# International Baccalaureate: Physics Extended Essay 

Investigatigation of the effect of initial launch velocity on the energy and momentum lost
in an non-ideal elastic collision system

## Research Question

How does varying initial velocities affect the energy loss in a two dimensional elastic collision between uniform disk-like flattened cylindrical masses in a non-ideal system with relation to the law of conservation of energy and momentum?

## Introduction

When I was younger, we went to an amusement park for one of my friend's birthdays. I was captivated by the bumper cars since they were so action-packed and exciting. Initially, I was having lots of fun but a few minutes later another person violently bumped into my car which made me hit my head onto the steering wheel and crack my tooth. At first, I grew extremely afraid of bumper cars but soon that fear sparked an interest in me. We both were in identical cars so why had I sustained such a blow and they had not? Over time, I became more aware of these collisions that I would see around me. My sister has been figure skating for some time which led to me observe all the bumping and falling taking place but this time the colliding people were not identical. The larger the difference in mass was, the harder my sister would fall. Recently, I became fond of billiards and started watching professional games which made me become even more fascinated by collisions. Many players had different styles which often corresponded to the angle of hit and the speed they hit the ball with. I decided on basing my extended essay topic on similar collisions to see how these variables actually affected collisions and why they did so.

In this essay the relationship between energy and momentum loss in non-ideal elastic collisions of uniform flattened cylindrical masses and the initial velocity at which the impacting mass is launched with will be investigated. The focus of the essay will be to look into the effects of the independent variable velocity on the dependent variables the losses of momentum and energy to see if these values are in correlation and if so to establish a relationship between them by discussing the possible reasons behind this correlation with regards to physics.

## Background Information

- Collisions: A collision is any situation where two ore more bodies exert forces onto one another in a period of time.
- Inelastic Collisions: An inelastic collision a collision where the kinetic energy is not conserved due to the internal frictions of the colliding bodies.
- Elastic Collisions: An elastic collision is a collision in which there is no net loss in kinetic energy in the system as a result of the collision. Both momentum and kinetic energy are conserved in elastic collisions.
- Conservation of Energy: The law of conservation of energy states that energy can neither be created nor destroyed - only converted from one form of energy to another. This means that a system always has the same amount of energy, unless it's added from the outside. This is particularly confusing in the case of non-conservative forces, where energy is converted from mechanical energy into thermal energy, but the
overall energy does remain the same. The only way to use energy is to transform energy from one form to another. ${ }^{1}$

$$
K_{i}+U_{i}=K_{f}+U_{f}
$$

Where ;
$\boldsymbol{K}_{\boldsymbol{i}}$ is initial kinetic energy
$\boldsymbol{U}_{\boldsymbol{i}}$ is initial potential energy
$\boldsymbol{U}_{f}$ is final potential energy
$\boldsymbol{K}_{f}$ is final kinetic energy

$$
\frac{1}{2} m_{x} V_{x i}^{2}+\frac{1}{2} m_{y} V_{y i}^{2}=\frac{1}{2} m_{x} V_{x f}^{2}+\frac{1}{2} m_{y} V_{y f}^{2}
$$

Where ;
$\frac{1}{2} m_{x} V_{x i}^{2}$ is the initial kinetic energy of body $\boldsymbol{x}$
$\frac{1}{2} m_{y} V_{y i}^{2}$ is the initial kinetic energy of body $\boldsymbol{y}$
$\frac{1}{2} m_{x} V_{x f}^{2}$ is the final kinetic energy of body $\boldsymbol{x}$
$\frac{1}{2} m_{y} V_{y f}^{2}$ is the final kinetic energy of body $\boldsymbol{y}$

[^0]- Conservation of Momentum: In a closed system (one that does not exchange any matter with its surroundings and is not acted on by external forces) the total momentum remains constant.

$$
m_{x} V_{x i}+m_{y} V_{y i}=m_{x} V_{x f}+m_{y} V_{y f}
$$

Where ;
$\boldsymbol{m}_{\boldsymbol{x}}$ is the mass of body $\boldsymbol{x}$
$\boldsymbol{m}_{\boldsymbol{y}}$ is the mass of body $\boldsymbol{y}$
$\boldsymbol{V}_{\boldsymbol{x} \boldsymbol{i}}$ is the initial velocity of body $\boldsymbol{x}$
$\boldsymbol{V}_{\boldsymbol{y} \boldsymbol{i}}$ is the initial velocity of body $\boldsymbol{y}$
$\boldsymbol{V}_{\boldsymbol{x}}$ is the final velocity of body $\boldsymbol{x}$
$\boldsymbol{V}_{\boldsymbol{y} f}$ is the final velocity of body $\boldsymbol{y}$
$\boldsymbol{m}_{\boldsymbol{x}} \boldsymbol{V}_{\boldsymbol{x} i}$ is the initial momentum of body $\boldsymbol{x}$
$\boldsymbol{m}_{\boldsymbol{y}} \boldsymbol{V}_{\boldsymbol{y} \boldsymbol{i}}$ is the initial momentum of body $\boldsymbol{y}$
$\boldsymbol{m}_{\boldsymbol{x}} \boldsymbol{V}_{\boldsymbol{x f}}$ is the final momentum of body $\boldsymbol{x}$
$\boldsymbol{m}_{\boldsymbol{y}} \boldsymbol{V}_{\boldsymbol{y f}}$ is the final momentum of body $\boldsymbol{y}$

## Proof:


$\overrightarrow{\boldsymbol{F}}_{x y} \Delta \mathrm{t}=\boldsymbol{m}_{x} \boldsymbol{V}_{x f}-\boldsymbol{m}_{\boldsymbol{x}} \boldsymbol{V}_{\boldsymbol{x i}} \quad$ change in momentum of mass $\boldsymbol{m}_{\boldsymbol{x}}$
$\overrightarrow{\boldsymbol{F}}_{\boldsymbol{y} \boldsymbol{x}} \Delta \mathrm{t}=\boldsymbol{m}_{\boldsymbol{y}} \boldsymbol{V}_{\boldsymbol{y f}}-\boldsymbol{m}_{\boldsymbol{y}} \boldsymbol{V}_{\boldsymbol{y} \boldsymbol{i}} \quad$ change in momentum of mass $\boldsymbol{m}_{\boldsymbol{y}}$

Combining the prior two equations $\overrightarrow{\boldsymbol{F}}_{x y} \Delta \mathrm{t}+\overrightarrow{\boldsymbol{F}}_{\boldsymbol{y} x} \Delta \mathrm{t}=\left(\boldsymbol{m}_{x} \boldsymbol{V}_{x f}-\boldsymbol{m}_{x} \boldsymbol{V}_{x i}\right)+\left(\boldsymbol{m}_{y} \boldsymbol{V}_{\boldsymbol{y f}}-\boldsymbol{m}_{y} \boldsymbol{V}_{\boldsymbol{y} i}\right)$

$$
=\left(\vec{F}_{x y}+\vec{F}_{y x}\right) \Delta t=\left(m_{x} V_{x f}+m_{y} V_{y f}\right)-\left(m_{x} V_{x i}-m_{y} V_{y i}\right)
$$

Since $\overrightarrow{\boldsymbol{F}}_{\boldsymbol{x} \boldsymbol{y}}$ and $\overrightarrow{\boldsymbol{F}}_{\boldsymbol{y} \boldsymbol{x}}$ are equal and opposite,

$$
\vec{F}_{x y}+\overrightarrow{\boldsymbol{F}}_{y x}=\mathbf{0}
$$

Thus, $\left(m_{x} V_{x f}+m_{y} V_{y f}\right)=\left(m_{x} V_{x i}-m_{y} V_{y i}\right)$

Momentum after collision $=$ Momentum before collision

Hence, momentum of an isolated system is conserved.

## - Energy Conversion from Potential to Kinetic

The energy in an isolated system is conserved as the mechanic energy is constant. For this reason it could be stated that if the total energy as stored as potential it will be converted to kinetic energy when the height is 0 , directly on the ground.

So, $\mathrm{Ep}=\mathrm{Ek} \quad \frac{1}{2} \mathbf{m} \boldsymbol{v}^{2}=\mathbf{m g h}$


#### Abstract

Aim

The aim of this experiment is to establish a relationship between varying velocities and the energy lost after a two dimensional collision where one object initially remains motionless in an non idealized media with regards to Newtons second law of conservation of energy thus the conservation of momentum.


## Hypothesis

I hypothise that the impacting object will overall determine the loss of energy depending on the speed it is emerging with. Higher velocity will result in a higher energy and momentum loss.

## Variables

- Dependent: Energy lost after the collision.
- Independent:
$>$ The initial velocity of which the impacting body is moving by.


## - Controlled Variables:

> The flat cylindrical bodies:The masses and subsequent volumes of uniform cylindrical masses of the same mateiral are kept constant and for the experiment the two bodies will have different diameters. A separate collision (Part A) will be performed to establish the range of error and accuracy. Two uniform disk-like clyinders with equal masses will be used as collisions between two bodies with equal masses results in the transfer of initial velocities. Therefore when the collision between one object with no initial
velocity and an object with a known velocity collides the static (impacted) body will move with the initial velocity of the impacting object. By performing this and comparing it with the theoric value the expected value of error could be derived.
$>$ The room at which the collisions were performed remained constant through the duration of the video taking process.
$>$ The point of collision was controlled by marking the placement of the mass.

The media at which the collisions were performed was kept constant as it would affect the friction coefficient.

## Materials

- A media with little friction which is created by coating a foam core with a thin layer of oil which is then absorbed by it.
- A motion capture camera will be utilized to record the experiments.
- A software to calculate the velocities of moving objects from videos which in this case is Tracker. The software will help quantify the initial and final velocities.
- Two cylindrical uniform geometric masses with equal masses and one with larger diameter made from the same material.

The masses used in the collisions:
> Two equal masses weighing $0,043 \mathrm{~kg} \pm 0,001$
$>$ One larger mass weighing $0,082 \mathrm{~kg} \pm 0,001$

The larger mass and one smaller mass is used to impact the other stationary smaller mass.

For the collision two masses of equal radii of
 9 cms and equal masses of 0.043 kgs were used. The larger body had a radius of 12 cms and a mass of 0.082 kgs .

- An inclined plane mechamism that can generate different initial velocities. The various heights were controlled by marking the stand and then placing the inclined plane onto the stand. Then with a ruler the inclined plane was adjusted to a parallel position to the marks.
$>$ The first sliding movement was observed at 15 cm so the minimum height was set to be 15 . Initially the heights were set with a difference of 5 cm but a difference of 5 cm didnt yield a significant change in the velocities so the difference was set at 10 cm . The final height was selected to e 40 cm due to the masses moving irregularly by skipping, hopping and spinning when the height was set to 45 cm .

The heights used in the collisions:
$>15 \mathrm{~cm}$
$>25 \mathrm{~cm}$
$>35 \mathrm{~cm}$
$>40 \mathrm{~cm}$


Image 1: inclined plane mechanism

## Pre Experiment



Image 2: markings on medium

The width of the media was 70 cm and the width of the inclined plane was 14 cm . In order to place the inclined plane in the middle of the media the inclined plane was set 28 cm from the corners. Then a line dividing the mass in half was drawn. The line was followed down the inclined plane and onto the media at the point where the mass is set to create the trajectory of movement.


Image3:birds eye view of set up

## Setup



## Methodology

## Part A (Identical bodies colliding to set the range of error in the momentum and energy

 lost)For this experiment two equal masses are used

1. Fix the inclined plane mechanism to the chequered media.
2. Adjust to the velocity favoured.
3. Place one disk onto the inclined plane and the other onto the media at the decided collision point.
4. Adjust the camera parallel to the inclined plane.
5. Push the body off the inclined plane and record via camera.
6. With the use of the software Tracker find the velocities.
7. Repeat these steps 3 times with velocities generated from heights of $15,25,35$ and 40 cm .
8. The collision point on the cylinder should be adjacent to the line where the centre of mass is.
9. This time record the collision from an overviweing angle.
10. With the use of the software Tracker find the initial and final velocities.

## Part B

1. Fix the inclined plane mechanism to the chequered media.
2. Adjust to the velocity favoured.
3. Place the smaller disk onto the inclined plane and the other onto the media at the decided collision point.
4. Adjust the camera parallel to the inclined plane.
5. Push the body off the inclined plane and record via camera.
6. With the use of the software Tracker find the velocities.
7. Repeat these steps 3 times with velocities generated from heights of $15,25,35$ and 40 cm .
8. This time record the collision from an overviweing angle.
9. With the use of the software Tracker find the initial and final(Right after the collision) velocities with the angle of deflection.(the angles can be calculated via the scaled media and a protractor built into the Tracker software) The deflection angle will be used to see if the trials provide a constant error as when two masses hit eachother at a point on the line passing through the centre of mass they will continue their movements on their prior trajectories.

## Data

## Raw Data Table

| Impacting Masses $(\mathrm{kg} \pm 0,001)$ | Heights(cm) | Recorded Initial <br> Velocity (m/s) |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { Mass 1 } \\ & (0,043) \end{aligned}$ | 15 cm | 1,39 |
|  |  | 1,55 |
|  |  | 1,32 |
|  | 25 cm | 1,93 |
|  |  | 1,72 |
|  |  | 1,91 |
|  | 35 cm | 2,41 |
|  |  | 2,23 |
|  |  | 2,14 |
|  | 40 cm | 2,49 |
|  |  | 2,83 |
|  |  | 2,61 |
| $\begin{aligned} & \hline \text { Mass 2 } \\ & (0,082) \end{aligned}$ | 15 cm | 1,49 |
|  |  | 1,58 |
|  |  | 1,44 |
|  | 25 cm | 2,13 |
|  |  | 1,96 |
|  |  | 2,00 |
|  | 35 cm | 2,39 |
|  |  | 2,64 |
|  |  | 2,45 |
|  | 40 cm | 2,87 |
|  |  | 2,95 |
|  |  | 2,86 |

Table 1: Heights and recorded initial velocities of the two impacting masses.

## Data Analysis

|  | Heights( cm) | Theoric Potential Energy(J) | Theoric Kinetic Energy(J) | ```Theoric Initial Velocity( \(\mathrm{m} / \mathrm{s}\) )``` | Recorded <br> Initial <br> Velocity( <br> $\mathrm{m} / \mathrm{s}$ ) | Mean recorded velocity( $\mathrm{m} / \mathrm{s}$ ) | Procedur <br> al <br> Uncertai nty |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Mass } 1 \\ (0,043 \mathrm{~kg} \pm 0, \\ 001) \end{gathered}$ | 15 cm | $\begin{gathered} 0,063 \pm 3 \\ 5 \% \end{gathered}$ | $\begin{gathered} 0,063 \pm 3 \\ 5 \% \end{gathered}$ | 1,7 $\pm 18 \%$ | 1,39 | 1,42 | $\pm 0,115$ |
|  |  |  |  |  | 1,55 |  |  |
|  |  |  |  |  | 1,32 |  |  |
|  | 25 cm | $\begin{gathered} 0,11 \pm 22 \\ \% \end{gathered}$ | $\begin{gathered} 0,11 \pm 22 \\ \% \end{gathered}$ | 2,2 $\pm 12 \%$ | 1,93 | 1,85 | $\pm 0,105$ |
|  |  |  |  |  | 1,72 |  |  |
|  |  |  |  |  | 1,91 |  |  |
|  | 35 cm | $\begin{gathered} 0,15 \pm 16 \\ \% \end{gathered}$ | $\begin{gathered} 0,15 \pm 16 \\ \% \end{gathered}$ | 2,6土9\% | 2,41 | 2,26 | $\pm 0,135$ |
|  |  |  |  |  | 2,23 |  |  |
|  |  |  |  |  | 2,14 |  |  |
|  | 40 cm | $\underset{\%}{0,2 \pm 14}$ | $\begin{gathered} 0,2 \pm 14 \\ \% \end{gathered}$ | $3 \pm 8 \%$ | 2,49 | 2,64 | $\pm 0,17$ |
|  |  |  |  |  | 2,83 |  |  |
|  |  |  |  |  | 2,61 |  |  |
| $\begin{gathered} \hline \text { Mass 2 } \\ (0,082 \mathrm{~kg} \pm 0, \\ 001) \end{gathered}$ | 15 cm | $\begin{gathered} 0,12 \pm 34 \\ \% \end{gathered}$ | $\begin{gathered} 0,12 \pm 34 \\ \% \end{gathered}$ | 1,8 $\pm 18 \%$ | 1,49 | 1,5 | $\pm 0,07$ |
|  |  |  |  |  | 1,58 |  |  |
|  |  |  |  |  | 1,44 |  |  |
|  | 25 cm | $\begin{gathered} 0,21 \pm 21 \\ \% \end{gathered}$ | $\begin{gathered} 0,21 \pm 21 \\ \% \end{gathered}$ | $2,4 \pm 11 \%$ | 2,13 | 2,03 | $\pm 0,085$ |
|  |  |  |  |  | 1,96 |  |  |
|  |  |  |  |  | 2,00 |  |  |
|  | 35 cm | $\begin{gathered} 0,28 \pm 15 \\ \% \end{gathered}$ | $\begin{gathered} 0,28 \pm 15 \\ \% \end{gathered}$ | 2,8土8\% | 2,39 | 2,49 | $\pm 0,125$ |
|  |  |  |  |  | 2,64 |  |  |
|  |  |  |  |  | 2,45 |  |  |
|  | 40 cm | $\underset{\%}{0,3 \pm 13}$ | $\begin{gathered} 0,3 \pm 13 \\ \% \end{gathered}$ | 3,3 $\pm 7 \%$ | 2,87 | 2,8 | $\pm 0,045$ |
|  |  |  |  |  | 2,95 |  |  |
|  |  |  |  |  | 2,86 |  |  |

Table 2: Mean and theoric initial velocities of the two masses mass1 and mass 2 with regards to varying heights and subsequent theoric potential and kinetic energies.

## Example Calculation of Potential Energy

Height of inclined plane: $0,15 \mathrm{~m} \pm 0,05$

Mass: $0,043 \mathrm{~kg} \pm 0,001$

Theoric potential energy: Utilising the equation $\mathrm{Ep}=\mathrm{mgh}$ by inserting the values measured

## Uncertainty Calculations

In order to calculate the uncertainty of the product the percentage uncertainty formula where the uncertainty is divided by the measured value and multiplied with 100 is used.
$\frac{0,001}{0,043} 100= \pm 2,3 \%$
$\frac{0,05}{0,15} 100= \pm 33 \%$

When multiplying values with percentage uncertainties the uncertainties are added.

So the total uncertainty is $\pm 35,3 \%$ but as the values have 2 significant figures the ucertainty is rounded to $\pm 35 \%$

## Example calculation of Kinetic Energy

Theoretically the kinetic energy should be equal to the potential energy as the law of conservation of energy states that the mechanic energy is constant meaning mgh $=\frac{1}{2} \mathrm{~m} v^{2}$

It is now possible to derive the theoric velocity as the kinetic energy is known.

Calculation of Theoric Initial Velocity
$\mathrm{Ek}=0,063 \pm 35 \%=\frac{1}{2} \mathrm{~m} v^{2}$
$\mathrm{Ek}=0,063 \pm 35 \%=\frac{1}{2}(0,043 \mathrm{~kg} \pm 0,001) v^{2}$
$0,126 \pm 35 \%=(0,043 \mathrm{~kg} \pm 0,001) v^{2}$
$2,9 \pm 37 \%=v^{2}$
$v=1,7 \pm 18 \%$

Uncertainty Calculation
$\frac{0,126 \pm 35 \%}{0,043 \pm 2,3 \%}=2,9 \pm 37 \%$

When the square root of a percentage uncertainty is being taken the percentage uncertainty is divided by 2 because square root in index form is to the power $1 / 2$.

## Example calculation of Mean Recorded Velocity

The mean is calculated by the arithmetic mean formula

$$
\bar{x}=\frac{\sum_{i=1}^{N} x_{i}}{N}
$$

For this particular case the arithmetic mean can be
found by

$$
\frac{(1,39+1,55+1,32)}{3}=1,42
$$

Finally the procedural uncertainty is found by subtracting the smallest value from the largest then dividing it by two.

So in this particular case the procedural uncertainty is $\pm 0,115$ as $\frac{(1,55-1,32)}{2}= \pm 0,115$

|  | Heights(cm) | $\begin{array}{c}\text { Theoric İnitial } \\ \text { Velocity(m/s) }\end{array}$ | $\begin{array}{c}\text { Mean recorded } \\ \text { velocity(m/s) }\end{array}$ | $\begin{array}{c}\text { Difference in the } \\ \text { values of } \\ \text { recorded and } \\ \text { theoric }\end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| velocities $(\mathrm{m} / \mathrm{s})$ |  |  |  |  |$]$.

Table 3: The difference of theoric and mean recorded velocities with reference to the varying heights of the inclined plane.

## Example Calculation of Difference

The difference between values is found by subtracting the mean recorded velocity from the theoric initial velocity.
$1,7 \pm 18 \%-1,42=0,28 \pm 18 \%$

| Collision <br> Points |
| :--- |

Table 4: Final recorded velocities of the two masses with reference to the heights.*mass 1 stands for the impacting mass and mass 2 stands for the mass hit.

|  | $\begin{aligned} & \text { Heights(c } \\ & \mathrm{m}) \end{aligned}$ | Collisi on Points | Initial momentu m of mass1 | Final momentum of mass1 | Final momentum of mass2 | Difference in total momentum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Collision withequalmasses $(0,043 \mathrm{~kg} \pm$0,001(mass1) $0,043 \mathrm{~kg} \pm$0,001(mass2)) | 15 | Centre of mass | $\begin{aligned} & 0,061 \pm 2, \\ & 3 \% \end{aligned}$ | $\begin{aligned} & \begin{array}{l} 0,012 \pm 2,3 \\ \% \end{array} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0,047 \pm 2,3 \\ & \% \end{aligned}$ | $\begin{aligned} & \hline 0,002 \pm 0,0 \\ & 03 \end{aligned}$ |
|  | 25 | Centre of mass | $\begin{aligned} & 0,079 \pm 2, \\ & 3 \% \end{aligned}$ | $\begin{aligned} & \begin{array}{l} 0,023 \pm 2,3 \\ \% \end{array} \\ & \hline \end{aligned}$ | $\begin{aligned} & \begin{array}{l} 0,054 \pm 2,3 \\ \% \end{array} \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline 0,002 \pm 0,0 \\ 03 \end{array}$ |
|  | 35 | Centre of mass | $\begin{aligned} & \hline 0,097 \pm 2, \\ & 3 \% \end{aligned}$ | $\begin{aligned} & \hline 0,029 \pm 2,3 \\ & \% \end{aligned}$ | $\begin{aligned} & 0,065 \pm 2,3 \\ & \% \end{aligned}$ | $\begin{aligned} & \hline 0,003 \pm 0,0 \\ & 03 \end{aligned}$ |
|  | 40 | Centre of mass | $\begin{aligned} & \hline 0,11 \pm 2,3 \\ & \% \end{aligned}$ | $\begin{aligned} & \begin{array}{l} 0,031 \pm 2,3 \\ \% \end{array} \\ & \hline \end{aligned}$ | $\begin{aligned} & 0,074 \pm 2,3 \\ & \% \end{aligned}$ | $\begin{aligned} & \hline 0,005 \pm 0,0 \\ & 03 \end{aligned}$ |
| $\begin{aligned} & \hline \begin{array}{l} \text { Collision with } \\ \text { different } \\ \text { masses }(0,043 \mathrm{~kg} \pm \\ 0,001 \\ \text { (mass2)0,082kg } \\ \pm 0,001(\text { mass } 1)) \end{array} \\ & \hline \end{aligned}$ | 15 | Centre of mass | $\begin{aligned} & 0,12 \pm 1,2 \\ & \% \end{aligned}$ | $\begin{aligned} & 0,012 \pm 1, \\ & 2 \% \end{aligned}$ | $\begin{aligned} & \text { 0,056 } \pm 2,3 \\ & \% \end{aligned}$ | $\begin{array}{\|l\|} \hline 0,052 \pm 0,0 \\ 03 \end{array}$ |
|  | 25 | Centre of mass | $\begin{aligned} & 0,17 \pm 1,2 \\ & \% \end{aligned}$ | $\begin{aligned} & 0,04 \pm 1,2 \\ & \% \end{aligned}$ | $\begin{aligned} & 0,064 \pm 2,3 \\ & \% \end{aligned}$ | $\begin{aligned} & \hline 0,066 \pm 0,0 \\ & 03 \end{aligned}$ |
|  | 35 | Centre of mass | 0,2 $\pm 1,2 \%$ | $\begin{aligned} & 0,058 \pm 1, \\ & 2 \% \end{aligned}$ | $\begin{aligned} & 0,073 \pm 2, \\ & 3 \% \end{aligned}$ | $\begin{aligned} & \hline 0,069 \pm 0,0 \\ & 03 \end{aligned}$ |
|  | 40 | Centre of mass | $\begin{aligned} & 0,23 \pm 1,2 \\ & \% \end{aligned}$ | $\begin{aligned} & 0,068 \pm 1, \\ & 2 \% \end{aligned}$ | $\begin{aligned} & 0,081 \pm 2,3 \\ & \% \end{aligned}$ | $\begin{aligned} & \hline 0,081 \pm 0,0 \\ & 03 \end{aligned}$ |

Table 5: The final momentums of both masses, the initial momentum of the impacting body and the momentum lost (the difference) with respect to the varying heights of the inclined plane. *mass 1 stands for the impacting mass and mass 2 stands for the mass hit.

## Example Calculation of Difference in Momentum

The momentum of a body is found by the multiplication of its mass and velocity which can be written as $\mathrm{p}=\mathrm{mv}$

Initial momentum of the impacting body: $(0,043 \mathrm{~kg} \pm 2,3 \%)(1,42)=0,061 \pm 2,3 \%$

Final momentum of the impacting body: $(0,043 \mathrm{~kg} \pm 2,3 \%)(0,28)=0,012 \pm 2,3 \%$

Intial momentum of the mass hit: $(0,043 \mathrm{~kg} \pm 2,3 \%)(0)=0$

Final momentum of the mass hit: $(0,043 \mathrm{~kg} \pm 2,3 \%)(1,1)=0,047 \pm 2,3 \%$

Momentum lost can be found by subtracting the final sum of momentums from the initial sum

$$
(0,061 \pm 2,3 \%+0)-(0,047 \pm 2,3 \%+0,012 \pm 2,3 \%)=0,002 \pm 0,003
$$

|  | Collis <br> ion <br> Points | Heights( cm) | Initial kinetic energy of mass1(J) | Final Kinetic Energy(J) |  | Total Kinetic Energy(J) | Kinetic Energy Lost(J) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Mass1 | Mass2 |  |  |
| $\begin{aligned} & \text { Collision with } \\ & \text { equal } \\ & \text { masses }(0,043 \mathrm{k} \\ & \mathrm{g} \pm 0,001 \\ & (\text { mass } 1) 0,043 \mathrm{k} \\ & \mathrm{~g} \pm 0,001 \\ & (\text { mass } 2)) \end{aligned}$ | Centr e of mass | 15 | $\begin{aligned} & 0,043 \pm \\ & 2,3 \% \\ & \hline \end{aligned}$ | $\begin{aligned} & 0,0017 \pm \\ & 2,3 \% \\ & \hline \end{aligned}$ | $\begin{aligned} & 0,026 \pm 2, \\ & 3 \% \end{aligned}$ | $\begin{aligned} & \hline 0,0277 \\ & \pm 0,002 \end{aligned}$ | $\begin{array}{\|l\|} \hline 0,0153 \\ \pm 0,003 \\ \hline \end{array}$ |
|  |  | 25 | $\begin{aligned} & 0,074 \pm \\ & 2,3 \% \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline 0,006 \pm 2, \\ 3 \% \end{array}$ | $\begin{aligned} & \hline 0,0341 \pm \\ & 2,3 \% \end{aligned}$ | $\begin{aligned} & 0,0401 \\ & \pm 0,002 \end{aligned}$ | $\begin{aligned} & 0,0339 \\ & \pm 0,003 \end{aligned}$ |
|  |  | 35 | $\begin{aligned} & 0,11 \pm 2, \\ & 3 \% \end{aligned}$ | $\begin{aligned} & 0,0096 \pm \\ & 2,3 \% \\ & \hline \end{aligned}$ | $\begin{aligned} & 0,0497 \pm \\ & 2,3 \% \\ & \hline \end{aligned}$ | $\begin{aligned} & 0,0593 \\ & \pm 0,002 \end{aligned}$ | $\begin{aligned} & 0,0507 \\ & \pm 0,003 \end{aligned}$ |
|  |  | 40 | $\begin{aligned} & 0,15 \pm 2, \\ & 3 \% \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \hline 0,0111 \pm \\ 2,3 \% \\ \hline \end{array}$ | $\begin{aligned} & 0,0636 \pm \\ & 2,3 \% \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline 0,0747 \\ \pm 0,002 \\ \hline \end{array}$ | $\begin{aligned} & 0,0753 \\ & \pm 0,003 \end{aligned}$ |
| $\begin{aligned} & \hline \text { Collision with } \\ & \text { different } \\ & \text { masses }(0,043 \mathrm{k} \\ & \mathrm{g} \pm 0,001 \\ & \text { (mass2) } 0,082 \mathrm{k} \\ & \mathrm{~g} \\ & \pm 0,001(\text { mass } 1) \\ & ) \end{aligned}$ | Centr e of mass | 15 | $\begin{aligned} & 0,09 \pm 1, \\ & 2 \% \end{aligned}$ | $\begin{aligned} & \hline 0,0009 \pm \\ & 1,2 \% \end{aligned}$ | $\begin{array}{\|l\|} \hline 0,0363 \pm \\ 2,3 \% \\ \hline \end{array}$ | $\begin{aligned} & 0,0372 \\ & \pm 0,002 \end{aligned}$ | $\begin{aligned} & \hline 0,0528 \\ & \pm 0,003 \\ & \hline \end{aligned}$ |
|  |  | 25 | $\begin{aligned} & 0,17 \pm 1, \\ & 2 \% \end{aligned}$ | $\begin{aligned} & \hline 0,0094 \pm \\ & 1,2 \% \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \hline 0,0477 \pm \\ 2,3 \% \\ \hline \end{array}$ | $\begin{aligned} & 0,0571 \\ & \pm 0,002 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0,1129 \\ & \pm 0,003 \\ & \hline \end{aligned}$ |
|  |  | 35 | $\begin{aligned} & 0,25 \pm 1, \\ & 2 \% \end{aligned}$ | $\begin{aligned} & \hline 0,0207 \pm \\ & 1,2 \% \end{aligned}$ | $\begin{aligned} & 0,0621 \pm \\ & 2,3 \% \end{aligned}$ | $\begin{aligned} & 0,0828 \\ & \pm 0,002 \end{aligned}$ | $\begin{aligned} & 0,1672 \\ & \pm 0,003 \end{aligned}$ |
|  |  | 40 | $\begin{array}{\|l} \hline 0,32 \pm 1, \\ 2 \% \\ \hline \end{array}$ | $\begin{aligned} & 0,0282 \pm \\ & 1,2 \% \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0,076 \pm 2, \\ & 3 \% \end{aligned}$ | $\begin{aligned} & \hline 0,1042 \\ & \pm 0,002 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0,2158 \\ & \pm 0,003 \\ & \hline \end{aligned}$ |

Table 6: Final kinetic energies of masses and the kinetic energy lost with respect to the initial kinetic energy of the impacting mass. *mass 1 stands for the impacting mass and mass 2 stands for the mass hit.

## Example Calculation of Energy Loss

Initial kinetic energy: $0,043 \pm 2,3 \%$

Final kinetic energy of mass $1: \frac{1}{2}(0,043 \mathrm{~kg} \pm 2,3 \%)(0,28)^{2}=0,0017 \pm 2,3 \%$

Final kinetic energy of mass $2: \frac{1}{2}(0,043 \mathrm{~kg} \pm 2,3 \%)(1,1)^{2}=0,026 \pm 2,3 \%$

Kinetic energy lost can be found by subtracting the initial kinetic energy by the final kinetic energy.
$(0,043 \pm 2,3 \%)-(0,0017 \pm 2,3 \%+0,026 \pm 2,3 \%)$

## Graphs



Graph 1: Varying velocities vs energy loss of part A
The slope uncertainty is found by subtracting the steepest worst fit slope from the least steep worst fit slope and then dividing it by two.

The uncertainty of the slope is
$\frac{(0,04417-0,04324)}{2}=0,000465$
The Pearson's correlation coefficient is the measure of a statistical relationship between two variables. The coefficient is found by the formula below.
$r=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^{2} \sum(y-\bar{y})^{2}}}$
The value of $R$ is 0.7226 .
This is a moderate positive correlation, which means there is a tendency for high X variable scores go with high Y variable scores.


Graph2: Varying velocities vs momentum loss of part A
The uncertainty of the slope is 0,004558
The value of R is 0.7456 .
This is a moderate positive correlation, which means there is a tendency for high X variable scores go with high Y variable scores.


Graph 3: Varying velocities vs energy loss of part B
The uncertainty of the slope is 0,0021
The value of R is 0.5648 .
This is a moderate positive correlation, which means there is a tendency for high X variable scores go with high Y variable scores.


Graph 4: Varying velocities vs momentum loss of part B
The uncertainty of the slope is 0,00825
The value of R is 0,62386 .
This is a moderate positive correlation, which means there is a tendency for high X variable scores go with high Y variable scores.

## Conclusion

In this experiment, the aim was to make a quantitative observation to derive an understanding of how different velocities affected momentum and energy losses in an imperfect system with regards to the theory. After the data was collected and processed it can be concluded that the results are in support of the initial hypothesis. This can be seen by considering the data in tables 2,5 and 6 and through the graphs.

Firstly, Part A or the calibration experiment lays the foundation for me to make these interpretations as it acts as quantifiable proof that the collisions and calculations are consistent with one another and the errors they have with regards to the theory.

The momentum and energy lost in the system with two equal masses were considerably lower than the one with two different (one larger) masses. This is due to the fact that the forces
exerted onto the bodies at the time of collision are less irregular since the masses are uniform. Theoretically, it would be expected for there to be no energy loss but due to factors such as air resistance, friction forces, and the general non-ideal system energy is lost. However the loss of energy is doesn't threaten the results of the experiment as it is within the range of $0,0153 \pm 0,003$ to 0,0753 .

Secondly, with reference to table 4, it can be said that the difference in the velocity of mass 1(impacting mass) the collisions being performed are comparative to one another. Ideally, the initial velocity of mass 1 would have been transferred to the stationary mass 2 but as this collision is being performed in a media with friction and a system with air friction it is unreasonable to expect such results. However, the data exhibits proportional values. The difference ranges from $-1,14$ to $-1,92$ which shows that the error isn't random.

Furthermore, the range of difference between recorded velocity and calculated theoric velocity can also be shown as a contributing factor to the overall uncertainty and error of the data analysis. Nevertheless other than one outlier $(0,57 \%)$ the general is in the range of $0,2818 \%-0,3711 \%$ as can be seen in table 3 . This means that the friction forces and other external forces have a consistently adverse effect on the results being produced and that the software the velocities were recorded on is accurate. The system exhibits consistent systematic error.

The distribution of points on the initial velocity vs momentum and energy loss graphs of both the parts of the experiments do not depict a clear straight line but the Pearson's coefficient shows that there is a moderate positive correlation constant in relation to a total correlation constant of 1 and no correlation coefficient of 0 . This correlation coefficient is enough to establish an accurate linear relationship between the initial launch velocity and the loss of energy and momentum in 2D collisions between flattened cylindrical uniform objects.

Finally, it can be seen that there is high precision in the data groups as the slope uncertainties of the energy loss graphs are 0,000465 and 0,0021 while the slope uncertainties of the momentum loss graphs are 0,004558 and 0,00825 .

## Evaluation

The deflection angles can be used to see if the trials provide a constant error as when two masses hit each other at a point on the line passing through the center of mass they will continue their movements on their prior trajectories. Ideally, there wouldn't be a deflection angle but it was observed that there were angles of deflection albeit being consistent with one another which is caused by the imperfect media and how as the mass is sliding down it derails from its path which can be seen by the table below.

|  | Collision <br> Points | Varying <br> Heights | Experimental angle of deflection(degrees) |  | Total angle of deflection (degrees) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\begin{array}{\|l} \text { M1 (impacting } \\ \text { mass) } \end{array}$ | M2 (hit mass) |  |
| Collision with equal | Centre of mass | 15 cm | 2,2 $\pm 0,5$ | 0,2 $\pm 0,5$ | 2,4 $\pm 1$ |
| masses ( $0,043 \mathrm{~kg} \pm 0,001$ |  | 25 cm | 18,8 $\pm 0,5$ | $-3,6 \pm 0,5$ | 22,4 $\pm 1$ |
| (mass1)0,043kg $\pm 0,001$ |  | 35 cm | $-23,5 \pm 0,5$ | 15,2 $\pm 0,5$ | 38,7 $\pm 1$ |
| (mass2)) |  | 40 cm | $-26,2 \pm 0,5$ | 22,2 $\pm 0,5$ | 48,4 $\pm 1$ |
| $\begin{aligned} & \text { Collision with different } \\ & \text { masses }(0,043 \mathrm{~kg} \pm 0,001 \\ & \text { (mass2)0,082kg } \\ & \pm 0,001(\text { mass }) \text { ) } \end{aligned}$ | Centre of mass | 15 cm | $0 \pm 0,5$ | 3,6 $\pm 0,5$ | 3,6 $\pm 1$ |
|  |  | 25 cm | $-13 \pm 0,5$ | 16,8 $\pm 0,5$ | 29,8 $\pm 1$ |
|  |  | 35 cm | $-33,6 \pm 0,5$ | $12 \pm 0,5$ | 45,6 $\pm 1$ |
|  |  | 40 cm | $-49 \pm 0,5$ | 8,8 $\pm 0,5$ | 57,8 $\pm 1$ |

## Example Calculation

The total deflection angle is found by adding the absolute values of the 2 deflection angles.

$$
|2,2|+|0,2|=2,4
$$

The uncertainty of deflection angle is 0,5 as the minimum degree measured is 1 degrees. By adding two degrees together the uncertainty is increased to 1 .

Due to the nature of my experiment, it was inevitable to eliminate errors but these errors if consistent wouldn't pose a threat to my hypothesis as these errors may be due to:

- As an experimenter, I apply a small force onto the masses as I hold them until I start recording. This force may in turn accelerate these masses. This force is also most likely to be inconsistent as such minor forces can not be controlled by the human body.
- The friction between the masses and the media with each trial causes a deformation on both the media and the masses. Unless for each trial new media and masses are used this can not be eliminated.
- There may be irregularities on the trajectory the masses are lunched from since the direction of the trajectory can differ a few millimeters for each trial as the path isn't constrained.
- More data could have been collected by using different masses that can move regularly without any skipping or spinning motion at higher velocities. This could help establish a stronger Pearson's coefficient which in turn will strengthen the linear correlation.
- The air resistivity will get larger as the speed of the object traveling increases which might lead to some error in the recordings for velocity.

Furthermore, more precise techniques and equipment for analysis could be used to procure data with higher accuracy. These may include;

- Motion detectors to record the velocities.
- A rail system was put in place on the inclined plane that won't allow any divergence from the path meant to follow.


## References

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[^0]:    ${ }^{1}$ J.M.K.C. Donev et al. (2021). Energy Education - Law of conservation of energy [Online]. Available: https://energyeducation.ca/encyclopedia/Law_of_conservation_of_energy. [Accessed: February 7, 2022].

