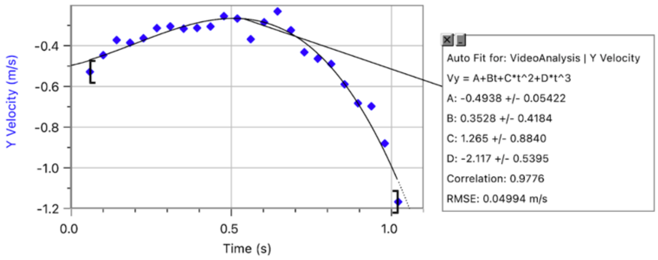


Table 9. Raw data table of 5th trial

	VideoAnalysis				
	Time (s)	X (m)	Y (m)	Vx (m/s)	Vy (m/s)
1	0.05857	0.8397	0.6672	-0.011	-0.529
2	0.1004	0.8398	0.6419	-0.027	-0.446
3	0.1422	0.8374	0.6278	-0.033	-0.372
4	0.1841	0.8374	0.6153	-0.045	-0.385
5	0.2259	0.8326	0.5937	-0.036	-0.363
6	0.2677	0.8342	0.5831	-0.022	-0.314
7	0.3096	0.8318	0.5697	-0.029	-0.305
8	0.3514	0.8305	0.5584	-0.014	-0.316
9	0.3932	0.8317	0.5425	-0.019	-0.313
10	0.4351	0.8288	0.5322	-0.022	-0.306
11	0.4769	0.8291	0.5142	-0.012	-0.255
12	0.5187	0.8291	0.5142	-0.028	-0.267
13	0.5606	0.8259	0.4967	-0.021	-0.368
14	0.6024	0.8264	0.4739	0.006	-0.285
15	0.6442	0.8264	0.4739	0.033	-0.231
16	0.6861	0.8303	0.4620	0.034	-0.324
17	0.7279	0.8303	0.4469	0.012	-0.432
18	0.7697	0.8304	0.4215	0.004	-0.464
19	0.8116	0.8304	0.4080	-0.005	-0.490
20	0.8534	0.8304	0.3848	-0.022	-0.590
21	0.8952	0.8299	0.3576	-0.060	-0.683
22	0.9371	0.8233	0.3232	-0.055	-0.698
23	0.9789	0.8235	0.3079	-0.013	-0.880
24	1.021	0.8245	0.2477	0.009	-1.167

4.4. Processed Data for the Main Experiment

To create a usable data gathering all trials together, we need to use a formula to get the mean of all (v-t) graphs provided. For this, the simple mean formula is used:

$$f_{avg}(x) = \frac{f_1(x) + f_2(x) + \cdots f_n(x)}{n}$$

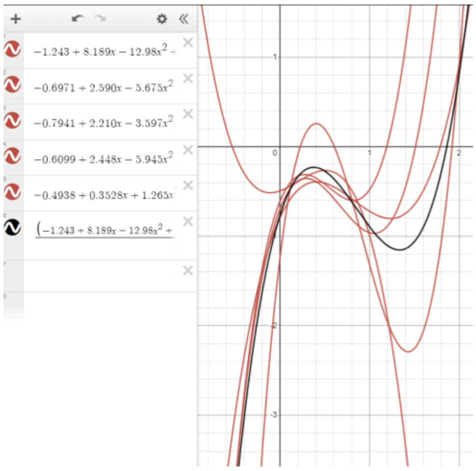
The mean velocity-time function is:

$$y = v_{(t)} = (-0.7676 + 3.1580t - 5.3864t^2 + 2.101t^3)ms^{-1}$$

An important thing to mention for investigating the movements is that for such experiment, mathematical derivation is used to find the graphs by hand rather than using applications to find x-t, x-t^2, x-t^3 graphs and so on as it will be more accurate to investigate a function obtained by hand.

During following graphs, error bars are not used as they are created using Desmos Graph Calculator; however, error percentages are given under each graph

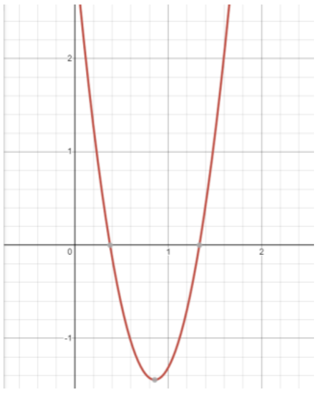
Graph 14. The mean velocity with respect to time function retrieved from the five trials prepared in Desmos Graphing Calculator (the black line among red ones)



By derivating this function, we are able to reach acceleration. The mean acceleration-time function is:

$$\frac{dy}{dt} = \frac{dv}{dt} = a_{(t)} = (3.1580 - 10.7728t + 6.303t^2) m\, s^{-2}$$

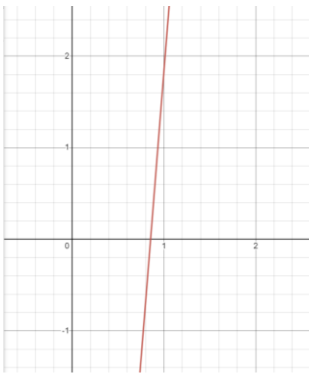
Graph 15. The mean acceleration with respect to time function retrieved from the derivative of the velocity with respect to time prepared in Desmos Graphing Calculator



The next derivation gives us jerk, the rate of change of acceleration per unit time.

$$\frac{d^2y}{dt^2} = \frac{d^2v}{dt^2} = jerk = j(t) = (-10.7728 + 12.606t)\ m\, s^{-3}$$

Graph 16. The mean jerk function retrieved from the derivative of the acceleration with respect to time prepared in Desmos Graphing Calculator



With the final derivation, we reach jounce.

$$jounce = \frac{dv^3}{dt^3} = 12.606\, m\, s^{-4}$$

Graph 17. The mean jounce function retrieved from the derivative of the jerk with respect to time prepared in Desmos Graphing Calculator

Graph 18. The demonstration of the all the functions obtained for velocity, acceleration, jerk and jounce in one figure with prepared in Desmos Graphing Calculator

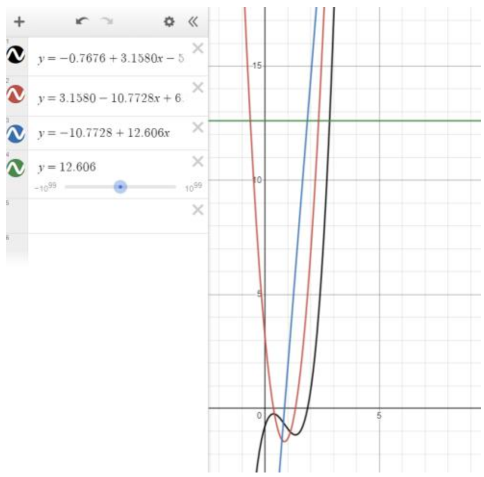


Table 10. All values of displacement used to interpret the resultant movement

$Velocity: v_{(t)} = (-0.7676 + 3.1580t - 5.3864t^2 + 2.101t^3) m\,s^{-1}$
$Acceleration: a_{(t)} = (3.1580 - 10.7728t + 6.303t^2) ms^{-2}$
$Jerk = j(t) = (-10.7728 + 12.606t)\ ms^{-3}$
$jounce = \ 12.606\ ms^{-4}$

5. DATA EVALUATION AND CONCLUSION

5.1.Interpretation of the Experimental Data Presented in Tables and Graphs

With the primary experiment, the dynamic friction coefficient is found. Even though it is not possible to evaluate the extensive mass change rate as it is not possible without specialized gear, the result achieved from that part of the experiment was that the change in the mass of part of chain on the surface of the table was 0.17 times as effective in increasing the velocity of the chain than the increase in free falling part does.

The result reached finally was consistent with the hypothetical formula, giving us a non-linear acceleration. As stated before, the acceleration is increasing because the force creating the acceleration is increasing and the resisting force is decreasing simultaneously and during this change, an increase in one causes a decrease in the other, making this change increase over time.

When we look at graph 17, it shows us that jerk is increasing over time even though it is negative at t=0 , and with the addition of graph 16, we can say that acceleration is decreasingly decreasing (first decreasing, and then becomes an increase). Thus, it is possible to interpret from the main formula that because of the dynamic friction coefficient, decreasing resistive force is decreasing at a slower pace than the motion inducing force, so the gravitational force was creating an increase in velocity.

The theoretical “positive velocity” part of the graph is created because of this decreasing resistive force. Even though it is not possible because mass could not be negative as well as a chain could not start magically flying, the graph is suitable for calculation until there is no part left on top of the table. It this case, a constant acceleration will be reached as the only force affecting on the chain will be gravity.

Another important part of the experiment was the correlation coefficients of trials of the main experiment’s functions. As a coefficient of 1 means totally accurate results and an experiments aims to get close to 1 as much as possible, our correlation coefficients which are ranging from 0.9776 to 0.7593 are proven to be mostly accurate, with some cases of high error margins being present. The possibilities for these margins will be discussed in the topic “5.2.Error Analysis and Sources of Errors”.

5.2.Error Analysis and Sources of Errors

In this investigation, the margins of error are not possible to find specifically, with the exception of coefficient of dynamic friction which is not sufficient to find the uncertainty of acceleration as rate of change of mass in not possible to measure. However, the correlation coefficients are another suitable way of measuring errors in such experiments. When looked at this investigation, it is seen that the coefficients are ranging from 0.9776 to 0.7593. This shows us that even though there is sufficient relation, random errors could impact the measurements largely.

Another justification for this are the root mean squared error margins. Even though they look small numerically, when the scale of the investigation is taken into account, it is possible to say that some amount of error is present which range from simple human errors such as reaction time and calculation limits to more systematic errors which could be exemplified as:

some of the force is lost due to horizontal changes even though it is theorized that all force will be applied vertically.

5.3.Strengths of the Experiment

One of the strengths of the experiment is the mathematical approach of the interpretation. Even in IB HL Physics curriculum the calculus-based application of physical structures are not widely found, and this experiment offers a deep understanding for the changing patterns of acceleration with respect to time and interpreting these types of motion with differential equations. Another strength is demonstrating an engineering model for real life examples of falling chain and its cascading motion while increase in acceleration in time.

5.4.Weaknesses of the Experiment and Suggestions for Improvement

When it comes to weaknesses of the experiment, for the analysis of the motion of the falling metal chain there is usage of video analysis tool of Logger Pro 3 Software by embedding the video in the software, however for more accurate analysis of the motion a force sensor could be used. And also, the dynamic friction coefficient of the wooden table is calculated with the measurements done by hitting the metal coin and observing its motion until it stops. Instead of this procedure a device called COF-Tester could be used directly measure the dynamic friction of the surface of the wooden table.

5.5.Further Investigations

As the topic of higher derivatives of displacement is a highly specific topic that is needed in areas of science where even the slight difference in any data is important, it is mostly overlooked by majority of people. An example that could be given such as the one used in this investigation is researching a rocket travelling through empty space, in which even the slightest change in a variable is crucial. This subject could be used to calculate the fuel that needs to be used as when fuel gets burnt, that thrust creates a movement, but also as fuel tank gets emptier, it also makes it easier to speed up. When looked at this scale, this topic would help us shape the volume of fuel tanks in spaceships or calculate the time we need to go from a planet to another.

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