Research Question:

The Effect of Rotational Inertia of a Wheel to the Energy

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Research Question

How does the mass of a pullback car (2kg, 5kg, 10kg) and rotational inertia of a wheel affect the angular velocity and therefore the amount of potential that is transformed to kinetic energy? (In Joules)

1. Introduction:

After learning about rotational systems in class it was valid for me to dig more into the working process of rotational systems. As soon as I started to learn about the topic, I remembered when I was a child it was my biggest joy to play with cars that used spring potential as energy source and therefore, I want to investigate the effect of mass on the velocity of a rotatable system. I did an internship this summer in automotive factory and there one of the biggest engineering challenges was to keep the car as light as possible. The mass of the object that is rotated has an impact on its angular velocity, however to specify its mass was the variable I wanted to investigate, since I had a hypothesis that it had a huge impact and was wondering whether it was correct or not. So, I will find the amount of energy that is transformed from spring to angular kinetic energy dependent on different masses by using a roller and spring force. Therefore, I came up with the question:

How does the change in mass of an object on a rotatable system affect the amount of spring potential energy that is transformed to angular kinetic energy?

2. Background Information:

System of a pullback car:

The main components of a pullback car are wheels and when the car is pulled back, it winds up an internal coil spring by engaging the motor with a clutch. When the car is released, the spring unwinds and propels the toy forward. The released energy turns the wheels and the car accelerates. This energy is caused by the potential energy created on the spring and its transformation to kinetic energy on the wheels.

Angular velocity:

Angular velocity is the vector measure of the rotation rate, which refers to how fast an object rotates or revolves relative to another point. It is given with the formula: $\omega = \frac{\theta}{t}$

Energy Calculations:

The law of conservation of energy states that the total energy of an isolated system remains constant. In this system the potential energy will be transferred to kinetic energy and some of the energy will be lost to the surrounding by becoming thermal energy.

Bifilar Pendulum

A bifilar pendulum consists of two masses suspended from a fixed point by two strings of different lengths. The unique feature of this type of pendulum is that it exhibits coupled oscillations, where the motions of the two masses are not independent but are affected by each other. This leads to a rich variety of dynamic behaviors, including stable and unstable motions, as well as resonant and non-resonant oscillations.

Bifilar pendulum can be used to measure the moment of inertia.

3. Theory:

At the beginning of a system that is made up of spring and attached to its end a roller there is rotational equilibrium when spring is pulled in the same amount, the only energy that can be found is spring potential energy. It represents the pullback car system by roller reflecting the projectile of wheels. Spring potential energy is the energy that is found in a spring and it has the formula $P.E = \frac{1}{2}kx^2$. When the apron is released, it is expected that the potential energy will transform to rotational kinetic energy on the roller. Therefore, the roller will start turning with velocity ω and the kinetic energy will be $K.E = \frac{1}{2}I\omega^2$ The comparison of these two energies on three different masses will show the relation between mass and potential energy that is transformed to kinetic energy.

In a pullback car system, there are two types of velocities: angular and linear. However, in this experiment my aim is to observe the angular velocity, therefore the system has no linear velocity and therefore it is an unmoving system.

3.1. Hypothesis:

As the masses increase the amount of energy that will be lost to the media will be higher.

4. Design

4.1 Variables

Dependents Variables	The angular velocity of the roller
Independent Variables	The mass on the roller

Table 1: Dependent and independent variables of the experiment 1

Controlled Variables:	**How to control**	**Effect on the experiment**
Type of roller	Using the same type of rollers of the same company and same diameters	No misleading information caused by the type of roller affecting the turning of the roller.
Type of spring	Using the same spring from the same company. Also, same diameters and thicknesses.	Since the spring causes the roller to turn, differences in two systems might prevent a correct discussion.
The tension on the spring	Pulling the spring the same amount and releasing from the same points.	Assuring the tensions are equal provides the information about the rotation affected only by the mass.
Same media	Carrying out the experiments in the same lab with the same temperature, pressure and light conditions.	It prevents the spring and roller to alter caused by the temperature conditions.
Same floor	By making the experiment on the same floor.	Since the rollers will be attached to the floor in order to make efficient measurements the floors must be the same.

Table 2: Controlled variables of the experiment 1

4.2 Materials

Material	Size (± uncertainty)	Quantity
Roller	8cm diameter	2
Spring	3cm diameter	2
Protractor	360° ± 0.5	1
Thermometer		1
Masses	0.5kg, 2kg, 5kg, 10kg	3
Double-sided tape	-	1
Ruler	30 cm±0.005	
Chronometer	-	1

Table 3: Materials of the experiment 1

4.3 Procedure

1) Firstly, two systems are created by attaching the rollers to the floor
2) Then the two identical springs are attached to the inner shell of the roller
3) Then the rulers are placed at the end of the two springs
4) Springs are pulled till the 25cm part of the ruler
5) Then the springs are released
6) The motion is observed for 30, 60, 90, 120 seconds and the angles turned are noted down.

4.4 Risk Assessment

The safety issues can be considered minimal. The velocity of the roller is expected not to be high so that there will not be a risk of collusion and an accident. However, attaching the roller firmly will avoid any minor injury that can be caused from the release of the roller from the system. At the same time wearing gloves while doing the experiment is a valid solution to prevent injuries the spring can cause on hands. Using safety googles will prevent any harm that can be caused by the unexpected release of the spring and release of the roller.

There are no environmental or ethical safety issues that should be considered, since the experiment does not include toxic substances.

5. Experimental Data

5.1 Raw Data Table

For the mass of 2 kg

Time (s) ± 0.005	**Angle that is turned Δ(rad)** ± 0.5	**Average Radians Δθ (rad)**
0	0,00	0
	0,00	
	0,00	
30	13.09	13.47
	14.15	
	13.18	
60	19.11	19.74
	21.13	
	18.99	
90	25.17	25.25
	24.86	
	25.73	
120	27.14	26.83
	26.90	
	26.46	

Table 4: Raw Data Values of the Angles Turned for the mass of 2 kg for the time period of 0, 30, 60, 90 and 120 seconds

For the mass of 5 kg

Time (s) ± 0.005	Angle that is turned Δ(rad) ± 0.5	Average Radians $\Delta\theta$ (rad)
0	0,00	0
	0,00	
	0,00	
30	8.33	8.47
	8.96	
	8.14	
60	13.82	13.46
	12.91	
	13.66	
90	16.89	16.23
	15.28	
	16.53	
120	18.35	18.85
	18.86	
	19.33	

Table 5: Raw Data Values of the Angles Turned for the mass of 5 kg for the time period of 0, 30, 60, 90 and 120 seconds

For the mass of 10 kg

Time (s) ± 0.005±0.008%	**Angle that is turned $\Delta\theta$ (rad)** ± 0.5	**Average Radians $\Delta\theta$ (rad) ± 1.07±2%**
0	0,00	0
	0,00	
	0,00	
30	6.58	6.57
	6.70	
	6.44	
60	10.23	10.52
	10.60	
	10.72	
90	12.71	12.63
	12.46	
	12.73	
120	13.35	13.3
	13.25	
	13.40	

Table 6: Raw Data Values of the Angles Turned for the mass of 10 kg for the time period of 0, 30, 60, 90 and 120 seconds

5.2 Sample Average Radian Calculation for the Mass of 2kg in 30 Seconds

$$\text{Average of } \boldsymbol{\Delta\theta} = \frac{\textbf{sum of three trials ofradians}}{\mathbf{3}}$$

$$\text{Average of } \boldsymbol{\Delta\theta} = \frac{\mathbf{13.09+14.15+13.18}}{\mathbf{3}} = \mathbf{13.47}$$

5.3 Sample Uncertainty in Radian Measurement

After obtaining data for all the trials at each mass, I decided to determine the largest differences between three trials of radian measurement and divided that by 2 to reach the average uncertainty for $\boldsymbol{\Delta\theta}$**.**

According to the data the largest difference between the maximum and the minimum value occurred at 60 seconds when the mass was 2kg. and $\Delta\theta$ values were 18.99 and 21.13.

For this reason, the uncertainty of the average $\Delta\theta$ can be measured as:

$$\text{Average uncertainty of } \boldsymbol{\Delta\theta} = \frac{21.13 - 18.99}{\mathbf{2}} = \pm\mathbf{1.07}$$

However, to obtain the kinetic energy the angular velocity (ω) must be calculated.

The formula used to calculate the angular velocity (ω) is given by $\omega = \frac{\text{average }\Delta\theta}{t}$

For this reason, the sample velocity (ω) calculation for time 30 and mass of 2 kg where

average Δθ is 13.47:

$$\frac{13.47}{30} = 0.45\text{rad/sec}$$

Uncertainty of it =0.008+2=2.008%

5.4 Tables of Angular Velocity (ω)

For the Mass of 2kg

Time (s) ± 0.005	velocity (ω) (rad/s)
0	0
30	0.45
60	0.33
90	0.28
120	0.22

Table 7: Average Angular Velocity Values for the mass of 2 kg for the time period of 0, 30, 60, 90 and 120 seconds

<u>For the Mass of 5 kg</u>

Time (s) ± 0.005	velocity (ω) (rad/s)
0	0.0
30	0.28
60	0.22
90	0.18
120	0.16

Table 8: Average Angular Velocity Values for the mass of 5 kg for the time period of 0, 30, 60, 90 and 120 seconds

<u>For the Mass of 10 kg</u>

Time (s) ± 0.005	velocity (ω) (rad/s)
0	0.0
30	0.22
60	0.18
90	0.14
120	0.11

Table 9: Average Angular Velocity Values for the mass of 10 kg for the time period of 0, 30, 60, 90 and 120 seconds

6. Calculation of the Potential Energy of the Spring

6.1 Derivation of k Constant of the Spring

In order to make a comparison between the potential and kinetic energy, firstly the potential energy coming from the spring must be measured. For this process the equation of $P.E = \frac{1}{2}kx^2$ must be used. k is the spring constant for the spring and it must be calculated by using the formula: $F = kx$. For this calculation I have used a cubical mass that was 0.5 kilograms and by expending the formula $k = Fx = \frac{mg}{x}$ I have reached this equation. I hanged the mass of half kilograms at the end of the spring and it stretched 27±0.005 cm on average which is 0.27 ± 0.005meters. Therefore:

$$k = \frac{mg}{x} = \frac{0.5 \pm 0.005 \times 9.81}{0.27 \pm 0.005} = \frac{18.16N}{m} \pm 2.85\%$$

6.2 Calculation of Potential Energy

After coming up with a k value, we can calculate the potential energy of the spring. Which is found with the formula $P.E = \frac{1}{2}kx^2$ which is $P.E = \frac{1}{2}18.16x^2$ and since the length of the spring this is pulled is 25 cm $x = 0.25 \pm 0.005 = 0.25 \pm 2\%$ For this reason:

$$P.E = \frac{1}{2}18.16(0.25)^2 = 0.5675 \text{ Joules}$$

To compare potential energy also the kinetic energy of the roller must be calculated. For this the equation $K.E = \frac{1}{2}I\omega^2$ can be used. However firstly the derivation of I is necessary.

5.7 Uncertainty of the Potential Energy

$$Uncetainty = 2.85\% + (2 \times 2)\% = 6.85\%$$

7. Derivation of Moment of Inertia for Roller

In addition to that the *I* constant for the roller must be known. For this reason, I calculated that by making a bifilar pendulum. Since the roller does not have a solid and proper shape as a cylinder or sphere, it must be calculated through experiment.

7.1 Procedure

1. Firstly, the mass of the roller is measured
2. A bar is connected to two rods that are vertically adjusted to the floor
3. After preparing two strings that are the same length which is 20cm, they are attached to the roller with equal distances from the center which is 0.15cm.
4. After that the roller is attached to the strings in a way its center of mass is equal to the bar's.
5. When the pendulum is prepared the time taken was recorded for 20 oscillations.
6. The Inertia is calculated using the data of oscillations.
7. The process is done for five trials.

7.2 Materials

Material	Size (± uncertainty)	Quantity
Roller	6cm in diameter	1
Bar		
Stopwatch	0.0005	1
Ruler	30cm (±0.05)	1
Scale	0.05	1
Rods x2	25cm	2
Strings	20cm	2

Table 10: Materials required for the Experiment of Bifilar Pendulum Used to Calculate the Moment of Inertia of the Roller

7.3 Calculations for Moment of Inertia

The formula for finding the moment of inertia is given by:

$$I = \frac{MgT^2b^2}{4\pi^2 L}$$

Where:

- I=inertia
- M=mass of rod
- T=rotational time period for one rotating oscillation
- b=length between where string attaches to rod and center of gravity
- L=length of string that suspends rod

In the experiment the rotational time for 20 oscillations is calculated, so the data to be more accurate and easing the measurement process by increasing the time period.

Trials	Time taken for 20 oscillations (s)	Therefore, the time required per one oscillation period (T) (s)
1	40.62	2.034
2	41.24	2.062
3	43.38	2.169
4	41.94	2.097
5	40.28	2.014

Table 11: The time required for 20 oscillations of five trials and time required per one oscillation

After T data is collected its average must be calculated:

$$\text{Average of } T = \frac{\textbf{sum of five trials of periods}}{\textbf{5}}$$

$$\text{Average of } T = \frac{\mathbf{2.034+2.062+2.169+2.097+2.014}}{\mathbf{5}} = \mathbf{2{,}0752}$$

The average T=**2,0752** Therefore now the moment of inertia can be calculated.

Uncertainty of the time chronometer is 0.0005

Known that the values are

M=0.3kg

b= 0.15m

L=0.2m

$$\frac{MgT^2b^2}{4\pi^2L} =$$

$$\frac{0.3 \times 9.81 \times (0.15)^2}{4\pi^2 0.2} = 0.0084$$

7.4 Uncertainty in Moment of Inertia

M=0.3±0.005=±1%

b= 0.15m±0.0005=±0.3%

L=0.2m±0.0005=±0.25%

T=**2,0752**±0.0005=0.024%

$$Uncertainty = 1 + 0.3 \times 2 + 0.25 + 0.024 \times 2 = 2.33\%$$

8. Energy Calculations

8.1 Sample Calculation for Angular Kinetic Energy

$$K.E = \frac{1}{2} I\omega^2$$

Since the value of I Is known the kinetic energy can be calculated for each mass. As the sample calculation the angular velocity of mass of 2 kg and 30 seconds is calculated as:

$$\frac{1}{2} \times 0.0084 \times (0.45)^2 = 0.00085 \text{ Joules}$$

8.2 Uncertainty of Angular Kinetic Energy

The uncertainty can be found by adding percentage uncertainties of the $I\ and\ \omega^2$ values.

$I = .0084 \pm 2.33\%$

ω values are independent however the average value is 0.45rad/sec and uncertainty

$= \pm 1.07$. Therefore, it has a 2.008% uncertainty.

The total percentage uncertainty is:

$$2.33 + 2 \times 2.008 = 6.3\%$$

8.3 Tables of Angular Velocity ($\boldsymbol{\omega}$)

<u>For the Mass of 2kg</u>

velocity (ω) (rad/s)	Angular kinetic energy (Joules)
0	0
0.45	0.00085
0.33	0.00046
0.28	0.00033
0.22	0.00020

Table 12: Angular Kinetic Energy Values Dependent on Angular Velocity for the Mass of 2 kg

<u>For the Mass of 5 kg</u>

velocity (ω) (rad/s)	Angular kinetic energy (Joules)
0.0	0
0.28	0.00033
0.22	0.00020
0.18	0.00014
0.16	0.00011

Table 13: Angular Kinetic Energy Values Dependent on Angular Velocity for the Mass of 5 kg

For the Mass of 10 kg

velocity (ω) (rad/s)	Angular kinetic energy (Joules)
0.0	0
0.22	0.00020
0.18	0.00014
0.14	0.00008
0.11	0.00005

Table 14: Angular Kinetic Energy Values Dependent on Angular Velocity for the Mass of 10 kg

8.4 The Calculation of Lost Energy

The potential energy stored in the spring was calculated below. To calculate the amount of lost to surrounding and friction, the kinetic energy in each mass should be extracted from the potential energy. Then for each mass the amount of lost energy can be observed in different velocities. This will provide a vivid effect of how the mass dictates the amount of kinetic energy that is transferred to potential and the amount that is lost to surrounding.

The calculation will be done by using the potential energy amount which is 0.5675 Joules for each mass and each value will be calculated as shown below:

Sample for mass of 2 kg and velocity of 0.45rad/s and 30 seconds.

The energy that is lost=$0.5675 - 0.00085$

=0.5667

8.5 Tables of the Results

Lost Energy of 2 kg Mass

Angular Velocity (ω) (rad/s)	Angular kinetic energy (Joules)	Lost energy (Joules)
0	0	0
0.45	0.00085	0.5667
0.33	0.00046	0.5670
0.28	0.00033	0.5671
0.22	0.00020	0.5673

Table 15: Different Angular Kinetic Energy and Lost Energy Values Dependent on Angular Velocity for the Mass of 2 kg

Lost Energy of 5 kg Mass

Angular Velocity (ω) (rad/s)	Angular kinetic energy (Joules)	Lost energy (Joules)
0.0	0	0
0.28	0.00033	0.5671
0.22	0.00020	0.5673
0.18	0.00014	0.5674
0.16	0.00011	0.5674

Table 16: Different Angular Kinetic Energy and Lost Energy Values Dependent on Angular Velocity for the Mass of 5 kg

Lost Energy of 10 kg Mass

Angular Velocity (ω) (rad/s)	Angular kinetic energy (Joules)	Lost energy (Joules)
0.0	0	0
0.22	0.00020	0.5673
0.18	0.00014	0.5674
0.14	0.00008	0.5674
0.11	0.00005	0.5675

Table 17: Different Angular Kinetic Energy and Lost Energy Values Dependent on Angular Velocity for the Mass of 10 kg

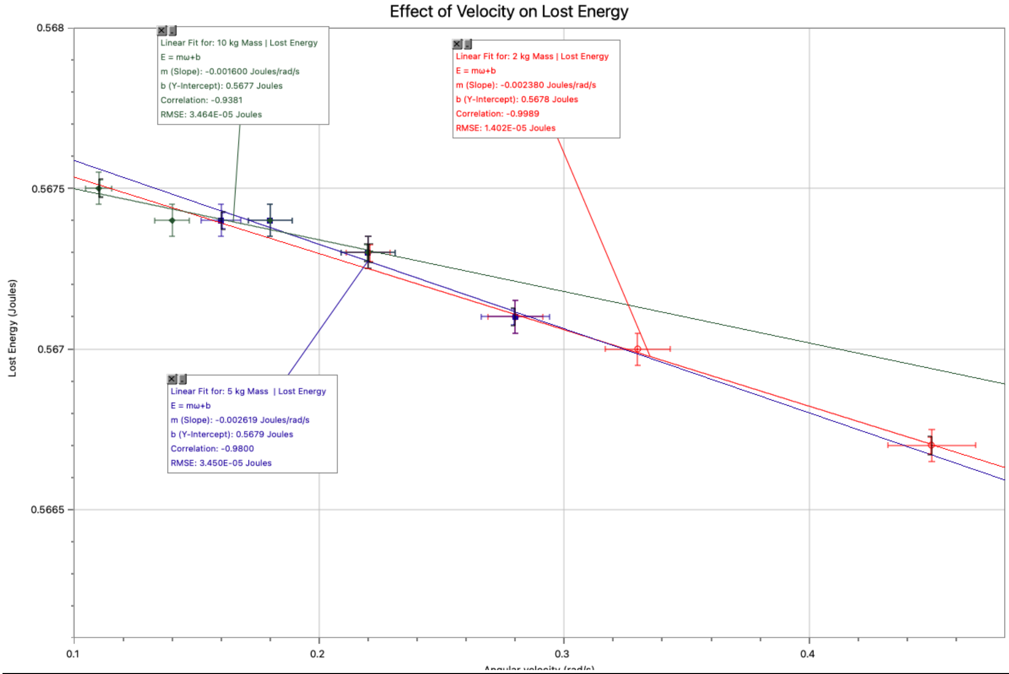
8.6 Uncertainty in the Lost Energy

The lost Energy is found by extracting the kinetic energy from potential energy. Therefore, the uncertainty can be found by adding the two uncertainties of these values. The uncertainty of potential energy is calculated as 6.85% and for the kinetic energy it is 6.3%.

Since it is more accurate to define these uncertainties on the graph. While constructing the graph these uncertainties are used and error bars are added. These bars will provide the graph to be linear, since the data obtained from the experiment might have the chance to include random errors, they will show these chances and create a best fit line.

9. Analysis

9.1 Graph of the Result



Graph 1: The Effect of Angular Velocity on Lost Energy

The *Graph 1* above displays the correlation between the mass on a rotating object and its energy amount that cannot be transferred to kinetic energy. It is evident that as the mass increases in each velocity the amount of energy lost to the media also increases. The greater the mass it the system becomes more open to transforming into other types of energies as friction or heat instead of angular velocity. For instance, this can be understood from the positions. If we observe the graph in four angular velocity periods. From 0.1 to 0.2 gap shows that in the beginning the system with the greater mass has the smallest lost energy. However, in the other time periods it is observed the system with 10 kg mass always has the highest amount of lost energy and its slope is the least steep, which means throughout the motion it has a high amount of lost energy.

When it comes to comparison of the 2 and 5 kg of masses, an unexpected result is observed considering the hypothesis. Even though the smaller mass has a smaller loss of energy in first two gaps, it is the observed the mass of 5 kg has the smallest lost energy through the last two gaps. At the same time, having observed the slopes of these two masses the 5 kg mass is observed to be steeper than the other. This result can be explained by the errors that were made in the process of experiments and the closeness amount of these two masses, and the slight difference of the slopes.

9.2 Mathematical Explanation

This can be explained by also how the principle of friction works. In a system, which is composed of heat, spring potential and angular kinetic energy; the total energy can be described as:

$$Total\ energy = P.E = K.E + Ff \times d$$

Energy lost to frictional force can be described as heat energy. Since the frictional force is type of a force its multiplication with distance gives the work done by frictional force which is in Joules.

Along with kinetic energy formula hosts mass in the formula also the frictional energy and heat energy include the mass of the object in their formulas. Considering the potential energy of the spring remains the same for each mass value, the total amount of energy stays the same however since the mass increases, all values of kinetic frictional and heat energies have to share that amount of energy with bigger values. Because of that when a rotational system is turning, as the mass of the system increases, the angular velocity decreases and this affects the kinetic energy.

If we were to open all of the formulas as:

$$Total\ energy = \frac{1}{2}kx^2 = K.E = \frac{1}{2}I\omega^2 + \mu \times m \times g \times d$$

$$= m(\frac{1}{4}R^2\omega^2 + \mu \times g \times d)$$

From this equality we can observe that the mass is a variable that change the result of the equation as the mass increases in the system it is expected to have smaller kinetic energy and loses more energy to the media. Also, the graph supports this by being linear.

10. Conclusion and Evaluation

In conclusion, an increase in the mass of a rotating system as a toy car, the energy lost to the environment increases. However, in my analysis I have come across a difference to this hypothesis while comparing the masses of 2 and 5 kilograms. I described this difference as a result of measurement mistakes might have occurred. On the other these two mass variables display that the small differences might not have a huge impact on the lost energy. As well as the variable of 10 kg and its comparison to other variables match with my hypothesis and aim, that difference the answer still remains uncertain to my research question. This is because the slope of the 5 kg mass is steeper than that of 2kg which was not a result that match with my hypothesis and conclusion.

The use of a roller offered simplicity in the carrying out process of the experiment and making necessary calculations. One of the disadvantages can be counted as roller not being in the form of wheel in terms of its outer surface. For this reason, it can also be inferred that this causing the experiment not being as accurate as a toy car model. At the same time, the roller being a much heavier material compared to a plastic toy car wheel it is also possible that it might have caused limitations. One other method that can be argued over is using a spring that is much bigger compared to one used in a pullback toy car. Essentially this also possesses limitations

The human reaction rate always presents difficulties while conducting the experiment in a number of tests where time or a change in time is being monitored, and these challenges result in random errors. Because of the accuracy of the data collected and the fact that all of the lines of best fit were able to pass over the error bars, my experiment was found to have relatively few random errors.

In addition, I want to emphasize that the mathematical analyses and estimates for the turning time of the roller did not take air resistance into account. This might have caused differences in the rotational velocity and kinetic energy. When the line of best fit for the experimental lost energy data was "above" the line of best fit for the lost energy values for the roller this was clear. It's crucial to note that, even if this may be the case, the results of my experiment still show that the mass of the system increases the lost energy. This will suffice for the purposes of comparing the stability, but for greater precision, I advise accounting for it.

While my research shows a connection between a toy car’s kinetic energy and spring potential energy, it can still be further investigated in more practical contexts. In a pull-back car system, it would be challenging to maintain a car system that decelerates steadily. If the investigation can be carried out in a more precise lab environment for bigger car systems. It could be efficient research for the automotive industry and spring systems.

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