International Baccalaureate Diploma Program Physics Extended Essay

Relationship Between the Current Supplied by the Power Source

And

The Back EMF Generated by a DC Motor in a Circuit

RQ: What is the relationship between the current supplied by the power source and the back emf generated by a spinning DC motor in a circuit?

May 2022

Word Count: 3724

Contents

1. Introduction

DC Motors are used in every aspect of life from electric toothbrushes to massive cooling systems. Most electrical gadgets use this technology to function. A curious property of DC Motors is the Back EMF that is created due to the internal magnetic field interactions of the DC Motors. As a DC motor starts to turn faster and faster during its operation, magnetic field interactions in the DC motor cause an internal voltage to appear and increase. This voltage increase –named Back Electro-Magnetic Force- opposes the applied voltage of the system thus, reducing the total electrical energy that is turned into mechanical energy. Such DC motors are used for electric or half diesel half electric trains. As the resultant EMF falls due to Back EMF. In the end, the acceleration of the train stops, as the drag of the train reaches the torque of the motors. I wanted to explore the working principles of trains to try to optimize their power. So, I have decided to explore the relationship between Back EMF created by DC motors and the current supplied to them. Thus, my research question is:

What is the relationship between the current supplied by the power source and the back emf generated by a spinning DC motor in a circuit?

2. Hypothesis and Theory

2.1 Magnetism and Magnets

Magnetism is caused by the motion of electric charges. Every substance is made of atoms that define its property. These atoms have a specific number of electrons, and these electrons are always in a spin. These electrons are negatively charged, and they spin around the nucleus of the atom, which contains both positively charged particles and neutral particles. The movement of these electrons around the nucleus creates an electric current, which results in a magnetic field –explained in more detail under the Electromagnetism section-. In most substances, equal numbers of electrons spin in different directions, canceling each other out and preventing the magnetic field to be formed. These substances are called weakly magnetic substances. However, in some substances, many electrons may be spinning in the same direction, creating a uniform magnetic field.[1] If electrons of many atoms are spinning in the same direction, they can create a magnetic domain. Magnetic domains are formed when the magnetic field of electrons of the atoms of certain substances align. They act like little magnets in the actual magnetic objects. However, these magnetic domains usually point in different directions, canceling each other out and causing the overall substance to be nonmagnetic. When these randomly pointing magnetic domains are forced to align with the help of an external magnet, the directions of the magnetic fields become uniform, and thus, a magnet is produced. Magnets produced in this manner are called permanent magnets.[2]

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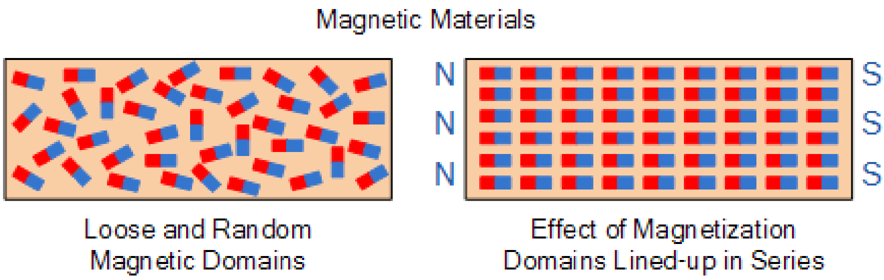
[1] National Geographic Society. (2012, October 9). *Magnetism*. National Geographic Society. Retrieved January 17, 2022, from https://www.nationalgeographic.org/encyclopedia/magnetism/#:~:text=Encyclopedic%20Entry%20Vocabulary-,Magnetism%20is%20the%20force%20exerted%20by%20magnets%20when%20they%20attract,of%20tiny%20units%20called%20atoms.&text=Their%20movement%20generates%20an%20electric,act%20like%20a%20microscopic%20magnet.

[2] Coey, J_M D. "Magnetic materials." *Journal of Alloys and Compounds* 326.1-2 (2001): 2-6.



[3]

2.2 Electromagnetism

Electromagnetism is a type of magnetism that is produced by an electric current or the movement of charged particles. It is a branch of physics, mainly exploring the interaction between electric fields and magnetic fields and electromagnetic forces. The electromagnetic force is one of the four fundamental interactions in nature along with strong interactions, weak interactions, and gravitation. It is the physical interaction between two charged particles and is used to explain many phenomena encountered in daily life. Even the bonds in an atomic structure consisting of protons, neutrons, and electrons are held together with electromagnetic attraction. It also defines the properties of chemical bonds and the movement of electrons in atoms thus, defining the chemical interactions. However, this paper will be interested in larger-scale interactions such as the movement of electrons in current-carrying wires. When a wire carries current, the charged particles, namely electrons, create a magnetic field around the wire, in relation to the direction of the current. This direction can be determined by using the "Fleming's

[3] "Magnetism, Magnetic Flux and Magnetic Materials." *Basic Electronics Tutorials*, 9 Feb. 2018, www.electronics-tutorials.ws/electromagnetism/magnetism.html.

Right-Hand Rule”[4]. This rule states that when the thumb shows the direction of the current or the opposite direction of the electron movement in a thumbs-up position, fingers curving inwards determines the direction of the magnetic field.

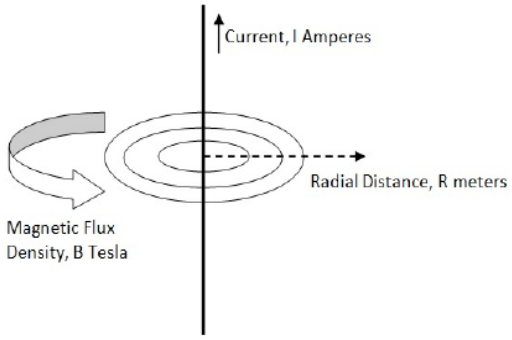


Fig. 1: Magnetic Field Pattern of a straight wire[5]

Then, this principle can be expanded when wires are bent in certain ways such as circular or rectangular wires. This expands upon the principle and makes it usable in a variety of ways. These bent wires produce two different magnetic fields in their center and their outer regions.

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[4] Tsokos, K. A. (2014). 5.4 Magnetic Fields. In *Physics for the IB diploma* (pp. 232–235). essay, Cambridge University Press.

[5] Macwan, I. (n.d.). *Magnetic field around a long straight conductor*. Retrieved February 28, 2022, from https://www.researchgate.net/figure/Magnetic-field-around-a-long-straight-conductor_fig1_262639489

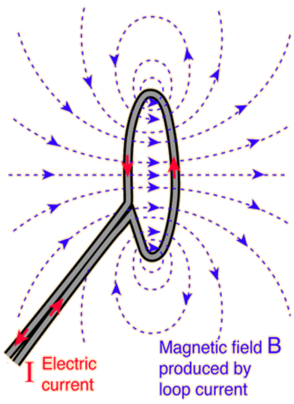


Fig. 2: Magnetic Field Pattern of a Circular Flat Wire[6]

Magnetic Flux is the number of magnetic lines passing through an area that is between two opposite magnetic poles. It is directly proportional to the area, magnetic field, and cosine of the angle of the surface.

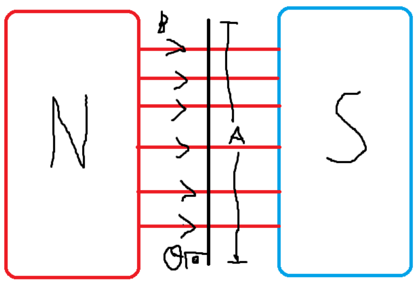


Fig. 3: Diagram of Magnetic Flux

So, according to this, the equation of magnetic flux is $\mathbf{\Phi = \beta * A * \cos(\theta)}$.[7] After the magnetic flux is found using this equation, magnetic flux linkage can be found by multiplying **Φ** with **N**, the winding count.

[6] *Magnetic field of current loop*. Magnetic Field of a Current Loop. (n.d.). Retrieved February 28, 2022, from http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/curloo.html#c2

[7] Tsokos, K. A. (2014). 11 Electromagnetic Induction. In *Physics for the IB diploma* (pp. 436). essay, Cambridge University Press.

Finally, Induced Electromagnetic Force, or induced EMF for short, is the main principle behind DC motors. It is created by magnetic induction –electromagnetic induction in this case- which is the production of an electromotive force across an area in a changing magnetic field, also known as the change in magnetic flux linkage. Induced EMF is mainly defined by two laws. One is Faraday's Law, which states that; induced EMF is equal to the negative rate of change in magnetic flux linkage. The other law, Lenz's Law decides the direction of the induced EMF and states that; induced EMF/current will be in such a direction as to oppose the change in magnetic flux that has created the current. As stated above, its magnitude in a certain amount of time can be calculated by using the change of magnetic flux linkage. This equation is as follows: $\boldsymbol{\varepsilon = -n\Delta\Phi / \Delta t}$ in which $\boldsymbol{n\Delta\Phi}$ represents the change in magnetic flux linkage and $\boldsymbol{\Delta t}$ represents the change in time and $\boldsymbol{\varepsilon}$ represents induced EMF.[8]

2.3 DC Motor Basic Principles

DC motors can be used in every aspect of daily life, making them one of the most common gadgets in mechanics. They are revolving electric motors, turning electrical energy into mechanical energy. Their main principle is built around magnetism and electromagnetism. One or more windings of coil wires, which are called armatures, are bent to form a circular or a rectangular system which is then aligned to the center of a stator that is covered with magnets either permanent or electrical. The ends of these armatures are bound to either a brush –called a brushed DC Motor- or they are without a

[8] Tsokos, K. A. (2014). 11 Electromagnetic Induction. In *Physics for the IB diploma* (pp. 434–441). essay, Cambridge University Press.

brush –brushless DC Motor-. Both of them are used to alternate the currents going through the coils thus keeping the motor in action. When an armature is powered, the current, thus the electromagnetic field created by it interacts with the fields of the magnets in the stator, creating an EMF that moves the armature to align with the domains of the magnetic fields. However, when the magnetic domains align, the motor stops and as a result, electromagnetic induction ends. Thus, the need for brushes or alternating currents to change the poles in order to keep the motor in Motion.

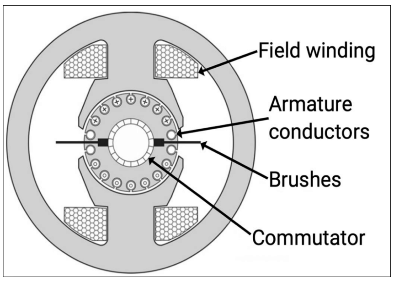


Fig. 4: Diagram of a Brushed DC Motor[9]

[9] Silbernagel, Cassidy. *Investigation of the Design, Manufacture and Testing of Additively Manufactured Coils for Electric Motor Applications*, *University of Nottingham*, page 29, ResearchGate, Oct. 2019, https://www.researchgate.net/publication/336922428_Investigation_of_the_design_manufacture_and_testing_of_additively_manufactured_coils_for_electric_motor_applications. Accessed 28 Feb. 2022.

2.4 Reasons, Calculation, and Results of Back EMF

Counter-electromotive force (counter EMF, CEMF), or as referred to in this paper as back-electromotive force (back EMF), is the electromotive force or voltage that counteracts the change in current which induced it. It is a form of induced EMF and just like the induced EMF, back EMF is also caused by magnetic induction. It follows Lenz's law, opposes the current which created it with a magnitude decided by Faraday's Law. It occurs in electric motors because of the movement of the armature or coil. As the armature moves within the magnetic field of the magnets/electromagnets that cover the inside of the stator, a magnetic induction occurs thus, back EMF is created.

Back EMF can be calculated in two different ways. One of them by a simple equation. For this calculation, the following equation will be used. $\boldsymbol{\varepsilon_b = (N * \Phi * W) / 60}$ [10] where $\boldsymbol{\varepsilon_b}$ represents back EMF, **N** is the number of armatures, **Φ** is the magnetic flux cutting the area of the armatures, and **W** is the rotational speed of armature in revolutions per minute time (rpm). The equation is divided by 60 to convert rpm to revolutions per second (rps). For the second way to calculate the back EMF, an experiment can be done by measuring the **Voltage from the Back EMF** and using it to calculate **Back EMF** by using Ohm's Law; **V = I * R**. **Voltage from the Back EMF is measured by using the following equation: Net Voltage = Supplied Voltage + Voltage from Back EMF** is measured by using the following equation: **Net Voltage = Supplied Voltage + Voltage from the Back**

[10]* C, V. (2021, May 31). *Back E.M.F of DC motor and equation*. Electrical Deck - All about Electrical & Electronics. Retrieved February 21, 2022, from https://www.electricaldeck.com/2020/02/back-emf-of-dc-motor-and-equation.html

EMF when the equation is derived as **Voltage from the Back EMF = Net Voltage – Supplied Voltage**. The collection of these data can be done by constructing an experiment. This paper will use the experiment to find the relation between the supplied current and the back EMF. The experiment can be done as the following: A simple circuit with an adjustable power supply with a display of the supplied voltage, a DC motor, and a voltmeter connected parallel to the DC motor to measure the voltage across. When the experiment is conducted, the displayed voltage supplied by the power supply is measured and accepted as the **Supplied Voltage**. The voltage is measured by the voltmeter that is connected parallel to the DC motor, which is the **Net Voltage**. By using these data, back EMF in that certain current can be calculated.

A crucial advantage of the back EMF, is it limits the motors to draw only as much current needed to give the torque needed for the load. This way, it prevents motors from burning while they are running at no load. Though, if the motor is running at load, increasing the load will reduce the rpm, thus reducing the back EMF –as can be seen by the equation above-, therefore allowing more current to pass through, increasing the torque to keep the rpm the same. Comparatively, when the load is decreased, the rpm will increase, the back EMF will increase, reducing the torque to keep the rpm the same. Conclusively, it can be said that the back EMF regulates the flow of armature current to meet the torque required by the load. This way, it forces the motor to run more efficiently and increases its lifespan.

2.5 Hypothesis

Relying on these pieces of information, the hypothesis of the paper is as follows: As can be inferred from the equations above, it can be understood that increase in current results in the increase of the back EMF of the system. However, the rate of change of the back EMF is always more than the rate of change of the supplied voltage due to the change of the supplied current. Thus, the increase in voltage resulting from the increase in current will be totally counteracted by the increase in back EMF. Consequently, after that point, the rpm of the DC motor will not increase or decrease at an ignorable rate.

3. Experiment Setup

3.1 Variables

Independent Variable: Voltage supplied by the power supply. (1V – 6V)

- The voltage will be measured in volts (V).
- The current will be measured with the use of an adjustable power supply and the voltage supplied will be calculated with the use of Ohm's Law; **V = I * R**.

Dependent Variable: Back Electromotive force created by the magnetic induction inside the DC motor.

- The voltage will be calculated in volts (V).
- It will be calculated by using the measured voltage supplied by the power supply and the measured net voltage across the DC motor by using the equation: Net Voltage = Supplied Voltage + Back EMF.

- To be able to calculate the value of this variable, the Net Voltage will be measured by a voltmeter connected parallel to the DC motor.

Controlled Variables:

1. The total resistance of the system.

- The experiment will be conducted in exactly the same system to eliminate the problem of total resistance changing and affecting the Supplied Voltage.

2. Load of the DC motor.

- As Back EMF is directly influenced by the rpm of the DC motor and the rpm of the DC motor is influenced by the load it carries thus, the experiment will be conducted without a load attached to the DC motor.

3.2 Materials

- Adjustable Power Supply (±0.05V) (0V – 10V)
- Copper wires (Resistance will be ignored throughout the experiment.)
- DC motor (Brushless or brushed does not matter.)
- Technomax Multimeter Model no: DT830D (±0.005 V) and (±0.05 Ω)

3.3 Methodology

1. The resistances of the DC motor and the resistances of wires connected to it are measured using a multimeter.
2. The adjustable power supply is connected to the DC motor in series to power the DC motor and adjust the current flowing through it.

3. The voltmeter is connected parallel to the DC motor to measure the potential difference between the two ends of the DC motor.

4. The predicted current of the circuit is calculated by using a supplied voltage of 1 volt, adjusted by the adjustable power supply. The calculation also assumes the DC motor has a constant resistance, measured in step one and applies it to Ohm's Law's -**V = I * R-** derived version; **I = V / R** where **V** is the voltage of the circuit, **I** is the current, and **R** is the resistance of the DC motor and the attached wires. The value of the calculation is recorded as Supplied Current.

5. The power supply is turned on, adjusted to the predetermined voltage.

6. While the DC motor is powered, the voltage across the DC motor is measured using the voltmeter that is connected parallel to the DC motor. The value of the measurement is recorded as Net Voltage.

7. The value of back EMF voltage is calculated using the following equation: **Net Voltage = Supplied Voltage + Back EMF voltage.**

8. The voltage from the Back EMF is then used to calculate the Back EMF of the DC motor, by using the derived version of Ohm's Law; **I = V / R.** The calculated magnitude is then recorded as the Back EMF of the DC motor.

9. The experiment is repeated four more times and the mean of the back EMF values of the five experiments are recorded.

10. The experiment will be repeated by changing the supplied voltage of the system to 2, 3, 4, 5, and 6 volts.

3.4 Safety

Just like most experiments, this experiment also has some safety issues, especially more as it requires the experimenter to deal with electricity. It is advised to turn the power supply off every time the experimenter interacts with the circuit. The experimenter should be careful to not hold any non-insulated open wires. The shaft of the DC motor should not be influenced by external forces as they can easily break. It is also advised that one should not hold the shaft of the DC motor while the DC motor is powered.

3.5 Experiment Setup

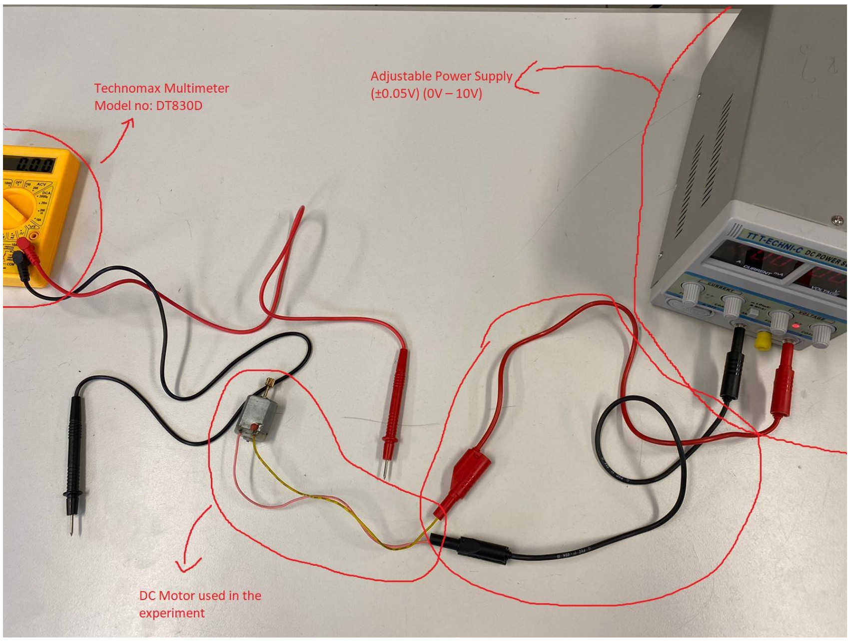


Fig. 5: Picture of experiment

3.6 Raw Data Table

Trials	Supplied Voltage (±0.05V)	Potential Difference Across DC Motor (±0.005V)
Trial 1	1.00	0.990
	2.00	1.860
	3.00	2.760
	4.00	3.880
	5.00	4.620
	6.00	5.700
Trial 2	1.00	0.920
	2.00	1.850
	3.00	2.750
	4.00	3.730
	5.00	4.700
	6.00	5.680
Trial 3	1.00	0.980
	2.00	1.870
	3.00	2.800
	4.00	3.700
	5.00	4.680
	6.00	5.650

	1.00	0.990
	2.00	1.860
	3.00	2.830
	4.00	3.800
	5.00	4.720
Trial 4	6.00	5.700
	1.00	0.920
	2.00	1.850
	3.00	2.760
	4.00	3.720
	5.00	4.620
Trial 5	6.00	5.600

Table of Supplied Voltage-Potential Difference Across DC Motor.

The data is acquired from the experiment trials. Data from all trials are listed in the table. The uncertainties of the data are the uncertainties of the measurement devices listed in the materials section.

4. Determining the Back EMF

4.1 Processed Data Table Regarding Supplied Voltage and Back EMF Voltage

Supplied Voltage (±0.05V)	Potential Difference Across DC Motor (V)	Voltage From Back EMF (V)
1.00	0.960 ±0.035	0.040 ±0.085
2.00	1.858 ±0.005	0.142 ±0.055
3.00	2.780 ±0.004	0.220 ±0.054
4.00	3.766 ±0.090	0.234 ±0.140
5.00	4.668 ±0.050	0.332 ±0.100
6.00	5.666 ±0.050	0.334 ±0.100

Table of supplied voltage-back EMF Voltage.

4.2 Calculation Regarding Supplied Voltage, Back EMF Voltage, and Uncertainty

The data of Potential Difference Across DC Motor is received by calculating the mean of the trials from the Table of Supplied Voltage-Potential Difference Across DC Motor. On the other hand, Voltage From Back EMF is calculated by subtracting Potential Difference Across DC Motor from Supplied Voltage. The uncertainties of all data are calculated specifically for each of the Supplied Voltage values. The uncertainties for Potential Difference Across DC Motor are found by subtracting the minimum datum acquired for a specific value of Supplied Voltage from the maximum datum of the same specific value and dividing the result by two. For example, the uncertainty of Potential Difference Across DC Motor when Supplied Voltage is set to 1.0V, is found by subtracting

0.920V from 0.990V which are the maximum and minimum data acquired for that specific Supplied Voltage value and dividing by two. The uncertainties for Voltage From Back EMF are found by adding uncertainties of Supplied Voltage and the specific value of Potential Difference Across DC Motor.

4.3 Processed Data Table Regarding Supplied Current and Back EMF

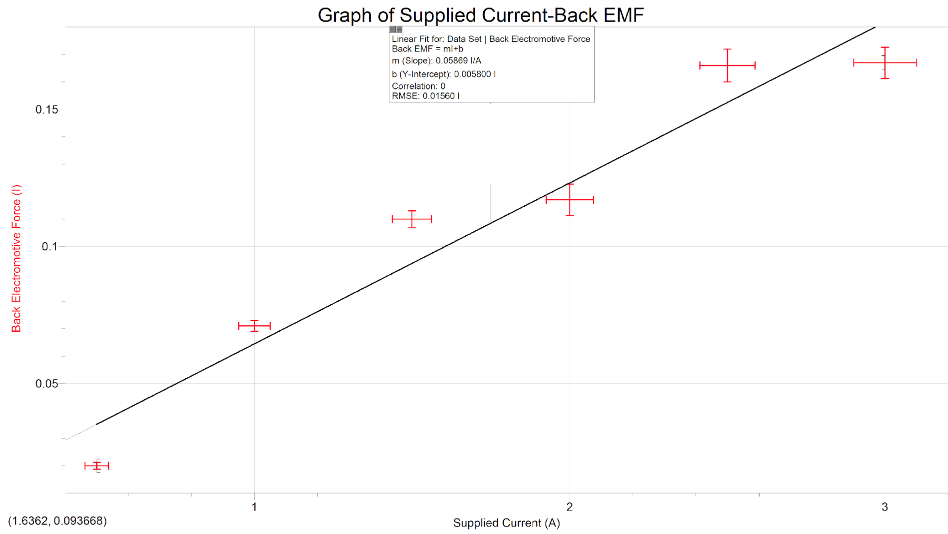
Supplied Current (A)	Back EMF (A)	Net Current (A)
0.50 ±7.50%	0.020 ±6.2%	0.480 ±0.080
1.00 ±5.00%	0.071 ±2.8%	0.929 ±0.079
1.50 ±4.16%	0.110 ±2.7%	1.390 ±0.092
2.00 ±3.75%	0.117 ±4.9%	1.883 ±0.148
2.50 ±3.50%	0.166 ±3.6%	2.334 ±0.142
3.00 ±3.33%	0.167 ±3.4%	2.833 ±0.154

Table of Supplied Current-Back EMF.

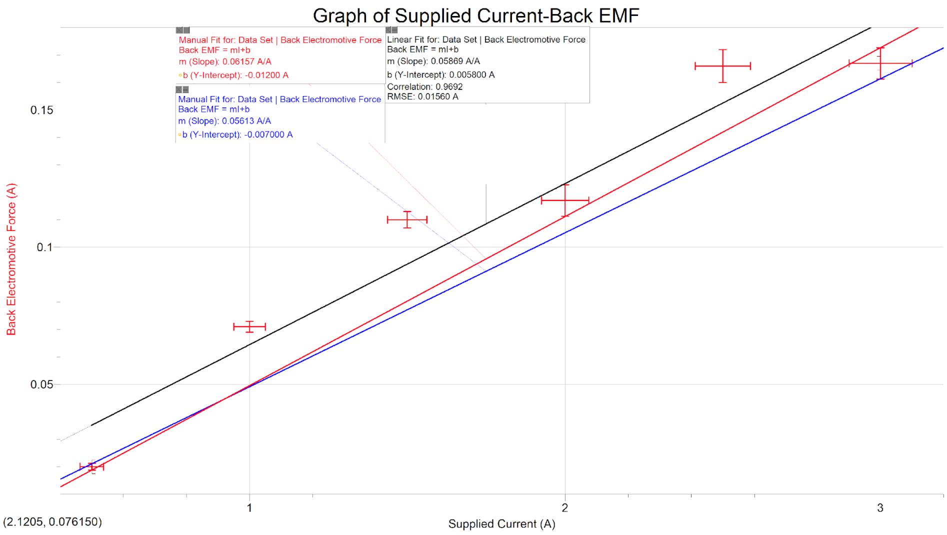
4.4 Calculation Regarding Supplied Current, Back EMF, and Uncertainty

The data for this table is acquired by calculating the current in the system by using Ohm's Law; **V = I * R** with respect to the data from the Table of supplied voltage-back EMF Voltage. The resistance of the system to be used in the equation specified above is measured with a multimeter specified in the materials section. The measured resistance is 2.00 ±0.05Ω. Thus, to be able to calculate the uncertainties for the data, all uncertainties from the Table of Supplied Voltage – Back EMF Voltage are converted into percentage uncertainties and then the percentage uncertainties are added with the percentage uncertainty of the measured resistance; ±2.5%Ω.

5. Graphs



This is the Graph of Supplied Current-Back EMF. The graph clearly shows the direct proportion between Back EMF and Supplied Current. Also, it can also be observed that the increase in Back EMF is at a constant rate. This result seems to be contradicting the hypothesis. Now, max and min slopes will be added to this graph to be able to calculate the uncertainty of the graph in general.



This graph, Supplied Current-Back EMF, is configured to calculate the uncertainty of the graph in general. the Black linear graph is the best fit, the blue graph blue is the minimum slope, and the red graph is the maximum slope. With the use of these graphs, the uncertainty of the graph overall can be calculated.

6. Analysis, Conclusion, and Evaluation

The hypothesis expected a positive correlation between supplied current and back EMF, and the data gathered supports this hypothesis. However, although the hypothesis was expecting a positive increase in the rate of the graph, it has a constant rate of increase. Therefore, the statement that the back EMF has a positive increase rate to the increase of Supplied Current is not justified by the experiment. This means that the recordings of this experiment can only be accepted as a rough estimation. This can be further proved by calculating the uncertainty of the experiment which will be done in this next section.

The uncertainty of the graph will be calculated by taking the absolute value of the subtraction of one of the worst gradients -min or max- from the best fit gradient. Then, dividing the acquired value by the best fit gradient and multiplying by 100. It can be formulated as follows:

Percentage Uncertainty = (|Best Gradient – Worst Gradient| / Best Gradient) * 100

Best Fit Gradient: 0.05869

Min Gradient: 0.05613

Max Gradient: 0.06157

Thus, with these data the percentage uncertainty is:

Percentage Uncertainty = (|0.059 – 0.098| / 0.059) * 100

Percentage Uncertainty = 4.9%

This result is unexpectedly high when it is considered that the claim of the experiment data is only a rough estimation. The results of the experiment seem to be reliable considering the low percentage uncertainty of the graph and low absolute uncertainties of the data from the tables. With such low uncertainty, the data acquired in this experiment can be considered to be universal for the equipment in the same range. However, all the error lines do not fit into the best fit line, suggesting that the experiment still gives some inaccurate data. The experiment clearly suggests that the increase in supplied current increases the value of back EMF, which is an accurate representation. The constant rate of change of the back EMF may be due to the errors in the experiment, resulting in such high percentage uncertainty.

The design and method of the experiment were suitable and sufficient for obtaining the results to come to a meaningful conclusion. However, the experiment was not enough to support the whole of the hypothesis. This is due to some of the weaknesses of the experiment such as the low amount of data. If the experiment was to be repeated in the future, it is advisable to collect more data. This way a more accurate result can be obtained. Another weakness of the experiment is, as it can be observed from the Raw Data Table, the values that were gathered are fluctuating. The main example for this is the sudden increase of the Back EMF when Supplied Current is 2.5 amperes. It can be observed that there was a stable increase up until that data. Another problem with that fluctuation is when Supplied Current is 3.0 amperes. This time, contradicting the previous sudden increase in the Back EMF, the increase in the Back EMF was considerably low. If the increase of Back EMF was to be continued with respect to the increase in Supplied Current = 2.5 amperes, the experiment results would support the full of the hypothesis as well. Although, this does not seem like a random error as all of the five trials of the experiment has approximately the same value, suggesting a systematic error. This may have two reasons: One of them is during the experiment, the voltage supplied by the power supply was always changing during the gathering of data. To prevent such a fluctuation in data, it is advised to use a more consistent power supply thus, reducing the fluctuations in data. The second probability is the resistance of the copper wires used in this experiment. Their resistance was not taken into account, making them suitable agents to manipulate the data of the experiment. If the resistance of the wires is too high to affect the experiment, the missing Back EMF could be transformed into heat before it can be calculated. If the experiment is to be repeated, it is advised to consider the resistance of all the wires used in the experiment instead of just the ones directly wired to the DC Motor.

Another error in the experiment is the y-intercept of the graph. As back EMF is created by the movement of the armature which is magnetized by being connected to a current inside a magnetic field, if there are no currents supplied, there should be no back EMF either. Without a current passing through the armature, there may be no movement inside the magnetic field thus, no back EMF is created. However, in the graph of *Supplied*

Current-back EMF, it can be seen that although the y-intercept should be zero, the best fit line shows it as 0.03A.

This experiment has shown that the increase in supplied current can be proved ineffective or inefficient in DC Motor after a certain amount. If the experiment is repeated in a more precise environment, the results can be used to optimize DC motors and their usage thus, increasing their lifetime.

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