# International Baccalaureate Diploma Program 

Physics High Level Extended Essay

Research Question: How does the diameter of marbles, number of marbles per magnet and distance between the magnets affect the velocity of the final marble in the Gauss gun system?

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## INTRODUCTION

I have always been interested in magnetic linear accelerators as I think that they will be very important in the future. I have met with magnetic linear accelerators after reading about the Gauss gun, and I was intrigued about the mechanism behind this rifle that enables the bullets to adduct to light speed.

The Gauss pistol is a simple device that converts magnetic energy into kinetic energy to accelerate a steel projectile. Gauss guns are made up of electromagnets that are attached to multiple steel marbles. When a second ball comes from the left at a low initial velocity, it encounters the magnet's attractive magnetic pull, collides with the magnet, and the final ball on the right is expelled with high velocity ${ }^{1}$. As the ball approaches the magnet, the force increases stronger, and the ball impacts the magnet at a higher velocity than before. After the impact the energy transfers to the final ball at the system giving it the kinetic and magnetic energy to accelerate.


Figure 1: Simple model of the Gauss Gun

The Gauss gun's physics are divided into three sections: acceleration of a ferromagnetic steel ball in the magnetic field formed by the magnet, momentum transmission into a chain of steel balls, similar to Newton's cradle, and ejection of the final ball that escapes the magnetic attraction.


Figure 2: Newton's Cradle
However, my interest peaked when I read an article about the possibility of sending spaceships or aircrafts with the usage of magnetic linear accelerators. As magnets do not need additional tools to function, they might even be used to launch spacecrafts outside the Earth as well where gravity or air does not exist. This can be achieved because magnets' power comes from the electromagnetic field, they create by themselves.

I believe that this extended essay was a very good opportunity for me to clear the doubt whether magnets could be used or not on a scientific manner. I designed three experiments to investigate how different factors affected the velocity of the final marble in the model of Gauss gun system (Figure 3). The system basically consists of two separate neodymium magnets and steel marble balls, but the number and the diameter change for each experiment. If a trend can be found in every experiment, then it is possible to understand how the velocity of the final marble in the system could be maximized. To understand the principle of this riffle I will use metal spheres instead of using bullets and magnets instead of using electromagnets make an experiment about. This led me to my research questions:

1. How does the diameter of the steel marbles in the system affect the velocity of the final marble?
2. How does the number of marbles per magnet in the system affect the velocity of the final marble?
3. How does the distance between the magnets' center affect the velocity of the final marble?

## HYPOTHESIS

For the first experiment, as the diameter of steel balls in the system increase the velocity of the final ball also increases as more energy is involved. Although steel balls with larger diameters need more energy to accelerate it can be omitted as it would not have much impact. Moreover, for the second experiment, as the number of steel balls increase per magnet the velocity of every steel ball per magnet should increase due to the magnetic force acting on the final ball per magnet will be lower as more balls are attached to the magnet. For the last experiment, the increasing distance between the magnets should have a positive impact until a certain point but due to surface friction to much increase between the magnets will have negative impact on the velocity of the final steel ball in the system.

## BACKGROUND INFORMATION

Rockets are being employed to transport people and payloads into space, as well as to deliver bombs over large distances. This method is extremely costly, as it necessitates a well-developed industry, advanced technology, expensive fuel, and complicated apparatus. The space elevator, hypersonic tube air rocket, cable space accelerator, circle launcher and space keeper, centrifugal launcher, and electrostatic linear accelerator are examples of means to attain space velocities
other than rockets. ${ }^{2}$. I will investigate how a simple and inexpensive method to launch into space may be affected. To better understand the investigation some terms and formulas should be known.

The Newton's cradle is a gadget that uses swinging spheres to explain the conservation of momentum and energy. When one of the end spheres is lifted and released, it collides with the stationary spheres, imparting a force that pulls the last sphere upward. ${ }^{3}$

Kinetic energy is the energy that an item or particle has as a result of its movement. When work is done on an object by exerting a net force, the object accelerates and gains kinetic energy as a result. Its formula is:

$$
K E=\frac{1}{2} m v^{2}
$$

Average velocity of an object is the displacement of an object over time and can be found by dividing displacement to the total time. Velocity is also a vector quantity. Its formula is:

$$
V_{a v}=\frac{X_{f}-X_{i}}{t_{f}-t_{i}}
$$

Where $X_{f}$ is final distance, $X_{i}$ is the initial distance, $t_{f}$ is the final time, and $t_{i}$ is the initial time.

Momentum which is a vector quantity is the product of the mass and velocity of an object. Where $m$ is the mass of an object and $v$ is the velocity of the object the formula of momentum for a single particle is:

$$
p=m v
$$

The momentum of a system can be found by vector summing the momenta of particles. For multiple particles the formula for momentum is:

$$
p=m_{1} v_{1}+m_{2} v_{2}
$$

Where two particles have masses $m_{1}$ and $m_{2}$ in kilograms, and velocities $v_{1}$ and $v_{2}$ respectively. Finally, for more than two particles the general equation is:

$$
p=\sum_{i} m_{i} v_{i}
$$

Newton's law of action and reaction, which states that every force has an equal and opposite reciprocating force, can be used to derive the law of conservation of momentum. In certain circumstances, moving charged particles can exert forces in non-opposite directions on one other. Despite this, the particles' and electromagnetic field's combined momentum is conserved. ${ }^{4}$.

An elastic collision is detected when no kinetic energy is converted to heat or another form of energy. Perfect elastic collisions can result when particles do not touch each other, such as in atomic or nuclear scattering, where electric repulsion keeps the objects apart. In an elastic collision, both momentum and kinetic energy are conserved ${ }^{5}$. Where m represents the mass in kilograms, $u$ is the velocity before the collision and $v$ is the velocity after the collision. The conservation of momentum is expressed by:

$$
m_{1} u_{1}+m_{2} u_{2}=m_{1} v_{1}+m_{2} v_{2}
$$

The formula for conservation of the total kinetic energy is:

$$
\frac{1}{2} m_{1} u_{1}^{2}+\frac{1}{2} m_{2} u_{2}^{2}=\frac{1}{2} m_{1} v_{1}^{2}+\frac{1}{2} m_{2} v_{2}^{2}
$$

If both masses are the same than this means that the bodies exchange their initial velocities with each other as:
$v_{1}=u_{2}$ and $v_{2}=u_{1}$
Average projectile acceleration can be found by:

$$
a=\frac{\mathcal{V}_{0}^{2}}{2 \ell}
$$

## EQUIPMENT

- Micrometer $( \pm 0.001)$
- Magnetometer $( \pm 0.01)$
- Ruler $( \pm 0.01)$
- 13 Steel ball radius 30 mm
- 13 Steel ball radius 20 mm
- 13 Steel ball radius 15 mm
- 13 Steel ball radius 12 mm
- 13 Steel ball radius 10 mm
- 3 Neodymium magnets $15 \mathrm{~mm} \times 10 \mathrm{~mm}$
- A mobile phone with camera that can record in slow motion
- Glue
- Box $20 \mathrm{~cm} \times 20 \mathrm{~cm} \times 20 \mathrm{~cm}$
- Chronometer $( \pm 0.001)$


## SAFETY

The neodymium magnets have strong magnetic forces between them so while trying to separate the magnets make sure two use gloves and push one of the neodymium magnets rather than pulling them to opposite sides. Moreover, also do not forget to wear googles during the experiments as sometimes due to collision some balls may burst into the air. In order to, eliminate the damage during the experiments put a box that would stop the final steel ball in the system immediately as it cause severe damage to the surroundings.


Figure 3: Experimental set up

## First Experiment: finding the relation between diameter of steel balls in the system and velocity of the final ball

## Variables:

Independent Variable:
Diameter of steel balls in the system is the independent variable. During the experiment balls with different diameters ( $10 \mathrm{~mm}, 12 \mathrm{~mm}, 15 \mathrm{~mm}, 20 \mathrm{~mm}$, and 30 mm ) will be used. The diameters of every steel ball are measured by a micrometer.

## Dependent Variable:

The dependent variable of the experiment is the velocity of the final steel ball in the system. The velocity is measured by making the necessary equations after a video is recorded in slow motion from the camera of the mobile phone.

## Controlled Variables:

Diameter of the final ball in the system, number of balls and magnets in the system, type of the magnets, the magnetic force outside the system and the place of the final ball in the system are the controlled variables. Diameter of the final ball is controlled to neglect the air friction while comparing velocities. Moreover, the number of balls and magnets as well as the type of the magnets are the same to make sure that the final ball's velocity only gets affected by number of balls. Furthermore, the magnetic force in the surrounding should be the same for every trial and it can be measure by a magnetometer. Finally, the place of the final ball in the system is controlled to eliminate extra surface friction.

## Procedure:

1. Put a 20 mm steel ball 20 cm away from the box to stop the motion
2. Attach one more 20 mm steel ball to the left of the ball that is 20 cm away from the box
3. Glue the bottom of the neodymium magnet and put it to the left of the second ball so that it does not rotate or move after the first trial
4. Measure 15 cm to the left and put two 20 mm steel balls
5. Then glue the bottom of one more magnet and attach it to the left of two balls.
6. Take one more 20 mm ball and put it 10 cm away from the magnet at the left side
7. As someone releases the ball without putting initial velocity start recording a video.
8. Record the time taken for the final 20 mm steel ball to move 20 cm by using a mobile phone's camera
9. Repeat this experiment at least for five times
10. Then go through the same procedure for different steel balls with different diameters. The first clause should be same for every experiment to neglect the final friction affect

## Raw Data:

| Diameter of steel ball <br> $(\mathrm{mm})( \pm 0.001)$ | Time taken for the ball to move 20cm (s) $( \pm 0.001)$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $1^{\text {st }}$ trial | $2^{\text {nd }}$ trial | $3^{\text {rd }}$ trial | $4^{\text {th }}$ trial | $5^{\text {th }}$ trial |
| 10.00 | 2.431 | 2.654 | 2.644 | 2.532 | 2.456 |
| 12.00 | 2.013 | 2.163 | 2.222 | 2.345 | 2.024 |
| 15.00 | 1.796 | 1.895 | 1.874 | 1.944 | 1.900 |
| 20.00 | 1.673 | 1.763 | 1.597 | 1.455 | 1.576 |
| 30.00 | 1.324 | 1.294 | 1.383 | 1.333 | 1.100 |

Table 1: Raw Data for the first experiment

## Processed data:

| Diameter of Steel Ball <br> $(\mathrm{mm})( \pm 0.001)$ | Average Time taken <br> for the ball to move <br> $20 \mathrm{~cm}(\mathrm{~s})$ | Displacement (m) <br> $( \pm 0.01)$ | Average Velocity <br> $(\mathrm{m} / \mathrm{s})$ |
| :--- | :--- | :--- | :--- |
| 10.000 | $2.5434 \pm 0.1115$ | 0.20 | $0.0786 \pm 9.38 \%$ |
| 12.000 | $2.1534 \pm 0.1660$ | 0.20 | $0.0928 \pm 12.71 \%$ |
| 15.000 | $1.8818 \pm 0.0740$ | 0.20 | $0.1063 \pm 8.93 \%$ |
| 20.000 | $1.6128 \pm 0.1540$ | 0.20 | $0.1240 \pm 14.55 \%$ |


| 30.000 | $1.2868 \pm 0.1415$ | 0.20 | $0.1554 \pm 15.99 \%$ |
| :--- | :--- | :--- | :--- |

Table 2: Processed Data for the first experiment

To understand how the diameter of steel balls in the system affect the velocity of the final ball the average time taken is needed. The average is used to minimalize random error during the process of the experiment. Then the average velocity is found by dividing the road taken by the final ball to the average time taken. To have it in SI units the road taken is transformed into meters.


Graph 1: Diameter of magnets vs velocity of the final ball in the system

Using the processed data above I got this graph. The graph clearly shows that there is a correlation between the diameter of the steel balls in the system and the velocity of the final ball. The graph has a strong correlation, 0.998 which is close to 1 . It is also plausible that the diameter and the square of velocity or the square root of diameter and velocity will have a linear relationship. Considering this graph, I can state that the relation between diameter of the steel balls in the system and the velocity of the final ball is positive.

## Evaluation of the First Experiment:

The line of best fit crosses the point $(0,0.0194)$ but normally it should cross the point 0,0 which is a systematic error because if the diameter of a steel ball is 0 then it means it should not exist. Thus, if a steel ball does not exist the magnetic linear accelerator does not function, and no velocity can be recorded.

According to the results although air resistance increases when a steel ball with a bigger diameter is used, the energy involved as a result of the bigger diameter can create enough energy to accelerate and eliminate the negative effect of friction.

## Second Experiment: finding the relation between number of steel balls per magnet and velocity of the final ball

## Variables:

## Independent Variable:

Number of steel balls per magnet is the independent variable. The number of steel ball attached per magnet will change from two to six steel balls. All balls are attached together from their center of gravity. By using a micrometer, it is made sure that identical steel balls are used.

## Dependent Variable:

The dependent variable of the experiment is the velocity of the final steel ball in the system. The velocity is measured by making the necessary equations after a video is recorded in slow motion from the camera of the phone.

## Controlled Variables:

Radius of the steel balls in the Gauss gun system, number of magnets in the system, type of magnets and steel balls in the system, the place of the final ball in the system, and the magnetic force outside of the system are the controlled variables. Firstly, the radius of the steel balls should be the same to make sure the velocity is only affected by the number of steel balls per magnet and not any other factors. Then, number of magnets and using identical magnets and steel balls is important to make sure there is equal magnetic field in the system. The place of the final ball is measured by a ruler, and it is controlled to eliminate the extra friction it would add to the equation. Finally, the outer magnetic force should be the same to have more accurate results.

## Procedure:

1. Put the final 20 mm steel ball of the system 20 cm away from the box to stop the motion
2. Attach one more 20 mm steel ball to the left of the final ball
3. Glue the bottom of the neodymium magnet and put it to the left of the second ball so that it does not rotate or move after the first trial
4. Measure 15 cm to the left and put two 20 mm steel balls
5. Then glue the bottom of one more magnet and attach it to the left of two balls.
6. Take one more 20 mm ball and put it 10 cm away from the magnet at the left side
7. As someone releases the ball without putting any energy start recording a video.
8. Record the time taken for the final 20 mm steel ball to move 20 cm
9. Repeat this experiment at least for five times
10. Then go through the same procedure by changing the number of steel balls per magnet to make the exploration

Raw Data:

| Number of steel ball <br> per magnet | Time taken for the ball to move 20cm (s) $( \pm 0.001)$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $1^{\text {st }}$ trial | $2^{\text {nd }}$ trial | $3^{\text {rd }}$ trial | $4^{\text {th }}$ trial | $5^{\text {th }}$ trial |
| 2 balls | 1.673 | 1.763 | 1.597 | 1.455 | 1.576 |
| 3 balls | 1.242 | 1.132 | 1.145 | 1.194 | 1.201 |
| 4 balls | 1.114 | 1.083 | 0.983 | 1.094 | 1.063 |
| 5 balls | 0.876 | 0.655 | 0.643 | 0.680 | 0.797 |
| 6 balls | 1.455 | 1.562 | 1.633 | 1.769 | 1.598 |

Table 3: Raw Data table for the second experiment

## Processed Data:

| Number of steel ball <br> per magnet | Average Time taken <br> for the ball to move 20 <br> $\mathrm{cm}(\mathrm{s})$ | Displacement (m) <br> $( \pm 0.01)$ | Average Velocity <br> $(\mathrm{m} / \mathrm{s})$ |
| :--- | :--- | :--- | :--- |
| 2 balls | $1.6128( \pm 0.1540)$ | 0.20 | $0.1240 \pm 14.5 \%$ |
| 3 balls | $1.1828( \pm 0.0550)$ | 0.20 | $0.1690 \pm 8.00 \%$ |
| 4 balls | $0.7302( \pm 0.1165)$ | 0.20 | $0.1874 \pm 11.1 \%$ |
| 5 balls $\pm 0.0655)$ | 0.20 | $0.2739 \pm 20.95 \%$ |  |
| 6 balls | $1.6034( \pm 0.1570)$ | 0.20 | $0.1247 \pm 14.79 \%$ |

Table 4: Processed Data table for the second experiment


Graph 2: Number of balls per magnet vs velocity of the final ball in the system

This is the graph of number of steel balls per magnet and the average velocity of the final steel ball in the system. The graph shows correlation of 1 which states that the shown equation is valid about the effect of number steel balls per magnet on velocity.

## Evaluation of the Second Experiment:

It may seem like as the number of balls attached to a magnet increase the velocity of the final ball in the system increases. However, when six balls are attached there is a huge drop down in the velocity because the system starts to push two metal balls instead of one. Thus, this slows the final ball as the energy is transferred into two metal balls instead of one. As a result, it is plausible that the magnetic force acting on the final ball did not get enough energy to accelerate.

## Third Experiment: finding the relation between the distance between the magnets and velocity of final ball

## Variables:

Independent Variable:

Distance between the two magnets is the independent variable of the third experiment. Distance is measured by a ruler from the centers of magnets, and they are $10,15,20,30$ and 50 cm .

## Dependent Variable:

The dependent variable of the experiment is the velocity of the final steel ball in the system. The velocity is measured by making the necessary equations after a video is recorded in slow motion from the camera of the phone.

## Controlled Variables:

The controlled variables in the Gauss gun system include the radius of the steel balls, the number of magnets in the system, the type of magnets and steel balls in the system, the location of the final ball in the system, and the magnetic force outside of the system. To begin, the steel balls' radius should be the same to ensure that the velocity is solely controlled by the quantity of steel balls per magnet and not by any other factors. Then, to ensure that the magnetic field in the system is equal, the number of magnets and the use of similar magnets and steel balls are critical. A ruler is used to measure the final ball's position, and it is controlled to eliminate the excess friction it would bring to the equation. Finally, for more precise findings, the outer magnetic force should be the same.

## Procedure:

1. Put the final 20 mm steel ball of the system 20 cm away from the box to stop the motion
2. Attach one more 20 mm steel ball to the left of the final ball that is 20 cm away
3. Glue the bottom of the neodymium magnet and put it to the left of the second ball so that it does not rotate or move after the first trial
4. Measure 10 cm to the left and glue the bottom of one more magnet to that place
5. Put two 20 mm steel balls to the right of the magnet
6. Take one more 20 mm ball and put it 10 cm away from the magnet at the left side
7. As someone releases the ball without putting any energy start recording a video.
8. Record the time taken for the final 20 mm steel ball to move 20 cm
9. Repeat this experiment at least for five times
10. Then go through the same procedure by changing the distance between the magnets ( $15 \mathrm{~cm}, 20 \mathrm{~cm}, 30 \mathrm{~cm}$ and 50 cm ) to understand the trend.

Raw Data:


Table 5: Raw Data table for the third experiment

## Processed Data:

| Distance between | Average Time taken | Displacement (m) | Average | Velocity |
| :--- | :--- | :--- | :--- | :--- |
| magnets | for the ball to move | $( \pm 0.01)$ | $(\mathrm{m} / \mathrm{s})$ |  |
| $(\mathrm{cm})( \pm 0.01)$ | 20 cm |  |  |  |


| 10 cm | $1.3054( \pm 0.0755)$ | 0.20 | $0.1532 \pm 10.78 \%$ |
| :--- | :--- | :--- | :--- |
| 15 cm | $1.6128( \pm 0.1540)$ | 0.20 | $0.1240 \pm 14.55 \%$ |
| 20 cm | $1.1062( \pm 0.1385)$ | 0.20 | $0.1714 \pm 17.52 \%$ |
| 30 cm | $1.0432( \pm 0.0043)$ | 0.20 | $0.1917 \pm 5.41 \%$ |
| 50 cm | $2.3056( \pm 0.1320)$ | 0.20 | $0.0867 \pm 10.72 \%$ |

Table 6: Processed Data table for the third experiment


Graph 3: Distance between magnets vs velocity of the final ball in the system

This is the graph which shows how the distance between the magnets affect the velocity of the final steel ball in the system. The graph shows correlation of 0.95 which is close to 1 and states that the shown equation is valid.

## Evaluation of the Third Experiment:

As it may be observed from the graph has some turning points. The highest velocity is observed when there is no space between the magnets and that is theatrically true as well because if two magnets are attached more electromagnetic force is created. Until a certain point the velocity
increases but after that due to both air resistance and surface friction the energy is lost so it cannot be transferred as kinetic energy to the final ball. Thus, this shows a decrease in velocity of the final ball after a certain point.

## ERROR CALCULATION:

The uncertainty in time is found by:

$$
\frac{\text { maximum time }- \text { minimum time }}{2}
$$

For example, in Table 1 the maximum reading for the steel ball with diameter 10 mm is 2.654 and the minimum reading is 2.431 .

$$
\frac{2.654-2.431}{2}=0.1115
$$

The uncertainty in time for 10 mm diameter ball is given by $2.5434 \pm 0.1115$.

To calculate error uncertainty should be written in percentage form and in this example percentage error is found by

$$
\frac{0.1115}{2.5434} \times 100=4.38 \%
$$

To calculate the error of velocity the percentage error in the displacement should also be found and it is done by

$$
\frac{0.01}{0.20} \times 100=5 \%
$$

Although velocity is found by dividing displacement to time the percentage errors are added so for this example the total uncertainty is

$$
5+4.38=9.38
$$

## EVALUATION

During the first experiment there are some possible errors both systematic and random. Random errors may occur while recording the time taken for the ball to move 20 cm which may be affected due to human reaction time. Then while putting the final steel ball 20 cm away from the box to stop the ball parallax error may be observed. Systematic errors can occur because of the phone I used to record, chronometer to record the time, and the micrometer to measure the diameter may mall function

The second experiment may have some systematic and random errors. Random error may occur due to the mall function of the magnet after too many steel balls are added. There is also the fact that the magnets may start to lose their properties after some time. The parallax error and the average human reaction time may also result in errors.

The final experiment may have some errors: both systematic and random. Firstly, the parallax error while putting the last ball and human based errors when using the chronometer are also present. Moreover, due to the burst of the metal balls the distance between the magnets may be decreased during the trials itself.

The results acquired from the experiments are accurate. This is because the measures and measurement devices I used were precise. The chronometer I used had an uncertainty of $\pm 0.001$ and the ruler had an uncertainty of $\pm 0.01$ as well which were both acceptable values. On the other hand, I still believe that this experiment could be more accurate and could give more information about the usage of magnetic linear accelerator in the field of space by:

- Using more precise equipment to minimalize systematic error as much as possible because using a more precise ruler would strongly reduce the uncertainty
- Conducting the experiment on a surface that is like the material used in spacecrafts launching areas
- Using more powerful magnets and larger magnets to understand if the trend continues in larger prototypes as well
- Limiting any external factors that may attract steel balls to eliminate systematic error


## CONCLUSION

The aim of this extended essay is accomplished as I understood the mechanism underlying the Gauss gun and the linear magnetic accelerator. I also analyzed how different factors affected the Gauss gun which was the main aim of the experiments. It is understood that as the diameter of the steel balls increased the velocity of the final ball increased. However, the distance between the magnets and number of steel balls per magnet have a positive effect only up to a certain point in the Gauss gun system.

I believe that this system should be tested in different conditions to fully understand if spacecrafts could be sent by using a magnetic linear accelerator. There are some other possible research areas to fully understand how a magnetic linear accelerator function. Another research may include the initial velocity of the first ball in the system, initial number of balls released at rest, and the number of magnets in the system as independent variable. Then, the researcher can use the same methods used above to understand the effects of those variables on the velocity of the final ball in the system.

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