MATHEMATICS EXTENDED ESSAY

"Mathematics in Art"

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ABSTRACT

Mathematics is one of the most important, beneficial and effective study, all over the world. Art is also has a great importance on a society as it includes visuality, emotions and creativity. It is thought that mathematics and art completely different kinds of studies, they have a strong interaction. As mathematics has an important role, its one of the use is in arts. The question that was researched and was used in this study is that "In what ways, does arts be affected by mathematics?" In this study, Fibonacci Numbers, Golden Ratio, Carla Farsi and the Music are studied.

First of all, after my researches, I agreed that Fibonacci Numbers and Golden Ratio are the most common relations between maths and art. These two, are not only related with arts, also, we can find them in our daily life, easily. These mathematical tools were used by lots of artists, for example Leonardo da Vinci. This study is exactly focused on the applications of these on arts. The applications of these two on other areas are also studied, basically, to emphasize their importance on life. The use of mathematics and also, to prove my study, Carla Farsi who is both a mathematician and an artist, are studied.

By taking all these points into consideration, the ways that arts is affected by mathematics and also, their relations are showed in this study.

Word Count : 230

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RESEARCH QUESTION :

In what ways, does arts be affected by mathematics?

1. INTRODUCTION

Mathematics and Art are the two different kinds of studies but there is an interaction between them. As many people think that they are opposed to each other, they have a connection and in arts, mathematical tools are used.

When you examine a human body, do you see a math in it? Most probably you don't. However, there is a golden ratio between the length of your face and the width of your face. The same ratio is used in arts, for example, in paintings. With these ratios or sequences, it can be declared that the art and the math appear together.

In this essay, under the question; " In what ways, does arts be affected by mathematics? ", the Fibonacci sequence and the Golden Ratio will be introduced with the examples of arts using this structure, Carla Farsi who is both a mathematician and an artist. Also, it will be introduced how mathematical tools are involved in music?

2. THE FIBONACCI SEQUENCE AND THE GOLDEN RATIO

2.1. The Fibonacci Sequence

The Fibonacci Sequence was firstly introduced by Leonardo of Pisa, known as Fibonacci, in 13^{th} century. His study was on the population of rabbits. Firstly he should have a pair of leverets with different sexes (one male, one female) are put together; one month later, rabbits become adult and are able to reproduce hence at next month a female rabbit can produce another pair of rabbits; he also assumed that rabbits will live forever and an adult pair always produces one couple rabbit every month from the time that they become adolescent. At the end of the year, the total number of rabbits is the main problem.

After a month, that they were put on the field, the pair become adolescent, however they don't produce a pair, therefore we have an only one couple of rabbits. For the next month the couple will have their leverets, so now there are 1 pair of adult rabbits and 1 pair of leverets in the field. At the end of the third month, the adult ones produce a new pair, the leverets become adolescents hence the number of pairs will be equal to three. For the following month, two adolescent pairs produce two new pairs and the newly-born pair become adult. Therefore, our field consists five pairs of rabbits. The terms of the sequence are given as,

$$\mathbf{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots}$$

Although the Fibonacci Sequence was introduced by Leonardo Fibonacci, it was known before in Europe. The ancient Indians applied this sequence to the metrical sciences.

2.2. The Golden Ratio and its Applications

2.2.1. The Golden Ratio

The Golden Ratio is a special type of ratio that can be seen on many structure of living organisms and many objects. It is not only observed in the part of a whole subjects, but also in arts and architecture for centuries. The Golden ratio gives the most compatible sizes of geometric figures. In nature, The Golden Ratio can be seen on the bodies of human beings, shells and branches of trees. For Platon, the keys of the cosmical physics are this ratio. Also, this ratio is widely believed that it is the most aesthetic ratio for a rectangle. The Golden Ratio is an irrational number just as pi or e and its approximate value is 1,618033988… To define the Golden Ratio, Φ or PHI is used.

The Golden Ratio has been used for many years for different purposes. For example for the region of the Acropolis, the approximate value of golden ratio can be seen on many of its proportions. Parthenon is a typical example of this. The facade of the Parthenon's including elements are said to be drawn rectangles. The ratio of the edges has an approximate value of 1,618033988… For many classical buildings, either the building itself or the elements of the buildings have a proportion which is approximately the same value. This information gives a result that their architects most probably knew the golden ratio and employed it in their buildings in a conscious way. On the other hand, the architects may use their senses and found a good proportion for their designs, and their proportions closely approximate the golden ratio. Beside this, some analyses can always be questioned on the ground that the investigator chooses the points from which measurements are made or where to superimpose golden rectangles, and the proportions that are observed are affected by the choices of the points.

Some scholars disagree with the idea that Greeks had an aesthetic association with golden ratio. For instance, Midhat J. Gazalé says, "It was not until Euclid, however, that the golden ratio's mathematical properties were studied. In the Elements (308 BC) the Greek mathematician merely regarded that number as an interesting irrational number, in connection with the middle and extreme ratios. Its occurrence in regular pentagons and decagons was duly observed, as well as in the dodecahedron (a regular polyhedron whose twelve faces are regular pentagons). It is indeed exemplary that the great Euclid, contrary to generations of mystics who followed, would soberly treat that number for what it is, without attaching to it other than its factual properties" In Keith Devlin's opinion, the claim that measurements of

Parthenon is not supported by actual measurements even though the golden ratio is observed. In fact, the entire story about the Greeks and golden ratio seems to be without foundation. The one thing we surely know that Euclid showed how to calculate its value, in his famous textbook Elements, that was written around 300 BC. Near-contemporary sources such as Vitruvius exclusively discuss proportions expressed in whole numbers, i.e. commensurate as opposed to irrational proportions.

2.2.2 The Golden Ratio In Great Pyramid of Giza:

In figure 1, we can see the golden ratio on the radius of a circle. Let us take a square namely FCGO, the edges of which have equal length with the length of the radius. T is the midpoint of the edge FC and A is the midpoint of GO. When we draw a perpendicular from T to A, we obtain a rectangle namely TCAO. The diagonal line of the rectangle is the AC line is one of the edges of the isosceles triangle ABC. Assuming that the height of the triangle is 1 unit, then the length of OB gives us the golden ratio. If looked up on a trigonometric table, it can be observed that the angle OCB is equal to 31"43' and the angle OBC has a measure 58"17'.

Figure 1 brings us a construction without losing its importance. The construction was too important for the monks of Egyptians.

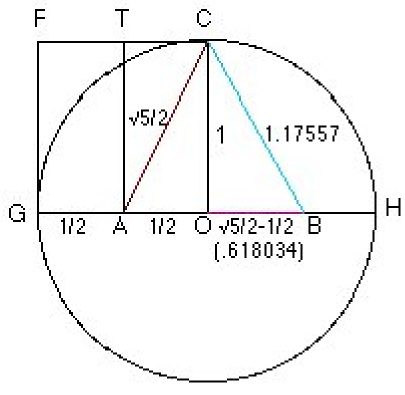

Figure 1: Golden Ratio In A Circle

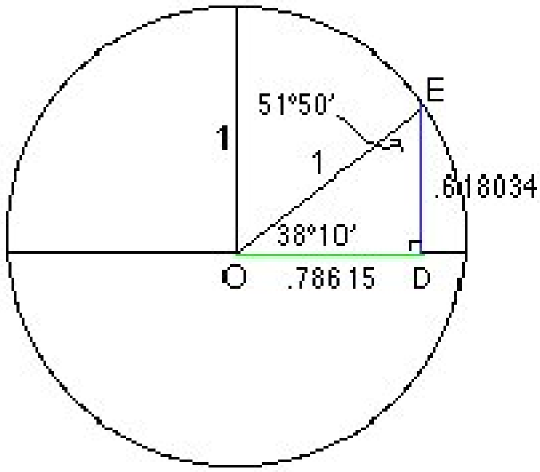

Figure 2: The right angle having an angle with a measure of 38"10'

In figure 2, the ratio of the length of ED over the radius of the circle is 0.618034. The 1-unit edge is the hypothenuse of the right triangle. Again using a trigonometric table we can find the angles as 38"10' and 51"50'. Using the Pythagorean Theorem, the length of OD will be approximated to 0.78615. There a two main points, the makes the construction special. Firstly, if we divide the length of ED (0.618034) by the length of OD (0.78615) the result is equal to the length of OD. By means of trigonometry this equality can be expressed as the tangent of 38"10' is equal to the cosine of 51"50'. Similarly, the sine of 38"10' is equal to the cotangent of 51"50'.Secondly and the most importantly, 4 times of the length of OD gives us 3.1446 which is nearly equal to the Pi. (3,1416) This invention exhibits the strange intersection of Pi and the golden ratio using a right triangle one of which has an angle with 38"10'

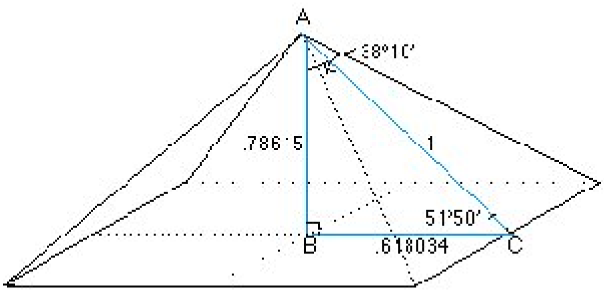

Figure 3: The structure of the pyramids

We can see the external lines of the Great Pyramid of Giza on Figure 3. Even conscious or not this pyramid consists a right traingle having an angle of 38"10'.

The tangent of the surface makes an angle of exactly 51"50'. The cross-section of the pyramids denotes that the length of BC is the 0.618034 times the radius, the length of AB is the 0.78615 times of the radius and length of AC is equal to the radius. The exact dimensions of the Great Pyramid of Giza are given as:

AB=146.6088m

BC=115.1839m

AC=186.3852m

It can be seen that the length of BC is the half of the length of the edge of the pyramid. Therefore, the circumference of the pyramid is 8 times the length of BC. The relative circumference of the pyramid will be 0.618034 x 8 = 4.9443. If we assume the relative height of the pyramid (0.78615) is the length of the radius of the circle, then the circumference of the circle is again 4.9443. This unexpected coherence comes true in a way;

1) For the triangle having an angle of 38"10' there is a relation 0.618034 ÷ 0.78615 = 0.78615. Thus the circumferenceof the pyramid can be written as 8 x 0.78618 x 0.78615

2) We stated that , 4 x 0.78615 gives an approximate value of Pi. In order to find the circumference of the circle we double the product of radius and Pi ;

$$C = 2 \times 0.78618 \times 4 \times 0.78615 = 8 \times 0.78618 \times 0.78615$$

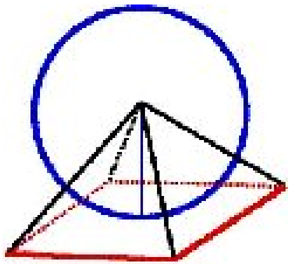

Figure 4: The pyramid and the circle having the equal circumferences

With the information given above, we can conclude that the Great Pyramid of Giza has the same circumference of the circumference of the horizontal cross-section with the circumference of the vertical cross-section. The exact length of the edges of Great Pyramid of Giza is 1/8 of the length of meridians with a measure of 1 minute at equator. In other words the twice of the circumference of the pyramids is equal to the length of meridians with a measure of 1 minute at equator. This shows that there exists a relationship between meridians and the golden ratio.

3. AN ARTIST AND A MATHEMATICIAN: CARLA FARSI

Carla Farsi is an academician at the University of Colorado at Boulder. She studies maths. She is different from the other mathematicians in that she is also an artist. In spite of the people's idea that a scientist can not be an artist since their fields are different, she can perform in both areas.

When anyone examines some works of Carla about her paintings, most probably, he or she can not find any maths in it. Even though some of her installations in particular are disordered, they appear impulsive. When asked the connection between art and maths, Carla replied that one of the main points is visualisation. She stated that it was used in both geometry and pictures since proof by visualisation could be possible for geometry and pictures could have an influence of a theorem. Using logic a mathematical theorem can be made. On the other hand, a piece of art should contain logic and meaning in its own structure. She thinks a mathematical problem can appear to be visual no matter how abstract it is.

Carla thinks that the maths will become more and more prominent with the advance of computers in aspects of visuality and artistical. She stated that the developments of the computers were so fast and it could not be predictable that what they could do for us in the future. She claimed that in future it could be sufficient to think about the images involved in a mathematical idea or proof, and a computer would compute the underlying equations for us. She does not believe that only drawing a picture is enough so we need a proper proof which computers can do. She presumes that a mathematician could simply present the computer with a picture, and the computer will be able to read off the maths in it. In her opinion, this method would result in more time that was spent by mathematicians on the creative aspects of maths and for the automatic parts computers would be used. As a result she claimed that maths may be closer to art than it appears now.

4. MATHS IN MUSIC

Throughout history, many mathematicians were interested in music. Most of us wonder that if many musicians were interested in mathematics. Certainly there are musicians who are interested in maths, but when a comparison is made mathematicians are more than the musicians. Examining the relationship between music and mathematics goes back to the ancient Greeks. In ancient Greece, music was as one of 4 main branches of mathematics.. The school of Pythagoras accepted music, arithmetic, geometry and astronomy at the same level. Wires with different lengths were obtained reveal the different sounds were found by

Pythagoras, in BC 6th Century. Pythagoras created the basics of the music that is still used today.

Pythagoras, divided a 12 unit wire and the octave has been achieved. Pythagoras found 5-intervalled system with 8 unit length of a wire and 4-intervalled system with 9 units of length of a wire. In ancient times, four voices heard together are called tetrachord and it is considered as the basic rule of the music theory. Thus tetrakord, were obtained from 6, 8, 9 and 12. These numbers have a relationship with golden ratio. According to Pythagoras rate, 5-intervalled system and 4-intervalled system differs by a full ton.

2 / 3 : 3 / 4 = 8 / 9 (5T-4T=2M)

So, full sound 8 / 9 with the multiplication gives us the shrill tone of a voice.

If we continue; 8/9.8/9 = 64/81 (2M +2 M =3M)

Pythagoras, obtained 1 full tone with 8/9 of the wire, full tone, but when added 6 more notes he could obtain an octave of the note, which is called the coma of Pythagoras. In this case, Pythagoras needed some changes in order that 12 equal half-ton tampere a system had been developed. 1 full ton was not shown as 8 /9 but with twelve half-ton.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987...

When we look at the sequence, the sum of last two numbers will give us the next number. The most important thing here is the ratio between the numbers. When we divide one of the consequent number to the other one we will obtain 0, 61803398......This ratio was used in some period of music history. The ratio is known as the golden ratio.

After some studies, golden ratio was used in some compositions in parts of melody, and rhythm. Bella Bartok is one of the composer who uses the golden ratio most. Bella Bartok constituted a Fibonacci sequence and used the terms of the sequence as the notes of his compositions. In his composition, "Music for strings, percussion and celeste" the most important part is the 55th measure of 89 measures. Both numbers are the elements of Fibonacci sequence. Another widely known composition is Hallelujah whose composer is Haendel. In the composition the most important part is “King of kings” which starts at 57th and 58th measure. It is approximately the 8/13 of the total measures. 8 and 13 are the Fibonacci numbers. When we calculate 57. 8/13 it will give us an approximate value of 34. At the 34th measure, the theme “The Kingdom of Glory” starts. 34 is again a Fibonacci number. Even though, no one knows that whether Haendel used these numbers with knowing the Fibonacci numbers or not, this composition is one of the most popular compositions. There are some opinons that Mozart also used the golden ratio. According to John F. Putz, the compositions of Mozart are genius works and they are the works that was done by a person who loves numbers. He concluded that Mozart knew the golden ratio and used it in his works.

4. CONCLUSION

All in all, the mathematical tools, like Golden Ratio, Fibonacci Numbers, are used in art and they form different types of works of art. Mathematical tools are not only used for equations, numerical problems, also they are used for the creation of works of art. Maths is not just formulas, but also, structure, symmetry and beauty. In this study, Fibonacci Sequence was worked. The formation and applications were studied. Also, the ratio of the two consequent terms of Fibonacci sequence, the Golden Ratio, was studied. Golden ratio can be seen on human body, in Egyptian pyramids and in some paintings such as Mona Lisa. After studying on the living combination of maths and art, Carla Farsi, to show that not only in paintings, also in music, maths is used, it was studied on the use of Fibonacci Numbers in music. To conclude, Maths is not only for numerical operations, it has a great range of use. One of these can be concluded as Arts.

5. BIBLIOGRAPHY

- "Art + Math = X ". 12 February 2009.

<http://plus.maths.org/issue37/features/farsi/index.html>

- Schattschneider, Doris. "Mathematics and Art – So Many Connections." 21 March 2009.

<http://www.mathaware.org/mam/03/essay3.html>

- "Golden Ratio" *Wikipedia: The Free Encyclopedia.* 12 February 2009.

<http://en.wikipedia.org/wiki/Golden_ratio>

- Sigler, Laurence. *Fibonacci's Liber Abaci*. Springer-Verlag, 2002.