International Baccalaureate Diploma Program Physics Extended Essay

How can the formula for energy stored in a capacitor be verified by discharging the energy stored in the capacitor onto a resistor connected in series by capturing the voltage across the capacitor against time?

Word Count: 3870

Research Question: How can the formula for energy stored in a capacitor be verified by discharging the energy stored in the capacitor onto a resistor connected in series by capturing the voltage across the capacitor against time?

Introduction:

I have had a remarkable affinity for consumer electronics for as long as I remember. Thanks to my father being an electronics engineer, I was acquainted with a wide variety of electronic equipment from early childhood. Every piece of electronics I was able to get my hands on would be taken apart; with every piece being rigorously inspected. Although all electronic equipment I took apart had different components and circuit layouts made to fit their specific purpose; one thing that was certain was that a capacitor - a component I did not know much about until learning about it in detail in the International Baccalaureate Diploma Program - would be there in the circuit.

Learning about capacitance in our Physics classes allowed me to understand why capacitors were needed and how they functioned. This knowledge helped me delve into the functions and uses of capacitors, leading me to conduct several experiments with capacitors myself, examining how different variables affect the way they charge and discharge. One thing that evoked the most curiosity in me was perhaps the least difficult to calculate, the energy stored inside a capacitor. I couldn't grasp how the equation for energy stored in a capacitor was derived from real life because the concept of energy usually involves some kind of time dependency, whereas the formula for energy stored in a capacitor does not involve time in any way. This led me to select the research question of this study as: **How can the formula for energy stored in a capacitor be verified by discharging the energy stored in the capacitor onto a resistor connected in series by capturing the voltage across the capacitor against time?**

The Aim of the Study:

The aim of this study is to charge a capacitor to a known voltage value and calculate the energy stored from the capacitance and charged voltage values. Afterwards, the capacitor will be completely discharged through a resistor of known resistance while recording the discrete resistor/capacitor voltage values against time. Then, for each record, the power dissipated on the resistor will be calculated and plotted against time. The area under the plotted graph of discrete power values will be calculated to find the energy dissipated on the resistor and compared with the result from the stored energy formula of a capacitor, verifying that the equation of energy stored in a capacitor is true. To improve the accuracy of the result, two resistors of different resistances will be used in two separate iterations of the experiment.

Background Information:

Capacitors:

A capacitor is an arrangement of two conductors separated by dielectric material or vacuum. This arrangement can store electric charges and energy and release said energy when needed, acting as a temporary energy reservoir.

When a voltage is applied on a capacitor, electric charge accumulates across the plates, creating an electric field between them. When a charged capacitor is connected to a resistor, this charge is released onto the resistor.

The Energy Stored in a Capacitor:

When a capacitor is charged to a voltage V_c; the energy E_c stored inside, in Joules, is given by the equation:

$$E_c = \frac{1}{2} \times Q \times V_c$$

Where Q is the charge stored in the capacitor. Q can be defined as $Q = C \times V_c$, where C is the capacitance, in farads, of the capacitor. Incorporating this into the original energy equation we derive:

$$E_c = \frac{1}{2} \times C \times V_c^2$$

Charging Phase of the Capacitor:

Relationship Between the Voltage Levels of the Components of the Circuit:

Consider the capacitor charging circuit illustrated in figure 1, under the heading "Setup of the Experiment". The relationship between the voltage of the power source, capacitor and resistor is given by the equation:

$$V_b = V_r + V_c$$

Where V_b is the voltage of the power source, V_r is the voltage on the resistor and V_c is the voltage on the capacitor.

Voltage of the Resistor:

The voltage of the resistor, V_R, can further be defined as $V_R = I_c \times R$, where I_C is the current flowing through the resistor (and the capacitor, hence the C instead of R) and R is the resistance of the resistor in ohms, Ω.

Capacitor current, I_c, is the derivative of capacitor charge with respect to time. This means that:

$$I_c = \frac{dQ}{dt} = C \times \frac{dV_c}{dt}$$

Substituting this to the equation of the resistor voltage, we get:

$$V_r = C \times \frac{dV_c}{dt} \times R$$

Substituting this new value for the resistor voltage into the original voltage equation, we get a differential equation in V_c:

$$V_b = RC \times \frac{dV_c}{dt} + V_c$$

With $\frac{dV_c}{dt} = -V_b \times \frac{-1}{RC} \times e^{\frac{-t}{RC}}$ also being obtained with some arithmetic rearrangement.

Voltage of the Capacitor:

The solution to the differential equation above, in terms of V_c, is an exponential function given by the equation:

$$V_c = (V_b - V_{c0}) \times \left(1 - e^{\frac{-t}{RC}}\right) + V_{c0}$$

Where V_{c0} is the initial voltage of the capacitor.

When the capacitor is fully discharged, $V_{c0} = 0$, the equation of the voltage of the capacitor becomes:

$$V_c = V_b \times \left(1 - e^{\frac{-t}{RC}}\right)$$

To show that the V_c equation is indeed the solution to the differential equation, it can be substituted into the differential equation, yielding:

$$V_b = RC \times \frac{dV_c}{dt} + V_b \times \left(1 - e^{\frac{-t}{RC}}\right),$$

Which, by incorporating the value of $\frac{dV_c}{dt}$ obtained from before, becomes:

$$V_b = RC \times (-V_b) \times \frac{-1}{RC} \times e^{\frac{-t}{RC}} + V_b \times \left(1 - e^{\frac{-t}{RC}}\right)$$

$$V_b = V_b \times e^{\frac{-t}{RC}} + V_b - V_b \times e^{\frac{-t}{RC}}$$

$$V_b = V_b$$

Proving that the V_c equation is the solution to the differential equation.

Discharging Phase of the Capacitor:

Relationship Between the Voltage Levels of the Components of the Circuit:

During discharging the DC source is cut from the rest of the circuit, like in figure 2. This means that the capacitor acts as the power source in the circuit, discharging stored energy onto the resistor. The new relationship between the voltage of the capacitor and resistor becomes:

$$V_c = V_r$$

Assuming the capacitor has been charged to voltage V_{cc}, during discharging the equation for capacitor voltage becomes:

$$V_c = V_{cc} \times e^{\frac{-t}{RC}}$$

Where V_{cc} represents the voltage the capacitor has been charged up to before discharging.

The Energy of the Resistor:

During discharging the energy of the resistor is given by the power of the resistor being integrated to the change in time, which is given by the equation:

$$E_r = \int P_r \, dt$$

Since power is equal to the product of the voltage and current, this integration equation can be modified as:

$$E_r = \int IV_r \, dt$$

As $I = \frac{V}{R}$, this integral can be expressed as:

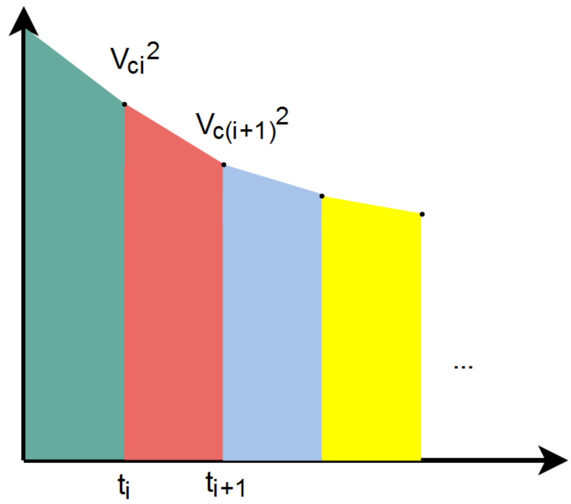
$$E_r = \frac{1}{R} \int V_r^2 \, dt$$

As $V_r = V_c = V_{cc} \times e^{\frac{-t}{RC}}$, we can calculate the energy of the resistor during discharging with the equation:

$$E_r = \frac{1}{R} \int \left(V_{cc} \times e^{\frac{-t}{RC}}\right)^2 dt$$

Approximating the Integral:

Discrete values of V_c will be recorded and the graph of V_c^2 against time will be plotted; with every plot point on the graph having coordinates (V_{ci}^2, t_i), V_{ci}^2 being the i^th value of V_c^2 and t_i being the i^th value of t. All plot points will be connected via straight lines to make the calculations straightforward.



Graph 1: Representative graph of V_c^2 against time.

The area under the line between two successive points is given by the equation:

$$\frac{V_{c(i+1)}^2 + V_{ci}^2}{2} \times (t_{i+1} - t_i)$$

Therefore, the approximate equation for the energy dissipated on the resistor is the total area under the curve divided by the resistance value of the resistor:

$$E_r = \frac{1}{R}\sum_{i=1}^{N-1} \frac{V_{c(i+1)}^2 + V_{ci}^2}{2} \times (t_{i+1} - t_i)$$

Whose result will be comparably similar to the integral equation, as the discrete values of V_c will be recorded as frequently as possible.

$$E_r = \frac{1}{R}\sum_{i=1}^{N-1} \frac{V_{c(i+1)}^2 + V_{ci}^2}{2} \times (t_{i+1} - t_i) \cong \frac{1}{R}\int \left(V_{cc} \times e^{\frac{-t}{RC}}\right)^2 dt$$

Relation Between the Energy of the Capacitor and the Energy of the Resistor:

During discharging, the energy stored in the capacitor is entirely discharged onto the resistor, meaning that the energy dissipated on the resistor will be equal to the energy in the capacitor. This is shown in the equation $E_r = E_c$.

Comparison and Verification of the Capacitor Energy Formula:

The E_c values from the experiment will be compared with the result gained from the formula of the energy stored in a capacitor, verifying the formula.

Variables:

Independent Variables	**The time taken for the capacitor to be fully discharged:** As all circuit components were kept constant during the experiment, the voltage of the capacitor is going to decrease as time passes.
Dependent Variables	**The voltage across the capacitor:** The voltage across the capacitor decreases over time when discharging.
Controlled Variables	**The resistance of the resistors:** The resistance of the resistors used have been kept constant, with two resistors of two resistance values being used for two discharging experiments to ensure reliability across results. **The charging voltage source:** A power supply having a constant voltage of 15.32V was applied to the circuit for charging the capacitor. **The capacitance of the capacitor:** Only one capacitor having 3300 micro farads of capacitance was used throughout the experiment.

Safety, Ethical and Environmental Considerations:

The safety issues involved with this study are regarding the risk of electrical shocks that is always present when handling electrical equipment. Although the 15.32V of direct current used in the experiment is not enough to be considered a shock hazard, the power supply and the experiment apparatus were handled with care so as to avoid possible electrical shocks. Another safety issue is the risk of electrical fires posed by bare wiring. This was countered by using insulated wires in the experiment.

There are no ethical issues involved with this study as no living organisms were used or harmed in the experiment.

There are no environmental issues involved with this study as no waste was generated for the experiment: all apparatus used were either present before the study, or were purchased with future reuse in mind.

Method:

Apparatus:

1. Multimeter (Fluke 107 600v CAT III Multimeter)
2. DC Power Supply (Toshiba Universal Laptop AC/DC Adapter 15V, 5A; Model: SADP-75PB A, Part No: PA3469U-1ACA)
3. Resistor of 1kΩ nominal resistance (±5%)
4. Resistor of 10kΩ nominal resistance (±5%)
5. Capacitor of 3300 μF nominal capacitance (±20%)
6. Singla Pole Double Throw Electrical Switch
7. Connecting Wires
8. Digital Stopwatch (±0.01 seconds)
9. Digital Video Camera

Accuracy of the Multimeter When Measuring DC Voltage:

The user manual of the multimeter states that with a voltage less than 6V the accuracy of the measurement is ±[0.5% + 0.003] and with a voltage between 6V and 60V the accuracy of the measurement becomes ± [0.5% + 0.03].

Accuracy of the Multimeter When Measuring Resistance:

The user manual of the multimeter states that, with a resistance of less that 4kΩ the accuracy is ± [0.5% + 2]; and with a voltage between 4kΩ and 40kΩ the accuracy becomes ± [0.5% + 20].

Reason for Using Components with Specific Capacitance and Resistance Values:

The reason resistors with resistances of 1kΩ and 10kΩ were used instead of components of different values is because these values were able to produce results slowly enough to be able to record a large enough number of discrete data points, while being quick enough to not unnecessarily consume time. A capacitor with capacitance of 3300μF was used so that the RC time constant at the integral equation could be large enough to be measured. For the 10kΩ resistor, RC time becomes 33 seconds, while for the 1kΩ resistor RC time becomes 3.3 seconds.

Reasons for Using a DC Power Supply Instead of a Battery:

There are several reasons for the use of a DC power supply instead of a conventional power source like a power bank or a battery. The first and most notable reason is that since conventional sources have only a finite amount of energy; it was better to ensure that there wouldn't be any complications related to the source of DC power running out. Another reason is that batteries might have had internal resistance which would have introduced further complications as that would mean that the battery would not have been able to charge the capacitor to its advertised voltage. Another reason is that with a battery or power bank, the voltage might have been unstable, meaning that there would be another variable affecting the rate of charging. Using a power supply would mitigate these effects, giving out reliable, stable voltage for the experiment.

To ensure that these handicaps of using batteries or power banks would not be replicated by the DC power supply, the voltage value was measured to be 15.32V using the multimeter. Additionally, a test was done to ensure that the voltage provided remained constant. This means that the power supply can be used as a reliable source of voltage.

Procedure:

1. Using the multimeter, measure the resistance of the two resistors, record the values.
2. Record the capacitance of the capacitor. (The capacitance value is stated on the capacitor. The capacitor used is out of the multimeter's measurement range, so it cannot be measured.)
3. Measure the voltage provided by the power supply, recording the value.
4. Using the connecting wires, connect the components of the circuit as shown in figure 1 in the "setup of the experiment" section. Ensure the capacitor is fully discharged. Make sure the designated negative terminal of the capacitor is connected to the negative terminal of the power supply.
5. Connect the negative (black/green) probe of the multimeter to the negative terminal of the capacitor. Connect the positive (red) probe of the multimeter to the positive terminal of the capacitor.
6. Set the multimeter to measure DC voltage.
7. Position the digital video camera so that the circuit, the digital stopwatch and the voltage value displayed by the multimeter are all in frame. Check that the video camera is recording at the highest possible number of frames per second through the settings.
8. Start the recording, followed by starting the stopwatch and flip the switch to begin charging the capacitor.
9. After the capacitor has been charged up to the highest level of voltage the power supply can output; start a new video, reset the stopwatch and flip the switch (figure 2) to begin discharging the capacitor.
10. After the capacitor has been fully discharged; stop the recording, and reset the stopwatch.
11. Repeat the steps with a new resistor/capacitor until all resistor-capacitor combinations are completed.
12. List the voltage measured in relation to time by going over the footage frame by frame, using either a pen and paper or a data processing program like Microsoft Excel.

Setup of the Experiment:

The following figures are sketches of the layout of the circuit in the experiment. Figure 1 depicts the flow of current when the switch is set to connect the power supply to the resistor and capacitor, charging the capacitor. Figure 2 depicts the flow of current when the switch is flipped to connect the capacitor and the resistance, resulting in discharging the capacitor.

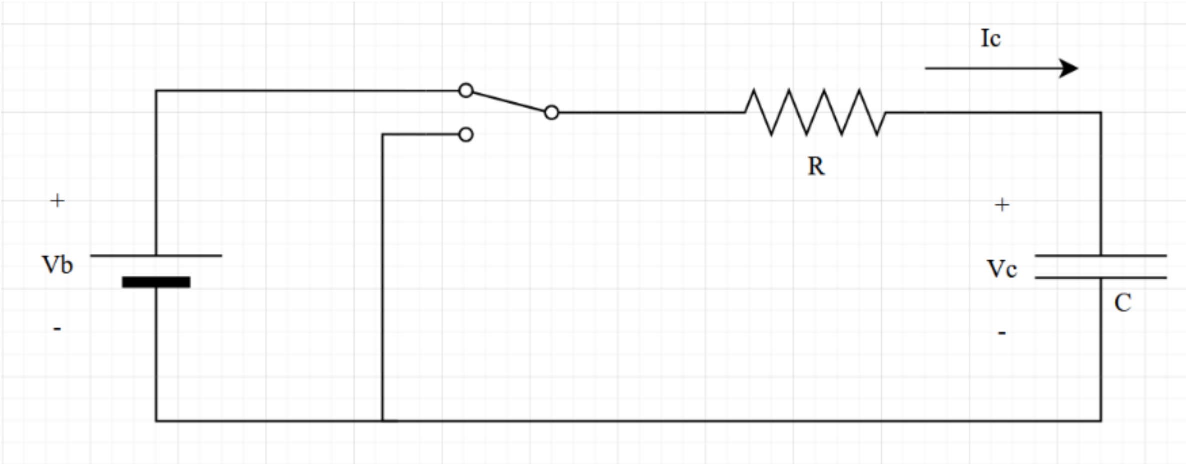


Figure 1: The layout of the circuit and the direction of current for when the capacitor is being charged.

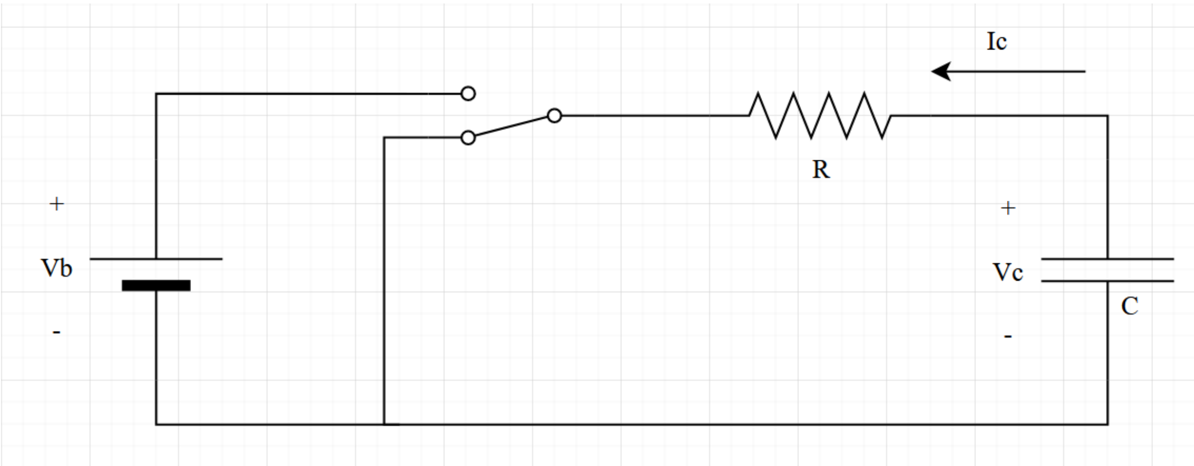


Figure 2: The layout of the circuit and the direction of current for when the capacitor is being discharged.

<u>Photograph of the Experiment:</u>

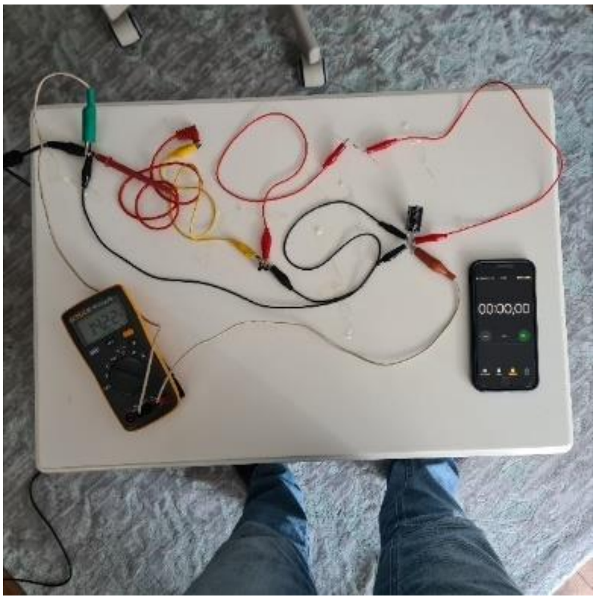


Figure 3: A photograph I've taken with my Samsung Galaxy S20 Smartphone, which I used as my digital video camera, on 31/01/2024, 12:07PM; displaying the setup of the circuit.

Raw Data Tables:

The tables for the raw data for discharging during two trials are presented. The data for charging won't be presented to not cause unnecessary clutter as they won't be used. Since voltage reading uncertainty depends on the value read, it is also calculated in the tables.

Table 1: The Raw Data Recorded for Discharging for Trial 1, 3300μF Capacitance, 10kΩ Resistance

Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)	Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)	Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)
0.00	14.89	0.104	40.97	4.198	0,024	80.57	1.323	0,010
0.37	14.70	0.104	41.50	4.16	0,024	80.92	1.313	0,010
0.66	14.55	0.103	41.74	4.123	0,024	81.35	1.302	0,010
0.86	14.42	0.102	42.06	4.086	0,023	81.66	1.291	0,009
1.07	14.27	0.101	42.37	4.049	0,023	81.88	1.28	0,009
1.48	14.14	0.101	42.72	4.013	0,023	82.10	1.269	0,009
1.68	14.00	0.100	43.01	3.977	0,023	82.46	1.258	0,009
2.07	13.87	0.099	43.29	3.941	0,023	82.81	1.248	0,009
2.31	13.74	0.099	43.39	3.906	0,023	83.09	1.238	0,009
2.61	13.60	0.098	43.68	3.872	0,022	83.24	1.227	0,009
2.98	13.48	0.097	44.05	3.837	0,022	83.70	1.217	0,009
3.31	13.35	0.097	44.41	3.803	0,022	83.92	1.207	0,009
3.50	13.22	0.096	44.91	3.769	0,022	84.18	1.197	0,009
3.80	13.10	0.096	45.10	3.735	0,022	84.51	1.187	0,009
4.10	12.98	0.095	45.41	3.702	0,022	85.01	1.177	0,009
4.48	12.86	0.094	45.67	3.669	0,021	85.35	1.168	0,009
4.86	12.74	0.094	45.95	3.636	0,021	85.49	1.157	0,009
5.19	12.61	0.093	46.21	3.604	0,021	85.75	1.148	0,009
5.45	12.50	0.093	46.51	3.572	0,021	86.09	1.138	0,009
5.76	12.38	0.092	46.78	3.540	0,021	86.31	1.129	0,009
5.98	12.27	0.091	47.07	3.508	0,021	86.70	1.120	0,009
6.34	12.15	0.091	47.31	3.477	0,020	87.02	1.111	0,009
6.66	12.04	0.090	47.58	3.447	0,020	87.31	1.101	0,009
6.91	11.93	0.090	48.09	3.416	0,020	87.54	1.092	0,008
7.17	11.81	0.089	48.35	3.385	0,020	87.79	1.083	0,008
7.49	11.71	0.089	48.69	3.355	0,020	88.22	1.074	0,008
7.70	11.59	0.088	48.98	3.326	0,020	88.41	1.066	0,008
8.12	11.48	0.087	49.32	3.296	0,019	88.84	1.056	0,008
8.42	11.38	0.087	49.57	3.267	0,019	89.08	1.048	0,008
8.69	11.27	0,086	49.88	3.238	0,019	89.41	1.040	0,008
8.94	11.17	0,086	50.11	3.210	0,019	89.72	1.030	0,008
9.32	11.06	0,085	50.31	3.181	0,019	90.09	1.022	0,008
9.62	10.96	0,085	50.76	3.153	0,019	90.35	1.013	0,008
9.90	10.86	0,084	51.07	3.125	0,019	90.62	1.005	0,008
10.11	10.76	0,084	51.42	3.097	0,018	90.95	0.997	0,008
10.42	10.66	0,083	51.68	3.070	0,018	91.18	0.989	0,008

Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)	Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)	Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)
10.88	10.56	0,083	51.86	3.043	0,018	91.39	0.981	0,008
11.11	10.46	0,082	52.16	3.016	0,018	91.66	0.973	0,008
11.42	10.36	0,082	52.44	2.99	0,018	92.18	0.965	0,008
11.69	10.27	0,081	52.76	2.963	0,018	92.35	0.957	0,008
11.88	10.18	0,081	53.02	2.937	0,018	92.60	0.949	0,008
12.32	10.09	0,080	53.29	2.911	0,018	92.91	0.941	0,008
12.57	9.99	0,080	53.71	2.886	0,017	93.17	0.940	0,008
12.80	9.90	0,080	53.97	2.86	0,017	93.69	0.926	0,008
13.04	9.80	0,079	54.28	2.835	0,017	93.90	0.918	0,008
13.44	9.71	0,079	54.58	2.81	0,017	94.59	0.903	0,008
13.71	9.63	0,078	54.90	2.785	0,017	95.01	0.900	0,008
13.96	9.53	0,078	55.07	2.76	0,017	95.37	0.897	0,007
14.44	9.52	0,078	55.39	2.736	0,017	95.88	0.873	0,007
14.69	9.36	0,077	55.77	2.712	0,017	96.56	0.868	0,007
14.92	9.28	0,076	56.08	2.688	0,016	97.31	0.861	0,007
15.17	9.19	0,076	56.26	2.665	0,016	97.75	0.826	0,007
15.48	9.11	0,076	56.79	2.641	0,016	98.65	0.821	0,007
15.84	9.02	0,075	57.02	2.619	0,016	99.30	0.813	0,007
16.09	8.94	0,075	57.31	2.596	0,016	99.86	0.803	0,007
16.47	8.86	0,074	57.61	2.572	0,016	100.45	0.790	0,007
16.75	8.78	0,074	57.80	2.550	0,016	100.76	0.784	0,007
17.04	8.70	0,074	58.17	2.528	0,016	101.34	0.771	0,007
17.30	8.61	0,073	58.38	2.506	0,016	101.88	0.759	0,007
17.58	8.60	0,073	58.79	2.484	0,015	102.65	0.747	0,007
17.99	8.46	0,072	59.00	2.463	0,015	103.12	0.735	0,007
18.26	8.45	0,072	59.31	2.441	0,015	104.09	0.718	0,007
18.57	8.31	0,072	59.69	2.420	0,015	104.94	0.701	0,007
18.98	8.23	0,071	59.89	2.398	0,015	105.52	0.690	0,006
19.55	8.08	0,070	60.16	2.377	0,015	106.19	0.679	0,006
19.81	8.00	0,070	60.62	2.357	0,015	107.28	0.658	0,006
20.15	7.99	0,070	60.92	2.336	0,015	108.31	0.642	0,006
20.34	7.97	0,070	61.08	2.316	0,015	109.21	0.627	0,006
20.67	7.78	0,069	61.51	2.295	0,014	109.69	0.617	0,006
21.25	7.75	0,069	61.66	2.276	0,014	110.72	0.603	0,006
21.60	7.57	0,068	62.16	2.257	0,014	111.38	0.594	0,006
22.22	7.54	0,068	62.42	2.236	0,014	112.10	0.580	0,006
22.49	7.52	0.068	62.69	2.217	0,014	112.68	0.571	0,006
22.88	7.48	0.067	62.94	2.198	0,014	113.36	0.562	0,006
23.21	7.23	0.066	63.20	2.179	0,014	114.11	0.554	0,006
23.75	7.20	0.066	63.66	2.160	0,014	115.10	0.537	0,006
23.98	7.18	0.066	63.90	2.141	0,014	116.16	0.524	0,006
24.39	7.15	0.066	64.22	2.121	0,014	117.06	0.512	0,006
24.66	7.11	0.066	64.44	2.103	0,014	118.28	0.497	0,005

Time (Seconds) (±0.01)	**V_c (V)**	**±Uncertainty ΔV_c (V)**	**Time (Seconds) (±0.01)**	**V_c (V)**	**±Uncertainty ΔV_c (V)**	**Time (Seconds) (±0.01)**	**V_c (V)**	**±Uncertainty ΔV_c (V)**
24.92	7.06	0.065	64.72	2.084	0,013	119.40	0.482	0,005
25.12	7.00	0.065	65.12	2.067	0,013	120.50	0.467	0,005
25.39	6.94	0.065	65.40	2.049	0,013	121.87	0.454	0,005
25.64	6.88	0.064	65.62	2.031	0,013	123.32	0.437	0,005
25.97	6.81	0.064	66.01	2.013	0,013	124.40	0.424	0,005
26.37	6.75	0.064	66.26	1.997	0,013	125.60	0.411	0,005
26.60	6.69	0.063	66.62	1.979	0,013	127.09	0.396	0,005
26.85	6.63	0.063	66.92	1.961	0,013	128.41	0.385	0,005
27.37	6.57	0,063	67.29	1.945	0,013	130.54	0.365	0,005
27.65	6.51	0,063	67.51	1.928	0,013	131.97	0.352	0,005
27.94	6.45	0,062	67.81	1.911	0,013	133.65	0.337	0,005
28.26	6.39	0,062	68.01	1.895	0,012	135.28	0.326	0,005
28.45	6.33	0,062	68.43	1.878	0,012	136.64	0.314	0,005
28.72	6.28	0,061	68.78	1.863	0,012	137.91	0.305	0,005
28.98	6.22	0,061	69.12	1.847	0,012	139.80	0.292	0,004
29.24	6.16	0,061	69.30	1.831	0,012	142.11	0.277	0,004
29.55	6.10	0,061	69.50	1.815	0,012	144.10	0.265	0,004
29.84	6.05	0,060	69.92	1.799	0,012	146.68	0.250	0,004
30.25	5.99	0,033	70.18	1.784	0,012	148.26	0.241	0,004
30.56	5.94	0,033	70.48	1.768	0,012	150.87	0.227	0,004
30.92	5.89	0,032	70.85	1.753	0,012	154.10	0.211	0,004
31.26	5.83	0,032	71.12	1.739	0,012	157.94	0.194	0,004
31.50	5.78	0,032	71.37	1.724	0,012	161.61	0.179	0,004
31.80	5.73	0,032	71.70	1.709	0,012	165.64	0.165	0,004
32.15	5.67	0,031	72.01	1.694	0,011	170.94	0.148	0,004
32.38	5.62	0,031	72.36	1.68	0,011	174.57	0.138	0,004
32.62	5.57	0,031	72.62	1.666	0,011	179.52	0.125	0,004
33.30	5.137	0,029	72.89	1.652	0,011	185.30	0.113	0,004
33.56	5.262	0,029	73.11	1.637	0,011	187.36	0.108	0,004
33.96	5.214	0,029	73.38	1.623	0,011	191.01	0.102	0,004
34.11	5.168	0,029	73.70	1.610	0,011	192.22	0.100	0,004
34.38	5.121	0,029	74.04	1.596	0,011	195.55	0.094	0,003
34.71	5.075	0,028	74.30	1.582	0,011	205.35	0.080	0,003
35.01	5.029	0,028	74.62	1.569	0,011	209.32	0.075	0,003
35.41	4.984	0,028	74.89	1.556	0,011	213.77	0.070	0,003
35.68	4.938	0,028	75.29	1.542	0,011	218.86	0.065	0,003
36.06	4.894	0,027	75.56	1.529	0,011	225.24	0.060	0,003
36.37	4.851	0,027	75.82	1.516	0,011	231.28	0.055	0,003
36.61	4.806	0,027	76.20	1.503	0,011	238.80	0.050	0,003
36.90	4.764	0,027	76.52	1.491	0,010	248.01	0.045	0,003
37.31	4.720	0,027	76.76	1.477	0,010	258.99	0.040	0,003
37.62	4.678	0,026	77.15	1.465	0,010	274.37	0.035	0,003
37.86	4.636	0,026	77.44	1.453	0,010	291.50	0.030	0,003

Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)	Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)	Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)
38.11	4.594	0,026	77.78	1.441	0,010	318.06	0.025	0,003
38.49	4.552	0,026	78.09	1.429	0,010	361.29	0.020	0,003
38.68	4.512	0,026	78.27	1.417	0,010	435.72	0.015	0,003
38.99	4.472	0,025	78.49	1.405	0,010	556.31	0.010	0,003
39.22	4.431	0,025	78.87	1.393	0,010	613.56	0.009	0,003
39.49	4.391	0,025	79.06	1.381	0,010			
39.94	4.352	0,025	79.34	1.369	0,010			
40.31	4.313	0,025	79.68	1.357	0,010			
40.50	4.274	0,024	80.18	1.346	0,010			
40.76	4.235	0,024	80.40	1.336	0,010			

Table 2: The Raw Data Recorded for Discharging for Trial 2, 3300μF Capacitance, 1kΩ Resistance

Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)	Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)	Time (Seconds) (±0.01)	V_c (V)	±Uncertainty ΔV_c (V)
0.00	15.29	0.106	8.07	1.357	0.010	15.75	0.181	0.004
0.13	14.60	0.103	8.40	1.246	0.009	16.15	0.170	0,004
0.48	13.26	0.096	8.68	1.143	0.009	16.33	0.159	0,004
0.73	12.05	0.090	8.93	1.050	0.008	16.74	0.149	0,004
1.01	10.96	0.085	9.15	0.966	0.008	16.97	0.140	0,004
1.37	9.98	0.080	9.55	0.888	0.007	17.34	0.131	0,004
1.60	9.08	0.075	9.79	0.817	0.007	17.50	0.130	0,004
1.92	8.27	0.071	10.08	0.752	0.007	17.87	0.128	0,004
2.18	7.53	0.068	10.37	0.693	0.006	18.25	0.109	0,004
2.59	6.86	0.064	10.73	0.639	0.006	19.31	0.102	0,004
2.85	6.25	0.061	10.98	0.590	0.006	20.07	0.095	0,003
3.10	5.70	0.032	11.39	0.544	0.006	20.73	0.086	0,003
3.75	4.583	0.026	11.57	0.503	0.006	21.59	0.074	0,003
4.23	4.323	0.025	11.81	0.465	0.005	22.71	0.061	0,003
4.39	3.946	0.023	12.23	0.430	0.005	23.49	0.056	0,003
4.62	3.603	0.021	12.55	0.398	0.005	24.52	0.047	0,003
5.10	3.290	0.019	12.73	0.369	0.005	25.88	0.041	0,003
5.39	3.006	0.018	13.08	0.342	0.005	26.87	0.036	0,003
5.67	2.747	0.017	13.47	0.317	0.005	29.63	0.028	0,003
6.01	2.512	0.016	13.68	0.295	0.004	32.70	0.023	0,003
6.24	2.297	0.014	13.92	0.274	0.004	35.79	0.019	0,003
6.63	2.102	0.014	14.43	0.255	0.004	42.33	0.014	0,003
6.94	1.924	0.013	14.59	0.238	0.004	51.91	0.010	0,003
7.11	1.762	0.012	14.87	0.222	0.004	83.35	0.005	0,003
7.37	1.614	0.011	15.20	0.207	0.004	144.94	0.002	0,003
7.61	1.480	0.010	15.48	0.194	0.004			

Analysis:

The Real Resistance, Capacitance and Voltage Values:

Before conducting the experiment, the real values of resistance, capacitance and voltage provided by the components of the resistors and DC source were measured using the multimeter respectively. These measured values will be used in the calculations instead of the advertised values for further accuracy.

The resistance of the 1kΩ resistor was measured as 978±6.89Ω and the 10kΩ resistor was measured as 9900±69.50Ω. The voltage of the DC power source was measured at a constant 15.32V. Due to the large capacitance value of the capacitor, the multimeter was unable to measure the capacitance. We will have to assume that the capacitor has a capacitance of exactly 3300µF ± 20% = 3300 ± 660µF as per manufacturer specifications.

The Uncertainty Values of Time, Capacitance, Resistance and Voltage:

$$\Delta t = \pm 0.01 \text{ sec}$$

$$\Delta C = \pm 20\% = 3300\mu F \times \frac{20}{100} = \pm 660\mu F$$

$$\Delta R_{1k\Omega} = \pm(0.5\% + 2\Omega) = \pm\left(978 \times \frac{0.5}{100} + 2\right) = \pm 6.89\Omega$$

$$\Delta R_{10k\Omega} = \pm(0.5\% + 20\Omega) = \pm\left(9900 \times \frac{0.5}{100} + 20\right) = \pm 69.5\Omega$$

$$\Delta V_c = \pm\left[0.5\% + \begin{cases} 0.003 & \text{if } V < 6 \\ 0.03 & \text{else} \end{cases}\right]$$

Trial 1: Resistance of 10kΩ, Capacitance of 3300µF:

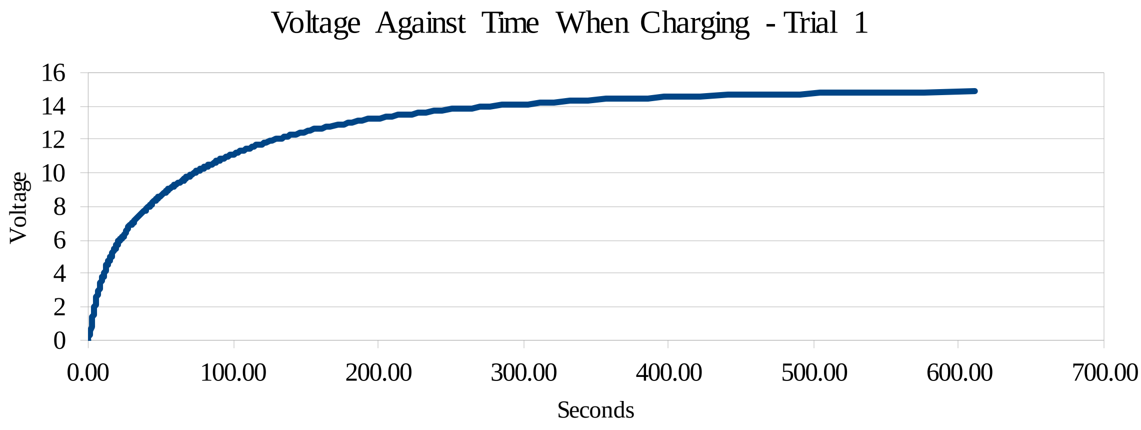
Charging Phase:

During charging, the energy accumulated on the capacitor plateaued at a maximum of 14.89±0.104V after ten minutes. Therefore, the value for the voltage on the capacitor will be taken as 14.89V instead of the theoretical maximum of 15.32V for this section.

Data:

Charging:

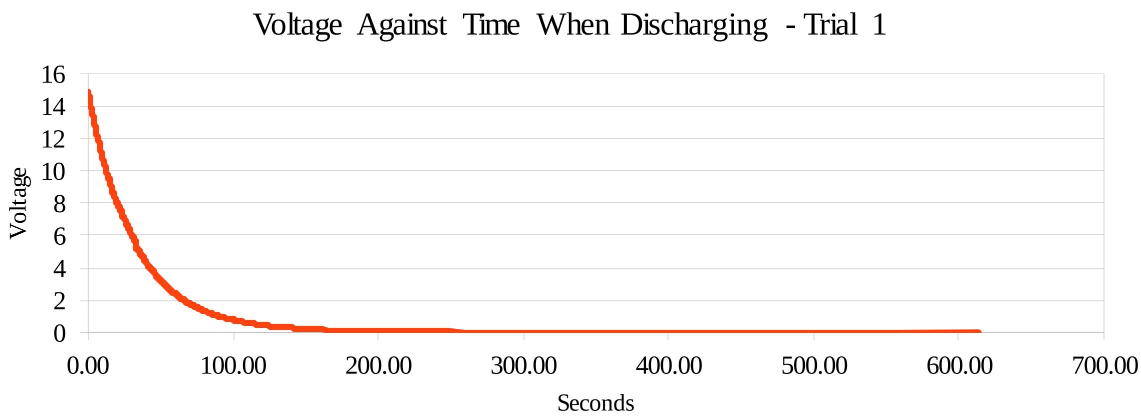
As the data on the discrete voltage values gathered during charging will not be used, they are not going to be included to not cause unnecessary clutter. Instead, the graph of the recorded values, created using Microsoft Excel, is presented here:



Graph 2: Graph of Voltage Against Time When Charging the Capacitor; Resistance of 10kΩ, Capacitance of 3300μF

Discharging:

The full data tables for the trials are presented at the Raw Data Tables section. The graph of these values, created using Microsoft Excel, is presented here:



Graph 3: Graph of Voltage Against Time When Discharging the Capacitor; Resistance of 10kΩ, Capacitance of 3300μF

<u>Energy Stored in the Capacitor:</u>

Using the conventional formula E_c is calculated as:

$$E_c = \frac{1}{2} \times C \times V_c^2 = \frac{1}{2} \times 3300 \times 10^{-6} \times 14.89^2 = 0.365825 \text{ J}$$

Calculation of Uncertainty:

$$\Delta V_c = \pm \left[0.5\% + \begin{cases} 0.003 & \text{if } V < 6 \\ 0.03 & \text{else} \end{cases}\right], V_c = 14.89$$

Therefore, $\Delta V_c = \pm \left[14.89 \times \frac{0.5}{100} + 0.03\right] = 0.104$

$$\Delta V_c^2 = \pm[2 \times V_c \times \Delta V_c] = \pm[2 \times 14.89 \times 0.10445] = 3.110521$$

With ΔE_c being:

$$\Delta E_c = \pm\frac{1}{2}(\Delta V_c^2 \times C + V_c^2 \times \Delta C)$$
$$= \pm\frac{1}{2}\left(3.110521 \times 3300 \times 10^{-6} + 221.712 \times \frac{20}{100} \times 10^{-6}\right)$$
$$= \pm 0.078297 \text{ J}$$

Incorporating the uncertainty into the value obtained from the conventional formula we receive:

$$E_c = 0.365825 \pm 0.078297 \text{ J}$$

Energy Dissipated on the Resistor:

The energy dissipated on the resistor during discharging, i.e. the area under the graph of discrete values of V_c^2 against time is calculated as:

$$E_r = \frac{1}{R}\sum_{i=1}^{N-1}\frac{V_{c(i+1)}^2 + V_{ci}^2}{2} \times (t_{i+1} - t_i) = \frac{1}{9900}\sum_{i=1}^{391-1}\frac{V_{c(i+1)}^2 + V_{ci}^2}{2} \times (t_{i+1} - t_i)$$

By using the data in Table 1, the summation equation and terms are calculated in the data processing tool Microsoft Excel which gives:

$$E_r = 0.361704 \text{ J}$$

Calculation of Uncertainty:

The uncertainty of the time difference in the summation equation is:

$$\Delta t_d = \Delta(t_{i+1} - t_i) = 0.01 + 0.01 = 0.02 \text{ sec}$$

The uncertainty of the square of the voltage is:

$$\Delta V_{ci}^2 = [2 \times V_{ci} \times \Delta V_{ci}]$$

For example, for the first two consecutive terms V_{c1}=14.89V and V_{c2}=14.70V;

$$\Delta V_{c1}^2 = [2 \times 14.89 \times 0.10445] = 3.110521 \text{ VV}$$

$$\Delta V_{c2}^2 = [2 \times 14.70 \times 0.10350] = 3.042900 \text{ VV}$$

The uncertainty of the square of the average voltage term in the summation equation is the arithmetic average of the uncertainties of the squares of the ith and (i+1)th terms; which is:

$$\Delta {V_{avi}}^2 = \Delta\frac{V_{c(i+1)}^2 + V_{ci}^2}{2} = \frac{\Delta V_{c(i+1)}^2 + \Delta V_{ci}^2}{2}$$

For example, for the first two consecutive data points the uncertainty of the square of the average voltage term in the summation equation is:

$$\Delta {V_{av1}}^2 = \frac{3.110521 + 3.042900}{2} = 3.0767105 \text{ VV}$$

The uncertainty of the multiplication of the time difference and the square of the average voltage is:

$$\Delta_i = \Delta V_{avi}{}^2\ (t_{i+1} - t_i) + \frac{V_{c(i+1)}^2 + V_{ci}^2}{2}\Delta t_d$$

For example, for i=1, the uncertainty of the inner term of the summation becomes:

$$\Delta_1 = 3.0767105\ \times\ (0.37 - 0) + \frac{14.70^2\ + 14.89^2}{2} \times 0.02 = 5.516403885\ \mathrm{VVsec}$$

The uncertainty of the summation equation is the sum of the all uncertainties of the inner terms:

$$\Delta \mathrm{Sum} = \sum_{i=1}^{391-1} \Delta_i = 287.5236\ \mathrm{VVsec}$$

All calculations were done in the word processing tool Microsoft Excel, using the data presented in Table 1.

Combining the uncertainty calculations, the uncertainty value of the energy dissipated on the resistor becomes:

$$\Delta E_r = E_r\left(\frac{\Delta \mathrm{Sum}}{\mathrm{Sum}} + \frac{\Delta R}{R}\right) = E_r\left(\frac{\Delta \mathrm{Sum}}{R\ E_r} + \frac{\Delta R}{R}\right) = \frac{1}{R}(\Delta \mathrm{Sum} + E_r \Delta R)$$
$$= \frac{1}{9900}(287.5236 + 0.361704 \times 69.50) = 0.031582\ \mathrm{J}$$

Incorporating the uncertainty into the value obtained from the summation equation we receive:

$$E_r = 0.361704 \pm 0.031582\ \mathrm{J}$$

Comparison of Results:

The trial yielded results such that the equation for the energy dissipated on the resistor has given a value $\frac{E_r}{E_c} = \frac{0.361704}{0.365825} \times 100 = 98.74\%$ within range of the value given by the conventional formula. The uncertainty values calculated were not used in this calculation as they are negligible enough to not majorly affect the results.

Trial 2: Resistance of 1kΩ, Capacitance of 3300µF:

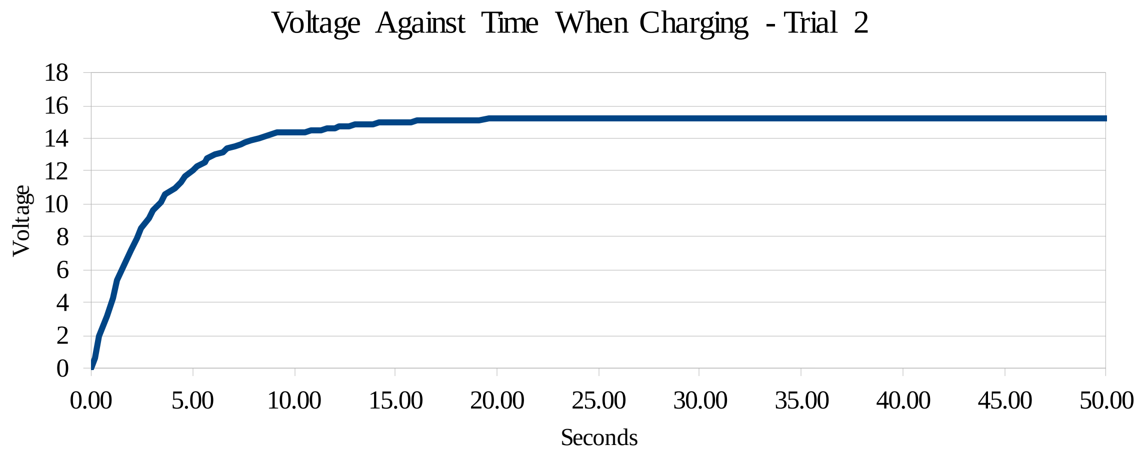
Charging Phase:

During charging, the energy accumulated on the capacitor plateaued at 15.29 V after ten minutes. Therefore, the value for the voltage on the capacitor will be taken as 15.29V instead of the theoretical maximum of 15.32V for this section.

Data:

Charging:

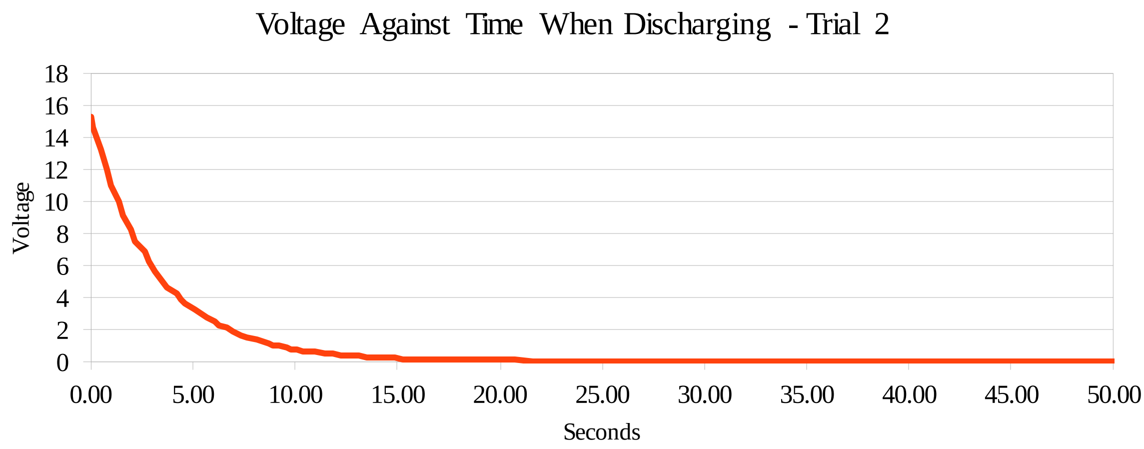
As the data on the discrete voltage values gathered during charging will not be used, they are not going to be included to not cause unnecessary clutter. Instead, the graph of the recorded values, created using Microsoft Excel, is presented here:



Graph 4: Graph of Voltage Against Time When Charging the Capacitor; Resistance of 1kΩ, Capacitance of 3300µF

Discharging:

The full data tables for the trials are presented at the Raw Data Tables section. The graph of these values, created using Microsoft Excel, is presented here:



Graph 4: Graph of Voltage Against Time When Discharging the Capacitor; Resistance of 1kΩ, Capacitance of 3300µF

<u>Energy Stored in the Capacitor:</u>

Using the conventional formula E_c is calculated as:

$$E_c = \frac{1}{2} \times C \times V_c^2 = \frac{1}{2} \times 3300 \times 10^{-6} \times 15.29^2 = 0.385744 \text{ J}$$

Calculation of Uncertainty:

$$\Delta V_c = \pm \left[0.5\% + \begin{cases} 0.003 & \text{if } V < 6 \\ 0.03 & \text{else} \end{cases} \right], V_c = 15.29$$

Therefore, $\Delta V_c = \pm \left[15.29 \times \frac{0.5}{100} + 0.03 \right] = 0.106$

$$\Delta V_c^2 = \pm[2 \times V_c \times \Delta V_c] = \pm[2 \times 15.29 \times 0.106] = 3.255241$$

With ΔE_c being:

$$\Delta E_c = \pm\frac{1}{2}(\Delta V_c^2 \times C + V_c^2 \times \Delta C)$$
$$= \pm\frac{1}{2}\left(3.255241 \times 3300 \times 10^{-6} + 233.784 \times \frac{20}{100} \times 10^{-6}\right)$$
$$= \pm 0.082520 \text{ J}$$

Incorporating the uncertainty into the value obtained from the conventional formula we receive:

$$E_c = 0.385744 \pm 0.082520 \text{ J}$$

Energy Dissipated on the Resistor:

The energy dissipated on the resistor during discharging, i.e. the area under a graph of the discrete values of V_c^2 against time is calculated as:

$$E_r = \frac{1}{R}\sum_{i=1}^{N-1} \frac{V_{c(i+1)}^2 + V_{ci}^2}{2} \times (t_{i+1} - t_i) = \frac{1}{978}\sum_{i=1}^{77-1} \frac{V_{c(i+1)}^2 + V_{ci}^2}{2} \times (t_{i+1} - t_i)$$
$$= 0.381929 \pm 0.035696 \text{ J}$$

In-depth workings have not been displayed for this series of calculations as the same workings have already been done using different values in Trial 1.

Comparison of Results:

The trial yielded results such that the equation for the energy dissipated on the resistor has given a value $\frac{E_r}{E_c} = \frac{0.381929}{0.385744} \times 100 = 99.01\%$ within range of the value given by the conventional formula. The uncertainty values calculated have not been used in this calculation as they are negligible enough to not majorly affect the result.

Conclusion and Evaluation:

The research question of this study is: "How can the formula for energy stored in a capacitor be verified by discharging the energy stored in the capacitor onto a resistor connected in series by capturing the voltage across the capacitor against time?"

As can be seen from the comparison of results, the values obtained from the trials are within close range of the conventional formula, which is given by:

$$E_c = \frac{1}{2} \times C \times V_c^2$$

where E_c is the energy stored in the capacitor, C is the capacitance of the capacitor and V_c is the voltage to which the capacitor is charged.

The total energy dissipated on the resistor connected to the capacitor when discharging, E_r, is equal to the energy stored in the capacitor before discharging. The energy dissipated on the resistor can be approximated using the equation:

$$E_r = \frac{1}{R}\sum_{i=1}^{N-1}\frac{V_{c(i+1)}^2 + V_{ci}^2}{2} \times (t_{i+1} - t_i)$$

where R is the resistance of the resistor connected, V_{ci}^2 is the square of the "i"th value of the voltage of the capacitor and t_i being the i^{th} value of the time.

The results gained from the summation equation for E_r were within close range of the energy value gained from the equation for E_c, verifying the conventional formula using the relation between the voltage of the capacitor and time; answering the research question.

Although the results obtained from the experiment verify the conventional formula for the energy stored in a capacitor, there is still room for evaluation on improving this study.

The biggest drawback was the lack of a way to record the values measured by the multimeter without manually going over video recordings of the trials. Beside the manual work it would save, an automated solution would be able to record more frequent data points. The most cost-effective and obvious way of doing this is to use a multimeter with storage functionality, which would store the data values recorded over the course of the trials.

Another drawback is that the leakage resistance of the capacitor used in the trials was not accounted for during the energy calculations, which would have brought the calculated values closer to the results of the conventional formula, giving a more accurate answer. To mitigate this drawback, a capacitor with a known value of leakage resistance should be used.

An improvement would be the use of components with smaller uncertainties, allowing the experimenter to not have to measure the exact value of capacitance or resistance for added accuracy before conducting the trials.

The principal strength of this study is the high number of raw data recorded. This increases the accuracy of the findings of the study, making it so that the results of the calculations for the energy dissipated on the resistor were in a close range to the real value of the energy stored in the capacitor yielded by the conventional formula.

Another strength is that all uncertainties that could influence the results of the study have been accounted for. Although the multimeter used in the case of this study was not able to measure the exact capacitance value of the capacitor used, the uncertainties of all other components were accounted for, having been incorporated in the calculations done to approximate the current dissipated onto the resistor.

A final strength is the lack of any systematic errors in the measurements made by the multimeter used to measure data values. There were no errors associated with the calibration of the devices used for measurement, meaning that there were no adverse effects on the accuracy of the results yielded from the experiment by devices of measurement.

With everything considered, the study yielded results that verified the correctness of the conventional formula for the energy stored in a capacitor, with the values gained from the trials matching the formula values by 98.74% and 99.01% respectively.