Calculating the optimum efficiency of a real-life thermoelectric generator and the maximum coefficient of performance of a thermoelectric refrigerator

Subject: Physics

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Abstract

With the high global warming levels, environmentally friendly devices have started to be the center of many researches. This essay specifically focuses on thermoelectric refrigeration systems since they are amongst the most efficient and environmentally friendly cooling systems. The use of thermoelectric refrigerators are limited to a small scale in daily life applications. The purpose of this essay is to discover the reason behind the limited use. Exploring how basic thermoelectric generators work and finding a real-life application to compare the theoretical maximum efficiency and the optimum efficiency of daily-life ideal materials is the first part of the investigation. The calculation of the theoretical minimum temperature needed in order to achieve the working current and the maximum coefficient of performance is the second part. These two parts form the flow of the essay in indicating the efficiency and utility of thermoelectric refrigerators.

Word count: 142

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1.Introduction

<u>Research Question:</u> *Investigating why it is not efficient yet to make a thermoelectric refrigerator in a large scale?*

The greenhouse effect, a naturally occurring effect which stabilizes Earth's atmosphere and keeps it warm, has become a harmful occurrence due to the excessive rise in carbon dioxide levels caused by human activity. In order to decrease this rise, researches have been trying to find environmentally friendly methods in a number of fields, one of them being the refrigeration systems. At present; vapor compression, sorption, thermoelectric and Stirling Cycles are the available cooling systems.[1] In all the refrigeration systems, vapor compression doesn't significantly affect global warming and is the most energy efficient with the first runner-up being thermoelectric systems.[2] Although vapor compression systems seem to be the most beneficial, they also produce the most noise thus making them unusable in auditory sensitive areas such as domestic use. So, the following option, thermoelectric systems can be identified to be the best candidate.

In 1821, Thomas Johann Seebeck discovered that when the ends of a circuit which consists of two dissimilar metals are conserved at two different temperatures, an electric current will form in the circuit. In 1834, Jean Charles Athanase Peltier discovered the inverse of the Seebeck

[1] J.R., H. C. (2011). Thermodynamic Comparison of Thermoelectric, Strling, and Vapor Compression Portable Coolers.

[2] Johan Mardini-Bovea, G. T.-D.-l.-H.-F.-M.-T. (2019). A Review to Refrigeration With Thermoelectric Energy Based on The Peltier Effect. *Dyna*, 9-18.

effect.[3] When an electric current flows in a circuit consisting of two dissimilar metals, a temperature difference will be created at the two junctions.

When the circuits with two dissimilar metals are combined, thermoelectric modules are formed. These modules work accordingly to the Peltier effect, when a current flows through the thermoelectric module, heat differences, thus, heat transfer occurs in between the circuits of the two dissimilar metals. Thermoelectric cooling systems consist of thermoelectric modules along with a power source. A thermoelectric cooling system which operates with a thermoelectric module is seen in (**Figure 1**). Thermoelectric coolers are used in many areas such as aerospace and the military. Although this technology is outstanding, it is said to have a low coefficient of performance (COP) particularly in large-scale applications.

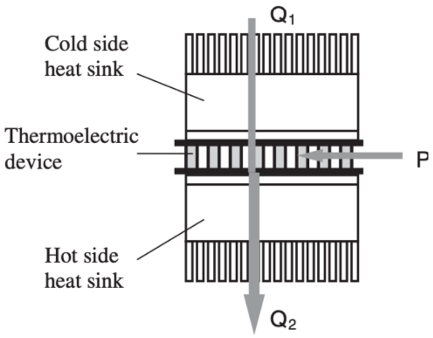


Figure 1: *Thermoelectric cooler. Where, Q_1 is the heat input, Q_2 is the heat which flows to the air, P is the electrical power supplied.*[4]

2.Thermoelectric Fundamentals

The Joule effect

This effect (also known as Joule heating) is the physical effect in which an energy conversion happens. The electrical energy of a current passing through a resistance is converted into

[3] Andresen, M. H. (2010). Generalized Performance Charateristics of Refrigeration and Heat Pump Systems. *Physics Research International*.

[4] Ma, S. B. (2004). Improving The Coefficient of Performance of Thermoelectric Cooling Systems: a Review. *International Journey of Energy Research*, 753-768.

thermal energy through resistive losses. This thermal energy can be observed through the rise in the temperature of the conductor. This effect is irreversible.

The Seebeck effect

This effect is the event that takes place between two dissimilar electrical conductors or semiconductors. When a temperature difference is created between them, a voltage difference is created. If the dissimilar metals are joined in the form of a loop, an electromotive force (emf), hence an electric current is created.

The reason for this voltage difference is the movement of the valance electrons. When one of the two conductors or semiconductors are heated, an electron flow from the hotter metal toward the cooler metal is created. This happens because electrons tend to migrate to the place with lower energy. Unlike the Joule effect, the Seebeck effect is reversible. The Seebeck coefficient is the ratio between emf and temperature change. The formula can be made up as: E_{emf} = -$\alpha \nabla$T, where α is Seebeck coefficient ΔT, is temperature gradient.

The Peltier effect

This effect also occurs between two dissimilar conductors. When a loop is created with the two conductors and a voltage is applied, an electric current is created. When the current passes through the junctions, a temperature difference is created. While one of the junctions becomes hot, the other one cools down. It can be phrased as the opposite of the Seebeck effect. If the cold junction is placed inside of an insulated area and the hot junction is placed outside, it can be used to cool the region. Like the Seebeck effect, Peltier effect is also reversible. The Peltier coefficient is the ratio between heat power and current. The formula can be made up as: $\mathrm{H} = \pi I$ where H is the heat power, π is the Peltier coefficient and I is current.

Coefficient of performance (COP)

This coefficient is the ratio of cooling power to supplied electrical power. High efficiency and low energy consumption leads to high COP. In refrigeration systems, the coefficient of performance usually exceeds one.

3. Heat transfer and thermoelectric generator

3.1 Heat transfer in a homogeneous conducting bar

As stated before, electrons move towards the place with the lowest energy. The same goes for heat as well. Heat flow always goes from higher temperature to lower temperature.

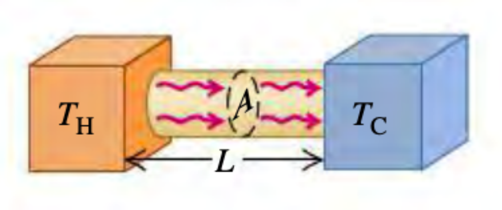


Figure 2: *Heat transfer in a homogenous conducting bar*[5]

A current I passes through (**Figure 2**) a conducting bar with length L, resistivity ρ, and thermal conductivity k. The left end of the bar is located at the coordinate x=0 and the right end of the bar can be excogitated to be located at the coordinate x=L. The temperature on x=0 is maintained at a temperature T_H and x=L at a lower temperature T_C . Therefore, heat flows from left to right. The Joule effect is neglected.

As a quantity of heat dQ is transferred through the rod in a time dt, the rate of heat flow can be recognized as dQ/dt. This rate is called the heat current and is symbolized by H. Thus, $H=$

[5] Hugh D. Young, R. A. (2012). *Sears and Zemansky's University Physics with Modern Physics.* United States of America: Pearson Education Inc.

dQ/dt. It is designated by experiments that the heat current is directly proportional to the cross-sectional area A of the rod and to the temperature difference dT $(T_H - T_C)$. Also, it is deducted that heat current is inversely proportional to the rod length L. These proportions can be stated in the form of a formula.

$$H = \frac{dQ}{dt} = kA\frac{dT}{L}$$

Where,

H: heat power (**J**)

L: length of rod (**m**)

A: cross-sectional area of rod ($\mathbf{m}^2$)

dt: time (**s**)

dT: temperature difference between ends (**K**)

k: thermal conductivity of rod material (**W/mK**)

The formula above can be converted into a function in order to find the temperature distribution when x varies at a steady rate.

$$H(x) = -kA\frac{dT(x)}{dx} \tag{1}$$

$T(x)$ is the temperature distribution and it can be found using conservation of energy. Work W is Force F times distance and it can be represented as:

$$W = F \times dx \tag{2}$$

In this case, $F = qE$. In words, Force equals charge times Electric field. Electric field also equals potential difference divided to distance. $E = \frac{V}{d}$. If these formulas are combined;

$$W = qE \times dx \tag{3}$$

$$W = q\frac{V}{dx} \times dx \tag{4}$$

$$W = qV \tag{5}$$

can be found.

Current I is the flow of charge q in a time t so it can be expressed as $I = \frac{q}{t}$. When this equation is also added to the previous formula; $W = VIt$ is deducted. Potential difference V is current I times resistance *R*. Basically, $V = IR$. Thus, $W = I^2Rt$

Power is the amount of energy transferred W per unit time t and is expressed as $P = \frac{W}{t}$. Therefore,

$$P = \frac{I^2Rt}{t} \tag{6}$$

$$P = I^2R \tag{7}$$

is deducted.

$$R = \rho\frac{L}{A} \tag{8}$$

Where,

R: resistance (**Ω**)

L: length (**m**)

A: cross-sectional area ($\mathbf{m}^2$)

ρ: resistivity ($\mathbf{\Omega} \times \mathbf{m}$)

$$P = I^2 \rho \frac{dx}{A} \tag{9}$$

$$-kA\frac{dT(x)}{dx} = I^2 \rho \frac{dx}{A} \tag{10}$$

$$-kA\frac{d^2T(x)}{dx^2} = \rho \frac{I^2}{A} \tag{11}$$

$$\frac{d^2T(x)}{dx^2} = -\rho \frac{I^2}{kA^2} \tag{12}$$

$T(x) = ax^2 + bx + c$ if we take the second derivative; T''$(x) = 2a$

$-\rho \frac{I^2}{2kA^2} = \mathrm{a}$ →

$$T(x) = -\rho \frac{I^2}{2kA^2} x^2 + bx + c \tag{13}$$

when x=0 ;

$$\mathrm{T}_H = c \tag{14}$$

when x=L ;

$$T(x) = -\rho \frac{I^2}{2kA^2} L^2 + bL + c \tag{15}$$

$$\mathrm{T}_C = -\rho \frac{I^2}{2kA^2} L^2 + bL + \mathrm{T}_H \tag{16}$$

$$\mathrm{T}_H - \mathrm{T}_C - \rho\frac{I^2}{2kA^2}L^2 = -bL \tag{17}$$

$$\frac{-(\mathrm{T}_H - \mathrm{T}_C)}{L} + \rho\frac{I^2}{2kA^2}L = b \tag{18}$$

Since $T(x)$ can be expressed as $T(x) = ax^2 + bx + c$;

$$T(x) = -\rho\frac{I^2}{2kA^2}\,x^2 + \left(-\frac{\mathrm{T}_H - \mathrm{T}_C}{L} + \rho\frac{I^2}{2kA}L\right)x + \mathrm{T}_H \tag{19}$$

Now that the temperature distribution which is the variable $T(x)$ is deducted, the heat current H can be found by writing the variables in place in equation (**1**).

The first derivative of $T(x)$ is ; $\frac{dT(x)}{dx} =$

$$-2\rho\frac{I^2}{2kA^2}\,x + \rho\frac{I^2}{2A}L - \frac{\mathrm{T}_H-\mathrm{T}_C}{L} \tag{20}$$

$$H(x) = -kA\left(-\rho\frac{I^2}{kA^2}\,x + \rho\frac{I^2}{2kA}L - \frac{\mathrm{T}_H - \mathrm{T}_C}{L}\right) \tag{21}$$

$$H(x) = \frac{\rho I^2}{A}x - \frac{\rho I^2 L}{2} + \frac{kA(\mathrm{T}_H - \mathrm{T}_C)}{L} \tag{22}$$

$$H(x) = \frac{\rho I^2}{A}\left(x - \frac{L}{2}\right) + \frac{kA}{L}(\mathrm{T}_H - \mathrm{T}_C) \tag{23}$$

Thus, the exact heat current at coordinates x=0 and x=L can be found.

$$H(0) = \frac{-\rho I^2 L}{2A} + \frac{kA}{L}(T_H - T_C) \tag{24}$$

$$H(L) = \frac{\rho I^2 L}{2A} + \frac{kA}{L}(T_H - T_C) \tag{25}$$

In order to have a more composed equation; we can identify

thermal conductance $K = \frac{kA}{L}$ and internal resistance $R = \frac{\rho L}{A}$. Therefore ;

$$H(0) = \frac{-RI^2}{2} + K(T_H - T_C) \tag{26}$$

and

$$H(L) = \frac{RI^2}{2} + K(T_H - T_C) \tag{27}$$

<u>3.2 Relation between Peltier and Seebeck Coefficients</u>

In order to understand a thermoelectric generator, the relationship between the Peltier and Seebeck effects must be known first.

The Joule effect is neglected for the following formulas.

Carnot cycle is the one complete cooling/heating cycle in which the cooled/heated material turns back to its original temperature. The cycle is named after Sadi Carnot who deducted the formula for the maximum possible efficiency of a thermoelectric generator in terms of the maximum and minimum temperatures during the cycle. In the equation, hot junction temperature is represented by T_H and cold junction temperature is represented by T_C. The energy which goes in the form of heat and comes out as mechanical work is called efficiency, represented as η. When written in equation form

$$\eta = \frac{T_H - T_C}{T_H} \tag{28}$$

is found.

Efficiency η also equals power output divided by power input. In order to find the power input, the Peltier effect is needed, since it consists of the heat power which appears in a junction. Because the heat flows from hot to cold; the heat power at the hot junction with the temperature T_H is the power input. When applied accordingly,

$$H_H = \pi_H I \tag{29}$$

is found. The power output is;

$$P = IV \tag{30}$$

$$V = E_{emf} \tag{31}$$

$$E_{emf} = \alpha \nabla T \tag{32}$$

$$P = \alpha \nabla T I \tag{32}$$

$$P = \alpha (T_H - T_C) I \tag{34}$$

Thus;

$$\eta = \frac{P}{H_H} = \frac{\alpha (T_H - T_C) I}{H_H} \tag{35}$$

where,

α : Seebeck coefficient (**V/K**)

V: potential difference (**V**)

P: power produced (**J**)

∇T: temperature gradient (**K**)

E_{emf}: energy of electromotive force (**V**)

If the two equations (**28**) and (**32**)are combined;

$$\frac{\alpha (T_H - T_C) I}{H_H} = \frac{T_H - T_C}{T_H} \tag{36}$$

is found

$$\frac{\alpha(T_H - T_C)I}{\pi_H I} = \frac{T_H - T_C}{T_H} \quad (37)$$

$$\frac{\alpha(T_H - T_C)}{\pi_H} = \frac{T_H - T_C}{T_H} \quad (38)$$

$$\pi_H(T_H - T_C) = T_H\alpha(T_H - T_C) \quad (39)$$

$$\pi_H = \alpha T_H \quad (40)$$

which displays the relationship between the Seebeck and Peltier coefficients is found.

3.3 Thermoelectric generator

Hereafter the Peltier coefficient π is taken to be equal to αT for all temperatures and the Joule effect must be included in consideration.

Two conductive bars A and B with the length L are used in (**Figure 4)**. Their cross-sectional areas, resistivities and thermal conductivities are A_A, A_B ; ρ_A, ρ_B ; k_A, k_B in order. The lower ends of the bars are connected to a load of resistance R_L.

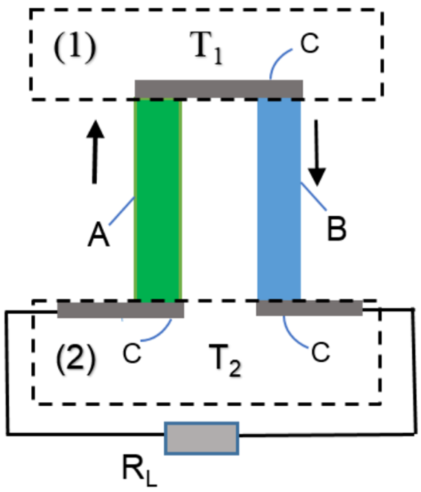


Figure 3: *Thermoelectric generator*[6]

Where,

R_L: a load of resistance

[6] Asian Physics Olympiad. (2018). Thermoelectric effects and their applications inthermoelectric generator and refrigerator. *19th APhO Q3*. Hanoi, Vietnam

$T_1 : T_H$

$T_2 : T_C$

A : homogenous conducting bar

B : homogenous conducting bar

Internal resistance is

$$R = R_A + R_B = \frac{\rho_A L}{A_A} + \frac{\rho_B L}{A_B} \tag{41}$$

Thermal conductance is

$$K = K_A + K_B = \frac{k_A A_A}{L} + \frac{k_B A_B}{L} \tag{42}$$

The efficiency of a thermoelectric generator is defined as $\eta = \frac{P_L}{H_H}$, where P_L is the power of the load. The ratio between the load R_L and internal resistance R is represented as $\frac{R_L}{R} = m$

$$\eta = \frac{P_L}{H_H} \tag{43}$$

When T_H is when x=0, the equation (**26**) is interpreted for a thermocouple,

$$H_H = K(T_H - T_C) + \alpha T_H I - \frac{RI^2}{2} \tag{44}$$

can be found

$$\eta = \frac{I^2 R_L}{K(\mathrm{T}_H - \mathrm{T}_C) + \alpha T_H I - \frac{RI^2}{2}} \tag{45}$$

$$\eta = \frac{\frac{I^2 R_L}{I^2 R}}{\frac{K(\mathrm{T}_H - \mathrm{T}_C)}{I^2 R} + \frac{\alpha T_H I}{I^2 R} - \frac{RI^2}{2I^2 R}} \tag{46}$$

$$\eta = \frac{m}{\frac{K(\mathrm{T}_H - \mathrm{T}_C)}{I^2 R} + \frac{\alpha T_H}{IR} - \frac{1}{2}} \tag{47}$$

$$I = \frac{V}{R} \tag{48}$$

$$V = E_{emf} \tag{49}$$

$$E_{emf} \ = \alpha \nabla T \tag{50}$$

$$I = \ \frac{\alpha(\mathrm{T}_H - \mathrm{T}_C)}{R + R_L} \tag{51}$$

$$I = \frac{\frac{\alpha(\mathrm{T}_H - \mathrm{T}_C)}{R}}{\frac{R + R_L}{R}} \tag{52}$$

$$I = \frac{\frac{\alpha(\mathrm{T}_H - \mathrm{T}_C)}{R}}{1 + m} \tag{53}$$

$$I = \frac{\alpha(\mathrm{T}_H - \mathrm{T}_C)}{R(1 + m)} \tag{54}$$

If we write I in place in the equation (**47**);

$$\eta = \frac{m}{\frac{K(\mathrm{T}_H - \mathrm{T}_C)}{\frac{\alpha^2(\mathrm{T}_H - \mathrm{T}_C)^2}{R^2(1+m)^2} R} + \frac{\alpha T_H}{\frac{\alpha(\mathrm{T}_H - \mathrm{T}_C)}{R(1+m)} R} - \frac{1}{2}} \tag{55}$$

$$\eta = \frac{m}{\frac{KR(1+m)^2}{\alpha^2(\mathrm{T}_H - \mathrm{T}_C)} + \frac{T_H(1+m)}{(\mathrm{T}_H - \mathrm{T}_C)} - \frac{1}{2}} \tag{56}$$

$$\eta = \frac{m(\mathrm{T}_H - \mathrm{T}_C)}{\frac{KR(1+m)^2}{\alpha^2} + T_H(1+m) - \frac{(\mathrm{T}_H - \mathrm{T}_C)}{2}} \tag{57}$$

To identify the efficiency of a thermoelectric generator, three properties of a thermocouple are essential. They are; low thermal conductivity (to retain heat at the junctions), low electrical resistance (to minimize the Joule heating) and a maintained large temperature gradient. These variables are combined in a singular quantity, figure-of-merit,

$$Z = \frac{\alpha^2}{KR} \tag{58}$$

If the figure of merit and the ideal Carnot cycle efficiency $(\mathbf{28})$ are incorporated to the efficiency formula;

$$\eta = \frac{\dfrac{m(\mathrm{T}_H - \mathrm{T}_C)}{\mathrm{T}_H}}{\dfrac{KR(1+m)^2}{\alpha^2 \mathrm{T}_H} + \dfrac{T_H(1+m)}{\mathrm{T}_H} - \dfrac{(\mathrm{T}_H - \mathrm{T}_C)}{2\mathrm{T}_H}} \tag{59}$$

$$\eta = \frac{m\eta_c}{\dfrac{(1+m)^2}{Z\mathrm{T}_H} + (1+m) - \dfrac{\eta_c}{2}} \tag{60}$$

If the deducted formula is interpreted, the direct proportion between the figure of merit Z and efficiency is found. In other words, this formula shows that when Z increases, the efficiency of the thermoelectric generator increases as well. The condition $ZT_H \geq 1$ can be used for material application in thermoelectric generators.

<u>3.4 The maximum efficiency</u>

When the electric power of the load is maximum, $P_L = P_{max}$. Since power and resistance is directly proportional, $R_L = R_{max}$ as well. Thus, in order for the efficiency of the thermocouple to be maximum, R_L must be equal to R and $m = 1$.

If $m = 1$ in the equation (**60**),

$$\eta_P = \frac{T_H - T_C}{\frac{4}{Z} + \frac{3T_H + T_C}{2}} \tag{61}$$

In order to find the derivate of the efficiency formula and equalize it to zero, it should be written in $\eta = ax^2 + bx - c$ format. To do that some components can be written as $\frac{1}{Z(\mathrm{T}_H - \mathrm{T}_C)} = a$ and $\frac{T_H}{(\mathrm{T}_H - \mathrm{T}_C)} = b.$

Thus, $\eta_c = \frac{1}{b}$ and (**60**) can be written as,

$$\eta = \frac{m\frac{1}{b}}{\frac{(1+m)^2}{Z\mathrm{T}_H} + (1+m) - \frac{1}{2b}} \tag{62}$$

$$\eta = \frac{m}{\frac{T_H(1+m)^2}{Z(\mathrm{T}_H - \mathrm{T}_C)\mathrm{T}_H} + b(1+m) - \frac{1b}{2b}} \tag{63}$$

$$\eta = \frac{m}{a(1+m)^2 + b(1+m) - \frac{1}{2}} \tag{64}$$

The efficiency is maximum $\eta = \eta_{\max}$ when the resistance ratio *m* takes some value which is denoted by M .

The equation $\frac{d\eta}{dm} = 0$ has the solution

$$M = \sqrt{1 + \frac{2b-1}{2a}} \tag{65}$$

When a and b are rewritten as the original variables;

$$M = \sqrt{1 + Z\frac{(\mathrm{T}_H + \mathrm{T}_C)}{2}} \tag{66}$$

is found

Ultimately, the expression for the maximum efficiency η_{max} can be deducted with combining the previous two equations.

$$\eta_{max} = \frac{(\mathrm{T}_H - \mathrm{T}_C)}{\mathrm{T}_H} \times \frac{(M-1)}{\left(M + \frac{\mathrm{T}_C}{\mathrm{T}_H}\right)} \tag{67}$$

3.5 The maximum figure of merit

Previously, the direct proportion between the figure of merit Z and efficiency was displayed. Ideally, the cross-sectional areas A_A, A_B of the bars A and B in the thermocouple are chosen so that the figure of merit has maximum value $Z = Z_{max}$.

According to (**58**), Z and KR are inversely proportional. Thus, when KR is smallest, Z is maximum. For the sake of simplification, $KR = y$ and $\frac{A_A}{A_B} = x$

$$(k_A A_A + k_B A_B)\left(\frac{\rho_A}{A_A} + \frac{\rho_B}{A_B}\right) = y \tag{68}$$

If the equation is multiplied by $\frac{A_B}{A_B}$,

$$\left(\frac{k_A A_A}{A_B} + \frac{k_B A_B}{A_B}\right)\left(\frac{\rho_A A_B}{A_A} + \frac{\rho_B A_B}{A_B}\right) = y \tag{69}$$

$$(k_A x + k_B)\left(\frac{\rho_A}{x} + \rho_B\right) = y \tag{70}$$

With this equation it is clearly displayed that the function y has the minimum $x = x_{max}$ where,

$$x_{max} = \sqrt{\frac{\rho_A k_B}{\rho_B k_A}} \tag{71}$$

If the cross-sectional area ratios are applicable in the equation above, then,

$$y_m = \left[(\rho_A k_A)^{\frac{1}{2}} + (\rho_B k_B)^{\frac{1}{2}}\right]^2 \tag{72}$$

$$Z_m = \frac{\alpha^2}{\left[(\rho_A k_A)^{\frac{1}{2}} + (\rho_B k_B)^{\frac{1}{2}}\right]^2} \tag{73}$$

<u>3.6 The optimum efficiency</u>

When the figure of merit $Z = Z_{max}$ and the electric power at the load is at its maximum value, the optimum efficiency η_{opt} of the thermoelectric generator is reached. In order to understand the findings more thoroughly, a concreate example with values could be given. Two ideal materials A and B could be listed as $Bi_2Te_{2.7}Se_{0.3}$ and $Bi_{0.5}Sb_{1.5}Te_3$ respectively. The resistivity for bar A ρ_A, is 1.0×10^{-5} and the thermal conductivity for bar A k_A, is 1.4. The resistivity for bar B ρ_B, is 1.0×10^{-5} and the thermal conductivity for bar B k_B, is 1.4. The thermocouple created by these bars is 0.02 meters long and its Seebeck coefficient is 420×10^{-6}. The hot and cold junctions of the thermocouple are kept at temperatures $T_H = 423K$ and $T_C = 303K$.

When the figure of merit found above is applied to the materials A and B,

$$\rho_A k_A = \rho_B k_B \tag{74}$$

since the materials have the same resistivity and thermal conductivity values. Thus,

$$Z_m = \frac{\alpha^2}{4\rho_A k_A} \tag{75}$$

When the values are put in,

$$Z_m = \frac{(420 \times 10^{-6})^2}{4 \times 1.0 \times 10^{-5} \times 1.4} \tag{76}$$

$$Z_m = 3.15 \times 10^{-3} \tag{77}$$

$$\eta_{opt} = \frac{T_H - T_C}{4{Z_m}^{-1} + \frac{3T_H + T_C}{2}} \tag{78}$$

Since $T_H = 423K$ and $T_C = 303K$

$$\eta_{opt} = \frac{423 - 303}{4\frac{1}{3.2 \times 10^{-3}} + \frac{3 \times 423 + 303}{2}} \tag{79}$$

$$\eta_{opt} = 5.84\% \tag{80}$$

The Carnot efficiency η_c (**28**),

$$\eta_c = \frac{120}{423} \tag{81}$$

$$\eta_c = 28.4\% \tag{82}$$

$$\frac{n_{opt}}{\eta_c} = 0.21 \tag{83}$$

According to (**67**) the maximum efficiency η_{max} of the thermocouple is,

$$\eta_{max} = \eta_c \times \frac{(M-1)}{\left(M + \frac{\mathrm{T}_C}{\mathrm{T}_H}\right)} \tag{84}$$

and according to (**66**)

$$M = \sqrt{1 + Z\frac{(T_H + T_C)}{2}} \tag{85}$$

$$M = \sqrt{1 + 3.2 \times 10^{-3}\frac{(423+303)}{2}} \tag{86}$$

$$M = 1.47 \tag{87}$$

$$\eta_{max} = 28.4 \times \frac{(1.47 - 1)}{\left(1.47 + \frac{303}{423}\right)} \tag{88}$$

$$\eta_{max} = 6.1\% \tag{89}$$

4. Thermoelectric refrigerator

4.1 The cooling power and the maximum temperature difference

Now that the basics of thermoelectricity are covered alongside thermocouples, thermoelectric refrigerators, which are the main point of this essay, can be further explained.

where,

$T_1 : T_H$

$T_2 : T_C$

A : homogenous conducting bar

B : homogenous conducting bar

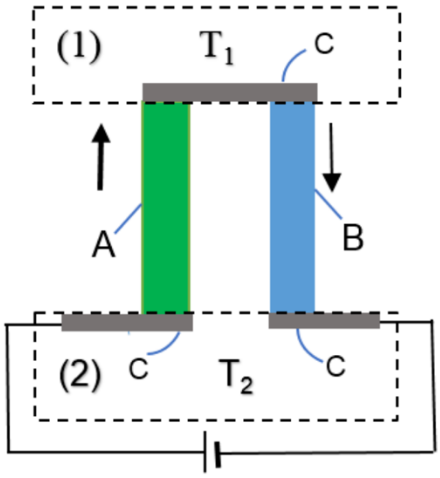


Figure 4: *Thermoelectric refrigerator*[7]

[7] Asian Physics Olympiad. (2018). Thermoelectric effects and their applications inthermoelectric generator and refrigerator. *19th APhO Q3*. Hanoi, Vietnam

As previously said, heat flows from hot to cold. Thus, in the thermoelectric refrigerator the heat current flows from heat source to the bars of the thermocouple.

With that in mind, the cooling power H_c can be written as (**44**). Thus,

$$H_c = K(T_H - T_C) + \alpha T_H I - \frac{RI^2}{2} \tag{90}$$

The maximum temperature difference ΔT can be expressed as

$$\Delta T_{max} = T_c - T_{H_{min}} \tag{91}$$

The maximum cooling power $H_{c_{max}}$ can be deducted by taking the derivative $\frac{dH_c}{dI}$ of the previous equation and equaling it to zero. The derivative is expressed as,

$$I_H = \frac{\alpha \mathrm{T}_H}{R} \tag{92}$$

$$H_{c_m} = \frac{\alpha^2 \mathrm{T}_H}{2R} - K(\mathrm{T}_H - \mathrm{T}_C)\#(93)$$

Since $\frac{dH_c}{dI} = 0,\ H_{c_m} = 0$. Thus,

$$\frac{\alpha^2 \mathrm{T}_H}{2R} = K(\Delta T) \tag{94}$$

$$\frac{\alpha^2 T_{H_{min}}}{2RK} = \Delta T_{max} \tag{95}$$

As previously stated in (**58**),

$$\frac{ZT_{H_{min}}}{2} = \Delta T_{max} \tag{96}$$

4.2 The working current

The thermocouple mentioned in part 1.6 is used as a thermoelectric refrigerator. The figure of merit is $Z_m = 3.15 \times 10^{-3}$ as found in (**77**). When $T_H = T_{H_{min}}$ and $T_C = 300K,$ the working current intensity is I_w. The lowest cooling temperature $T_{H_{min}}$ can be found if the figure of merit is put in (**96**).

$$0 = T_{H_{min}}{}^2 + \frac{2}{Z_m} T_{H_{min}} - \frac{2}{Z_m} T_C \tag{97}$$

$$T_{H_{min}} = \frac{1}{Z_m}\left(\sqrt{1 + 2Z_m T_C} - 1\right) \tag{98}$$

If $Z_m = 3.15 \times 10^{-3}$ found in (**77**) and the value of T_C are written in place,

$$T_{H_{min}} = \frac{1}{3.15 \times 10^{-3}}\left(\sqrt{1 + 2\ \times 3.15 \times 10^{-3} \times\ 300} - 1\right) \tag{99}$$

$$T_{H_{min}} = 2.22 \times 10^2 \tag{100}$$

For the sake of simplification, the cross-sectional areas of the bars A and B are taken to be equal. Thus, $A_A = A_B = 10^{-4} m^2$. The internal resistance is,

$$R = \frac{\rho_A L}{A_A} + \frac{\rho_B L}{A_B} \tag{101}$$

$$R = \frac{2\rho_A L}{A_A} \tag{102}$$

$$R = 4.0 \times 10^{-3} \Omega \tag{103}$$

If the values are put in the equation (**92**),

$$I_w = \frac{4.2 \times 10^{-4} \times 2.22 \times 10^{2}}{4.0 \times 10^{-3}} \tag{104}$$

$$I_W = 23.31\ A \tag{105}$$

is found.

4.3 The coefficient of performance

The coefficient of performance β is used for determining the performance of the thermoelectric refrigerator when the ΔT is less than ΔT_{max}.

$$\beta = \frac{H_c}{P} \tag{106}$$

where,

H_c : cooling power

P: supplied electrical power

According to the law of conservation of energy, the electrical power supplied, P converts and is equal to the Joule heat and Peltier's heat.

If equations (**7**) and (**40**) are written accordingly,

$$P = \alpha(T_C - T_H)I + RI^2 \tag{107}$$

is found.

The equation for coefficient of performance β is deducted when (**90**) and (**107**) are written in place in (**106**)

$$\beta = \frac{\alpha T_H I - K(T_C - T_H) - \frac{RI^2}{2}}{\alpha(T_H - T_C)I + RI^2} \tag{108}$$

$$\beta = -\frac{1}{2} + \frac{\alpha(T_H + T_C)I - 2K(T_C - T_H)}{2[\alpha(T_H - T_C) + RI]I} \tag{109}$$

When the coefficient of performance has its maximum value β_{max}, the current intensity is I_β.

In order to find I_β, the derivative of (**109**) $\frac{d\beta}{dI} = 0$ leads to,

$$-\alpha R(T_H + T_C)I^2 + 4K(T_C - T_H)RI + 2K\alpha(T_C - T_H)^2 = 0 \tag{110}$$

$$\frac{-2\alpha R(T_H + T_C)I^2}{-2\alpha R(T_H + T_C)} + \frac{2K(T_C - T_H)RI}{-\alpha R\frac{(T_H + T_C)}{2}} + \frac{2K\alpha}{-\alpha R\frac{(T_H + T_C)}{2}}(T_C - T_H)^2 = 0 \tag{111}$$

$$I^2 - \frac{2K(T_C - T_H)I}{\alpha\frac{(T_H + T_C)}{2}} - \frac{2K}{R\frac{(T_H + T_C)}{2}}(T_C - T_H)^2 = 0 \tag{112}$$

If we label T_M as the mean of the two junction temperatures,

$$T_M = \frac{(T_H + T_C)}{2} \tag{113}$$

$$I^2 - \frac{2K(T_C - T_H)I}{\alpha T_M} - \frac{2K}{RT_M}(T_C - T_H)^2 = 0 \tag{114}$$

(**114**) has the solution,

$$I_\beta = \frac{K(T_C - T_H)}{\alpha T_M}\left\{\sqrt{1 + ZT_M} + 1\right\} \tag{115}$$

with (**58**) taken into consideration.

(**115**) can be re arranged to be,

$$I_\beta = \frac{\alpha(T_C - T_H)}{R\left\{\sqrt{1+ZT_M} - 1\right\}} \tag{116}$$

If (**116**) written place in (**108**)

$$\beta_{max} = \frac{\alpha T_H \left[\frac{\alpha(T_C - T_H)}{R\left\{\sqrt{1+ZT_M} - 1\right\}}\right] - K(T_C - T_H) - \frac{R\left[\frac{\alpha(T_C - T_H)}{R\left\{\sqrt{1+ZT_M} - 1\right\}}\right]^2}{2}}{\alpha(T_H - T_C)\left[\frac{\alpha(T_C - T_H)}{R\left\{\sqrt{1+ZT_M} - 1\right\}}\right] + R\left[\frac{\alpha(T_C - T_H)}{R\left\{\sqrt{1+ZT_M} - 1\right\}}\right]^2} \tag{117}$$

$$\beta_{max} = \frac{T_H\left[\sqrt{1+ZT_M} - T_C/T_H\right]}{(T_C - T_H)\left[\sqrt{1+ZT_M} + 1\right]} \tag{118}$$

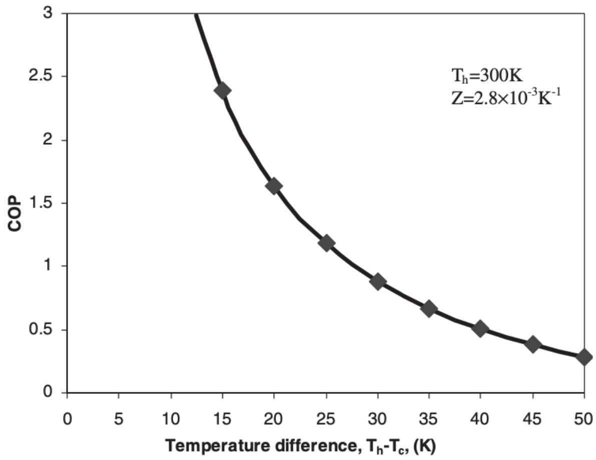
Thus, the maximum value of the coefficient of performance is deducted.

5. Analysis

The equation for the maximum value of coefficient of performance β_{max}, is ultimately indicated by equation (**118**). With the ratios and proportions, it is displayed that the COP will improve if the T_C is increased and the T_H is decreased, leading in a smaller temperature difference.

(**Graph 1**) displays the change in COP due to temperature difference, a commercially available module is investigated at an average domestic use condition which is T_H = 300K. A temperature difference about 25-30K between the ambient and the cabinet is generally needed in order to procure a suitable cooling performance in domestic use thermoelectric refrigerators*. As it can be spotted in (**Graph 1**), these conditions leads to the maximum value of COP to be nearly 0.9-1.2 for commercially available modules.

The COP of the thermoelectric refrigeration systems is heavily affected by the efficiency of the occurring heat exchange. When the heat exchange is highly efficient, the temperature difference between the hot and cold sides of the thermoelectric refrigerator is significantly smaller. Whereas when the heat exchange isn't as efficient, the temperature difference is greater which ultimately leads to a lower COP. Thus, the efficiency of the heat exchange system is directly proportional to the COP value.

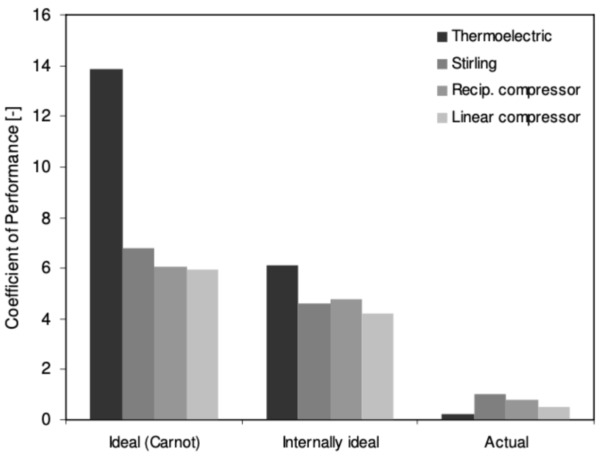


Graph 1: *COP as a function of temperature difference across the module at hot side temperature T_H = 300K.*[8]

In (**Graph 2**), it can be clearly spotted that thermoelectric refrigeration systems are in lead in both Ideal (Carnot) and Internally ideal calculations, thus, making them the most suitable environmentally friendly and efficient system in theory. However, when the theory is applied to daily use, the results indicated that thermoelectric refrigeration systems had the lowest coefficient of performance. This signifies that thermoelectric refrigeration systems aren't as efficient as they could be in real life. This research's results align with the calculations in this

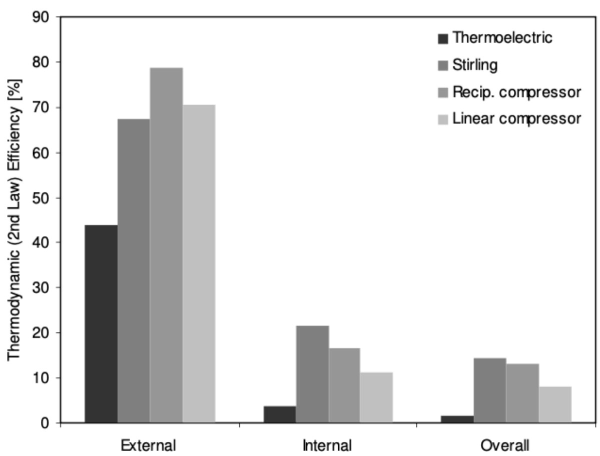
[8] Ma, S. B. (2004). Improving The Coefficient of Performance of Thermoelectric Cooling Systems: a Review. *International Journey of Energy Research*, 753-768.

paper which displays that the use of thermoelectric refrigerators are limited to a small scale due to efficiency reasons.



Graph 2: *Coefficient of performance calculations in Ideal(Carnot), internally ideal and actual applications.*[9]

In (**Graph 3**) which is taken from the same study as (**Graph 2**), the efficiencies of the refrigeration systems are compared. Thermoelectric refrigerators are in last place both internal and externally with quite low efficiencies. The internal efficiency is significantly lower than the external. The overall efficiency is the lowest in thermoelectric refrigerators too. This graph clearly displays the shortcoming of thermoelectric refrigerators in real life use which is low efficiency.



Graph 3: *Efficiency percentage calculations in Internal, External and Overall applications*

[9] J.R., H. C. (2011). Thermodynamic Comparison of Thermoelectric, Strling, and Vapor Compression Portable Coolers.

6. Conclusion & Evaluation

The aim of this paper was to find the reason behind the limited usage of thermoelectric refrigerators in large scales. In the end, a conclusion was found and thus the investigation was successful. The ultimate result is that thermoelectric refrigerators aren't used in large scales because of the materials that are currently available commercially. In order to have a high efficiency and a high COP, a small temperature difference is needed. As it can be seen in (**Graph 1**), the thermoelectric refrigerators that are made with the current usable materials, have a COP nearly 0.9-1.2 at maximum because the temperature difference of the commercial thermoelectric refrigerators are about 25-30K, which isn't ideal. In (**Graph 2**) it can be spotted that thermoelectric refrigerators aren't as efficient in practice compared to the theoretical calculations. The difference between the calculations clearly showcase the shortcoming of the available materials efficiency wise.

In section ***3.6***, two commercially available materials are used for calculations, $Bi_2Te_{2.7}Se_{0.3}$ and $Bi_{0.5}Sb_{1.5}Te_3$. The efficiency of those materials are calculated in (**89**) as 6.1%. This result aligns with (**Graph 3**) in indicating the low efficiency of the commercially available materials

In order to use thermoelectric refrigerators in large scales, new materials need to be sought out. When said ideal materials are found, the COP will increase and the theoretical calculations will align with the commercial use. With higher efficiency, thermoelectric generators will be more widely used. Switching the domestic refrigeration systems with thermoelectric refrigerators will result in a more environment friendly cooling system since they don't significantly affect global warming. Ultimately, the negative effect human activities have on the rising levels in global warming and the greenhouse effect will decrease.

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