

International Baccalaureate

Extended Essay

Observation of the changing the water strike radius effect on hydropower efficiency

Research Question: What is the relationship between the water strike radius of a hydropower turbine and the efficiency of a steady water flow for 90 seconds with a constant water strike?

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Introduction

Hydropower is an essential renewable energy source that utilizes water's force to generate electricity. One of the critical components of hydropower generation is the hydropower turbine, which converts the kinetic energy of water for rotating the hydropower turbine into electrical energy. However, the efficiency of hydropower generation is affected by several factors such as the water strike radius of the hydropower turbine.

The water strike radius refers to the distance between the center of the turbine and the point where the water strikes the turbine blades. In practice, the water strike radius can be adjusted by varying the size and shape of the turbine blades or by changing the water flow rate. Nevertheless, the effect of changing the water strike radius on efficiency in a steady water flow for a prolonged period is not well understood.

Therefore, this study aims to investigate the relationship between the water strike radius of a hydropower turbine and the efficiency in a steady water flow for 90 seconds with a constant mass and velocity of water strike. The findings of this research will provide insights into the optimal design and operation of hydropower turbines, which can improve the efficiency and sustainability of hydropower generation.

The energy in a hydropower turbine is the product of the rotation of the turbines. Since the motion of the turbines is rotational, the hydropower turbine will be examined as circular in the experiment. The moment of inertia of an object also referred to as rotational inertia, is the basic form of the rotational mass of an object. Kinetic energy is needed to determine the moment of inertia of an object. Kinetic energy's formula is the product of one-half the mass of an object and velocity squared.¹ However, when an external force source, such as water, travels in a circular

motion, the rotational kinetic energy must be considered. Rotational kinetic energy is defined by one-half the first derivative of time concerning angular displacement squared multiplied by the radius squared times the object's mass. The inertia is the sum of the points of mass multiplied by the corresponding radius. In my experiment, I set up a water wheel and changed the strike point of the water. As a result, the radius of the water wheel was changed, and took the system's water flow and mass constant. The energy production will change by changing the object's radius where the water strikes. A ruler measures the water strike radius; the values are 7 cm, 6cm, 5cm, 4cm, and 3cm. Change in the distance was selected to see the difference better with sufficient data. Change in voltage is changed with the velocity of the turbine which is measured by an optical tachometer. The water flow is measured by a water flow sensor. The system's mass is measured with a scale, and in every trial, the same system is used, so the system's mass is conserved. Five shots are taken at each water strike radius to increase precision. Every trial will be done in 90 seconds and measured by a chronometer. The data is collected in 15 seconds, 30 seconds, 45 seconds, 60 seconds, 75 seconds, and 90 seconds.

Variables

The investigation is based on the “To what extent does the change in the water strike radius of the hydropower turbine change the efficiency in a steady water flow for 90 seconds with a constant mass and velocity of water strike?”. The main objective is the changing the water strike radius. As a result, the experiment's independent variable is the water strike radius. The distance was measured by using a ruler.

The dependent variable for the experiment is the efficiency of the water turbine. One trial of the experiment will be done in 90 seconds, so the efficiency is based on the rotation per 90 seconds which is the frequency of the turbine. The water turbine's velocity changes the system's period, so the velocity is measured by an optical tachometer. An optical tachometer is a measurement device that measures rotations per minute (RPM).

The controlled variables are incidence angle (β), the time taken for 1 trial, the mass of the turbine, water resource height, and the water rate. The time in every experiment is measured by a chronometer which is 1 minute. If the incidence angle changes the power formula, have two variables. As a result, the incidence angle (β) is taken at 45° in the experiment. If the time taken for 1 trial changes, the turbine's rotations will change, leading to the experiment being wrong. Another controlled variable mass of the turbine is measured with a scale and used the same turbine in every experiment. Mass is one of the components of the moment of inertia and if the mass is not conserved the velocity of the turbine will be inaccurate. The height of the water resource is another controlled variable and is measured by a ruler in every experiment. Which is taken 25 cm in the experiment. If the height of the water resource is different, the water's gravitational energy will change, affecting the turbine's velocity.

Water rate is also a controlled variable measured by a water flow sensor. If the water rate is not constant, it will be chaotic and unpredictable, leading to the velocity of the turbine being inaccurate. Also, the Reynolds Number is taken under 1000 for laminar flow conditions. The exact amount of mass of water will strike with a constant speed. The water rate is taken at $12.0 \text{ m}^3/\text{s}$. As a result, the angular momentum will be conserved. The other type of variable is the uncontrolled variable. In the experiment, the wind is the uncontrolled variable because the experiment is done in backyards to minimize water waste.

Theory

The moment of inertia is one-half the first derivative of time concerning angular displacement squared multiplied by the radius squared times the object's mass. We have a thick hollow cylinder (wheel) and a rod (axle) for a water wheel. The inertia is defined as the sum of the points of mass multiplied by the corresponding radius squared.² However, in the experiment, water strikes a fixed point so the moment of inertia will be the product of mass and velocity squared:

$$I = \Sigma m_i r_i^2$$

$$I = mr^2$$

Formula 1 Moment of Inertia

For a body rotating about some axis, its kinetic energy is then:

$$EK = \frac{1}{2} I \omega^2$$

Formula 2 Kinetic Energy

Water strikes the turbine with a constant mass and speed in the experiment. As a result, momentum will be conserved. The angular momentum of a rigid body with the moment of inertia I rotating about a fixed axis with angular speed ω . The body is a point mass, so angular momentum is defined as with the unit $kgms^{-2}$:

$$L = I\omega$$

$$L = mr^2\omega$$

$$L = m(\omega r)r = mvr$$

Formula 3 Angular Momentum³

Power is the rate at which work is being done or the rate of energy that is being transformed with the unit watt. Work is the product of force in the direction of displacement and the distance traveled. When we combine the power and the work formula, power is the product of force and displacement per unit of time.⁴ The force unit is the newton. The displacement is a meter. The time is second.

$$P = \Delta W / \Delta t$$

$$\Delta W = F \Delta x$$

$$\Delta P = F \Delta x / \Delta t$$

Formula 4 Power Formula

If the potential energy formula is combined with the power formula needs to be modified because the mass of the water is unknown. A density of a substance is the mass per volume which also means that the mass is the product of density and volume and can be used in the formula. Moreover, the water's volumetric flow rate is a fluid volume that transitions per time. When the power formula is modified, power is the product of the density of the liquid, flow rate of the liquid, gravitational acceleration, and height of the water source.⁵ The SI unit of height is m, the density of the liquid is kgm^{-3} , and the flow rate of the water is m^3s^{-1} .

$$P = \frac{mgh}{t} \quad Q = \frac{v}{t}$$

$$\rho = \frac{m}{v}, \quad m = \rho v$$

$$P = \frac{(\rho v)gh}{t} = \rho Qgh$$

Formula 5 Modified Power Formula

If the torque formula is modified, it can be used in the power formula. Torque is the product of the perpendicular force and the distance from the origin if we change the perpendicular force with the product of mass and acceleration as the equation 5. After that, acceleration can be changed to velocity per time. Furthermore, mass can be changed as equation 6. The new power equation will be the product of the density of the water, the volume of the water, the distance from the origin, and the velocity per time. Volume per time is the water flow equation, which we can change in the torque equation. As a result, the torque equation is the product of the density of the water, the velocity of the water, the distance from the origin, and the water flow. As in figure 1, the velocity is able to separate into equations as the radius of inner and outer and velocity inner and outer. This multiplication is the change in distance (displacement) in the equation. The equations can be summarized as:

$$T = F d$$

$$T = m a d$$

$$a = \frac{V}{t}$$

$$T = \rho v \left(\frac{V}{t} \right) d$$

$$T = \rho Q (vd)$$

Formula 6 Modified Torque Formula

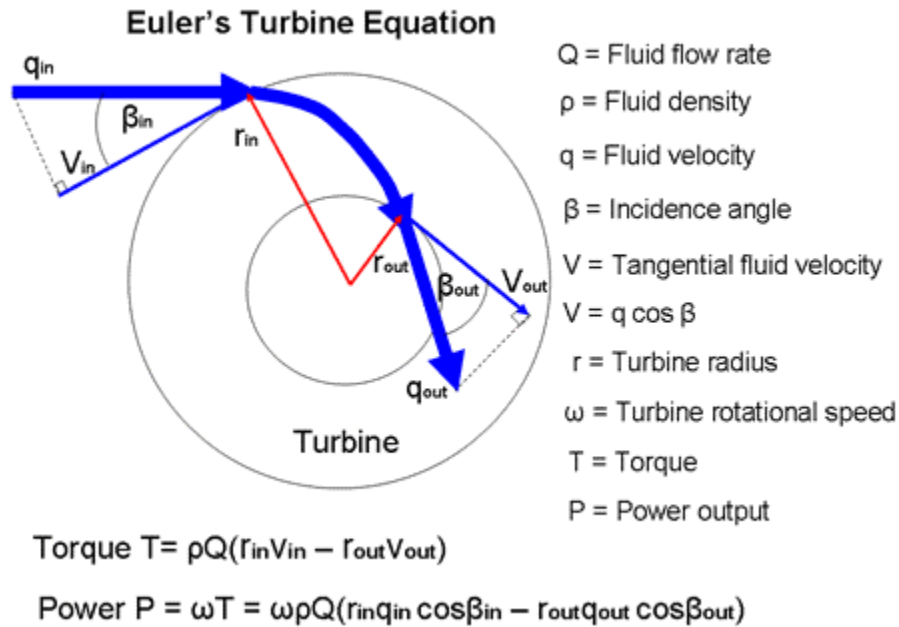


Figure 1 Euler's⁶ Turbine Equation

In the investigation, the change in efficiency of energy production by the turbine is the one looking. The efficiency equation is the ratio of input power between output power times 100. The output power is the power of the hydropower turbine. The input value will be calculated from the potential energy.

$$\frac{\text{input}}{\text{output}} \times 100$$

Formula 7 Efficiency Equation

Hypothesis

The conservation of the power to energy is directly proportional to the angular velocity. Since the mass and velocity of the water strikes are constant, angular momentum will be conserved. When the distance of the water striking radius decreases, the efficiency of the water turbine will increase.

Methodology

There are two techniques used in this investigation. One of them is observing the difference in the experiment by using the formula. The other one is observing the difference in the experiment by modeling the hydropower turbine. There are two techniques for this investigation because the procedure will give the literal value and the model is going to give real-life data.



Figure 2 Hydropower Turbine Model

Material List

- Hose
- Arduino Uno⁷
- Water Flow Sensor(±0.1)
- Cardboard
- Computer
- Saw
- Gloves
- Source of Water
- Chronometer (±0.1)
- Ruler (30 cm) (±0.1)
- Measuring tape (±0.5)
- Hot Glue
- 25x25x25 Wood Stick x2
- 45x25x25 Wood Stick
- Strip
- USB cable
- Jumper Cable
- 20x15x15 Box
- Spray Paint 400 ml (White)
- Optical Tachometer⁸ (±0.1) (RPM)



Figure 3 Optical Tachometer

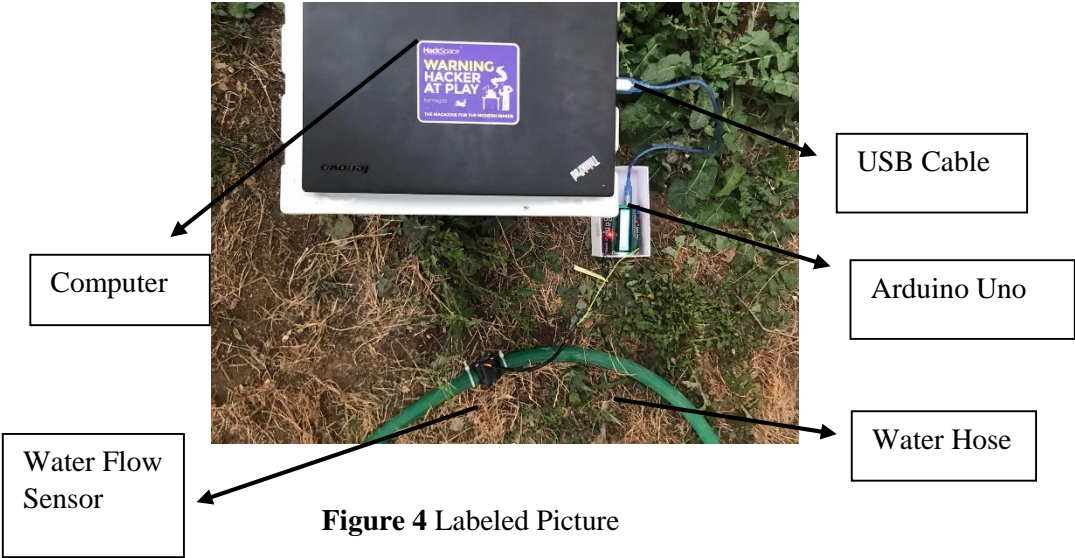


Figure 4 Labeled Picture

Procedure

The model of the experiment is based on some steps. The first step is the setup of the water turbine. A water tank is made for the base of the turbine with a height of 10 cm. Then two equally sized wood pieces were attached to the sides of the tank by screwing. The height of these wood pieces is 30 centimeters and drilled holes two both of the woods with a radius of 1 cm and a pipe is placed to connect the woods. Furthermore, 45 cm of wood was screwed to the head of the system and drilled into a hole with a radius of 2 cm. The cardboard is cut into eight square shapes for the water turbine blades. A cylindrical object is found in the trash and the cardboard is glued to the cylindrical object via forty-five degrees. The cylindrical object is placed in the middle of the pipe and rings are placed near the cylindrical object to prevent gliding. The setup is painted with blue and white spray paint since the materials are mostly waste materials the setup is not clear.

The other step for the experiment is making a device to measure the flow rate of water. The device is basically made by using Arduino Uno and a water flow sensor. The water flow sensor is connected to the Arduino Uno by jumper cables. Three jumper cable is used.

Another step is taking data from the model. The water flow sensor is connected to the measurement device. A computer is connected to the measurement device for power supply via a USB cable. The water flow sensor is placed in the hose and squeezed to prevent waste. The edge of the water edge is placed through the hole in the 45 cm wood piece. The water hose's angle changes as the water strike radius changes. The data is collected in the experiment in 15 seconds, 30 seconds, 45 seconds, 60 seconds, 75 seconds, and 90 seconds. For every water strike radius 5 trials are done. The water strike radius has 5 values 7 cm, 6cm, 5cm, 4cm, and 3cm. In conclusion, 150 data will be collected in the model.

Risk Assessment and Ethical Considerations

The experiment does not include any risk to living organisms. In the experiment, water is used a lot. However, to minimize the water waste, a water tank is used to hold water, and then the water is used for irrigation of the garden. All the health and safety devices are taken for the experiment. When the water turbine is set up, wood is cut into pieces. The saw is a hazardous device since it is sharp and scatters sparkles. As a result, gloves and protective glasses are used in the setup part.

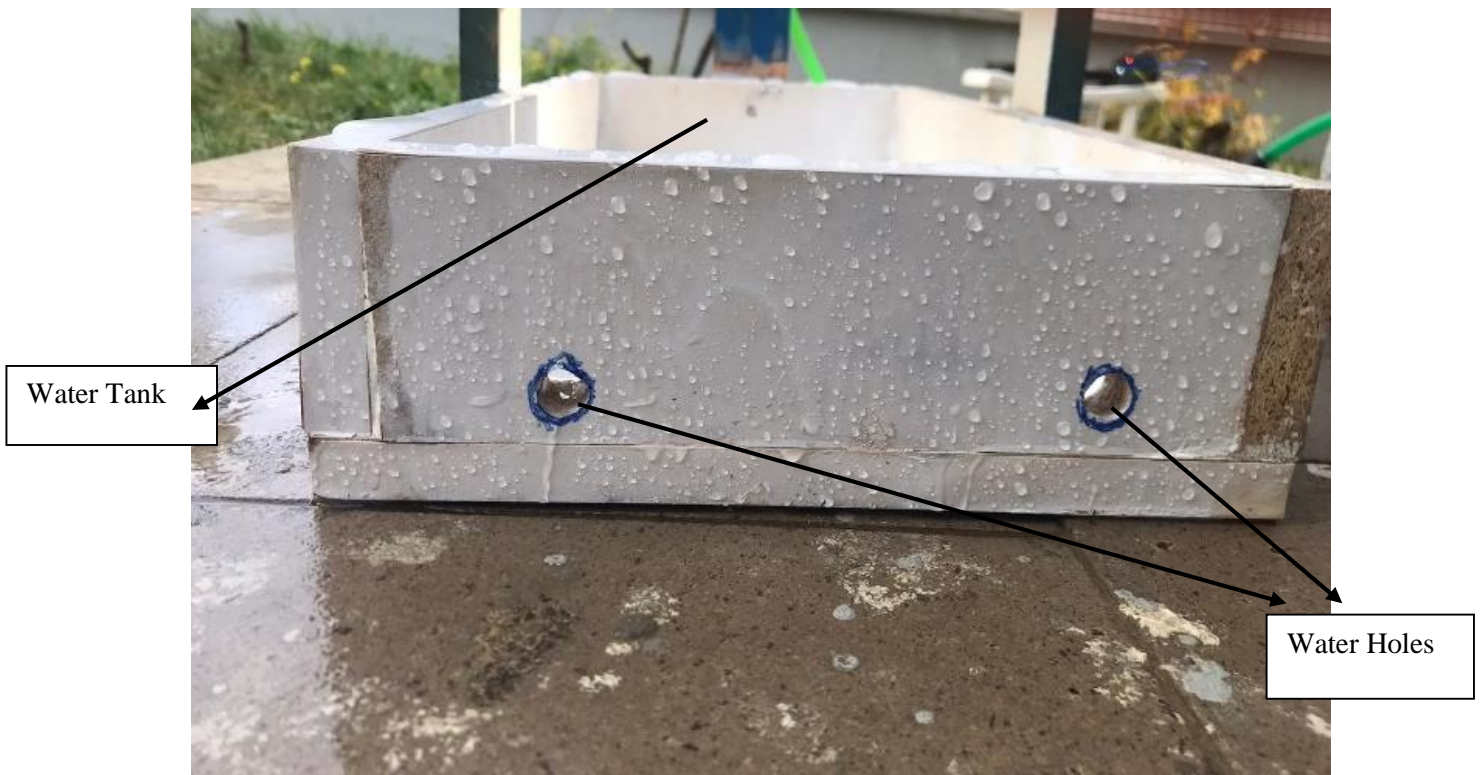


Figure 5 Labeled Picture

Analysis

Raw Data Table

Radius (cm) (± 0.1)	Trials	Rpm (± 0.1) at Time(s) (± 0.1)					
		15.0	30.0	45.0	60.0	75.0	90.0
7.0	1 st trial	106.3	117.3	124.2	126.3	128.3	129.3
	2 nd trial	107.4	120.2	124.1	126.2	127.7	129.1
	3 rd trial	106.1	118.5	124.2	126.3	127.9	128.6
	4 th trial	107.2	119.7	125.3	127.1	128.1	130.3
	5 th trial	106.5	118.8	123.4	126.1	128.0	129.2
6.0	1 st trial	100.3	107.3	113.2	115.2	118.0	121.2
	2 nd trial	100.4	105.2	112.2	116.2	118.2	120.1
	3 rd trial	102.1	107.5	112.1	114.6	117.9	120.6
	4 th trial	101.2	109.7	113.2	115.3	117.8	119.9
	5 th trial	100.5	106.8	112.3	116.0	118.1	120.2
5.0	1 st trial	97.4	100.2	103.5	107.1	110.2	111.3
	2 nd trial	97.3	100.5	104.1	106.7	109.2	110.2
	3 rd trial	98.2	101.4	104.2	106.9	109.9	110.1
	4 th trial	99.1	101.5	103.8	107.2	110.1	111.0
	5 th trial	99.0	100.4	103.9	107.1	109.1	109.4
4.0	1 st trial	75.4	77.4	78.5	78.9	78.8	78.9
	2 nd trial	75.8	76.6	78.2	78.7	78.9	79.1
	3 rd trial	76.6	77.2	78.1	79.0	79.1	79.2
	4 th trial	76.2	76.9	78.3	78.8	79.2	79.2
	5 th trial	76.0	76.9	78.4	78.6	79.0	79.1
3.0	1 st trial	50.1	51.3	52.1	52.9	53.4	53.6
	2 nd trial	50.4	51.2	52.6	53.2	53.5	53.7
	3 rd trial	50.3	51.4	52.3	52.9	53.2	53.5
	4 th trial	50.2	51.6	52.3	53.1	53.1	53.3
	5 th trial	50.0	51.5	52.2	52.9	53.3	53.4

Table 1 Raw Data Table

Data Processing

There are 150 data collected from the setup. A mean is a number expressing the average or central value of data. Each trial means found for reaching the revolutions per minute (rpm). Equation 1 is an example calculation in which the water strike radius is 7 cm and the time is 90 seconds. The mean for all data can be summarized by equation 1:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

x_i = The values that x takes

N = The number of population

\bar{x} = The sample of mean

Formula 8 Mean Formula

$$\bar{x} = \frac{(129.3 + 129.1 + 128.6 + 130.3 + 129.2)}{5}$$

$\bar{x} = 129.3$ (1 Decimal Places)

Equation 1 Example Mean Calculation

Radius (cm) (± 0.1)	Rpm (± 0.1) at Time(s) (± 0.1)					
	15.0	30.0	45.0	60.0	75.0	90.0
7 cm	106.7	118.9	124.2	126.4	128.0	129.3
6 cm	100.9	107.3	112.6	115.5	118.0	120.4
5 cm	98.2	100.8	103.9	107.0	109.7	110.4
4 cm	76.0	77.0	78.3	78.8	79.0	79.1
3 cm	50.2	51.4	52.3	53.0	53.3	53.5

Table 2 Processed Data Table

Calculation

The equation is organized according to the Euler formula for a hydropower turbine. The angular velocity is a vector quantity equal to the angular displacement per unit of time. The revolutions per minute(rpm) which is velocity is equal to the angular velocity multiplied by the radius of the hydropower turbine. The angular velocity is divided into sixty because changes from a second to a minute. Equation 2 is an example of this process.

$$T = \rho Q (vd)$$

$$T = (1)(12.0) \left(\frac{129.3}{90} \right) (7) = 120.7 (1 \text{ Decimal Places})$$

Equation 2 Example calculation of Torque

Furthermore, the equation's torque value obtained from equation 2 is utilized to calculate the power generated by the hydro turbine, which is the product of its rotational speed and torque. An illustration of the power calculation for a hydropower turbine operating at the height of 25cm and a rotational speed of approximately 12.0 RPM is provided in equation 3.

$$P = \frac{\omega}{60} T$$

$$P = \frac{129.3}{60} \times 120.7 = 260.1 (1 \text{ Decimal Places})$$

Equation 3 Example of Calculation Power

An example calculation of the percentage ratio for the efficiency of the hydro turbine model can be derived from Equation 4, which involves dividing the result into the literal value of each data and multiplying it by 100.

$$efficiency = \frac{\frac{\omega}{60} T}{\rho Q_1 g h_1} \times 100$$

$$efficiency = \frac{260.1}{1 \times 12.0 \times 25} \times 100 = \%86.7$$

Equation 4 Example Calculation Of Efficiency

Error Propagation

During the experiment, measurement devices were used, which introduced uncertainties. The photo tachometer, measuring tape, and water flow rate measurement sensor were employed at various stages of the investigation. To account for uncertainties resulting from multiplication and division, Equation 5 was used for error propagation, where fractional uncertainties were added in quadrature. To simplify calculations, it is recommended to convert all fractional uncertainties into percentage uncertainty before applying the formula. The uncertainty values were obtained from the datasheet, and the final result obtained from the formula was multiplied by a coverage factor to increase reliability. The coverage factor is used to expand the uncertainty derived from the combined standard uncertainty.

$$u = \sqrt{((U(\text{flowrate sensor}))^2 \times ((U(\text{measuring tape}))^2 \times ((U(\text{photo tachometer}))^2))$$

Formula 9 Uncertainty Calculation Formula

$$u = \sqrt{10^2 \times 0.05^2 \times 0.05^2}$$

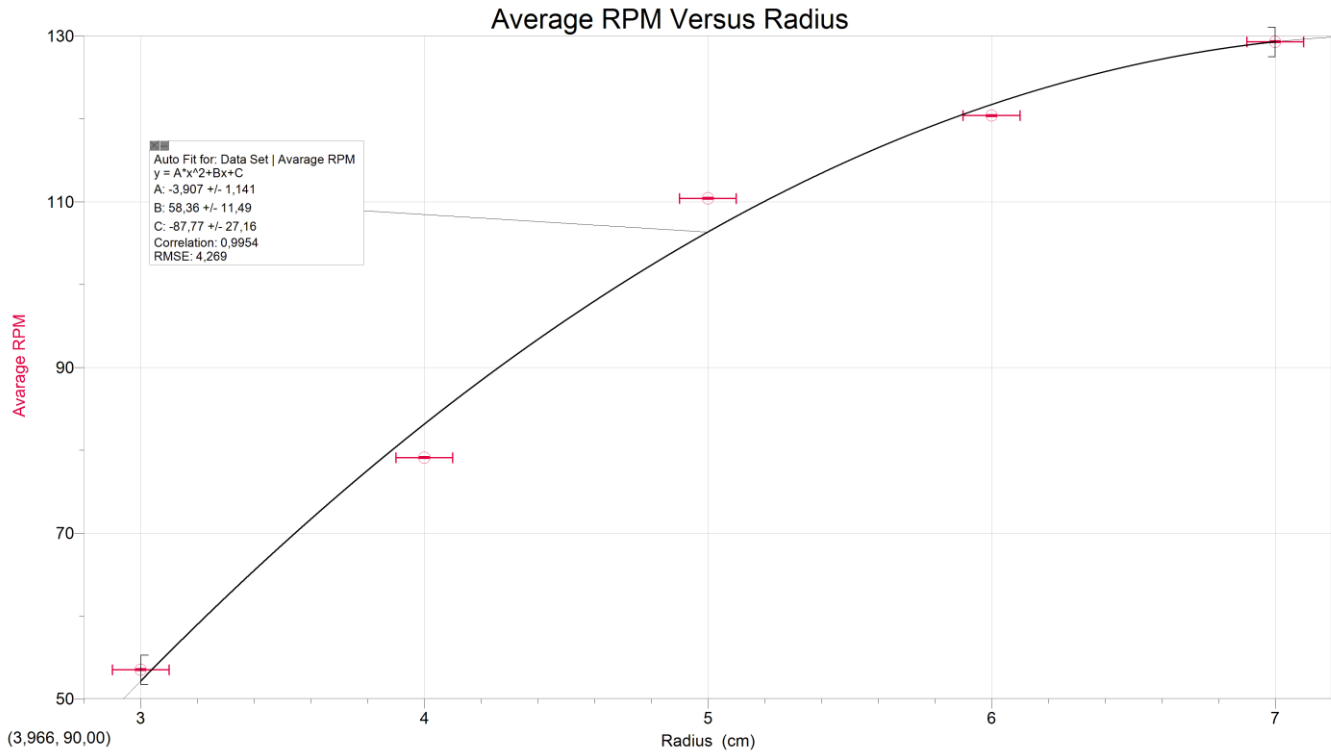
% ± 10 (1 Significant figure)

% ± 10 x 2

$u = \% \pm 20$

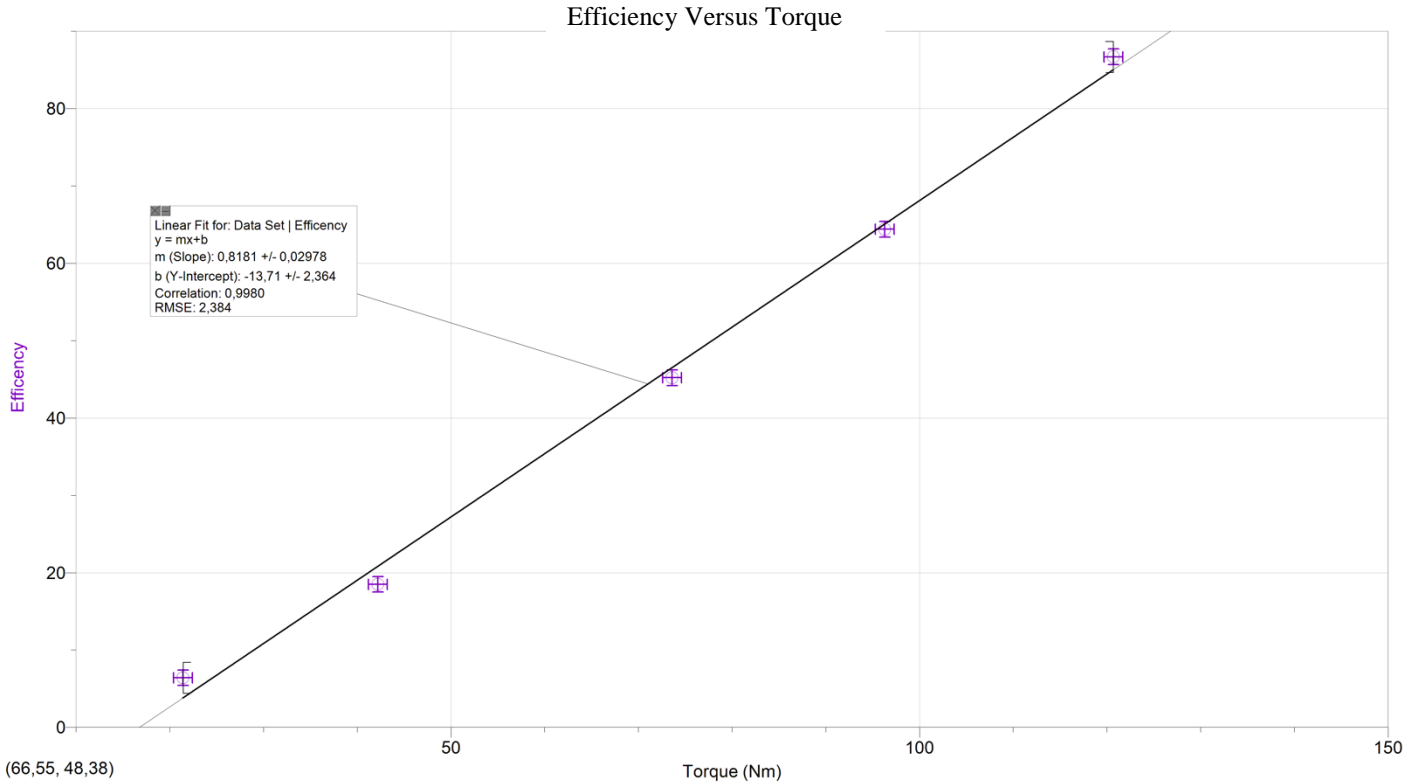
Equation 5 Calculation of the Uncertainty

Graph Analysis



Graph 1 Average Rpm Versus Radius at 90 seconds

Graph 1 is plotted according to table 2. The uncertainty bars are small but not negligible. Radius has a proportional relationship via efficiency. As a result, the increase in water strike radius, RPM value will increase. The correlation coefficient is 0.9954, so the data points are strongly moderate and positive correlation with the equation. The Rpm changes due to the radius of the water strike.



Graph 2 Efficiency Versus Torque Graph

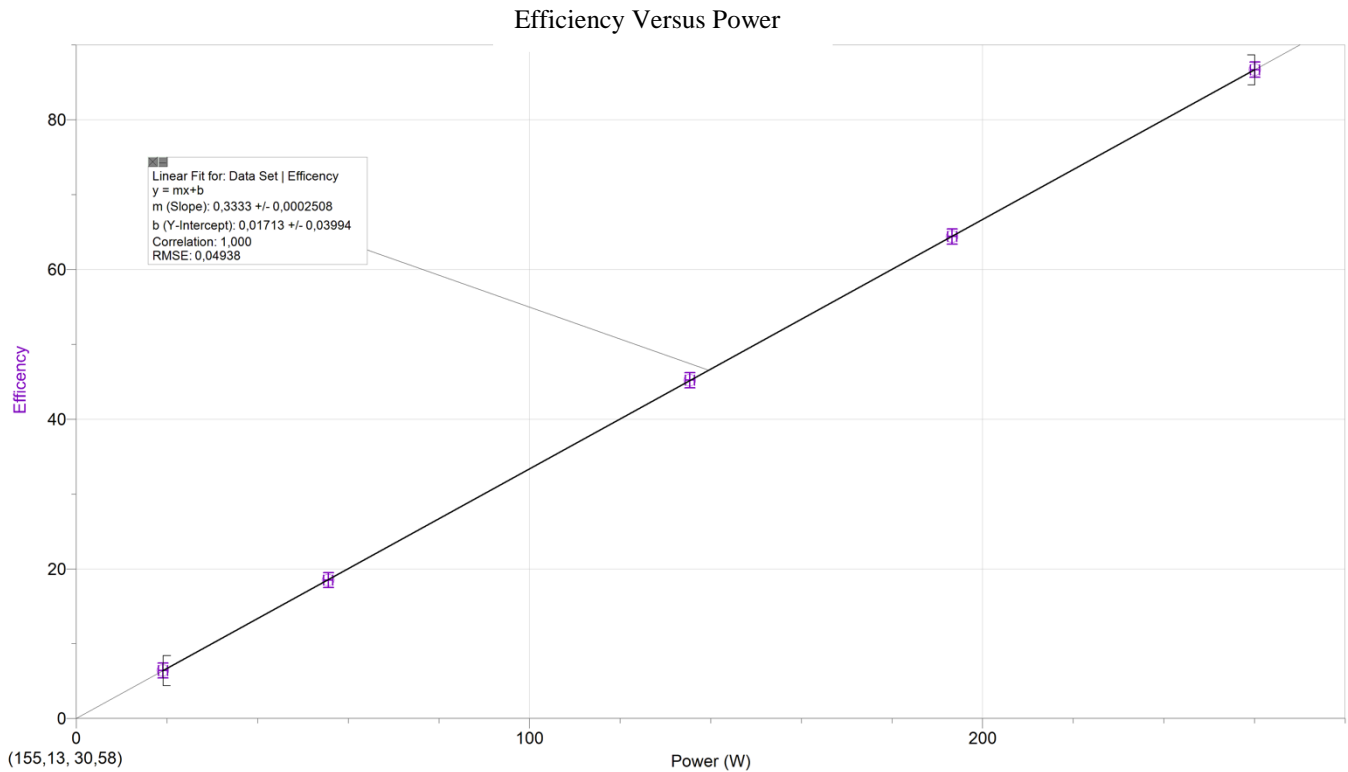
The data of graph 2 is taken in table 8. The formula of the graph is a linear formula. Looking trend of the points torque has a promotional relationship via efficiency in experiment. As the torque increases, efficiency increases. The correlation coefficient is 0.9980, which shows that the data points are strongly moderate and positive correlation. The percentage error in slope is the percentage between the uncertainty of the slope and slope. Which can be formula and equation can be summarized as:

$$\frac{\text{Uncertainty in slope}}{\text{slope}} \times 100$$

Formula 10 Percentage Uncertainty in Slope

$$\frac{0.02978}{0.8181} \times 100 = \%36.4 \text{ (in 3 significant figure)}$$

Equation 6 percentage Uncertainty in efficiency versus torque graph



Graph 3 Efficiency Versus Power Graph

The graph was constructed based on the data obtained from table 7 and follows a linear formula. The graph's trend suggests that power has a promotional relationship with efficiency in the experiment. As power increases, efficiency increases, indicating a positive correlation between power and efficiency. The correlation coefficient of 1.000 shows that the data points are strongly and positively correlated, with a high degree of moderation. A correlation coefficient of 1.000 suggests that the relationship between the variables is perfect, meaning that there is a direct relationship between the variables that can be represented by a straight line on the graph. The percentage error in the slope equation is the:

$$\frac{0.0002508}{0.3333} \times 100 = \%0.07 \text{ (in 3 significant figure)}$$

Equation 7 percentage Uncertainty in efficiency versus Power graph

Conclusion and Evaluation

Conclusion

The investigation aimed to observe the relationship between the water strike radius of a hydropower turbine and the efficiency of a steady water flow for 90 seconds with a constant water strike. After conducting extensive research and experimentation, it can be concluded that there is a direct relationship between the water strike radius of a hydropower turbine and the efficiency of a steady water flow for 90 seconds with a constant water strike. The water strike radius refers to the distance between the center of the turbine and the outer edge of the water flow that strikes the turbine. The turbine's efficiency refers to the amount of energy extracted from the water flow and converted into electricity. As the water strike radius increases, the efficiency of the turbine also increases. This is because a larger water strike radius means more water flows through the turbine, which results in more energy being extracted. The result of the experiment is strongly moderated with my hypothesis according to the correlation coefficient of **Graph 1, Graph 2, and Graph 3**. In conclusion, the water strike radius of a hydropower turbine and the efficiency of a steady water flow for 90 seconds with a constant water strike are directly related. Understanding this relationship is crucial for optimizing the efficiency of hydropower turbines and ensuring that they are operating at their maximum potential. Further research in this area could lead to the development of more efficient hydropower turbines, which could significantly impact the renewable energy sector.

Evaluation

The resource's uncertainty of approximately $\pm 20\%$ is a significant concern, primarily since it arises from several limitations in the experimental setup. One such limitation is the turbine model's inadequacy within its class, leading to slow RPM readings for some data. A Pelton hydropower turbine could be used instead to improve the accuracy of the readings. Another limitation is the gravitational acceleration's inconsistency worldwide, which must be considered while calculating the resource's uncertainty. The value of local gravitational acceleration must be used instead of the constant 9.81 to account for this variation. The time intervals in the data set are too close to each other, leading to aberrations in the data. The solution to this limitation is to collect data every thirty seconds. This strategy can reduce the uncertainty arising from this limitation. Water droplets falling on the photo tachometer screen or signal light beam can lead to inaccurate readings. The solution is to move the photo tachometer farther from the turbine, which can reduce the error, but it may introduce more uncertainty in measuring RPM. Lastly, the time intervals are insufficient for recording the water flow rate and RPM in the book. This limitation results in data aberrations that must be acknowledged. Two solutions to this random uncertainty issue include taking data every thirty seconds or gathering flow rate data every fifteen seconds. In conclusion, the investigation has several limitations and sources of error and uncertainty that must be addressed. By acknowledging and mitigating these limitations, the accuracy and reliability of the experimental results can be improved, and the resource's uncertainty can be reduced.

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Appendix

Radius (cm) (± 0.1)	Torque (Nm) (± 0.1)
7 cm	120.7
6 cm	96.3
5 cm	73.6
4 cm	42.2
3 cm	21.4

Table 3 Radius Versus Torque

Radius (cm) (± 0.1)	Power (W) (± 0.1)
7 cm	260.1
6 cm	193.3
5 cm	135.4
4 cm	55.6
3 cm	19.1

Table 4 Radius Versus Power

Radius (cm) (± 0.1)	Efficiency (± 0.1)
7 cm	86.7
6 cm	64.4
5 cm	45.2
4 cm	18.5
3 cm	6.4

Table 5 Radius Versus Efficiency

Torque (Nm) (± 0.1)	Power (W) (± 0.1)
120.7	260.1
96.3	193.3
73.6	135.4
42.2	55.6
21.4	19.1

Table 6 Torque Versus Power

Power (W) (± 0.1)	Efficiency(± 0.1)
260.1	86.7
193.3	64.4
135.4	45.2
55.6	18.5
19.1	6.4

Table 7 Power Versus Efficiency

Torque (Nm) (± 0.1)	Efficiency(± 0.1)
120.7	86.7
96.3	64.4
73.6	45.2
42.2	18.5
21.4	6.4

Table 8 Torque Versus Efficiency