<u>Physics Extended Essay</u>

Investigating the Relationship Between Amount of Water Inside a Can and Its Velocity

Word Count: 3993

Research Question: How does the amount of water inside a can changes the velocity of the can that is left to roll down an inclined plane?

Introduction

The specific investigation that this research is intending is to investigate the motion of a circular surfaced object (cans, in this case) filled up with different amounts of water. The purpose of this work is to observe and understand the pattern that rotational motion of cans with different amounts of water inside them would create. The essay is particularly based on how and why this motion occurs while comparing the patterns seen in the experiment to the theoretical results obtained. Initially, what made me tempted to write my extended essay about observation of cans filled with different amount of water was observing my mother perform what is called an "egg spin test" while she was preparing breakfast. It is a test to see whether an egg is raw or boiled performed by spinning the egg on a flat surface. If you stop the egg from spinning by gently pressing down on top of it with your finger and it starts spinning again right after you remove your finger from the top surface of the egg, it is raw. Otherwise it is boiled. The reason why it starts spinning again if you stop the raw egg is because of inertia, a property that all objects possess, simply put it is what keeps them doing what they are doing or in the words of Isaac Newton "*An object at rest remains at rest, and an object in motion remains in motion at constant speed and in a straight line unless acted on by an unbalanced force."* When the raw egg is spinning both the shell and the yolk inside are spinning, when you stop it, you can only stop the movement of the shell, thus, the egg continues spinning because of the yolk which keeps spinning inside. The reason why this doesn't work with a boiled egg is because what is inside a boiled egg is solid and when you stop the egg there is nothing inside that would make it continue spinning. Since most of the time solids have particles much closer to each other than liquids do, they are denser, which is why I thought it might be an interesting idea to test and see how liquids with different densities would make a difference in the particular experiment being conducted. I think instinctively, it is safe to say that water could behave almost like an inviscid fluid, as the can rolls down from the top of the setup the water would incline as a non-rotating body while the can would just rotate around it also having rotational inertia. The case for a can completely filled with water, assuming no slipping occurs between the surface and the can should be very similar to the movement of a solid cylinder. And if the can is completely empty it should act very similar to a hollow cylinder rolling down an inclined plane.

Background Knowledge

Gravitational Potential Energy is what an object has due to the position it has in a gravitational field. Gravitational potential energy occurs to an object's position relative to a specified point of zero height. The relevant gravitational potential energy usage is for the object to be near the surface of the Earth the flat surface mentioned is set as the "zero of gravitational potential energy" in this case, so the gravitational potential energy will be calculated accordingly. The force required to lift an object to a specific *height* is equal to its *weight*, thus the gravitational potential energy is equal to the objects weight times its height from the "zero of gravitational potential energy". Earth's gravitational constant is taken as $9.81\ m/s^2$.

$$PE_{gravitational} = weight * height = mgh$$

Rotational Kinetic Energy (also known as *Angular Kinetic Energy*) is a type of energy which exists due to the objects rotation. It is directly proportional to the square of the magnitude and rotational inertia of the angular velocity. This is why it can be disclosed as half of the angular velocity of the object and moment of inertia around the axis of rotation. If K_r represents *Rotational Kinetic Energy*.

$$K_r = \frac{1}{2} I\omega^2$$

Translational Kinetic Energy is dependent on mass and velocity. An extended object that's in motion is in possession of this energy. It is directly proportional to the square of its velocity and the mass of the body. For this reason as the velocity increases, its *Translational Kinetic Energy* will increase. If the velocity doesn't change, the *Translational Kinetic Energy* doesn't change. If the object is at rest, the *Translational Kinetic Energy* is zero.

$$\mathrm{K_t} = \frac{1}{2}\mathrm{mv}^2$$

it is a part of the total kinetic energy of an *Extended Object*, the other part being *Translational Kinetic Energy*. The sum of *Rotational Kinetic Energy* and *Translational Kinetic Energy* of the center of mass is the complete kinetic energy of the *Extended Object*, in this case, the cans.

The Law of Conservation of Energy states that energy can neither be created nor destroyed – only converted from one form of energy to another. In this particular situation it will help determine the velocity of the cans, because the initial *Gravitational Potential Energy* is supposed to be equal to the total *Kinetic Energy* by the end of the inclined plain (disregarding air resistance)

Velocity of The Center of Mass of A Rolling Object That Is Not Slipping is known to be the angular velocity of the object multiplied with the radius of the object. This can be understood when for instance, thinking about a sphere rolling on the ground, if it is not slipping, the bottom point of the sphere would have zero velocity, because it never moves, since the object is rolling, only the bottom of the object is always touching the ground and the bottom point is constantly changing because it is rolling. For this reason, the only thing that actually has a value of velocity in the sphere would be its center of mass. in this research knowing this is required because otherwise it wouldn't be possible to solve the equation that is obtained for velocity.

$$V_{cm} = r\omega$$

Arithmetic Mean will be used to calculate average AQI value of every quarter-year of every city that is used in this research for graphing. The mathematical formula of *Arithmetic Mean* is shown as:

$$Average = \frac{1}{n}\sum_{i=1}^{n} a_i = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

Considerations

The inclined plane that the cans will be dropped down from has to be sturdy enough not to be deformed by the mass of the cans, for this reason a sturdy piece of wood is used, this way the weight of the cans won't be able to deform the inclined plane.

The wood will be placed on top of something so that the angle can be modified if it is required, which could make it unstable and move around as it is being touched, to make it more stable a heavy apparatus is used to go through the back of the wood which is specifically cut for the apparatus, this way the plane won't be able to move unless an unusually strong force is applied because of the friction between the apparatus and the ground would be higher than the forces the inclined plane possibly has to deal with.

For the calculations to be accurate and for the initial gravitational potential energy to always be the same the cans should be dropped exactly from the same spot every time, for this purpose a ramp is implemented at the back of the inclined plane. This way, when the cans are rolled up to the highest point of the inclined plane excluding the ramp they always have equal gravitational potential energy.

The theoretical calculations of the experiment are done in a way that assumes there is no slipping between the cans and the surface, this is mostly the case, but just to make sure the slipping is minimized, all cans are attached a thick rubber band on the top and the bottom of them. Rubber is an elastic material, making it a material that could cause friction easily, by making the cans wear rubber bands the friction between the inclined plane and the cans the increased friction should further minimize the slipping, making the experiment more accurate.

Variables

Independent Variable

- Amount of water used

Dependent Variable

- Time it takes for the jars to cease contact with inclined plane

Controlled Variables

- Cans
- Density of the liquid (kg/m^3)
- Slope of the surface (°)

How and Why The Variables Were Controlled

Cans were controlled because different cans would have different masses and different radii which would cause inconsistent velocity calculations, to control the cans, cans with identical shape, size and weight were used.

Density of the liquid was controlled because different densities of water could cause inconsistencies within the data because they would result in different masses, to control density of the liquid, all the water used was drawn from the same source, the sink.

Slope of the surface was kept constant because changing the slope would change the initial height of the inclined plane, which would increase the initial gravitational potential energy and would result in an increase of velocity overall, it would result in inconsistencies. The slope of the surface was controlled by making sure the angle between the inclined plane and the box it is connected to stayed the same every time before doing the experiment.

Apparatus

- 3 Identical cans
- Water
- 6 Rubber bands
- Beaker (±0.5mL)
- Stopwatch (±0.01s)
- An inclined plane
- Ruler (±0.05cm)
- Angle Gauge (±0.5°)

Methodology

1. Set up the tray with a 6.50° angle utilizing the angle gauge
2. Use the Beaker to fill the cans with equal volumes of water, honey and molasses
3. Seal the cans and wrap rubber bands near both ends of each can

4. Note down the temperature value you read on the thermometer, start the stopwatch and let the can with water fall freely from the top of the tray simultaneously
5. Stop time stopwatch as soon as the can hits the flat surface right below the tray and write down the value
6. Repeat steps 4 and 5 10 times
7. Repeat steps 4 to 6 Repeat the process until all (110, 220, 330, 440, 550, 660) mL and then also do it for the empty can.

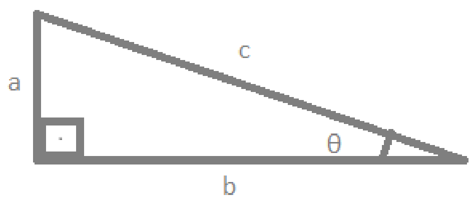


Figure 1: Diagram for the inclined plane

The values for the diagram (Fig. 1) are as follows:

a= 3.7cm b=61.6cm c=62.00cm and θ=6.50°

Figure 2: Picture of the inclined plane   Figure 3: Another angle of the inclined plane

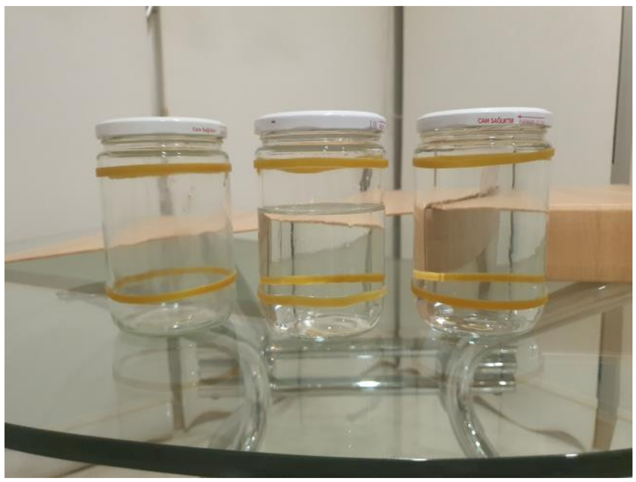


Figure 4: Picture of an empty, partially filled and completely filled can

<u>Raw Data</u>

This section includes the data obtained solely by experimentation and measurement, no data interpretations or calculations were done to obtain these values. These values will be used while interpreting the data.

Water Inside the Can (mL)	Attempt 1 (s)	Attempt 2 (s)	Attempt 3 (s)	Attempt 4 (s)	Attempt 5 (s)
0 (Empty)	1.47	1.60	1.60	1.48	1.54
110	1.49	1.55	1.53	1.56	1.52
220	1.42	1.52	1.46	1.42	1.51
330	1.45	1.40	1.39	1.45	1.33
440	1.35	1.28	1.28	1.36	1.31
550	1.31	1.21	1.21	1.28	1.31
Full (660)	1.20	1.11	1.10	1.12	1.19

Table 1: This table shows how long it took for an Empty can, a can filled with 200ml of water, a can filled with 400ml of water and a can completely filled with water (660ml) to fall from an inclined planed

For the purpose of calculating the uncertainty percentage values of the time of how long it takes for the cans to roll down the average of the times were taken using the arithmetic mean method.

Water Inside the Can (mL)	Average Time (s)	Uncertainty Percentage for time
0 (Empty)	1.54	±0.65%
110	1.53	±0.65%
220	1.47	±0.68%
330	1.40	±0.71%
440	1.32	±0.76%
550	1.26	±0.79%
660 (Full)	1.14	±0.88%

Table 2: This table shows the average time it took for the cans to roll down and the uncertainty percentages of these values

Theoretical Calculations

To be able to determine the theoretical average velocities of the rolling cans different calculations should be done for each one, because there are different *moment of inertia* to consider. But, for all of the cans the energies in work are the same. Initially, there is a *gravitational potential energy* and because of the existence of *the law of conservation of energy* by the end of the incline the *gravitational potential energy (at the top)* will become the sum of *translational kinetic* energy, *rotational kinetic energy* and *gravitational potential energy (at the bottom)*:

$$PE_{g1} = KE_t + KE_r + PE_{g2}$$

Gravitational potential energy at the bottom can be ignored because the flat surface that the cans fall into is considered as “zero of gravitational potential energy”, so the height of the bottom is 0, if you plug 0 in as the height of the *gravitational potential energy* you should get 0.

$$PE = mgh$$

As can be seen from the formula when the height (h) is given 0 the resultant will be 0, therefore the initial formula obtained with the help of *the law of conservation of energy* can be shortened as:

$$PE_{g1} = KE_t + KE_r$$

Opening this up to its components, formulas for the corresponding energies can be used to get:

$$mgh = \frac{1}{2}m{v_{cm}}^2 + \frac{1}{2}I\omega^2$$

To solve this equation for velocity, it is required to leave v alone, to achieve this mathematically velocity must be left as the only unknown value. From our experiment we know what the values of mass and height are, Inertia can be figured out through known information of mass and radius that are talked a little further in the essay, the only thing left to consider is the angular velocity (ω). Assuming this experiment is not slipping with respect to the inclined plane. If the formula for this occasion mentioned earlier is rearranged to give the value of angular velocity and the angular velocity on the formula given above is replaced with it the only variable left unknown would be velocity of the center of mass, which would allow to solve for v.

Rearranged formula:

$$\omega = \frac{v_{cm}}{r}$$

Substitution of angular velocity:

$$mgh = \frac{1}{2}m{v_{cm}}^2 + \frac{1}{2}I(\frac{v_{cm}}{r})^2$$

The final step to create the formula which would allow us to obtain the theoretical value of velocity when a can rolls down an inclined plane is to calculate the inertia that will be in work throughout the process. Moment of inertia of the can which isn't filled with any water can be assumed to be the same as the moment of inertia of a hollow cylinder, as they are the same shape. The moment of inertia of all the other cans can be obtained by substituting certain variables in the equation that gives the moment of inertia of a hollow cylinder which is:

$$\frac{1}{2}M({R_1}^2 + {R_2}^2)$$

M represents the total mass. R_1 is the inner radius and R_2 is the outer radius of the cylinder as can be seen below.

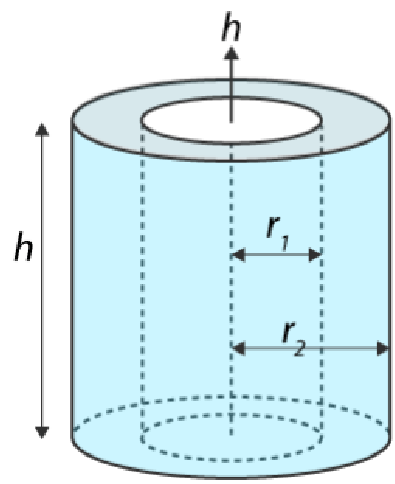


Figure 5: Representative cylinder with variable R_1 and R_2[3]

When the final form of the initial formula is rewritten with the inertia it becomes:

$$mgh = \frac{1}{2}m{v_{cm}}^2 + \frac{1}{2}[\frac{1}{2}M({R_1}^2 + {R_2}^2)](\frac{v_{cm}}{r})^2$$

When both sides of the equation are divided by m, m's can be cancelled:

$$gh = \frac{1}{2}{v_{cm}}^2 + \frac{1}{2}[\frac{1}{2}({R_1}^2 + {R_2}^2)](\frac{v_{cm}}{r})^2$$

First the right side of the equation can be taken into a bracket of v^2, then the formula can be rearranged we arrive at the final formula:

$$v = \sqrt{\frac{R^2gh}{\frac{1}{2} + \frac{1}{4} * ({R_1}^2 + {R_2}^2)}}$$

For an empty can R_1 and R_2 would be exactly the same value, since there is no water inside the can to make R_1 smaller. A can fully filled with water would have 0 as its R_1 value because there is no empty space left inside the can, but the R_2 would remain the same as before. And for the cans which are partially filled with water the R_1 can be calculated by first calculating how much space water takes up in the can and then extracting it from the total value to see what R_1 becomes.

To simply show how the calculations are done an example of a fully filled can, completely empty can and a partially filled can will be shown, and together with these calculated values rest of the values calculated will be exhibited in a table.

Before the calculations the volume of the can and the radius of the can should be known. The volume of the can is already known as 66cl (660ml) and the radius of the can could be calculated from the circumference of the can by utilizing the formula that gives the circumference of a circle, mathematically written as:

$$Circumference = 2\pi r$$

Where r is radius. Pi is assumed to be 3.14 as it would give an estimate that is good enough, and the circumference is measured to be 27.5cm(±0.05cm) so if the formula is rewritten as:

$$r = \frac{Circumference}{2\pi}$$

The resulting value for the radius is 4.37898089… which is rounded up as 4.38cm(±0.05cm)

For all the calculations R is 4.38, g is considered 9.81, pi is considered 3.14, h is 0.037 because height of the plane is 3.7cm, R_2 is 4.38 and R_1 depends on how much space water occupies inside the can.

The R_1 value for the empty can is already known as 4.38 (same as R_2 as it is not occupied by any form of water) so when the values are substituted at the formula that gives us the velocity the value would be: 0.3969m/s.

For the can which is fully filled with water the R_1 is known to be 0, because there is no space at all to be considered an inner radius. When the calculations are done by substituting R_1 as 0 and R_2 as 4.38 the resulting velocity is: 0.4288m/s.

Before calculating the velocity of a partially filled can the formula for the volume of a cylinder should be known as it will be utilized, for the cans used in this experiment the formula can be written as:

$$\pi * 4.38^2 * 10.956 = 660\sim$$

In this formula 4.38 is the radius and 10.956 is the height of the can.

For the partially filled cans the R_2 values are the same but the R_1 should be calculated, in this case, for example the can filled with 200ml of water occupies 200cm^3 inside the can, so 660 – 200 = 460cm^3. When put into the formula above while leaving r as blank we should get:

$$\pi * r^2 * 10.956 = 460$$

When the formula is arranged to leave r alone and the square root of both sides is taken the r is left to be: 3.66. This is the equivalent of R_1 for this particular can, the can filled with 200ml of water, when the same calculations as the others are done (except using 3.66 as the value of R_1) the velocity of this can is found as: 0.4703m/s

Information about. To leave less space to human error, and for simplicities sake the calculations were done on excel utilizing the same formula written in a suitable form:

“=SQRT((9.81*4.38*0.037)/(1/2+1/2*((1/2)*(R_1^2+4.38^2))))” SQRT stands for square root, other numbers are just the values of everything that is constant in this experiment for all occasions which was explained below the formula above, R_1 was replaced with the suitable R_1 value corresponding to how much empty space the can had inside it as shown above.

Water Inside the Cans (mL)	R_1 (cm)	R_2 (cm)	Velocity (m/s)
0 (empty)	4.38	4.38	0.3969
110	4.00	4.38	0.4135
220	3.58	4.38	0.4325
330	3.10	4.38	0.4544
440	2.53	4.38	0.4801
550	1.79	4.38	0.5106
660 (full)	0	4.38	0.5479

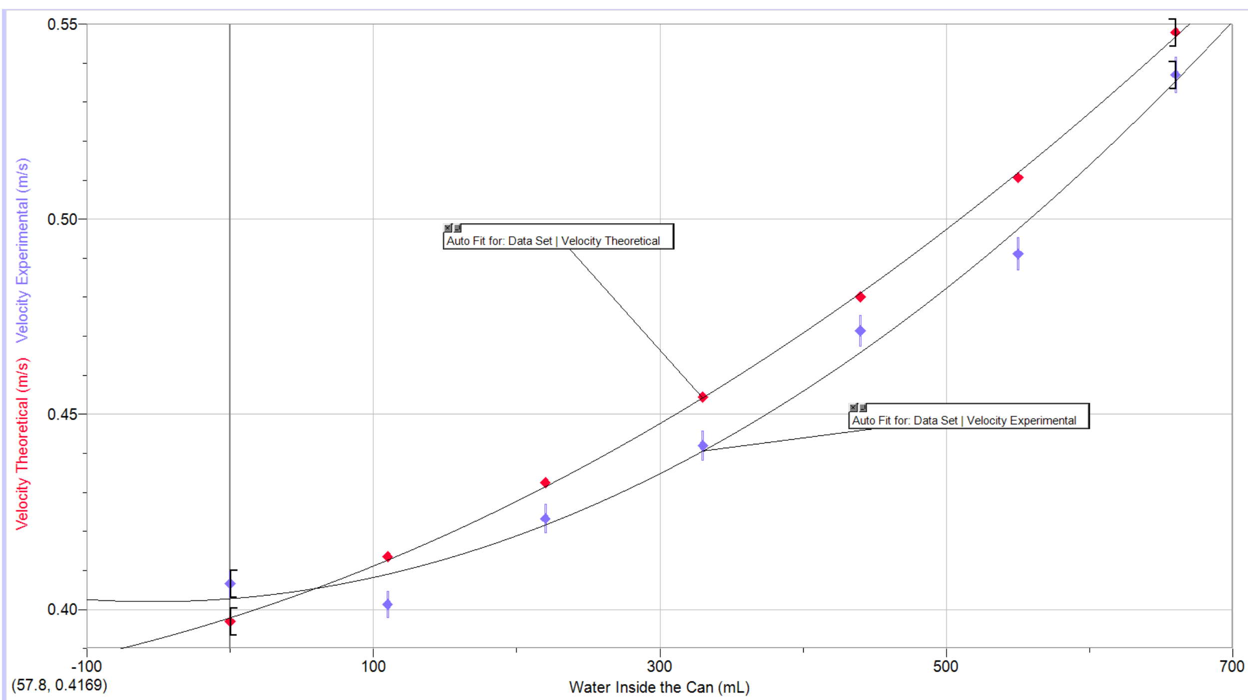
Table 2: Theoretical velocity values of velocity according to the calculated R_1 values of the cans filled with different amounts of water

The observed values of time can be used to calculate the velocities by asking the question “if the can rolled down in (1.47 seconds) and 0.62 meters then how much would it roll down in 1 second?” The answer is given by dividing 0.62 by 1.47 which is the rate of speed of occurrence, velocity. By changing (1.47 seconds) with other time values it is possible to find all velocities. To make things easier excel was used to do the calculations. The following formula was used: “=0.62/H2” H2 was replaced by whatever the time it took the related can to roll down the incline. After all the velocities for all 5 trials of cans were calculated the average velocity was calculated through utilization of arithmetic mean. All the numbers are rounded according to 4 significant figures. To calculate the uncertainty values, the uncertainty value of the ruler together with 62 and the stopwatch together with different values of times were turned into percentage uncertainties and added up. For instance 12.99% (uncertainty for time) + 0.08% (uncertainty for length) = 13.07%.

Water Inside The Can (mL)	1st Trial velocity (m/s)	2nd Trial velocity (m/s)	3rd Trial velocity (m/s)	4th Trial velocity (m/s)	5th Trial velocity (m/s)	Average velocity (m/s)	Uncertainty Percentage
None (0)	0.4366	0.3875	0.3875	0.4189	0.4026	0.4066	±0.73%
110	0.4161	0.4000	0.4052	0.3974	0.3875	0.4013	±0.73%
220	0.4366	0.4079	0.4247	0.4366	0.4106	0.4233	±0.76%
330	0.4276	0.4429	0.4460	0.4276	0.4662	0.4420	±0.79%
440	0.4593	0.4844	0.4844	0.4559	0.4733	0.4714	±0.84%
550	0.4733	0.5124	0.5124	0.4844	0.4733	0.4911	±0.87%
Full (660)	0.4960	0.5082	0.5124	0.5000	0.4844	0.5002	±0.96%

Table 3: Velocity and uncertainty data for the cans with different amounts of water

<u>Interpretation of Data</u>



Graph 1: Theoretical and experimental value of velocity with relation to water inside the can

The mark in the experimental data where there is 110ml of water inside the can shows how in the experiment an amount that occupies 16.67% (110ml) actually slows down the can rolling down the incline, which is inconsistent with the theoretical data. After that point, as values of water inside the can increase the velocity also increases consistently.

Similarity: In the experimental velocity data, the velocity value is constantly increasing as the amount of water inside the can increases, this is seen to be true for the experimental data after for water values above 110.

Explanation: Water has a low viscosity[5], which makes it so that it doesn't slow the can down by flowing backwards and makes it so the mass is closer to the axis of rotation. Distance from the axis of rotation matters because it is a component of rotational inertia, as the distance increases there will be more inertia, resulting in a lower velocity.

Similarity: Both the theoretical and the experimental data fit a quadratic best fit line

Explanation: As the water inside the can changes the inner radius of the can also changes (decreases), the inner radius is squared in the formula derived for calculating the velocity, which gives the values a quadratic shape when a best fit line is drawn.

Difference: Theoretical velocity always increases as the water in the can increases, however this is only consistent after 110ml with the experimental data.

Explanation: A probability is that, since the water amount is small it occupies very little amount of space inside the can and when the can is rolling down the small amount of water ends up fluctuating in the way it flows, this results in the center of mass of the can to change constantly, slowing it down.

Concluding Thoughts

Despite the experimental results not matching the predicted values, it seems that this is because of a systematic error. When an offset is used the experimental values mostly align with the theoretical data, however saying this is more than a rough estimate would be a far fetch from truth.

Also, all the other relationships between data are close the relationship between the first and the second recorded velocity seem to not match what is seen with the theoretical data, the velocity seems to decrease when the can has a small amount of water inside it while the theoretical data predicts it will be increasing always as the water amount increases, this is most likely because the assumptions that were made for the experiment regarding physical calculations don't fit what is actually happening for particularly when there is 110ml of water inside the can.

It can be concluded from the experimental data that as the water amount inside the can increases the velocity of which it rolls down an inclined plane without slipping increases if the water amount occupies 33.34% (220ml) of the can or higher. an amount that occupies 16.67% (110ml) of the can however, is seen to slow it down, values for particularly smaller or higher amounts cannot be known without educated guesses or rough estimations through this experiment.

<u>Evaluation</u>

What was done wrong?

Issue: *Systematic error* is seen when the experimental values and theoretical values are compared. To make it clear "Systematic errors are caused by imperfect calibration of measurement instruments or imperfect methods of observation, or interference of the environment with the measurement process, and always affect the results of an experiment in a predictable direction."[6] This could have been caused because of imperfect observation with the beaker or a systematic delay with the stopwatch.

Solution: To solve this issue the used digital devices and methods of observation should be went over. I would check the stopwatch to see if it has a particular offset or if it has considerable delay between the button that starts and stops the time, in the case it does if the values are known the values could be manually added or subtracted to the value. If it can't be calculated a better stopwatch would be chosen. To make sure the volume values of water was recorded correctly I would make sure to keep my eye-level parallel to the water level, which I neglected.

Issue: Theoretical data couldn't predict how when 16.67% of the can was occupied with water the velocity would be slower than the initial value with no water at all, most likely, because the assumed physical situations doesn't apply to this particular situation.

Solution: The calculations could have been made more precise by adding in the calculations for the way the water would flow and the changes of center of mass within the can instead of relying solely on assumptions.

Multiple Issues: Uncertainty is too high and the time being recorded by a stopwatch through instantaneous human reaction could have caused inaccuracies in results

Solution: Both these issues can be solved with a common solution. Instead of using a stopwatch, a high fps camera could have been used to record the can rolling down. Utilizing a software to analyze the footage frame by frame a much more accurate value for the time it takes the can to roll down can be obtained also lowering uncertainty.

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