Application of Prisoner's Dilemma to the Event of Cheating in an Exam Mathematics

Internal Assessment

## INTRODUCTION

Last year, while I was trying to decide what major I wanted to study in college, I took several classes from a course about economics. In one of those courses, there was a lecture where game theory was explained and later on, I found out that a mathematician named John F. Nash won a Nobel prize for game theory in 1994. The theory intrigued me and I decided to explore it further as part of my mathematics research. I decided to apply it to a relatable event for students and use their perception of the theory as data for my research.

Personal interest is shown C+

I handed out a survey that I implemented where my peers were asked their genders, program they study in and a question where they were asked to answer how they would act in a hypothetical event. In this hypothetical event they are explained that they had cheated on an exam with their friend, and they had got caught. The teacher interrogated them separately and they had a choice between cooperation and denial, but they were unaware of their friends' answer and they were explained that their friends answer affects the consequence of their answer.

By analyzing the outcome of this survey, I studied whether Prisoner's Dilemma, a game theory application, can predict the average behavior of my schoolmates effectively.

## BACKGROUND INFORMATION

Game Theory is the field which observes how firms and people behave strategically in the context of a game. For example, rival firms in a specific field, like telecommunication, want to maximize their profits. Their success on this reading depends on the strategy that is followed by the rival firms.

Prisoner's Dilemma is one of the introductory games of the Game Theory and it is applied to many fields of study. One of these fields is psychology and the research that examined behavioral consistency within the same mixedmotive game, by presenting participants with a series of one-shot Prisoner's Dilemma Games ${ }^{1}$. Another field that can be an example is economics and the research about the application of the Prisoner's Dilemma onto the electricity market models in order to detect the iterative collusive strategy ${ }^{2}$. Moreover, prisoner's dilemma has also been applied to the field of international relations where a study on Saudi-Iranian relations was conducted by the use of the theory. An example for the environmental sciences could be the research where the existence of a prisoner's dilemma for retailers under a medium investment efficiency because although green investment enhances demand, it also increases

[^0]the unit cost and subsequently the retail price is verified ${ }^{3}$. As a more daily example the use of the theory in the field of sports could be given where a one-shot prisoner's dilemma game to measure ingroup cooperation among Japanese baseball fans ${ }^{4}$. Prisoner's dilemma can also be applied to social events. For instance, research was conducted on the schoolchildren in Brazil to analyze the effect of gender at group size on cooperation ${ }^{5}$. Another study dealing with the choices of high school children and the effect of gender on cooperation was carried out by collecting a very large data set containing 1200 responses ${ }^{6}$. the last two research given were the base inspiration for my research since I also examined relation between the responses of children with their gender and the program they study at my school.

In Prisoner's Dilemma, there are two people that were arrested by the police. The evidence is not sufficient to convict them, therefore the police interrogator suggests the following deal to each of them: If one of them testifies that the other one is guilty (defect), he will be set free and the other one will be imprisoned for 3 years if the other prisoner stays silent. However, if both prisoners defect and testifies against each other, they will both be imprisoned for 2 years. If they both keep silent (cooperate), each of them will serve 1 year in prison. Each prisoner knows that the same offer that was made to them, has also been made to the other convict. The fact that both prisoners are aware of the offers made to each other is important for the Prisoner's Dilemma because without that common knowledge, the choice of the prisoners could vary. Common knowledge, where players know about offers made to other players, is a prerequisite in Game Theory, and makes sure that the model is completely specified, and its analysis is coherently commenced. In addition to common knowledge, another significant concept is complete information. Complete information is a game in which knowledge about other players conditions is available to all players. Therefore, complete information is also a prerequisite of Game Theory since common knowledge completes the concept of complete information.

The strategic game of Prisoner's Dilemma can be represented as a function $G$, which depends on the number of players, their decisions and their utility function.
$\mathrm{G}=(\mathrm{N}, \mathrm{A}, \mathrm{U})$
$\mathrm{N}=\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$

N represents the number of players. In Prisoner's Dilemma there are 2 players and they are demonstrated by $P_{1}$ and $P_{2}$.

[^1]$\mathrm{A}=(\mathrm{C}, \mathrm{D})$

A represents the available decisions that can be taken by the players. In Prisoner's Dilemma there are two options defect (D) and cooperate (C).
$\mathrm{U}=\left(\mathrm{U}_{1}, \mathrm{U}_{2}\right)$

In Prisoner's Dilemma, a utility function, which is represented by $U$, maps player decisions to payoffs. $U_{1}$ is the utility function for Player 1 and $U_{2}$ is the utility function for Player 2. In other words, the players will get payoffs denoted by the utility function, where $\mathrm{U}_{\mathrm{i}}: \mathrm{A} \rightarrow \mathrm{R}$. " i " represents the players' number which could be 1 or 2 in Prisoner's Dilemma. The payoff, represented by R, can be any real value but since players are awarded years in prison (they get punishment rather than reward), the payoffs take a minus sign, to denote that more years are worse (non-preferable) to less years.

In Game Theory, if a player's decision doesn't change according to the other player's decision, then this decision is called the dominant strategy for that player. When the best choice for a player is independent from the other player's decision, then the best choice is dominant strategy. When both players have a dominant strategy and follow it, the game is concluded in a Nash Equilibrium which is considered the final outcome and endpoint of the game. As an application of Game Theory, concepts of dominant strategy and Nash Equilibrium are applicable for Prisoner's Dilemma too.

Table 1: The Matrix Representation of The Prisoner's Dilemma

|  |  | $\boldsymbol{P}_{\mathbf{2}}$ |  |
| :---: | :---: | :---: | :---: |
|  | $U_{1}, U_{2}$ | C | D |
| $\boldsymbol{P}_{\mathbf{1}}$ | C | $-1,-1$ | $-3,0$ |

The numbers in the shaded cells represent the utility functions of each player $\left(U_{1}, U_{2}\right)$. "C" signifies the decision to cooperate, and "D" signifies the decision to defect. $P_{1}$ and $P_{2}$ represents the players and at the intersection of their choices their utility functions (years of sentence) are given. The number before the comma represents the utility function of $P_{1}$ and the number after the comma represents the utility function of $P_{2}$.

For the case where Player 1 chooses to cooperate with the police; Player 2 has two possible outcomes. If she/he chooses to also cooperate she/he will receive a one-year sentence, on the other hand, if she/he chooses to defect she/he will not receive a sentence and will be let free. This means that the choice to defect is more reasonable for Player 2 when Player 1 cooperates. When the other case is observed (Player 1 chooses to defect), Player 2 also has two possible
outcomes. If she/he chooses to also defect she/he will receive two-year sentence, on the other hand, if she/he chooses to cooperate she/he receives a three-year sentence. It is again observed that the choice to defect is more reasonable for Player 2. The outcomes of these observations point out that the choice of effect is more reasonable in both cases for Player 2, hence the dominant strategy for Player 2 is to defect.

Similarly, for the case where Player 2 chooses to cooperate with the police; Player 1 has two possible outcomes. If she/he chooses to also cooperate she/he will receive a one-year sentence, on the other hand, if she/he chooses to defect she/he will not receive a sentence and will be let free. This means that the choice to defect is more reasonable for Player 1 when Player 2 cooperates. When the other case is observed (Player 2 chooses to defect), Player 1 also has two possible outcomes. If she/he chooses to also defect she/he will receive two-year sentence, on the other hand, if she/he chooses to cooperate she/he receives a three-year sentence. It is again observed that the choice to defect is more reasonable for Player 1. The outcome of these observations points out that the choice of effect is more reasonable in both cases for Player 1, hence the dominant strategy for Player 1 is to defect.

Nash Equilibrium happens in a game when both players follow their dominant strategy. Hence, when both players follow their dominant strategy and choose to defect, they game concludes with a 2-year sentence for each, which corresponds to the Nash Equilibrium for this game scenario.

In this assessment, I applied Prisoner's Dilemma to my schoolmates in a hypothetical scenario where they are caught cheating in an exam. First, I determined the dominant case and the Nash Equilibrium in my scenario. Then, I handed out a survey to collect my friends' behavior in such a case and finally studied survey results to observe if majority they behaved according to expectations set of by the Prisoner's Dilemma.

## DATA \& METHODOLOGY

First of all, I discussed with my mathematics teacher and decided how many points she would break from students if they were to cheat in an exam. We decided upon breaking 20 points from each student if they both denied cheating and breaking 40 points from each student if they both confess. For the case where one student confesses and the other denies cheating; we decided to break 60 points from the one that denies and no points from the one that confesses. Note that denying to cheat corresponds to cooperation in original Prisoner's Dilemma scenario and confessing to cheat corresponds to defect in the original Prisoner's Dilemma scenario. And our numbers are directly proportional to values in Prisoner's Dilemma; they are 20 times the corresponding values in there.

So, in our scenario, when Student1 denies cheating (cooperation in original Prisoner's Dilemma scenario), Student2 will lose 20 points if she/he also denies cheating or will lose no points if she/he chooses to confess (defect in original Prisoner's Dilemma scenario). On the other hand, when Student1 confesses cheating; Student2 will lose 60 points if she/he denies cheating or will lose 40 points if she/he confesses. In both situations, defect (confessing to cheat) results in less penalty, so it is the choice with better outcome for Student2 hence defect is the dominant strategy for Student2.

Similarly, when Student2 denies cheating, Student1 will lose 20 points if she/he also denies cheating or will lose no points if she/he confesses cheating. On the other hand, when Student2 confesses cheating; Student1 will lose 60 points if she/he denies cheating or will lose 40 points if she/he chooses to confess. In both situations defect (confessing to cheat) is the better choice for Student1 hence defect is the dominant strategy for Student1.

Table 2 provides a summary of our scenario. As both students have a dominant strategy, survey answers are expected to accumulate on defect. The intersection point where they both students defect (confesses to cheating) and get a 40 points penalty represents the Nash Equilibrium for our scenario.

Table 2: The Adjusted Version of Prisoner's Dilemma with respect to the Survey

|  |  | Student 2 chooses |  |
| :---: | :---: | :---: | :---: |
|  | $U_{1}, U_{2}$ | C | D |
| Student 1 chooses | C | $-20,-20$ | $-60,0$ |
|  | D | $0,-60$ | $-40,-40$ |

In order to collect the choices of my schoolmates, I presented the information in Table 2 to 12 th graders in my school using the survey in Table3. In addition to their choice, I also collected their gender and the program to further analyse student choices.

Table 3: The Survey Conducted to the 12th Grade Students at TED Ankara College

| Mathematics Internal Assessment Survey: Prisoners Dilemma |
| :--- |
| 1. The program you're in: |
| A) National program |
| B) IB |
| C) Science Program |
| 2. Your gender: |
| A) Male |
| B) Female |
| 3. You and your friend cheated in an exam and your teacher noticed it. The teacher talks to you and your |
| friend separately and you are both unaware of each other's testimonies. You are aware that if both of you |
| accept that you cheated, the teacher will award your honesty and will only break 40 points from each of |
| your exams. If you both deny that you cheated, the teacher will break 20 points from each of your exams. |
| If you accept and your friend denies, the teacher will break no points from your exam and 60 from your |
| friend's exam. If you deny and your friend accepts, 60 points will be broken from you and no points will be |
| broken from your friend. Remember that neither you nor your friend know how the other one will act. |
| In this situation, how would you act? |
| A) Accept that you cheated |
| B) Deny that you cheated |

Information collected in the survey about the students are given in Table 4. As shown in Table 4, data from 144 students was collected. The majority were women with 93 women and 51 men. 63 of the students who participated in the survey were in the IB program, 60 were in the National program and 42 were in the Science program. Amongst the students in the IB program 57 students were females and 6 students were males; amongst the students in the national program 30 students were females and 30 students were males; amongst the students from the science program 6 students were female and 15 students were male.

Table 4: Distribution of The Students with respect to Their Genders \& The Program They Are In

| Program /Gender | Female | Male | Total |
| :---: | :---: | :---: | :---: |
| National Program | 30 | 30 | $\mathbf{6 0}$ |
| IB | 57 | 6 | $\mathbf{6 3}$ |
| Science | 6 | 15 | $\mathbf{2 1}$ |
| Total | $\mathbf{9 3}$ | $\mathbf{5 1}$ | $\mathbf{1 4 4}$ |

The answers of each gender and program is shown in Table 5. Amongst the 93 women that participated in the survey 39 of them chose to cooperate whilst 54 chose to defect. Amongst the 51 men who participated in the survey 20 decided upon cooperating whilst 31 decided upon defecting. Data from 63 IB students were collected in which 29 decided to cooperate and 34 chose to defect. Data from 60 national program students were collected 24 accepted to cooperate whilst 36 chose to defect. Amongst the 21 science students 6 cooperated whilst 15 defected.

Table 5: The Answer Distribution according to Gender and Program

| Program /Gender | Female |  | Male |  | Total |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C | D | C | D | C | D |
|  | 12 | 18 | 12 | 18 | $\mathbf{2 4}$ | $\mathbf{3 6}$ |
| IB | 27 | 30 | 2 | 4 | $\mathbf{2 9}$ | $\mathbf{3 4}$ |
| Science | 0 | 6 | 6 | 9 | $\mathbf{6}$ | $\mathbf{1 5}$ |
| Total | $\mathbf{3 9}$ | $\mathbf{5 4}$ | $\mathbf{2 0}$ | $\mathbf{3 1}$ | $\mathbf{6 6}$ | $\mathbf{7 8}$ |

## RESULT

The probability is the most probable event in a case. In order to analyze and compare the distribution of the answers from the students, with respect to their programs and gender besides generally, the distribution of the responses of the students in each program were converted into probabilities. by the usage of Equation 1.

$$
P c=\frac{C}{(C+D)}
$$

## Equation 1

Pc: the probability of cooperate
C: the total amount of students who chose to 'cooperate'
D: the total amount of students who chose to 'defect'

$$
P d=\frac{D}{(C+D)}
$$

Equation 2

Pd : the probability of defect

For example, the probability of the cooperate answer from the females in the national program is calculated by using Equation 1 as shown below:

$$
P c=\frac{12}{12+18}=0.400
$$

Similarly, the probability of the defect answer from the females in the national program is calculated by using Equation 2 as shown below:

$$
P d=\frac{18}{12+18}=0.600
$$

The remaining probabilities can be found in Table 6.

| Program /Gender | Female |  | Male |  | Total |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C | D | C | D | C | D |
| National Program | 0.400 | 0.600 | 0.400 | 0.600 | 0.400 | 0.600 |
| IB | 0.474 | 0.526 | 0.333 | 0.667 | 0.460 | 0.540 |
| Science | 0.000 | 1.000 | 0.400 | 0.600 | 0.286 | 0.714 |
| Total | 0.419 | 0.581 | 0.392 | 0.608 | 0.458 | 0.542 |

Whilst all the other students have made similar decisions, the female students from the science program are observed to differ from them. While all genders in every program the majority of the students chose to defect, all females from the science program have chosen to defect. Hence, the data taken from the female science students were excluded in order to make the results more accurate and reasonable. The probability of the answer 'cooperate' deviates between 0.333 and 0.474 when the data from the science program is excluded. The probability of the answer 'defect' deviates between 0.526 and 0.667 when the data from the science program is excluded. In order to determine whether the students' decisions were coherent among programs and genders or not, the standard deviation of cooperate and defect for each gender and program is calculated by the usage of Equation 3.
$S D=\sqrt{\sum_{i=1}^{n} \frac{\left(X_{i}-\bar{X}\right)^{2}}{n-1}}$

## Equation 3

SD: Standard Deviation of the Observed Data
$X_{i}$ : The probability of the Observed Data
$\bar{X}$ : The average of the Observed Data
n : The amount of the Observed Data

For example, the standard deviation of females in who chose to cooperate is calculated using Equation 3.
$\bar{X}=\frac{(0.400+0.474+0)}{3}=0.291$
$S D=\sqrt{\frac{(0.400-0.291)^{2}+(0.474-0.291)^{2}+(0-0.291)^{2}}{3-1}} \approx 0.255$

These calculations were applied to all data from all genders and programs both separately as cooperate and defect and in total. In addition to this it was also applied without including the data from the females in the science program. The results can be seen on Table 7 below.

Table 7: Standard Deviation for Gender with respect to Observed Data

|  | Female | Male | All |
| :---: | :---: | :---: | :---: |
| C | 0.255 | 0.039 | 0.089 |
| D | 0.255 | 0.039 | 0.089 |
| C without Females in Science Program | - | 0.039 | 0.034 |
| D without Females in Science Program | - | 0.039 | 0.034 |

The values of standard deviation are small enough hence, it can be stated that other than the data from the females in the science program, the results are parallel to my predictions and the standard deviation results are coherent.

If an event has 2 possible outcomes, the distribution of this event can be represented by using binomial distribution. In other words, binomial distribution is applied to events which have two discrete probability outcomes. In the cheating event there are also two discreet outcomes: to cooperate or to defect. Hence the probabilities of the outcomes from the event can be displayed by binomial distribution graphs. In order to calculate the probability of an " X " value in binomial distribution Equation 4 is used.
$P(X)=\frac{n!}{(n-X)!X!} \times(p)^{X} \times(q)^{n-X}$

## Equation 4

n : total number of samples
p : the probability of a student choosing to cooperate
X: the number of students choosing to cooperate
q : the probability of a student choosing to defect

For example, the probability of 66 students among all the students who participated in the survey choosing to cooperate is calculated below.

$$
P(66)=\frac{144!}{(144-66) \times 66!} \times 0.45833^{66} \times 0.54167^{78}=6.513339519 \times 10^{154} \approx 6.513 \times 10^{154}
$$

Average of binomial distribution is calculated with the use of Equation 5.

Average $=n \times p$
Equation 5

For example, the average of students who chose to cooperate among all is calculated below.

Average $=144 \times 0.45833=65.99952 \approx 66$

Mode of binomial distribution is calculated by the use of Equation 6.

Mode $=(n+1) \times p$

## Equation 6

For example, the mode of the students who chose to cooperate among all is calculated below.

Mode $=(144+1) \times 0.45833=66.45785 \approx 66.458$

Variance of the binomial distribution is calculated with the use of Equation 7.
$S^{2}=n \times p \times(1-p)$
Equation 7
$S^{2}$ : Variance

For example, the variance of the students who chose to cooperate among all is calculated below.
$S^{2}=144 \times 0.45833 \times(1-0.45833)=0.2482636 \approx 0.248$

Skewness of the binomial distribution is calculated with the use of Equation 9.

Skewness $=\frac{1-(2 \times p)}{(n \times p \times(1-p))^{1 / 2}}$

For example, the skewness of the students who chose to cooperate among all is calculated below.

Skewness $=\frac{1-(2 \times 0.45833)}{(144 \times 0.45833 \times(1-0.45833))^{1 / 2}}$

The binomial distribution of the probability of cooperate for each gender, program and the general are analyzed below.


Figure 1: Binomial Distribution for All Students
When the graph is analyzed visually it is observed that there are sharp drops to nearly zero probability at the two ends of the graph of distribution. One of these drops occurs at 59 and the other occurs at 99 . This indicates that amongst the 144 people, the possibility of 59 or less people to defect while 85 or more people cooperate and the possibility of 99 or more people to defect while 45 or more people cooperate is close to none. Among these 144 people the most likely outcome is for between 77 and 81 people to defect and between 67 and 63 people to cooperate since the possibility of these amounts of people are above $\% 6$ and the possibility of $\% 6$ distinguishes from other possibilities significantly. With the use of equations $4,5,6,7,8,9$ certain analysis were made about the binomial distribution graph. The average is 78.048 and the mode is 78.59 . The variance is 0.458 . The skewness is -0.169 . values between 77 and 81 are most likely to be observed in the binomial distribution graph. Since the skewness is between -0.5 and 0.5 the distribution could be described as symmetrical.


Figure 2: Binomial Distribution for Females
When the graph is analyzed visually it is observed that there are sharp drops to nearly zero probability at the two ends of the graph of distribution. One of these drops occurs at 39 and the other occurs at 70 . This indicates that amongst the 93 people, the possibility of 39 or less people to defect while 54 or more people cooperate and the possibility of 70 or more people to defect while 23 or more people cooperate is close to none. Among these 93 people the most likely outcome is for between 52 and 58 people to defect and between 41 and 35 people to cooperate since the possibility of these amounts of people are above $\% 6$ and the possibility of $\% 6$ distinguishes from other possibilities significantly. The average is 54.033 and the mode is 54.614 . The variance is 22.640 . The skewness is -0.004. Values between 52 and 58 are most likely to be observed in the binomial distribution graph. Since the skewness is between -0.5 and 0.5 the distribution could be described as symmetrical.


Figure 3: Binomial Distribution for Males
When the graph is analyzed visually it is observed that there are sharp drops to nearly zero probability at the two ends of the graph of distribution. One of these drops occurs at 21 and the other occurs at 43. This indicates that amongst the 51 people, the possibility of 21 or less people to defect while 30 or more people cooperate and the possibility of 43 or more people to defect while 8 or more people cooperate is close to none. Among these 51 people the most likely outcome is for between 30 and 35 people to defect and between 21 and 16 people to cooperate since the possibility of these amounts of people are above $\% 6$ and the possibility of $\% 6$ distinguishes from other possibilities significantly. The average is 31.008 and the mode is 31.616 . The variance is 12.155 . The skewness is -0.009 . Values between 30 and 35 are most likely to be observed in the binomial distribution graph. Since the skewness is between -0.5 and 0.5 the distribution could be described as symmetrical.


Figure 4: Binomial Distribution for IB Program
When the graph is analyzed visually it is observed that there are sharp drops to nearly zero probability at the two ends of the graph of distribution. One of these drops occurs at 22 and the other occurs at 48 . This indicates that amongst the 63 people, the possibility of 22 or 13 less people to defect while 41 or more people cooperate and the possibility of 48 or more people to defect while or more people cooperate is close to none. Among these 63 people the most likely outcome is for between 31 and 39 people to defect and between 32 and 24 people to cooperate since the possibility of these amounts of people are above $\% 6$ and the possibility of $\% 6$ distinguishes from other possibilities significantly. The average is 34.020 and the mode is 34.560 . The variance is 15.650 . The skewness is -0.003 . Values between 31 and 394 are most likely to be observed in the binomial distribution graph. Since the skewness is between -0.5 and 0.5 the distribution could be described as symmetrical.


Figure 5: Binomial Distribution for National Program
When the graph is analyzed visually it is observed that there are sharp drops to nearly zero probability at the two ends of the graph of distribution. One of these drops occurs at 25 and the other occurs at 49 . This indicates that amongst the 60 people, the possibility of 25 or less people to defect while 35 or more people cooperate and the possibility of 49 or more people to defect while 11 or more people cooperate is close to none. Among these 60 people the most likely outcome is for between 34 and 41 people to defect and between 26 and 19 people to cooperate since the possibility of these amounts of people are above $\% 6$ and the possibility of $\% 6$ distinguishes from other possibilities significantly. The average is 36 and the mode is 36.600 . The variance is 14.400 . The skewness is 0.007. Values between 34 and 41 are most likely to be observed in the binomial distribution graph. Since the skewness is between -0.5 and 0.5 the distribution could be described as symmetrical.


Figure 6: Binomial Distribution for Science Program
When the graph is analyzed visually it is observed that there are sharp drops to nearly zero probability at the two ends of the graph of distribution. One of these drops occurs at 9 and the other occurs at 20. This indicates that amongst the 21 people, the possibility of 9 or less people to defect while 12 or more people cooperate and the possibility of 20 or more people to defect while 1 or more people cooperate is close to none. Among these 21 people the most likely outcome is for between 13 and 19 people to defect and between 8 and 2 people to cooperate since the possibility of these amounts of people are above $\% 6$ and the possibility of $\% 6$ distinguishes from other possibilities significantly. The average is 14.994 and the mode is 15.708 . The variance is 4.288 . The skewness is 0.050. Values between 13 and 19 are most likely to be observed in the binomial distribution graph. Since the skewness is between -0.5 and 0.5 the distribution could be described as symmetrical.

In order to visualize all the choices of the students in a single graph, the scatter plot of cooperate and defect data of each gender in each group was created (Figure 1). The data points represent the choices of each gender in each program. The $y$-axis represents the probability of defect while the $x$-axis represents the probability of cooperate. The probabilities of the two choices are shown in a single graph by the placement at the intersections of the data points. For example, the data point in the blue circle in Figure 1 demonstrates the choices of the female students in the IB program. The probability of defect is 0,526 and the probability of cooperate is 0,474 . Each program was analyzed both separated into genders and generally, also the total of all data was also analyzed, hence 12 data points (all National, all IB, all Science, National female, National male, IB female, IB male, Science female, Science male, all females, all males, total) are present. However, some data points cannot be observed visually since data points with equal probabilities overlap with each other.


Figure 7: Scatter Plot of Cooperate and Defect Data

The data point in the green circle represents the female choices in the science classes. This data can be taken in consideration as an outlier, which means an observation that is detached from other values of a sample, since it lies an abnormal distance from the other data points. The reason for this may be that there are a very limited number of females in the science program who attended the survey. Hence, it is most probable that they have influenced each other's choices. In addition to this the data points in the red circle represents the male students in the IB program and the total of data collected from the science program.

Similar to the previous case with the females in science program, only 6 males attended the survey from the IB classes. This one again creates the possibility that they may have influenced each other's opinions and hence cause a result that was not expected in the beginning of the survey. The reason the data from the science program is inside the red circle is because the females in the science program are the outliers. Other than these specific cases all the choices are as expected, and the majority of the students have decided upon the dominant strategy (defect)

## CONCLUSION

In this assessment, the concept, rules and constraints of Prisoner's Dilemma was applied to a cheating case and the behavior of $12^{\text {th }}$ grade students from different programs and genders were analyzed. In order to get data, a survey was handed out to 144 students from the National, IB and Science programs. Two choices of defect and cooperate were offered to the students and the outcomes were analyzed. The dominant strategy of the cheating case was determined to be defect. To better understand the behavior of the students, the binomial distribution of probabilities of the cooperate choice was graphed and analyzed.

When all the data is analyzed, it is observed that most of the students chose to defect. Among the 144 students approximately 78 of them have chosen to defect. When the data from the females is analyzed, it is observed that most of the females chose to defect. Out of 93 females, approximately 54 of them have chosen to defect. When the data from males is analyzed, it is observed that more than half of the males have also chosen to defect. Besides, the probability of males choosing defect is observed to be higher than the probability of females choosing to defect since out of the 51 males, approximately 31 of them have chosen to defect.

When the programs are analyzed separately, the expected outcome is also reached. In the IB program, out of the 63 students who participated in the survey, approximately 34 have chosen to defect. In the National program, 60 students had participated in the survey and 36 of them have chosen to defect. In the Science program, out of the 21 students who participated in the survey, approximately 15 students chose to defect. The students in the science program were observed to have a tendency to choose defect, this may be because they were less in number and might have been influenced by each other.

The main goal of this assessment was to determine whether or not Prisoner's Dilemma could be applied to a cheating scenario that students may come across in their school life. After discussions with my teacher, the points that would be taken from the students were decided and the presence of a dominant strategy was expected. Based on the points taken from the students that were decided before the dominant strategy was determined to be defect/defect. When the data was analyzed, it was observed that the students' actions were harmonious with the initial predictions and the students tended to defect just like expected independent to the program and the gender of the student.

In conclusion, Prisoner's Dilemma is appliable to a cheating scenario in high school and when it is applied, the students are observed to act with respect to the dominant strategy of Prisoner's Dilemma. Hence if a scenario like this occurs, the students are expected to defect according to Prisoner's Dilemma.

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